Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.1-Sine/69-4.1.12-e-x-^m-a+b-sin-c+d-x^n-^p

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September 5, 2023 Compiled on September 5, 2023 at 4:43pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [357]. This is test number [69].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (357)	0.00 (0)
Mathematica	97.76 (349)	2.24 (8)
Fricas	85.43 (305)	14.57 (52)
Maxima	75.63 (270)	24.37 (87)
Maple	68.63 (245)	31.37 (112)
Giac	51.26 (183)	48.74 (174)
Mupad	36.13 (129)	63.87 (228)
Sympy	31.37 (112)	68.63 (245)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

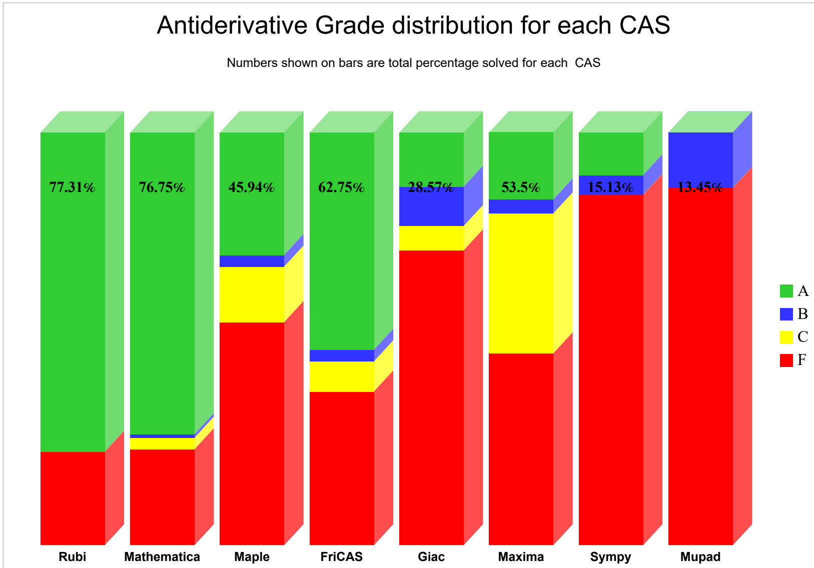
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

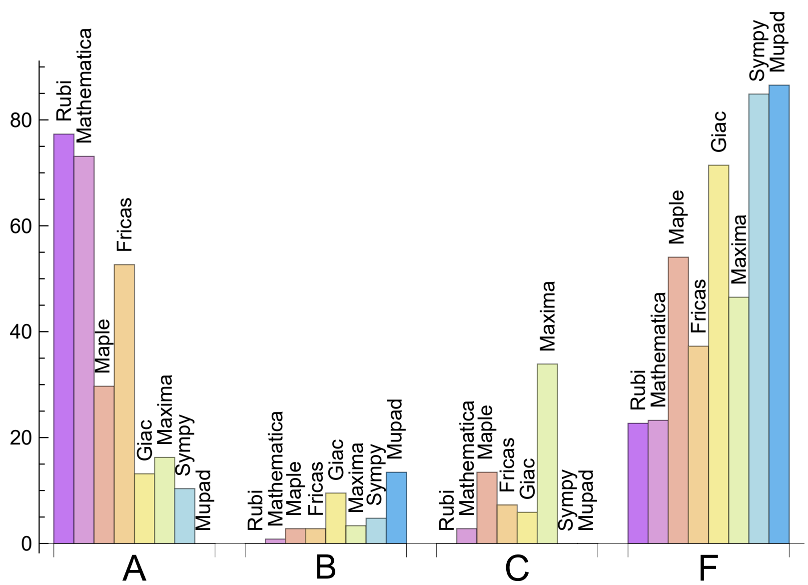
System	% A grade	% B grade	% C grade	% F grade
Rubi	77.311	0.000	0.000	22.689
Mathematica	73.109	0.840	2.801	23.249
Fricas	52.661	2.801	7.283	37.255
Maple	29.692	2.801	13.445	54.062
Maxima	16.246	3.361	33.894	46.499
Giac	13.165	9.524	5.882	71.429
Sympy	10.364	4.762	0.000	84.874
Mupad	0.000	13.445	0.000	86.555

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	8	25.00	75.00	0.00
Fricas	52	100.00	0.00	0.00
Maxima	87	93.10	2.30	4.60
Maple	112	100.00	0.00	0.00
Giac	174	95.40	0.00	4.60
Sympy	245	85.31	14.69	0.00
Mupad	228	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.13
Fricas	0.28
Maple	0.39
Maxima	1.19
Giac	1.68
Mathematica	2.23
Mupad	5.87
Sympy	11.19

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	38.55	1.08	22.00	1.10
Sympy	117.14	1.92	29.00	1.07
Mathematica	127.67	0.97	85.00	0.96
Rubi	136.07	1.00	96.00	1.00
Fricas	144.43	1.30	72.00	1.02
Maple	150.43	1.20	56.00	1.00
Giac	289.23	1.80	32.00	1.11
Maxima	410.60	12.66	90.50	1.10

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

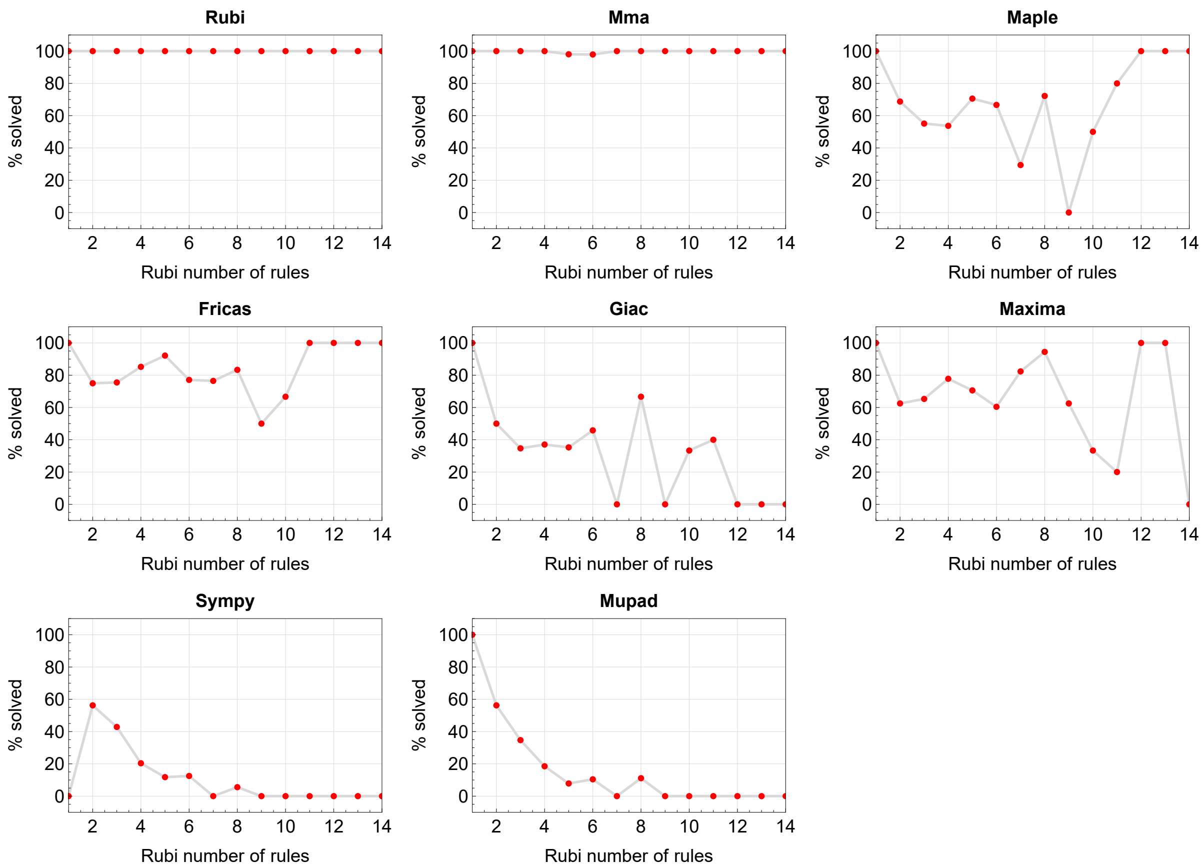


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

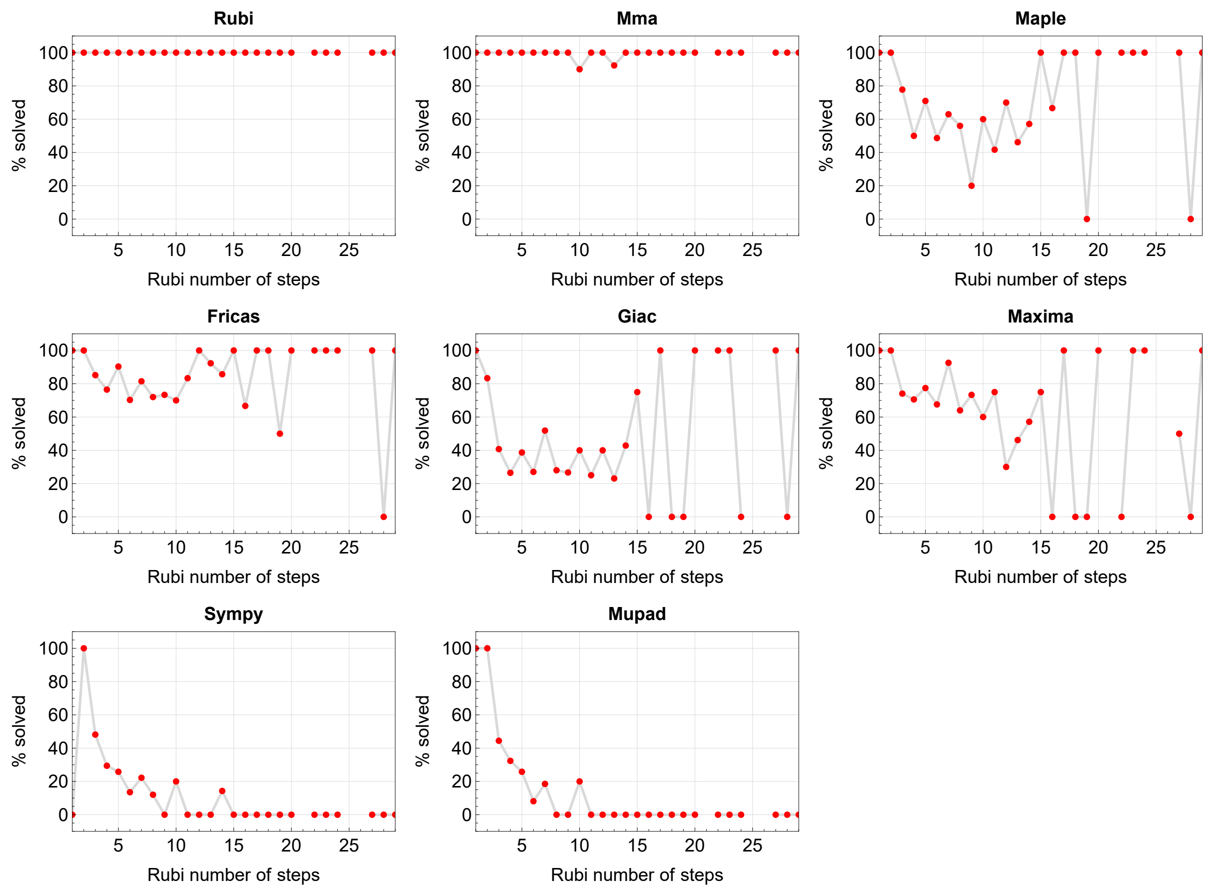


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

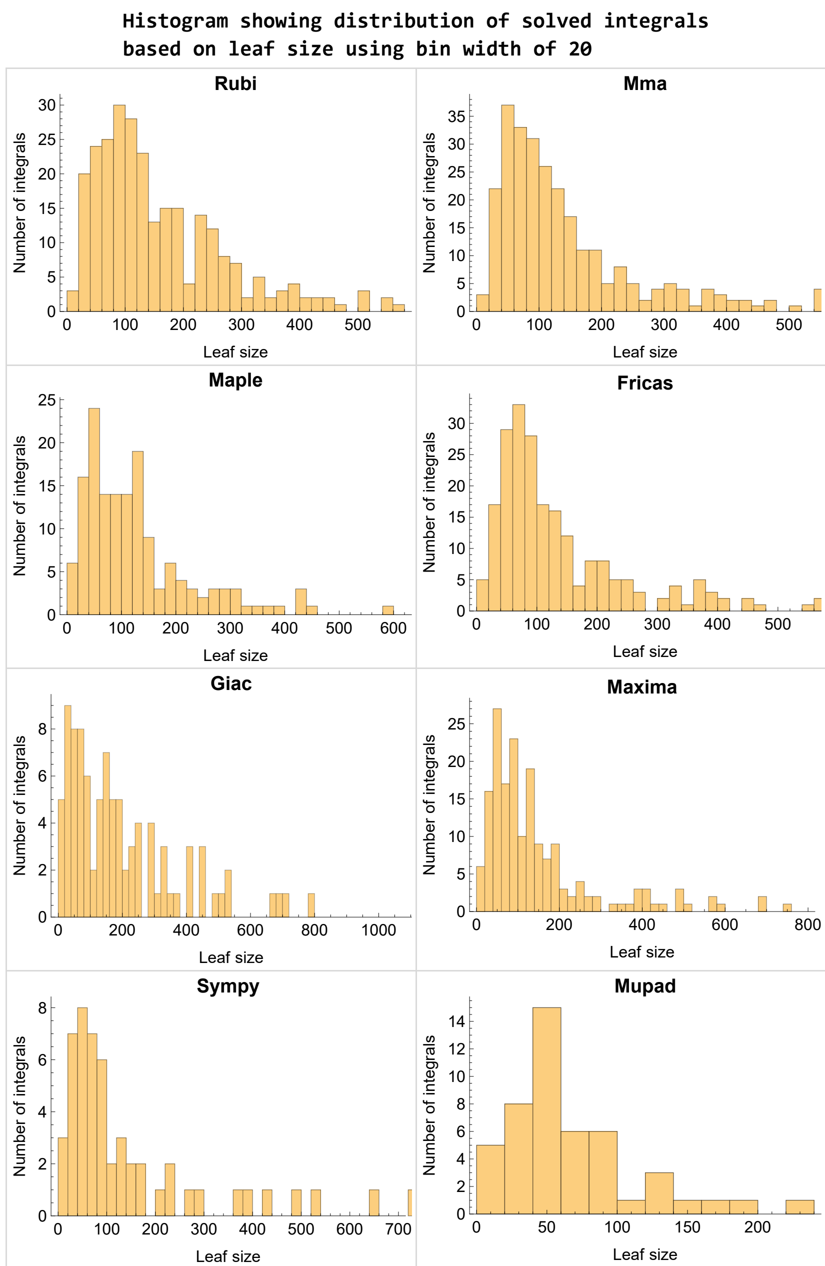


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

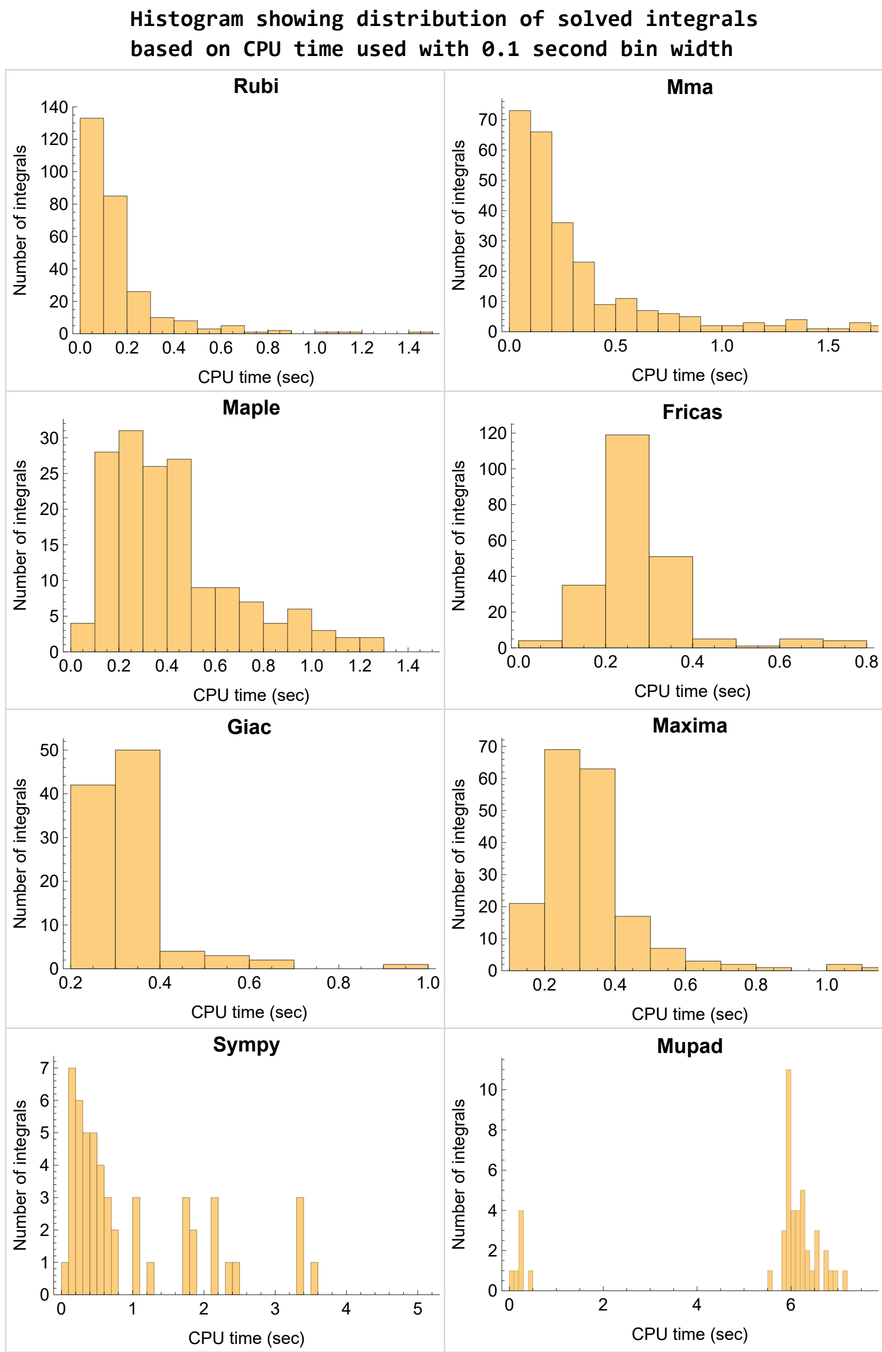


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

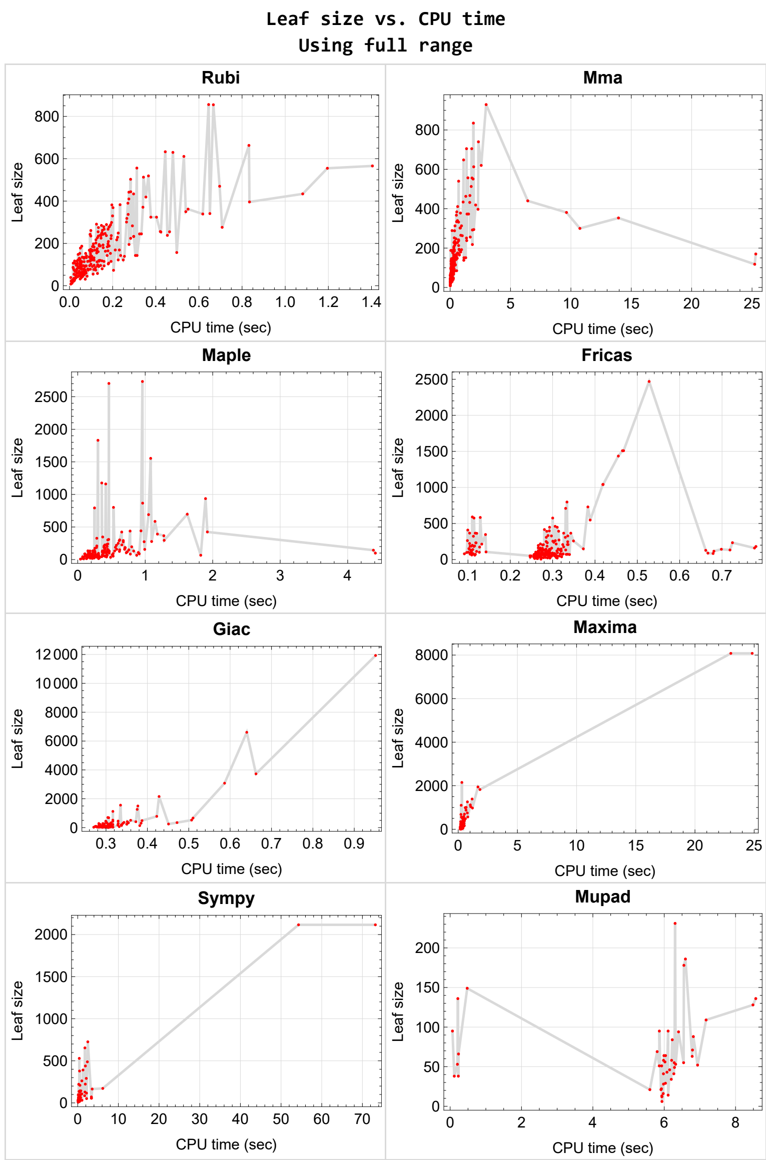


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 55, 56, 83, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 101, 102, 129, 130, 132, 134, 157, 158, 163, 164, 169, 170, 175, 176, 180, 181, 185, 186, 195, 196, 205, 206, 215, 216, 225, 226, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {15, 16, 17, 26, 27, 220, 221, 315, 323, 331, 339, 347, 355}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima abs_integrate was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error Exception raised: NotImplementedError.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function LeafSize was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
```

```
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

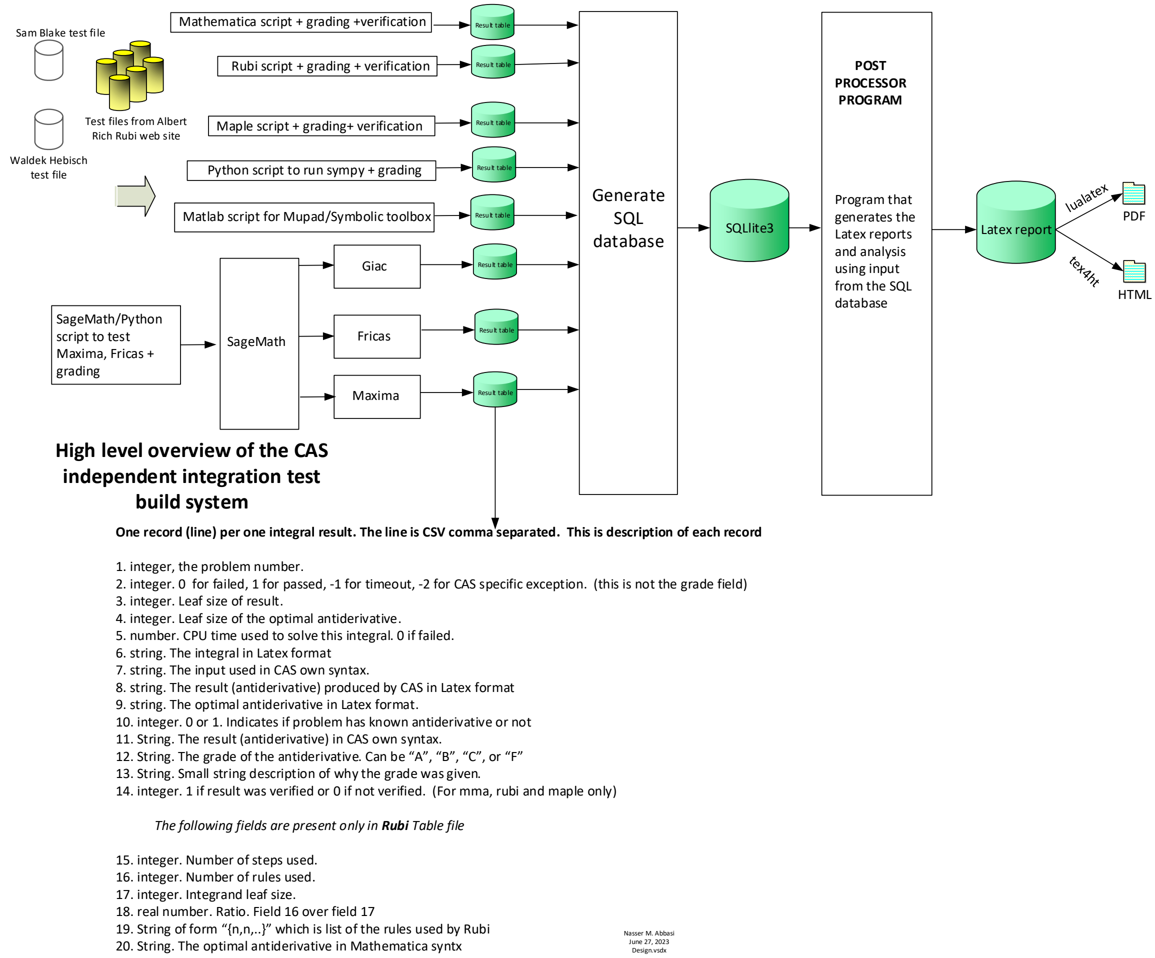
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 89, 90, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 165, 166, 167, 168, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 187, 188, 189, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 214, 217, 218, 219, 220, 221, 222, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 89, 90, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 165, 166, 167, 168, 171, 172, 173, 174, 177, 178, 179, 182, 184, 187, 188, 189, 192, 194, 198, 199, 202, 204, 207, 208, 209, 213, 214, 218, 219, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

B grade { 183, 193, 203 }

C grade { 190, 191, 197, 210, 211, 212, 217, 220, 221, 222 }

F normal fail { 200, 201 }

F(-1) timedout fail { 282, 283, 285, 286, 304, 308 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 31, 32, 33, 34, 37, 45, 57, 58, 69, 70, 82, 90, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 135, 136, 137, 138, 145, 146, 147, 148, 149, 150, 151, 152, 156, 159, 160, 161, 162, 177, 178, 179, 189, 197, 198, 199, 200, 209, 212, 213, 214, 217, 218, 219, 222, 223, 224, 288, 289, 290, 291, 292, 294, 295, 296, 297 }

B grade { 153, 154, 155, 187, 188, 190, 191, 201, 207, 208 }

C grade { 15, 16, 17, 26, 27, 127, 139, 142, 165, 166, 167, 168, 210, 211, 220, 221, 293, 298, 311, 312, 313, 314, 315, 316, 317, 319, 320, 321, 322, 323, 324, 325, 331, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 347, 348, 349, 355 }

F normal fail { 35, 36, 43, 44, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 131, 133, 140, 141, 143, 144, 171, 172, 173, 174, 182, 183, 184, 192, 193, 194, 202, 203, 204, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 310, 318, 326, 327, 328, 329, 330, 332, 333, 334, 342, 350, 351, 352, 353, 354, 356, 357 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 45, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 90, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 135, 136, 137, 138, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 165, 166, 167, 168, 171, 172, 173, 174, 177, 178, 179, 187, 188, 189, 192, 193, 194, 197, 198, 199, 202, 204, 207, 208, 209, 212, 213, 214, 217, 218, 219, 222, 223, 224, 227, 228, 229, 230, 231, 237, 238, 245, 246, 247, 248, 255, 256, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 310, 311, 312, 313, 314, 318, 319, 321, 334, 335, 336, 337, 338, 342, 343, 345 }

B grade { 35, 36, 43, 44, 81, 89, 182, 183, 184, 203 }

C grade { 190, 191, 200, 201, 210, 211, 220, 221, 315, 316, 317, 320, 322, 323, 324, 325, 331, 339, 340, 341, 344, 346, 347, 348, 349, 355 }

F normal fail { 131, 133, 139, 140, 141, 142, 143, 144, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 249, 250, 251, 252, 253, 254, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 326, 327, 328, 329, 330, 332, 333, 350, 351, 352, 353, 354, 356, 357 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 12, 13, 14, 23, 24, 25, 33, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 107, 115, 122, 124, 125, 126, 127, 128, 145, 146, 189, 193, 194, 204, 209, 231, 238, 245, 255, 311, 312, 313, 314, 319, 321, 336, 343, 345 }

B grade { 37, 82, 187, 188, 192, 202, 203, 207, 208, 335, 337, 338 }

C grade { 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 34, 59, 60, 71, 72, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 123, 135, 136, 137, 138, 153, 154, 155, 156, 165, 166, 167, 168, 197, 198, 199, 212, 213, 214, 217, 218, 219, 222, 223, 224, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 258, 288, 289, 290, 294, 295, 315, 316, 317, 320, 322, 323, 324, 325, 331, 339, 340, 341, 344, 346, 347, 348, 349, 355 }

F normal fail { 35, 36, 43, 52, 53, 54, 81, 98, 99, 100, 131, 133, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 159, 160, 161, 162, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 190, 191, 200, 201, 210, 211, 220, 221, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 291, 292, 293, 296, 297, 298, 310, 318, 326, 327, 328, 329, 330, 332, 333, 334, 342, 350, 351, 352, 353, 354, 356, 357 }

F(-1) timedout fail { 45, 90 }

F(-2) exception fail { 44, 89, 93, 94 }

Giac

A grade { 2, 3, 4, 12, 13, 14, 15, 23, 24, 25, 26, 33, 34, 37, 45, 57, 58, 59, 69, 70, 71, 82, 90, 106, 107, 108, 115, 116, 117, 122, 124, 125, 126, 127, 128, 187, 188, 189, 208, 209, 227, 228, 229, 230, 231, 291, 296 }

B grade { 1, 5, 6, 16, 17, 27, 60, 72, 103, 104, 105, 109, 110, 111, 112, 113, 114, 118, 197, 198, 199, 207, 217, 218, 219, 288, 289, 290, 292, 293, 294, 295, 297, 298 }

C grade { 7, 8, 9, 18, 19, 20, 28, 29, 31, 32, 153, 154, 155, 156, 165, 166, 167, 168, 212, 213, 214 }

F normal fail { 10, 11, 21, 22, 30, 35, 36, 43, 44, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 119, 120, 121, 123, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 159, 160, 161, 162, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 190, 191, 192, 193, 194, 200, 201, 202, 203, 204, 210, 211, 220, 221, 222, 223, 224, 232, 233, 234, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

F(-1) timedout fail { }

F(-2) exception fail { 235, 236, 237, 238, 239, 240, 241, 249 }

Mupad

A grade { }

B grade { 1, 2, 3, 9, 12, 13, 14, 23, 24, 25, 33, 37, 45, 57, 58, 69, 70, 82, 90, 107, 108, 109, 110, 115, 116, 117, 118, 121, 122, 124, 125, 126, 127, 128, 153, 154, 155, 156, 162, 168, 189, 209, 311, 312, 313, 314, 319, 321 }

C grade { }

F normal fail { }

F(-1) timedout fail { 4, 5, 6, 7, 8, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 43, 44, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 103, 104, 105, 106, 111, 112, 113, 114, 119, 120, 123, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 159, 160, 161, 165, 166, 167, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 187, 188, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 210, 211, 212, 213, 214, 217, 218, 219, 220, 221, 222, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 310, 315, 316, 317, 318, 320, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 9, 12, 13, 14, 23, 24, 25, 29, 31, 33, 34, 57, 58, 69, 70, 106, 107, 108, 109, 110, 114, 122, 124, 125, 126, 128, 187, 188, 189, 209, 311, 312, 313, 319 }

B grade { 7, 8, 28, 32, 37, 45, 82, 90, 115, 116, 117, 118, 127, 145, 146, 314, 321 }

C grade { }

F normal fail { 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 30, 35, 36, 43, 44, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 103, 104, 105, 111, 112, 113, 119, 120, 121, 123, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 165, 166, 167, 168, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 210, 211, 212, 213, 214, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 250, 251, 252, 253, 254, 255, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 310, 315, 316, 317, 318, 320, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 164, 181, 185, 186, 206, 227, 234, 235, 247, 248, 249, 256, 257, 258, 277, 281, 282, 283, 284, 285, 286, 287, 298, 299, 300, 302, 303, 304, 305, 306, 307, 308, 309, 350, 351, 352 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	51	47	47	51	65	128	53
N.S.	1	1.00	0.89	0.82	0.82	0.89	1.14	2.25	0.93
time (sec)	N/A	0.050	0.102	0.182	0.247	0.293	0.404	0.289	0.204

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	37	40	49	75	38
N.S.	1	1.00	1.00	0.89	0.84	0.91	1.11	1.70	0.86
time (sec)	N/A	0.031	0.005	0.135	0.236	0.267	0.231	0.286	0.115

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	41	22	21	23	31	26	21
N.S.	1	1.00	1.64	0.88	0.84	0.92	1.24	1.04	0.84
time (sec)	N/A	0.015	0.037	0.134	0.238	0.262	0.085	0.282	5.598

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	29	50	27	0	32	0
N.S.	1	1.00	0.94	0.94	1.61	0.87	0.00	1.03	0.00
time (sec)	N/A	0.023	0.068	0.145	0.299	0.273	0.000	0.305	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	47	57	49	0	99	0
N.S.	1	1.00	0.91	0.89	1.08	0.92	0.00	1.87	0.00
time (sec)	N/A	0.069	0.094	0.164	0.328	0.266	0.000	0.305	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	86	65	58	63	0	204	0
N.S.	1	1.00	1.16	0.88	0.78	0.85	0.00	2.76	0.00
time (sec)	N/A	0.087	0.114	0.191	0.330	0.289	0.000	0.305	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	125	89	92	103	488	165	0
N.S.	1	1.00	1.03	0.74	0.76	0.85	4.03	1.36	0.00
time (sec)	N/A	0.097	0.195	0.177	0.255	0.260	2.374	0.310	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	104	68	75	86	223	145	0
N.S.	1	1.00	1.02	0.67	0.74	0.84	2.19	1.42	0.00
time (sec)	N/A	0.049	0.154	0.140	0.257	0.290	1.868	0.294	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	61	48	53	67	66	100	56
N.S.	1	1.00	0.82	0.65	0.72	0.91	0.89	1.35	0.76
time (sec)	N/A	0.032	0.115	0.103	0.247	0.307	0.201	0.291	6.021

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	91	66	81	78	0	0	0
N.S.	1	1.00	1.03	0.75	0.92	0.89	0.00	0.00	0.00
time (sec)	N/A	0.052	0.146	0.155	0.401	0.276	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	119	83	82	98	0	0	0
N.S.	1	1.00	1.04	0.73	0.72	0.86	0.00	0.00	0.00
time (sec)	N/A	0.068	0.171	0.169	0.393	0.290	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	122	111	106	121	209	284	149
N.S.	1	1.00	0.75	0.68	0.65	0.74	1.28	1.74	0.91
time (sec)	N/A	0.166	0.248	0.254	0.264	0.290	0.547	0.283	0.478

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	92	89	87	84	136	165	95
N.S.	1	1.00	0.90	0.87	0.85	0.82	1.33	1.62	0.93
time (sec)	N/A	0.099	0.149	0.217	0.241	0.301	0.316	0.283	6.110

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	52	52	52	53	95	57	51
N.S.	1	1.00	0.90	0.90	0.90	0.91	1.64	0.98	0.88
time (sec)	N/A	0.034	0.176	0.216	0.238	0.270	0.128	0.277	5.861

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	74	74	71	157	108	68	0	77	0
N.S.	1	1.00	0.96	2.12	1.46	0.92	0.00	1.04	0.00
time (sec)	N/A	0.072	0.123	0.577	0.362	0.262	0.000	0.290	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	115	115	116	219	124	112	0	226	0
N.S.	1	1.00	1.01	1.90	1.08	0.97	0.00	1.97	0.00
time (sec)	N/A	0.160	0.190	0.573	0.368	0.274	0.000	0.286	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	169	169	158	271	128	156	0	448	0
N.S.	1	1.00	0.93	1.60	0.76	0.92	0.00	2.65	0.00
time (sec)	N/A	0.212	0.312	0.675	0.364	0.276	0.000	0.299	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	234	185	207	216	0	329	0
N.S.	1	1.00	0.95	0.75	0.84	0.87	0.00	1.33	0.00
time (sec)	N/A	0.165	0.377	0.279	0.349	0.308	0.000	0.302	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	191	138	171	176	0	283	0
N.S.	1	1.00	0.96	0.70	0.86	0.89	0.00	1.43	0.00
time (sec)	N/A	0.111	0.343	0.242	0.331	0.326	0.000	0.292	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	147	98	129	134	0	195	0
N.S.	1	1.00	0.96	0.64	0.84	0.88	0.00	1.27	0.00
time (sec)	N/A	0.075	0.217	0.181	0.336	0.282	0.000	0.294	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	184	134	170	159	0	0	0
N.S.	1	1.00	0.98	0.72	0.91	0.85	0.00	0.00	0.00
time (sec)	N/A	0.110	0.341	0.223	0.458	0.298	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	226	173	176	206	0	0	0
N.S.	1	1.00	0.95	0.72	0.74	0.86	0.00	0.00	0.00
time (sec)	N/A	0.143	0.455	0.254	0.455	0.278	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	75	85	79	79	143	138	94
N.S.	1	1.00	0.64	0.73	0.68	0.68	1.22	1.18	0.80
time (sec)	N/A	0.092	0.201	0.200	0.252	0.280	0.771	0.283	6.402

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	58	66	60	58	92	94	66
N.S.	1	1.00	0.73	0.84	0.76	0.73	1.16	1.19	0.84
time (sec)	N/A	0.054	0.115	0.312	0.250	0.305	0.396	0.276	0.234

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	27	26	46	26	28
N.S.	1	1.00	1.00	0.79	0.82	0.79	1.39	0.79	0.85
time (sec)	N/A	0.021	0.021	0.321	0.232	0.278	0.156	0.294	5.980

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	55	55	51	125	89	47	0	47	0
N.S.	1	1.00	0.93	2.27	1.62	0.85	0.00	0.85	0.00
time (sec)	N/A	0.059	0.054	0.618	0.376	0.311	0.000	0.286	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	91	91	90	185	97	90	0	186	0
N.S.	1	1.00	0.99	2.03	1.07	0.99	0.00	2.04	0.00
time (sec)	N/A	0.136	0.100	0.701	0.370	0.292	0.000	0.308	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	159	132	143	147	439	259	0
N.S.	1	1.00	0.85	0.70	0.76	0.78	2.34	1.38	0.00
time (sec)	N/A	0.138	0.303	0.182	0.338	0.313	1.888	0.295	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	117	99	112	120	129	185	0
N.S.	1	1.00	0.76	0.65	0.73	0.78	0.84	1.21	0.00
time (sec)	N/A	0.048	0.157	0.158	0.345	0.310	0.510	0.312	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	167	130	152	147	0	0	0
N.S.	1	1.00	0.99	0.77	0.90	0.88	0.00	0.00	0.00
time (sec)	N/A	0.092	0.300	0.253	0.463	0.320	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	58	97	51	116	97	0
N.S.	1	1.00	0.89	0.82	1.37	0.72	1.63	1.37	0.00
time (sec)	N/A	0.032	0.050	0.250	0.338	0.310	2.161	0.298	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	75	78	117	73	291	125	0
N.S.	1	1.00	0.89	0.93	1.39	0.87	3.46	1.49	0.00
time (sec)	N/A	0.045	0.112	0.367	0.334	0.323	2.106	0.285	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	50	55	52	95	52	55
N.S.	1	1.00	1.00	0.75	0.82	0.78	1.42	0.78	0.82
time (sec)	N/A	0.029	0.030	0.407	0.252	0.298	0.668	0.280	6.544

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	39	54	38	51	39	0
N.S.	1	1.00	0.93	0.89	1.23	0.86	1.16	0.89	0.00
time (sec)	N/A	0.061	0.079	0.322	0.294	0.321	2.184	0.281	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	362	362	289	0	0	1435	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	3.96	0.00	0.00	0.00
time (sec)	N/A	0.550	0.157	0.000	0.000	0.455	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	245	188	0	0	1041	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	4.25	0.00	0.00	0.00
time (sec)	N/A	0.325	0.053	0.000	0.000	0.419	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	48	8078	208	165	63	128
N.S.	1	1.00	1.00	1.00	168.29	4.33	3.44	1.31	2.67
time (sec)	N/A	0.043	0.063	0.161	24.834	0.324	3.557	0.293	8.488

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	19	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.06	0.83	1.11	1.11
time (sec)	N/A	0.016	1.089	0.044	0.441	0.347	2.380	0.305	6.103

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	23	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.28	0.94	1.11	1.11
time (sec)	N/A	0.017	0.985	0.049	0.447	0.274	3.219	0.324	6.112

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.018	0.555	0.045	0.470	0.287	1.856	0.301	5.963

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14
time (sec)	N/A	0.004	0.022	0.042	0.417	0.322	0.799	0.296	5.963

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	23	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.28	0.94	1.11	1.11
time (sec)	N/A	0.018	0.433	0.047	0.454	0.278	3.015	0.305	6.433

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	663	663	513	0	0	2469	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	3.72	0.00	0.00	0.00
time (sec)	N/A	0.832	1.743	0.000	0.000	0.528	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	324	324	302	0	0	1509	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	4.66	0.00	0.00	0.00
time (sec)	N/A	0.377	0.817	0.000	0.000	0.465	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	131	0	366	2116	144	178
N.S.	1	1.00	1.00	1.44	0.00	4.02	23.25	1.58	1.96
time (sec)	N/A	0.072	0.201	0.239	0.000	0.343	54.296	0.284	6.552

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	3466	45	17	20	20
N.S.	1	1.00	1.11	1.00	192.56	2.50	0.94	1.11	1.11
time (sec)	N/A	0.016	4.684	0.298	4.106	0.308	26.566	0.444	6.275

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	3475	51	19	20	20
N.S.	1	1.00	1.11	1.00	193.06	2.83	1.06	1.11	1.11
time (sec)	N/A	0.016	6.935	0.228	4.117	0.337	39.901	0.553	6.377

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	922	46	17	20	20
N.S.	1	1.00	1.11	1.00	51.22	2.56	0.94	1.11	1.11
time (sec)	N/A	0.016	3.026	0.208	0.600	0.296	40.495	0.363	6.016

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	3381	43	15	16	16
N.S.	1	1.00	1.14	1.00	241.50	3.07	1.07	1.14	1.14
time (sec)	N/A	0.004	3.122	0.223	4.013	0.296	18.265	0.344	5.979

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	3486	51	19	20	20
N.S.	1	1.00	1.11	1.00	193.67	2.83	1.06	1.11	1.11
time (sec)	N/A	0.016	5.365	0.288	4.145	0.327	44.262	0.372	6.085

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10
time (sec)	N/A	0.017	0.796	0.059	0.723	0.359	16.095	0.558	6.386

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	444	444	373	0	0	323	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.284	1.397	0.000	0.000	0.113	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	551	0	0	198	0	0	0
N.S.	1	1.00	1.97	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.159	1.900	0.000	0.000	0.110	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	149	0	0	98	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.068	0.345	0.000	0.000	0.106	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.017	0.687	0.062	0.521	0.285	0.434	0.302	6.101

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	3886	48	19	22	22
N.S.	1	1.00	1.10	1.00	194.30	2.40	0.95	1.10	1.10
time (sec)	N/A	0.015	1.192	0.233	8.109	0.290	1.445	0.342	6.202

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	37	40	49	75	38
N.S.	1	1.00	1.00	0.89	0.84	0.91	1.11	1.70	0.86
time (sec)	N/A	0.030	0.009	0.191	0.198	0.291	0.400	0.274	0.229

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	41	22	21	23	31	26	21
N.S.	1	1.00	1.64	0.88	0.84	0.92	1.24	1.04	0.84
time (sec)	N/A	0.018	0.038	0.137	0.289	0.288	0.113	0.270	5.909

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	29	0	50	27	0	32	0
N.S.	1	1.00	0.94	0.00	1.61	0.87	0.00	1.03	0.00
time (sec)	N/A	0.024	0.077	0.000	0.386	0.309	0.000	0.279	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	48	0	57	49	0	99	0
N.S.	1	1.00	0.91	0.00	1.08	0.92	0.00	1.87	0.00
time (sec)	N/A	0.066	0.094	0.000	0.391	0.316	0.000	0.287	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	124	0	109	82	0	0	0
N.S.	1	1.00	1.11	0.00	0.97	0.73	0.00	0.00	0.00
time (sec)	N/A	0.056	0.176	0.000	0.207	0.112	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	108	0	93	63	0	0	0
N.S.	1	1.00	1.19	0.00	1.02	0.69	0.00	0.00	0.00
time (sec)	N/A	0.039	0.113	0.000	0.198	0.109	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	120	0	89	72	0	0	0
N.S.	1	1.00	1.19	0.00	0.88	0.71	0.00	0.00	0.00
time (sec)	N/A	0.054	0.170	0.000	0.186	0.118	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	143	0	91	101	0	0	0
N.S.	1	1.00	1.10	0.00	0.70	0.78	0.00	0.00	0.00
time (sec)	N/A	0.067	0.291	0.000	0.185	0.127	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	124	0	110	80	0	0	0
N.S.	1	1.00	1.17	0.00	1.04	0.75	0.00	0.00	0.00
time (sec)	N/A	0.037	0.146	0.000	0.181	0.111	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	138	0	85	61	0	0	0
N.S.	1	1.00	1.68	0.00	1.04	0.74	0.00	0.00	0.00
time (sec)	N/A	0.018	0.084	0.000	0.175	0.116	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	120	0	90	80	0	0	0
N.S.	1	1.00	1.19	0.00	0.89	0.79	0.00	0.00	0.00
time (sec)	N/A	0.035	0.160	0.000	0.191	0.110	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	126	146	0	91	101	0	0	0
N.S.	1	1.00	1.16	0.00	0.72	0.80	0.00	0.00	0.00
time (sec)	N/A	0.049	0.290	0.000	0.181	0.101	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	92	92	87	84	143	165	95
N.S.	1	1.00	0.86	0.86	0.81	0.79	1.34	1.54	0.89
time (sec)	N/A	0.182	0.178	0.525	0.206	0.305	0.538	0.283	5.864

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	52	52	53	99	57	51
N.S.	1	1.00	0.87	0.87	0.87	0.88	1.65	0.95	0.85
time (sec)	N/A	0.077	0.100	0.225	0.204	0.304	0.179	0.290	5.928

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	108	70	0	79	0
N.S.	1	1.00	0.89	0.00	1.35	0.88	0.00	0.99	0.00
time (sec)	N/A	0.151	0.131	0.000	0.323	0.286	0.000	0.292	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	116	0	124	112	0	226	0
N.S.	1	1.00	0.95	0.00	1.02	0.92	0.00	1.85	0.00
time (sec)	N/A	0.249	0.192	0.000	0.326	0.287	0.000	0.309	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	249	339	0	234	186	0	0	0
N.S.	1	1.00	1.36	0.00	0.94	0.75	0.00	0.00	0.00
time (sec)	N/A	0.226	0.610	0.000	0.232	0.108	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	193	193	251	0	199	141	0	0	0
N.S.	1	1.00	1.30	0.00	1.03	0.73	0.00	0.00	0.00
time (sec)	N/A	0.155	1.352	0.000	0.220	0.104	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	229	332	0	187	173	0	0	0
N.S.	1	0.99	1.44	0.00	0.81	0.75	0.00	0.00	0.00
time (sec)	N/A	0.223	0.695	0.000	0.226	0.128	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	285	283	292	0	194	232	0	0	0
N.S.	1	0.99	1.02	0.00	0.68	0.81	0.00	0.00	0.00
time (sec)	N/A	0.292	1.843	0.000	0.234	0.119	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	237	339	0	240	182	0	0	0
N.S.	1	1.00	1.43	0.00	1.01	0.77	0.00	0.00	0.00
time (sec)	N/A	0.194	0.598	0.000	0.218	0.114	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	183	228	0	192	139	0	0	0
N.S.	1	1.00	1.25	0.00	1.05	0.76	0.00	0.00	0.00
time (sec)	N/A	0.114	0.614	0.000	0.212	0.111	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	227	225	334	0	188	189	0	0	0
N.S.	1	0.99	1.47	0.00	0.83	0.83	0.00	0.00	0.00
time (sec)	N/A	0.146	0.535	0.000	0.225	0.116	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	275	294	0	193	233	0	0	0
N.S.	1	0.99	1.06	0.00	0.70	0.84	0.00	0.00	0.00
time (sec)	N/A	0.134	1.957	0.000	0.231	0.119	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	245	188	0	0	1041	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	4.25	0.00	0.00	0.00
time (sec)	N/A	0.333	0.117	0.000	0.000	0.419	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	49	8078	208	172	64	136
N.S.	1	1.00	1.00	0.96	158.39	4.08	3.37	1.25	2.67
time (sec)	N/A	0.052	0.068	0.178	23.030	0.335	6.161	0.295	8.567

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	19	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.06	0.83	1.11	1.11
time (sec)	N/A	0.018	1.033	0.046	0.465	0.291	2.487	0.320	6.627

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	23	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.28	0.94	1.11	1.11
time (sec)	N/A	0.020	0.924	0.063	0.467	0.299	4.425	0.338	6.285

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.012	0.438	0.044	0.452	0.285	2.156	0.324	5.932

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	23	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.28	0.94	1.11	1.11
time (sec)	N/A	0.018	0.440	0.047	0.500	0.308	2.712	0.336	6.055

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14
time (sec)	N/A	0.005	0.015	0.045	0.457	0.311	0.742	0.302	5.898

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	23	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.28	0.94	1.11	1.11
time (sec)	N/A	0.018	0.465	0.049	0.491	0.344	3.987	0.307	6.041

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	324	324	302	0	0	1509	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	4.66	0.00	0.00	0.00
time (sec)	N/A	0.403	0.846	0.000	0.000	0.468	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	91	131	0	366	2116	146	186
N.S.	1	1.00	0.97	1.39	0.00	3.89	22.51	1.55	1.98
time (sec)	N/A	0.075	0.144	0.240	0.000	0.326	73.187	0.294	6.595

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	2696	45	17	20	20
N.S.	1	1.00	1.11	1.00	149.78	2.50	0.94	1.11	1.11
time (sec)	N/A	0.016	6.888	0.289	4.214	0.309	25.038	0.442	6.334

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	2705	51	19	20	20
N.S.	1	1.00	1.11	1.00	150.28	2.83	1.06	1.11	1.11
time (sec)	N/A	0.015	10.075	0.289	4.242	0.316	50.273	0.598	6.527

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	0	44	15	18	18
N.S.	1	1.00	1.12	1.00	0.00	2.75	0.94	1.12	1.12
time (sec)	N/A	0.009	4.337	0.217	0.000	0.324	24.651	0.367	6.043

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	0	51	19	20	20
N.S.	1	1.00	1.11	1.00	0.00	2.83	1.06	1.11	1.11
time (sec)	N/A	0.015	7.519	0.327	0.000	0.309	35.661	0.371	6.203

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	2171	43	15	16	16
N.S.	1	1.00	1.14	1.00	155.07	3.07	1.07	1.14	1.14
time (sec)	N/A	0.004	5.192	0.235	1.840	0.323	16.636	0.315	5.957

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	2171	51	19	20	20
N.S.	1	1.00	1.11	1.00	120.61	2.83	1.06	1.11	1.11
time (sec)	N/A	0.015	8.380	0.297	1.958	0.324	49.111	0.358	6.369

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10
time (sec)	N/A	0.015	0.770	0.053	0.727	0.331	17.069	0.539	6.377

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	442	442	373	0	0	347	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.275	1.524	0.000	0.000	0.142	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	285	285	556	0	0	214	0	0	0
N.S.	1	1.00	1.95	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.148	1.810	0.000	0.000	0.133	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	149	0	0	106	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.063	0.319	0.000	0.000	0.143	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.020	0.644	0.063	0.708	0.262	0.432	0.306	5.975

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	2505	48	19	22	22
N.S.	1	1.00	1.10	1.00	125.25	2.40	0.95	1.10	1.10
time (sec)	N/A	0.020	1.206	0.321	3.702	0.286	1.464	0.360	6.123

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	73	86	66	0	400	0
N.S.	1	1.00	0.90	0.94	1.10	0.85	0.00	5.13	0.00
time (sec)	N/A	0.096	0.053	0.512	0.255	0.258	0.000	0.300	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	57	76	56	0	251	0
N.S.	1	1.00	0.87	0.95	1.27	0.93	0.00	4.18	0.00
time (sec)	N/A	0.062	0.038	0.404	0.237	0.292	0.000	0.296	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	38	58	34	0	132	0
N.S.	1	1.00	1.00	1.19	1.81	1.06	0.00	4.12	0.00
time (sec)	N/A	0.049	0.019	0.235	0.250	0.308	0.000	0.291	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	43	21	17	42	0
N.S.	1	1.00	1.00	1.05	2.05	1.00	0.81	2.00	0.00
time (sec)	N/A	0.019	0.036	0.221	0.249	0.294	0.426	0.300	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	14	14	14	12
N.S.	1	1.00	1.00	1.08	1.00	1.17	1.17	1.17	1.00
time (sec)	N/A	0.012	0.013	0.089	0.189	0.265	0.286	0.304	5.934

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	34	50	33	29	48	29
N.S.	1	1.00	1.00	1.17	1.72	1.14	1.00	1.66	1.00
time (sec)	N/A	0.017	0.004	0.226	0.237	0.247	0.380	0.286	6.035

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	38	46	51	44	46	106	46
N.S.	1	1.00	0.84	1.02	1.13	0.98	1.02	2.36	1.02
time (sec)	N/A	0.032	0.041	0.281	0.234	0.267	0.510	0.302	6.152

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	55	50	52	61	191	64
N.S.	1	1.00	1.00	0.90	0.82	0.85	1.00	3.13	1.05
time (sec)	N/A	0.049	0.006	0.260	0.231	0.247	0.715	0.313	5.993

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	86	96	99	96	0	442	0
N.S.	1	1.00	0.89	0.99	1.02	0.99	0.00	4.56	0.00
time (sec)	N/A	0.114	0.123	0.308	0.248	0.276	0.000	0.317	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	76	87	77	0	283	0
N.S.	1	1.00	1.00	1.17	1.34	1.18	0.00	4.35	0.00
time (sec)	N/A	0.068	0.113	0.290	0.234	0.259	0.000	0.330	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	52	66	45	0	153	0
N.S.	1	1.00	1.00	1.27	1.61	1.10	0.00	3.73	0.00
time (sec)	N/A	0.056	0.064	0.250	0.233	0.265	0.000	0.316	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	36	51	31	31	65	0
N.S.	1	1.00	0.86	0.97	1.38	0.84	0.84	1.76	0.00
time (sec)	N/A	0.030	0.047	0.283	0.229	0.256	1.045	0.332	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	32	23	25	34	262	29	22
N.S.	1	1.00	1.03	0.74	0.81	1.10	8.45	0.94	0.71
time (sec)	N/A	0.018	0.043	0.192	0.190	0.266	1.018	0.317	5.923

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	43	42	68	60	391	77	41
N.S.	1	1.00	0.84	0.82	1.33	1.18	7.67	1.51	0.80
time (sec)	N/A	0.026	0.056	0.306	0.237	0.269	1.300	0.314	5.978

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	54	56	69	72	654	153	64
N.S.	1	1.00	0.62	0.64	0.79	0.83	7.52	1.76	0.74
time (sec)	N/A	0.044	0.092	0.392	0.228	0.256	1.769	0.306	6.037

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	65	67	68	90	726	255	84
N.S.	1	1.00	0.61	0.63	0.64	0.84	6.79	2.38	0.79
time (sec)	N/A	0.052	0.135	0.437	0.228	0.287	2.483	0.308	6.226

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	59	127	74	0	0	0
N.S.	1	1.00	1.01	0.74	1.59	0.92	0.00	0.00	0.00
time (sec)	N/A	0.035	0.091	0.173	0.237	0.262	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	43	21	0	0	0
N.S.	1	1.00	1.00	0.88	1.72	0.84	0.00	0.00	0.00
time (sec)	N/A	0.015	0.037	0.184	0.229	0.311	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	61	47	98	64	0	0	55
N.S.	1	1.00	0.81	0.63	1.31	0.85	0.00	0.00	0.73
time (sec)	N/A	0.021	0.072	0.181	0.237	0.276	0.000	0.000	6.281

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	20	17	13
N.S.	1	1.00	1.00	0.93	0.87	1.13	1.33	1.13	0.87
time (sec)	N/A	0.011	0.013	0.130	0.185	0.263	0.493	0.307	5.932

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	89	65	74	85	0	0	0
N.S.	1	1.00	0.92	0.67	0.76	0.88	0.00	0.00	0.00
time (sec)	N/A	0.031	0.112	0.232	0.233	0.272	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	8	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	1.00	0.75	0.75
time (sec)	N/A	0.006	0.010	0.041	0.177	0.272	0.103	0.305	5.937

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	23	15	15	15	29	15	14
N.S.	1	1.00	1.10	0.71	0.71	0.71	1.38	0.71	0.67
time (sec)	N/A	0.012	0.020	0.309	0.190	0.258	0.172	0.291	6.111

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.008	0.018	0.069	0.182	0.263	0.103	0.299	5.974

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	41	19	30	37	379	30	34
N.S.	1	1.00	0.59	0.28	0.43	0.54	5.49	0.43	0.49
time (sec)	N/A	0.025	0.035	0.218	0.187	0.255	0.452	0.302	6.199

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	62	59	47	51	71	47	58
N.S.	1	1.00	0.71	0.68	0.54	0.59	0.82	0.54	0.67
time (sec)	N/A	0.040	0.040	0.364	0.182	0.249	3.359	0.293	6.207

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.012	1.030	0.101	1.314	0.255	11.578	0.962	6.018

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10
time (sec)	N/A	0.014	1.191	0.094	1.426	0.283	34.564	6.070	5.859

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	88	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.122	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	20	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.91	1.09	1.09
time (sec)	N/A	0.031	1.176	0.124	1.335	0.284	9.809	0.943	5.951

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	148	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	0.583	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.08
time (sec)	N/A	0.040	1.290	0.124	1.427	0.292	26.359	5.970	6.119

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	91	23	0	0	0
N.S.	1	1.00	0.92	0.96	3.64	0.92	0.00	0.00	0.00
time (sec)	N/A	0.023	0.049	0.244	0.326	0.272	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	40	100	36	0	0	0
N.S.	1	1.00	0.86	0.93	2.33	0.84	0.00	0.00	0.00
time (sec)	N/A	0.038	0.069	0.427	0.323	0.267	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	52	181	50	0	0	0
N.S.	1	1.00	0.81	0.78	2.70	0.75	0.00	0.00	0.00
time (sec)	N/A	0.059	0.083	0.525	0.408	0.261	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	66	189	62	0	0	0
N.S.	1	1.00	0.84	0.84	2.39	0.78	0.00	0.00	0.00
time (sec)	N/A	0.064	0.080	1.823	0.412	0.292	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	95	74	0	0	0	0	0
N.S.	1	1.00	1.09	0.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.016	0.061	0.172	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	94	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.166	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	177	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.206	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	118	110	0	0	0	0	0
N.S.	1	1.00	1.08	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.146	0.256	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	129	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	0.360	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	237	225	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	0.400	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	36	32	32	56	0	0
N.S.	1	1.00	0.86	1.03	0.91	0.91	1.60	0.00	0.00
time (sec)	N/A	0.020	0.051	0.313	0.187	0.259	3.384	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	29	35	29	29	56	0	0
N.S.	1	1.00	0.85	1.03	0.85	0.85	1.65	0.00	0.00
time (sec)	N/A	0.020	0.047	0.487	0.205	0.283	3.354	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	47	44	0	47	0	0	0
N.S.	1	1.00	1.02	0.96	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.054	0.054	0.356	0.000	0.293	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	58	66	0	53	0	0	0
N.S.	1	1.00	0.87	0.99	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.075	0.097	0.874	0.000	0.281	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	95	99	0	95	0	0	0
N.S.	1	1.00	0.84	0.88	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.128	0.130	4.410	0.000	0.275	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	68	65	0	68	0	0	0
N.S.	1	1.00	0.87	0.83	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.069	0.092	0.432	0.000	0.299	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	82	89	0	88	0	0	0
N.S.	1	1.00	0.86	0.94	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.093	0.126	0.912	0.000	0.285	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	141	144	0	143	0	0	0
N.S.	1	1.00	0.85	0.87	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.178	0.201	4.381	0.000	0.292	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	173	586	974	255	0	507	231
N.S.	1	1.00	0.78	2.63	4.37	1.14	0.00	2.27	1.04
time (sec)	N/A	0.192	0.674	1.145	1.200	0.301	0.000	0.360	6.305

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	117	291	564	162	0	333	136
N.S.	1	1.00	0.78	1.94	3.76	1.08	0.00	2.22	0.91
time (sec)	N/A	0.097	0.420	0.615	0.809	0.266	0.000	0.354	0.216

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	120	271	80	0	235	58
N.S.	1	1.00	0.96	1.74	3.93	1.16	0.00	3.41	0.84
time (sec)	N/A	0.045	0.156	0.394	0.495	0.261	0.000	0.350	5.980

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	42	53	45	0	143	41
N.S.	1	1.00	1.00	1.08	1.36	1.15	0.00	3.67	1.05
time (sec)	N/A	0.006	0.016	0.073	0.194	0.262	0.000	0.307	6.270

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	31	29	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.72	1.61	1.11	1.11
time (sec)	N/A	0.008	2.488	0.085	0.380	0.279	0.900	0.395	6.308

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	42	31	20	20
N.S.	1	1.00	1.11	1.00	1.11	2.33	1.72	1.11	1.11
time (sec)	N/A	0.008	4.478	0.076	0.386	0.270	2.289	2.555	6.673

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	440	277	0	398	0	0	0
N.S.	1	1.00	1.31	0.82	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.272	6.427	1.095	0.000	0.292	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	265	196	0	262	0	0	0
N.S.	1	1.00	1.14	0.84	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.297	0.880	0.800	0.000	0.306	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	95	101	0	130	0	0	0
N.S.	1	1.00	0.79	0.84	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.141	0.326	0.464	0.000	0.271	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	52	0	73	0	0	52
N.S.	1	1.00	1.00	0.87	0.00	1.22	0.00	0.00	0.87
time (sec)	N/A	0.039	0.025	0.223	0.000	0.272	0.000	0.000	6.936

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	31	26	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.72	1.44	1.11	1.11
time (sec)	N/A	0.016	1.085	0.100	0.356	0.302	18.534	0.331	7.359

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	42	0	20	20
N.S.	1	1.00	1.11	1.00	1.11	2.33	0.00	1.11	1.11
time (sec)	N/A	0.013	16.817	0.111	0.373	0.304	0.000	0.474	13.261

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	218	690	1824	328	0	521	0
N.S.	1	1.00	0.64	2.02	5.35	0.96	0.00	1.53	0.00
time (sec)	N/A	0.651	1.841	1.050	1.837	0.336	0.000	0.507	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	151	438	1038	208	0	347	0
N.S.	1	1.00	0.59	1.71	4.05	0.81	0.00	1.36	0.00
time (sec)	N/A	0.421	1.144	0.776	1.053	0.337	0.000	0.472	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	114	209	483	131	0	249	0
N.S.	1	1.00	0.93	1.71	3.96	1.07	0.00	2.04	0.00
time (sec)	N/A	0.226	0.435	0.283	0.608	0.277	0.000	0.451	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	87	69	89	0	151	95
N.S.	1	1.00	0.81	1.05	0.83	1.07	0.00	1.82	1.14
time (sec)	N/A	0.049	0.039	0.108	0.205	0.294	0.000	0.338	0.065

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	32	31	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.60	1.55	1.10	1.10
time (sec)	N/A	0.009	5.597	0.104	0.422	0.287	0.880	0.596	6.182

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	43	32	22	22
N.S.	1	1.00	1.10	1.00	1.10	2.15	1.60	1.10	1.10
time (sec)	N/A	0.009	6.031	0.109	0.425	0.251	2.220	6.312	6.632

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	434	434	353	0	0	591	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.297	13.941	0.000	0.000	0.110	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	280	300	0	0	409	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.185	10.744	0.000	0.000	0.100	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	235	235	381	0	0	263	0	0	0
N.S.	1	1.00	1.62	0.00	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.134	9.634	0.000	0.000	0.099	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	115	0	0	113	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.024	0.096	0.000	0.000	0.120	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	44	44	22	22
N.S.	1	1.00	1.10	1.00	1.10	2.20	2.20	1.10	1.10
time (sec)	N/A	0.010	15.280	0.115	0.478	0.259	0.971	2.773	6.531

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	55	46	22	22
N.S.	1	1.00	1.10	1.00	1.10	2.75	2.30	1.10	1.10
time (sec)	N/A	0.010	43.558	0.120	0.499	0.260	2.511	3.954	6.438

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	467	274	0	392	0	0	0
N.S.	1	1.00	1.26	0.74	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.340	1.636	0.984	0.000	0.286	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	242	150	0	233	0	0	0
N.S.	1	1.00	1.22	0.76	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.178	0.694	0.622	0.000	0.287	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	100	80	0	137	0	0	0
N.S.	1	1.00	0.95	0.76	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.050	0.116	0.263	0.000	0.316	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	51	27	22	22
N.S.	1	1.00	1.10	1.00	1.10	2.55	1.35	1.10	1.10
time (sec)	N/A	0.009	2.367	0.145	0.556	0.266	18.540	0.908	6.718

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	62	0	22	22
N.S.	1	1.00	1.10	1.00	1.10	3.10	0.00	1.10	1.10
time (sec)	N/A	0.009	27.212	0.148	0.557	0.274	0.000	1.251	9.764

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	330	330	620	0	0	576	0	0	0
N.S.	1	1.00	1.88	0.00	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.199	2.572	0.000	0.000	0.114	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	235	235	705	0	0	367	0	0	0
N.S.	1	1.00	3.00	0.00	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	0.099	1.357	0.000	0.000	0.104	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	203	0	0	185	0	0	0
N.S.	1	1.00	1.90	0.00	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.017	0.301	0.000	0.000	0.105	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	74	0	22	22
N.S.	1	1.00	1.10	1.00	1.10	3.70	0.00	1.10	1.10
time (sec)	N/A	0.008	3.186	0.148	0.676	0.263	0.000	5.108	7.234

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	85	0	22	22
N.S.	1	1.00	1.10	1.00	1.10	4.25	0.00	1.10	1.10
time (sec)	N/A	0.008	21.854	0.161	0.707	0.281	0.000	7.273	14.050

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	138	1161	1101	196	529	701	0
N.S.	1	1.00	0.34	2.83	2.69	0.48	1.29	1.71	0.00
time (sec)	N/A	0.275	1.128	0.415	0.249	0.266	0.384	0.304	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	85	347	348	86	221	218	0
N.S.	1	1.00	0.46	1.88	1.88	0.46	1.19	1.18	0.00
time (sec)	N/A	0.107	0.412	0.370	0.205	0.269	0.247	0.317	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	61	62	44	65	44	43
N.S.	1	1.00	0.93	1.13	1.15	0.81	1.20	0.81	0.80
time (sec)	N/A	0.021	0.043	0.124	0.194	0.308	0.206	0.293	6.078

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	238	793	0	250	0	0	0
N.S.	1	1.00	1.00	3.33	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.454	1.372	0.249	0.000	0.285	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	397	1831	0	416	0	0	0
N.S.	1	1.00	1.17	5.40	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.618	2.309	0.300	0.000	0.281	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	382	382	419	0	694	366	0	0	0
N.S.	1	1.00	1.10	0.00	1.82	0.96	0.00	0.00	0.00
time (sec)	N/A	0.196	2.110	0.000	0.525	0.117	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	291	705	0	375	223	0	0	0
N.S.	1	1.00	2.42	0.00	1.29	0.77	0.00	0.00	0.00
time (sec)	N/A	0.125	1.787	0.000	0.364	0.104	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	123	0	112	75	0	0	0
N.S.	1	1.00	1.07	0.00	0.97	0.65	0.00	0.00	0.00
time (sec)	N/A	0.049	0.109	0.000	0.216	0.105	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	29	34	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.32	1.55	1.00	1.00
time (sec)	N/A	0.011	13.568	0.097	0.681	0.268	6.106	0.436	6.324

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	40	36	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.82	1.64	1.00	1.00
time (sec)	N/A	0.009	17.216	0.096	0.890	0.262	41.574	0.486	6.290

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	611	611	557	696	877	393	0	6606	0
N.S.	1	1.00	0.91	1.14	1.44	0.64	0.00	10.81	0.00
time (sec)	N/A	0.530	1.471	1.623	0.660	0.314	0.000	0.640	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	367	295	407	203	0	2158	0
N.S.	1	1.00	1.22	0.98	1.35	0.67	0.00	7.17	0.00
time (sec)	N/A	0.264	0.517	1.281	0.427	0.276	0.000	0.428	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	99	84	124	104	0	413	0
N.S.	1	1.00	1.05	0.89	1.32	1.11	0.00	4.39	0.00
time (sec)	N/A	0.077	0.055	0.780	0.284	0.275	0.000	0.351	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	0	441	0	308	0	0	0
N.S.	1	1.00	0.00	1.60	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.708	0.000	0.937	0.000	0.330	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	0	2734	0	448	0	0	0
N.S.	1	1.00	0.00	7.81	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.539	0.000	0.959	0.000	0.311	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	390	390	463	0	993	584	0	0	0
N.S.	1	1.00	1.19	0.00	2.55	1.50	0.00	0.00	0.00
time (sec)	N/A	0.271	1.295	0.000	0.589	0.129	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	251	835	0	503	364	0	0	0
N.S.	1	1.00	3.33	0.00	2.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.150	1.925	0.000	0.351	0.121	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	166	0	151	152	0	0	0
N.S.	1	1.00	1.44	0.00	1.31	1.32	0.00	0.00	0.00
time (sec)	N/A	0.056	0.292	0.000	0.220	0.102	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	59	32	22	22
N.S.	1	1.00	1.09	0.91	1.00	2.68	1.45	1.00	1.00
time (sec)	N/A	0.009	11.035	0.180	0.899	0.264	28.566	0.601	5.879

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	70	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	3.18	0.00	1.00	1.00
time (sec)	N/A	0.010	15.252	0.178	1.161	0.284	0.000	0.758	5.952

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	633	633	256	2704	2151	333	0	1558	0
N.S.	1	1.00	0.40	4.27	3.40	0.53	0.00	2.46	0.00
time (sec)	N/A	0.444	1.665	0.462	0.308	0.289	0.000	0.335	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	147	801	681	142	0	453	0
N.S.	1	1.00	0.51	2.78	2.36	0.49	0.00	1.57	0.00
time (sec)	N/A	0.190	0.533	0.530	0.227	0.270	0.000	0.329	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	134	120	58	94	82	69
N.S.	1	1.00	0.76	1.58	1.41	0.68	1.11	0.96	0.81
time (sec)	N/A	0.046	0.077	0.151	0.193	0.272	0.284	0.302	5.803

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	118	327	0	448	0	0	0
N.S.	1	1.00	0.30	0.83	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.834	25.223	0.283	0.000	0.293	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	555	555	180	1176	0	730	0	0	0
N.S.	1	1.00	0.32	2.12	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	1.196	0.872	0.356	0.000	0.383	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	513	432	366	561	308	0	780	0
N.S.	1	1.00	0.84	0.71	1.09	0.60	0.00	1.52	0.00
time (sec)	N/A	0.343	1.621	1.276	0.267	0.285	0.000	0.423	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	213	175	248	159	0	409	0
N.S.	1	1.00	0.88	0.72	1.02	0.65	0.00	1.68	0.00
time (sec)	N/A	0.156	0.701	0.406	0.206	0.271	0.000	0.372	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	114	86	92	98	0	172	0
N.S.	1	1.00	0.88	0.66	0.71	0.75	0.00	1.32	0.00
time (sec)	N/A	0.045	0.110	0.103	0.205	0.283	0.000	0.329	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	19	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.86	1.00	1.00
time (sec)	N/A	0.009	63.335	0.102	1.033	0.258	1.700	0.386	6.182

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	33	20	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.50	0.91	1.00	1.00
time (sec)	N/A	0.010	58.438	0.105	1.705	0.259	10.580	0.395	6.575

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	855	855	929	936	1003	576	0	11931	0
N.S.	1	1.00	1.09	1.09	1.17	0.67	0.00	13.95	0.00
time (sec)	N/A	0.667	2.980	1.894	0.668	0.300	0.000	0.952	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	540	391	458	258	0	3727	0
N.S.	1	1.00	1.29	0.93	1.09	0.62	0.00	8.89	0.00
time (sec)	N/A	0.354	0.701	1.181	0.447	0.349	0.000	0.662	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	133	108	138	121	0	663	0
N.S.	1	1.00	0.98	0.79	1.01	0.89	0.00	4.88	0.00
time (sec)	N/A	0.092	0.072	0.898	0.425	0.266	0.000	0.510	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	434	434	170	156	0	548	0	0	0
N.S.	1	1.00	0.39	0.36	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	1.081	25.311	0.993	0.000	0.389	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	566	566	313	1554	0	798	0	0	0
N.S.	1	1.00	0.55	2.75	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	1.404	0.939	1.081	0.000	0.334	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	630	630	613	424	1260	461	0	0	0
N.S.	1	1.00	0.97	0.67	2.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.479	1.950	1.921	0.778	0.306	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	378	225	584	247	0	0	0
N.S.	1	1.00	1.19	0.71	1.84	0.78	0.00	0.00	0.00
time (sec)	N/A	0.266	0.851	0.654	0.466	0.336	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	146	105	219	143	0	0	0
N.S.	1	1.00	1.04	0.74	1.55	1.01	0.00	0.00	0.00
time (sec)	N/A	0.079	0.105	0.227	0.319	0.284	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	36	19	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.64	0.86	1.00	1.00
time (sec)	N/A	0.011	63.411	0.191	0.825	0.305	2.925	0.458	6.430

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	47	20	22	22
N.S.	1	1.00	1.09	0.91	1.00	2.14	0.91	1.00	1.00
time (sec)	N/A	0.010	46.459	0.185	1.102	0.277	22.101	0.518	6.537

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	289	289	226	0	175	234	0	486	0
N.S.	1	1.00	0.78	0.00	0.61	0.81	0.00	1.68	0.00
time (sec)	N/A	0.171	0.514	0.000	0.382	0.725	0.000	0.387	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	111	0	193	143	0	286	0
N.S.	1	1.00	0.55	0.00	0.96	0.71	0.00	1.42	0.00
time (sec)	N/A	0.122	0.290	0.000	0.373	0.698	0.000	0.340	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	97	0	156	128	0	192	0
N.S.	1	1.00	0.61	0.00	0.98	0.80	0.00	1.20	0.00
time (sec)	N/A	0.097	0.235	0.000	0.379	0.662	0.000	0.337	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	70	0	129	84	0	90	0
N.S.	1	1.00	0.82	0.00	1.52	0.99	0.00	1.06	0.00
time (sec)	N/A	0.044	0.097	0.000	0.363	0.679	0.000	0.303	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	23	46	0	37	0
N.S.	1	1.00	1.00	0.00	0.55	1.10	0.00	0.88	0.00
time (sec)	N/A	0.034	0.111	0.000	0.217	0.269	0.000	0.310	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	85	0	126	0	0	0	0
N.S.	1	1.00	0.71	0.00	1.05	0.00	0.00	0.00	0.00
time (sec)	N/A	0.098	0.184	0.000	0.374	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	175	175	115	0	129	0	0	0	0
N.S.	1	1.00	0.66	0.00	0.74	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	0.227	0.000	0.386	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	267	184	0	129	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.48	0.00	0.00	0.00	0.00
time (sec)	N/A	0.179	0.337	0.000	0.376	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	267	175	0	386	0	0	0	0
N.S.	1	1.00	0.66	0.00	1.45	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	0.609	0.000	0.520	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	227	227	160	0	424	0	0	0	0
N.S.	1	1.00	0.70	0.00	1.87	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	0.382	0.000	0.505	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	72	0	129	89	0	0	0
N.S.	1	1.00	0.81	0.00	1.45	1.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.045	0.000	0.371	0.667	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	23	46	0	0	0
N.S.	1	1.00	1.00	0.00	0.52	1.05	0.00	0.00	0.00
time (sec)	N/A	0.040	0.105	0.000	0.218	0.262	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	96	0	487	0	0	0	0
N.S.	1	1.00	0.72	0.00	3.66	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.153	0.000	0.482	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	168	133	0	380	0	0	0	0
N.S.	1	1.00	0.79	0.00	2.26	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	0.229	0.000	0.501	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	126	87	0	126	0	0	0	0
N.S.	1	1.00	0.69	0.00	1.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	0.197	0.000	0.379	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	247	208	0	129	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.52	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	0.316	0.000	0.381	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	168	131	0	172	0	0	0	0
N.S.	1	1.00	0.78	0.00	1.02	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.194	0.000	0.372	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	88	0	155	0	0	0	0
N.S.	1	1.00	0.76	0.00	1.34	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.204	0.000	0.378	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	42	0	31	64	0	0	0
N.S.	1	1.00	0.93	0.00	0.69	1.42	0.00	0.00	0.00
time (sec)	N/A	0.032	0.112	0.000	0.197	0.256	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	72	0	171	116	0	0	0
N.S.	1	1.00	0.79	0.00	1.88	1.27	0.00	0.00	0.00
time (sec)	N/A	0.049	0.068	0.000	0.377	0.681	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	172	172	113	0	1389	160	0	0	0
N.S.	1	1.00	0.66	0.00	8.08	0.93	0.00	0.00	0.00
time (sec)	N/A	0.098	0.167	0.000	1.173	0.777	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	112	0	1943	181	0	0	0
N.S.	1	1.00	0.52	0.00	8.95	0.83	0.00	0.00	0.00
time (sec)	N/A	0.122	0.262	0.000	1.667	0.780	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	237	0	1120	0	0	0	0
N.S.	1	1.00	0.79	0.00	3.75	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.701	0.000	1.021	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	262	228	0	749	0	0	0	0
N.S.	1	1.00	0.87	0.00	2.86	0.00	0.00	0.00	0.00
time (sec)	N/A	0.153	0.312	0.000	0.770	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	168	113	0	129	0	0	0	0
N.S.	1	1.00	0.67	0.00	0.77	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	0.294	0.000	0.399	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	90	0	126	0	0	0	0
N.S.	1	1.00	0.74	0.00	1.03	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	0.232	0.000	0.375	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	164	136	0	383	0	0	0	0
N.S.	1	1.00	0.83	0.00	2.34	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	0.273	0.000	0.487	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	96	0	487	0	0	0	0
N.S.	1	1.00	0.68	0.00	3.45	0.00	0.00	0.00	0.00
time (sec)	N/A	0.098	0.185	0.000	0.499	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	44	0	31	64	0	0	0
N.S.	1	1.00	0.94	0.00	0.66	1.36	0.00	0.00	0.00
time (sec)	N/A	0.046	0.119	0.000	0.210	0.270	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	72	0	129	133	0	0	0
N.S.	1	1.00	0.76	0.00	1.36	1.40	0.00	0.00	0.00
time (sec)	N/A	0.064	0.072	0.000	0.382	0.719	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	237	165	0	408	0	0	0	0
N.S.	1	1.00	0.70	0.00	1.72	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.768	0.000	0.498	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	277	192	0	405	0	0	0	0
N.S.	1	1.00	0.69	0.00	1.46	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	0.944	0.000	0.505	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.007	10.993	0.054	0.438	0.276	8.020	0.320	6.135

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	503	503	384	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.550	0.000	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	369	369	288	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.365	0.000	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	192	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.330	0.000	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	121	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	0.078	0.000	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.008	1.027	0.049	0.502	0.265	1.878	0.338	6.331

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12
time (sec)	N/A	0.008	0.905	0.049	0.508	0.274	7.610	0.325	6.314

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	519	519	411	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.366	0.621	0.000	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	383	383	313	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.402	0.000	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	255	215	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.133	0.388	0.000	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	126	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.196	0.000	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	25	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.25	1.10	0.85	1.10	1.10
time (sec)	N/A	0.013	2.165	0.048	0.482	0.286	4.458	0.328	6.146

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	27	22	19	22	22
N.S.	1	1.00	1.10	1.00	1.35	1.10	0.95	1.10	1.10
time (sec)	N/A	0.013	1.835	0.043	0.489	0.275	19.634	0.319	6.073

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	856	856	648	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.645	1.114	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	556	556	434	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	1.075	0.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	261	261	247	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.099	0.479	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	62	52	19	24	24
N.S.	1	1.00	1.09	1.00	2.82	2.36	0.86	1.09	1.09
time (sec)	N/A	0.015	3.245	0.326	1.041	0.301	18.645	0.384	6.223

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	67	52	20	24	24
N.S.	1	1.00	1.09	1.00	3.05	2.36	0.91	1.09	1.09
time (sec)	N/A	0.016	2.811	0.371	1.035	0.283	51.056	0.382	6.065

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	0	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09
time (sec)	N/A	0.018	2.040	0.066	0.982	0.255	0.000	0.347	6.107

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.011	1.782	0.098	0.861	0.290	103.165	0.353	5.884

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.004	0.189	0.084	0.560	0.261	41.919	0.322	6.071

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	23	19	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.05	0.86	1.09	1.09
time (sec)	N/A	0.019	1.508	0.092	0.524	0.259	106.656	0.326	5.842

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	27	0	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.23	0.00	1.09	1.09
time (sec)	N/A	0.019	1.328	0.066	0.527	0.275	0.000	0.332	5.948

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	0	22	1509	54	0	24	24
N.S.	1	1.00	0.00	1.00	68.59	2.45	0.00	1.09	1.09
time (sec)	N/A	0.017	0.000	0.304	4.797	0.272	0.000	0.403	5.915

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	0	20	1476	52	0	22	22
N.S.	1	1.00	0.00	1.00	73.80	2.60	0.00	1.10	1.10
time (sec)	N/A	0.011	0.000	0.273	4.587	0.276	0.000	0.425	6.055

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	1403	51	0	20	20
N.S.	1	1.00	1.11	1.00	77.94	2.83	0.00	1.11	1.11
time (sec)	N/A	0.005	7.925	0.296	1.468	0.279	0.000	0.347	6.012

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	0	22	5041	53	0	24	24
N.S.	1	1.00	0.00	1.00	229.14	2.41	0.00	1.09	1.09
time (sec)	N/A	0.018	0.000	0.369	24.272	0.288	0.000	0.366	5.969

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	0	22	5369	59	0	24	24
N.S.	1	1.00	0.00	1.00	244.05	2.68	0.00	1.09	1.09
time (sec)	N/A	0.016	0.000	0.280	33.204	0.286	0.000	0.363	5.859

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.015	1.843	0.113	1.935	0.271	0.000	171.343	5.808

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	150	188	258	185	0	1263	0
N.S.	1	1.00	0.67	0.84	1.15	0.83	0.00	5.64	0.00
time (sec)	N/A	0.283	0.421	0.558	0.292	0.287	0.000	0.375	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	79	109	153	108	0	520	0
N.S.	1	1.00	0.67	0.92	1.30	0.92	0.00	4.41	0.00
time (sec)	N/A	0.150	0.183	0.404	0.258	0.283	0.000	0.317	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	50	43	65	40	0	137	0
N.S.	1	1.00	1.32	1.13	1.71	1.05	0.00	3.61	0.00
time (sec)	N/A	0.054	0.034	0.317	0.233	0.296	0.000	0.382	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	83	129	0	100	0	169	0
N.S.	1	1.00	0.81	1.25	0.00	0.97	0.00	1.64	0.00
time (sec)	N/A	0.180	0.157	0.468	0.000	0.264	0.000	0.295	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	85	131	0	129	0	339	0
N.S.	1	1.00	0.90	1.39	0.00	1.37	0.00	3.61	0.00
time (sec)	N/A	0.130	0.538	0.547	0.000	0.270	0.000	0.312	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	151	423	0	328	0	1501	0
N.S.	1	1.00	0.65	1.82	0.00	1.41	0.00	6.44	0.00
time (sec)	N/A	0.298	1.290	0.652	0.000	0.297	0.000	0.377	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	252	249	321	240	0	1125	0
N.S.	1	1.00	0.99	0.98	1.26	0.94	0.00	4.43	0.00
time (sec)	N/A	0.424	0.342	0.599	0.305	0.303	0.000	0.317	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	105	100	137	103	0	305	0
N.S.	1	1.00	1.12	1.06	1.46	1.10	0.00	3.24	0.00
time (sec)	N/A	0.160	0.094	0.424	0.250	0.267	0.000	0.385	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	195	302	0	217	0	362	0
N.S.	1	1.00	0.76	1.18	0.00	0.85	0.00	1.42	0.00
time (sec)	N/A	0.464	0.303	0.622	0.000	0.271	0.000	0.343	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	263	292	0	271	0	686	0
N.S.	1	1.00	1.35	1.50	0.00	1.39	0.00	3.52	0.00
time (sec)	N/A	0.276	1.003	0.675	0.000	0.315	0.000	0.306	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	470	470	740	866	0	710	0	3078	0
N.S.	1	1.00	1.57	1.84	0.00	1.51	0.00	6.55	0.00
time (sec)	N/A	0.696	2.341	0.962	0.000	0.331	0.000	0.587	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	35	0	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.59	0.00	1.09	1.09
time (sec)	N/A	0.019	0.833	0.247	2.614	0.275	0.000	1.219	6.304

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	24	0	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.20	0.00	1.10	1.10
time (sec)	N/A	0.011	0.407	0.232	0.615	0.260	0.000	1.125	6.077

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	18	12	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.29	0.86	1.14	1.14
time (sec)	N/A	0.004	0.028	0.097	0.335	0.285	27.605	1.129	5.942

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	24	0	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.20	0.00	1.10	1.10
time (sec)	N/A	0.011	0.035	0.007	0.597	0.265	0.000	1.003	0.004

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	35	0	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.59	0.00	1.09	1.09
time (sec)	N/A	0.019	0.040	0.007	2.603	0.281	0.000	1.290	0.002

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	0	22	1281	63	0	24	24
N.S.	1	1.00	0.00	1.00	58.23	2.86	0.00	1.09	1.09
time (sec)	N/A	0.018	0.000	0.303	24.077	0.297	0.000	7.348	6.423

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1103	52	0	22	22
N.S.	1	1.00	1.10	1.00	55.15	2.60	0.00	1.10	1.10
time (sec)	N/A	0.010	16.276	0.355	2.100	0.288	0.000	5.685	6.460

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	974	47	0	16	16
N.S.	1	1.00	1.14	1.00	69.57	3.36	0.00	1.14	1.14
time (sec)	N/A	0.004	2.048	0.180	0.567	0.294	0.000	3.644	6.316

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1103	52	0	22	22
N.S.	1	1.00	1.10	1.00	55.15	2.60	0.00	1.10	1.10
time (sec)	N/A	0.011	1.349	0.010	2.106	0.270	0.000	4.931	0.003

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	0	22	1281	63	0	24	24
N.S.	1	1.00	0.00	1.00	58.23	2.86	0.00	1.09	1.09
time (sec)	N/A	0.019	0.000	0.007	24.281	0.293	0.000	7.523	0.002

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	26	0	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.18	0.00	1.09	1.09
time (sec)	N/A	0.018	1.228	0.192	0.708	0.306	0.000	0.493	6.321

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	94	0	0	80	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.193	0.121	0.000	0.000	0.092	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	47	151	146	74	129	0	109
N.S.	1	1.00	0.49	1.57	1.52	0.77	1.34	0.00	1.14
time (sec)	N/A	0.144	0.271	0.401	0.345	0.284	1.780	0.000	7.174

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	40	133	99	64	107	0	88
N.S.	1	1.00	0.54	1.80	1.34	0.86	1.45	0.00	1.19
time (sec)	N/A	0.121	0.248	0.366	0.315	0.265	1.011	0.000	6.817

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	30	117	60	55	70	0	63
N.S.	1	1.00	0.67	2.60	1.33	1.22	1.56	0.00	1.40
time (sec)	N/A	0.081	0.163	0.363	0.307	0.261	0.620	0.000	6.784

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	105	31	43	49	0	49
N.S.	1	1.00	1.00	4.20	1.24	1.72	1.96	0.00	1.96
time (sec)	N/A	0.011	0.059	0.720	0.288	0.290	0.423	0.000	6.288

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	55	55	36	106	42	57	0	0	0
N.S.	1	1.00	0.65	1.93	0.76	1.04	0.00	0.00	0.00
time (sec)	N/A	0.109	0.110	0.385	0.327	0.261	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	51	102	229	70	0	0	0
N.S.	1	1.00	0.66	1.32	2.97	0.91	0.00	0.00	0.00
time (sec)	N/A	0.112	0.198	0.380	0.345	0.258	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	69	123	256	90	0	0	0
N.S.	1	1.00	0.59	1.06	2.21	0.78	0.00	0.00	0.00
time (sec)	N/A	0.136	0.209	0.395	0.345	0.299	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	138	0	0	98	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.190	0.333	0.000	0.000	0.097	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	38	135	32	67	85	0	71
N.S.	1	1.00	0.66	2.33	0.55	1.16	1.47	0.00	1.22
time (sec)	N/A	0.130	0.066	0.453	0.309	0.273	1.707	0.000	6.793

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	105	240	73	134	0	0	0
N.S.	1	1.00	0.68	1.55	0.47	0.86	0.00	0.00	0.00
time (sec)	N/A	0.126	0.292	0.446	0.320	0.294	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	119	16	51	63	0	53
N.S.	1	1.00	1.00	3.84	0.52	1.65	2.03	0.00	1.71
time (sec)	N/A	0.073	0.076	0.735	0.300	0.283	0.642	0.000	6.321

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	80	157	51	120	0	0	0
N.S.	1	1.00	0.68	1.34	0.44	1.03	0.00	0.00	0.00
time (sec)	N/A	0.035	0.091	0.443	0.317	0.320	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	73	73	47	128	47	67	0	0	0
N.S.	1	1.00	0.64	1.75	0.64	0.92	0.00	0.00	0.00
time (sec)	N/A	0.070	0.095	0.434	0.380	0.273	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	105	232	76	134	0	0	0
N.S.	1	1.00	0.78	1.72	0.56	0.99	0.00	0.00	0.00
time (sec)	N/A	0.100	0.250	0.431	0.380	0.281	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	67	119	52	86	0	0	0
N.S.	1	1.00	0.68	1.21	0.53	0.88	0.00	0.00	0.00
time (sec)	N/A	0.130	0.252	0.457	0.383	0.267	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	142	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.497	0.399	0.000	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	129	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	0.258	0.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	129	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	0.247	0.000	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	129	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	0.216	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	119	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.059	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	73	73	47	131	144	70	0	0	0
N.S.	1	1.00	0.64	1.79	1.97	0.96	0.00	0.00	0.00
time (sec)	N/A	0.204	0.158	0.829	0.465	0.340	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	110	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	0.200	0.000	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	114	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.174	0.213	0.000	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	142	0	0	112	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.231	0.495	0.000	0.000	0.108	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	79	208	286	111	0	0	0
N.S.	1	1.00	0.48	1.26	1.73	0.67	0.00	0.00	0.00
time (sec)	N/A	0.127	0.300	0.390	0.333	0.275	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	69	190	219	95	0	0	0
N.S.	1	1.00	0.50	1.37	1.58	0.68	0.00	0.00	0.00
time (sec)	N/A	0.103	0.288	0.388	0.393	0.276	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	55	174	162	82	0	0	0
N.S.	1	1.00	0.70	2.20	2.05	1.04	0.00	0.00	0.00
time (sec)	N/A	0.085	0.184	0.396	0.513	0.287	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	47	158	116	60	0	0	0
N.S.	1	1.00	0.85	2.87	2.11	1.09	0.00	0.00	0.00
time (sec)	N/A	0.021	0.084	0.694	0.287	0.273	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	99	99	50	121	52	63	0	0	0
N.S.	1	1.00	0.51	1.22	0.53	0.64	0.00	0.00	0.00
time (sec)	N/A	0.131	0.119	0.381	0.328	0.288	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	65	112	265	80	0	0	0
N.S.	1	1.00	0.76	1.30	3.08	0.93	0.00	0.00	0.00
time (sec)	N/A	0.120	0.152	0.431	0.330	0.318	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	85	137	296	104	0	0	0
N.S.	1	1.00	0.71	1.15	2.49	0.87	0.00	0.00	0.00
time (sec)	N/A	0.136	0.215	0.393	0.339	0.293	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	209	189	0	0	130	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.180	0.770	0.000	0.000	0.107	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	67	200	47	96	0	0	0
N.S.	1	1.00	0.74	2.20	0.52	1.05	0.00	0.00	0.00
time (sec)	N/A	0.106	0.279	0.436	0.297	0.311	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	113	309	99	149	0	0	0
N.S.	1	1.00	0.58	1.58	0.51	0.76	0.00	0.00	0.00
time (sec)	N/A	0.137	0.281	0.459	0.297	0.373	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	55	182	28	72	0	0	0
N.S.	1	1.00	0.85	2.80	0.43	1.11	0.00	0.00	0.00
time (sec)	N/A	0.064	0.131	0.704	0.287	0.332	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	93	224	76	119	0	0	0
N.S.	1	1.00	0.63	1.51	0.51	0.80	0.00	0.00	0.00
time (sec)	N/A	0.040	0.077	0.402	0.302	0.320	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	115	115	60	145	55	73	0	0	0
N.S.	1	1.00	0.52	1.26	0.48	0.63	0.00	0.00	0.00
time (sec)	N/A	0.084	0.144	0.454	0.349	0.336	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	107	301	90	133	0	0	0
N.S.	1	1.00	0.81	2.28	0.68	1.01	0.00	0.00	0.00
time (sec)	N/A	0.096	0.199	0.445	0.359	0.331	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	79	134	64	100	0	0	0
N.S.	1	1.00	0.49	0.83	0.40	0.62	0.00	0.00	0.00
time (sec)	N/A	0.133	0.182	0.450	0.352	0.289	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	194	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.734	0.000	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	188	161	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.501	0.000	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	188	168	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	0.507	0.000	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	188	160	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.143	0.524	0.000	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	149	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.221	0.000	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	121	121	63	149	153	78	0	0	0
N.S.	1	1.00	0.52	1.23	1.26	0.64	0.00	0.00	0.00
time (sec)	N/A	0.114	0.187	0.757	0.471	0.325	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	180	180	125	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.326	0.000	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	184	184	129	0	0	0	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.175	0.354	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [177] had the largest ratio of [.6999999999999999956]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	16	0.250
2	A	5	4	1.00	16	0.250
3	A	4	3	1.00	14	0.214
4	A	5	4	1.00	16	0.250
5	A	7	6	1.00	16	0.375
6	A	8	6	1.00	16	0.375
7	A	7	6	1.00	16	0.375
8	A	6	5	1.00	16	0.312
9	A	4	3	1.00	12	0.250
10	A	6	5	1.00	16	0.312
11	A	7	6	1.00	16	0.375
12	A	10	8	1.00	18	0.444
13	A	7	6	1.00	18	0.333
14	A	2	2	1.00	16	0.125
15	A	9	6	1.00	18	0.333
16	A	13	8	1.00	18	0.444
17	A	15	8	1.00	18	0.444
18	A	13	8	1.00	18	0.444
19	A	11	8	1.00	18	0.444
20	A	8	5	1.00	14	0.357
21	A	11	8	1.00	18	0.444
22	A	13	8	1.00	18	0.444
23	A	7	5	1.00	14	0.357
24	A	4	4	1.00	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	2	1.00	12	0.167
26	A	8	4	1.00	14	0.286
27	A	12	6	1.00	14	0.429
28	A	10	5	1.00	14	0.357
29	A	8	4	1.00	10	0.400
30	A	9	5	1.00	14	0.357
31	A	6	3	1.00	10	0.300
32	A	7	4	1.00	14	0.286
33	A	3	2	1.00	12	0.167
34	A	8	6	1.00	12	0.500
35	A	11	7	1.00	18	0.389
36	A	9	6	1.00	18	0.333
37	A	4	4	1.00	16	0.250
38	N/A	0	0	1.00	18	0.000
39	N/A	0	0	1.00	18	0.000
40	N/A	0	0	1.00	18	0.000
41	N/A	0	0	1.00	14	0.000
42	N/A	0	0	1.00	18	0.000
43	A	19	11	1.00	18	0.611
44	A	12	9	1.00	18	0.500
45	A	6	6	1.00	16	0.375
46	N/A	0	0	1.00	18	0.000
47	N/A	0	0	1.00	18	0.000
48	N/A	0	0	1.00	18	0.000
49	N/A	0	0	1.00	14	0.000
50	N/A	0	0	1.00	18	0.000
51	N/A	0	0	1.00	20	0.000
52	A	13	5	1.00	20	0.250
53	A	9	5	1.00	20	0.250
54	A	5	3	1.00	18	0.167
55	N/A	0	0	1.00	20	0.000
56	N/A	0	0	1.00	20	0.000
57	A	5	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
58	A	4	3	1.00	16	0.188
59	A	5	4	1.00	16	0.250
60	A	7	6	1.00	16	0.375
61	A	6	4	1.00	16	0.250
62	A	5	3	1.00	14	0.214
63	A	6	4	1.00	16	0.250
64	A	7	5	1.00	16	0.312
65	A	6	4	1.00	16	0.250
66	A	4	2	1.00	12	0.167
67	A	6	4	1.00	16	0.250
68	A	7	5	1.00	16	0.312
69	A	7	6	1.00	18	0.333
70	A	2	2	1.00	18	0.111
71	A	9	6	1.00	18	0.333
72	A	13	8	1.00	18	0.444
73	A	11	7	1.00	18	0.389
74	A	9	5	1.00	16	0.312
75	A	11	7	0.99	18	0.389
76	A	13	7	0.99	18	0.389
77	A	11	7	1.00	18	0.389
78	A	8	4	1.00	14	0.286
79	A	11	7	0.99	18	0.389
80	A	13	7	0.99	18	0.389
81	A	9	6	1.00	18	0.333
82	A	4	4	1.00	18	0.222
83	N/A	0	0	1.00	18	0.000
84	N/A	0	0	1.00	18	0.000
85	N/A	0	0	1.00	16	0.000
86	N/A	0	0	1.00	18	0.000
87	N/A	0	0	1.00	14	0.000
88	N/A	0	0	1.00	18	0.000
89	A	12	9	1.00	18	0.500
90	A	6	6	1.00	18	0.333
91	N/A	0	0	1.00	18	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
92	N/A	0	0	1.00	18	0.000
93	N/A	0	0	1.00	16	0.000
94	N/A	0	0	1.00	18	0.000
95	N/A	0	0	1.00	14	0.000
96	N/A	0	0	1.00	18	0.000
97	N/A	0	0	1.00	20	0.000
98	A	13	5	1.00	20	0.250
99	A	9	5	1.00	20	0.250
100	A	5	3	1.00	18	0.167
101	N/A	0	0	1.00	20	0.000
102	N/A	0	0	1.00	20	0.000
103	A	7	5	1.00	12	0.417
104	A	6	5	1.00	10	0.500
105	A	5	5	1.00	8	0.625
106	A	3	3	1.00	12	0.250
107	A	2	2	1.00	12	0.167
108	A	3	3	1.00	12	0.250
109	A	4	3	1.00	12	0.250
110	A	5	3	1.00	12	0.250
111	A	9	6	1.00	14	0.429
112	A	8	8	1.00	12	0.667
113	A	6	6	1.00	10	0.600
114	A	5	4	1.00	14	0.286
115	A	3	3	1.00	14	0.214
116	A	3	3	1.00	14	0.214
117	A	5	5	1.00	14	0.357
118	A	5	4	1.00	14	0.286
119	A	5	5	1.00	8	0.625
120	A	3	3	1.00	12	0.250
121	A	4	4	1.00	12	0.333
122	A	2	2	1.00	12	0.167
123	A	5	5	1.00	12	0.417
124	A	2	2	1.00	12	0.167
125	A	3	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
126	A	3	3	1.00	6	0.500
127	A	5	5	1.00	8	0.625
128	A	7	5	1.00	8	0.625
129	N/A	0	0	1.00	18	0.000
130	N/A	0	0	1.00	20	0.000
131	A	3	3	1.00	20	0.150
132	N/A	0	0	1.00	22	0.000
133	A	5	5	1.00	22	0.227
134	N/A	0	0	1.00	24	0.000
135	A	3	3	1.00	12	0.250
136	A	5	4	1.00	14	0.286
137	A	8	4	1.00	14	0.286
138	A	8	4	1.00	14	0.286
139	A	3	2	1.00	8	0.250
140	A	5	3	1.00	10	0.300
141	A	8	3	1.00	10	0.300
142	A	3	2	1.00	12	0.167
143	A	5	3	1.00	14	0.214
144	A	8	3	1.00	14	0.214
145	A	3	3	1.00	16	0.188
146	A	3	3	1.00	16	0.188
147	A	5	5	1.00	16	0.312
148	A	7	6	1.00	18	0.333
149	A	12	6	1.00	18	0.333
150	A	6	5	1.00	16	0.312
151	A	8	6	1.00	18	0.333
152	A	14	6	1.00	18	0.333
153	A	10	8	1.00	18	0.444
154	A	7	6	1.00	18	0.333
155	A	5	4	1.00	16	0.250
156	A	1	1	1.00	10	0.100
157	N/A	0	0	1.00	18	0.000
158	N/A	0	0	1.00	18	0.000
159	A	16	11	1.00	18	0.611

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	12	10	1.00	18	0.556
161	A	8	7	1.00	16	0.438
162	A	3	3	1.00	10	0.300
163	N/A	0	0	1.00	18	0.000
164	N/A	0	0	1.00	18	0.000
165	A	14	10	1.00	20	0.500
166	A	11	8	1.00	20	0.400
167	A	7	6	1.00	18	0.333
168	A	3	3	1.00	12	0.250
169	N/A	0	0	1.00	20	0.000
170	N/A	0	0	1.00	20	0.000
171	A	14	9	1.00	20	0.450
172	A	10	7	1.00	20	0.350
173	A	8	5	1.00	18	0.278
174	A	3	2	1.00	12	0.167
175	N/A	0	0	1.00	20	0.000
176	N/A	0	0	1.00	20	0.000
177	A	18	14	1.00	20	0.700
178	A	12	11	1.00	18	0.611
179	A	5	5	1.00	12	0.417
180	N/A	0	0	1.00	20	0.000
181	N/A	0	0	1.00	20	0.000
182	A	13	10	1.00	20	0.500
183	A	8	5	1.00	18	0.278
184	A	3	2	1.00	12	0.167
185	N/A	0	0	1.00	20	0.000
186	N/A	0	0	1.00	20	0.000
187	A	14	3	1.00	22	0.136
188	A	8	3	1.00	20	0.150
189	A	3	3	1.00	14	0.214
190	A	8	4	1.00	22	0.182
191	A	10	6	1.00	22	0.273
192	A	12	9	1.00	22	0.409
193	A	9	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
194	A	4	3	1.00	14	0.214
195	N/A	0	0	1.00	22	0.000
196	N/A	0	0	1.00	22	0.000
197	A	23	5	1.00	22	0.227
198	A	14	5	1.00	20	0.250
199	A	6	5	1.00	14	0.357
200	A	13	5	1.00	22	0.227
201	A	10	6	1.00	22	0.273
202	A	14	8	1.00	22	0.364
203	A	8	3	1.00	20	0.150
204	A	4	3	1.00	14	0.214
205	N/A	0	0	1.00	22	0.000
206	N/A	0	0	1.00	22	0.000
207	A	20	4	1.00	22	0.182
208	A	11	4	1.00	20	0.200
209	A	4	3	1.00	14	0.214
210	A	11	4	1.00	22	0.182
211	A	13	6	1.00	22	0.273
212	A	17	10	1.00	22	0.454
213	A	10	8	1.00	20	0.400
214	A	5	5	1.00	14	0.357
215	N/A	0	0	1.00	22	0.000
216	N/A	0	0	1.00	22	0.000
217	A	29	5	1.00	22	0.227
218	A	17	5	1.00	20	0.250
219	A	7	5	1.00	14	0.357
220	A	16	5	1.00	22	0.227
221	A	13	6	1.00	22	0.273
222	A	24	13	1.00	22	0.591
223	A	15	12	1.00	20	0.600
224	A	7	7	1.00	14	0.500
225	N/A	0	0	1.00	22	0.000
226	N/A	0	0	1.00	22	0.000
227	A	9	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
228	A	7	4	1.00	27	0.148
229	A	6	4	1.00	27	0.148
230	A	4	4	1.00	27	0.148
231	A	3	3	1.00	27	0.111
232	A	6	6	1.00	27	0.222
233	A	7	6	1.00	27	0.222
234	A	9	6	1.00	27	0.222
235	A	9	8	1.00	27	0.296
236	A	8	8	1.00	27	0.296
237	A	5	5	1.00	27	0.185
238	A	4	4	1.00	27	0.148
239	A	6	6	1.00	27	0.222
240	A	7	7	1.00	27	0.259
241	A	7	7	1.00	27	0.259
242	A	9	6	1.00	27	0.222
243	A	7	6	1.00	27	0.222
244	A	6	6	1.00	27	0.222
245	A	3	3	1.00	27	0.111
246	A	4	4	1.00	27	0.148
247	A	6	4	1.00	27	0.148
248	A	7	4	1.00	27	0.148
249	A	11	9	1.00	27	0.333
250	A	10	9	1.00	27	0.333
251	A	8	7	1.00	27	0.259
252	A	7	7	1.00	27	0.259
253	A	8	8	1.00	27	0.296
254	A	6	6	1.00	27	0.222
255	A	4	4	1.00	27	0.148
256	A	5	5	1.00	27	0.185
257	A	9	9	1.00	27	0.333
258	A	10	9	1.00	27	0.333
259	N/A	0	0	1.00	18	0.000
260	A	14	5	1.00	16	0.312
261	A	11	5	1.00	16	0.312
262	A	8	5	1.00	14	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
263	A	3	2	1.00	12	0.167
264	N/A	0	0	1.00	16	0.000
265	N/A	0	0	1.00	16	0.000
266	A	16	6	1.00	20	0.300
267	A	13	6	1.00	20	0.300
268	A	10	6	1.00	18	0.333
269	A	4	2	1.00	16	0.125
270	N/A	0	0	1.00	20	0.000
271	N/A	0	0	1.00	20	0.000
272	A	28	10	1.00	22	0.454
273	A	19	10	1.00	20	0.500
274	A	8	4	1.00	18	0.222
275	N/A	0	0	1.00	22	0.000
276	N/A	0	0	1.00	22	0.000
277	N/A	0	0	1.00	22	0.000
278	N/A	0	0	1.00	20	0.000
279	N/A	0	0	1.00	18	0.000
280	N/A	0	0	1.00	22	0.000
281	N/A	0	0	1.00	22	0.000
282	N/A	0	0	1.00	22	0.000
283	N/A	0	0	1.00	20	0.000
284	N/A	0	0	1.00	18	0.000
285	N/A	0	0	1.00	22	0.000
286	N/A	0	0	1.00	22	0.000
287	N/A	0	0	1.00	24	0.000
288	A	23	6	1.00	20	0.300
289	A	15	6	1.00	18	0.333
290	A	6	5	1.00	12	0.417
291	A	12	6	1.00	20	0.300
292	A	7	6	1.00	20	0.300
293	A	15	6	1.00	20	0.300
294	A	27	11	1.00	20	0.550

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	12	8	1.00	14	0.571
296	A	22	6	1.00	22	0.273
297	A	12	8	1.00	22	0.364
298	A	27	11	1.00	22	0.500
299	N/A	0	0	1.00	22	0.000
300	N/A	0	0	1.00	20	0.000
301	N/A	0	0	1.00	14	0.000
302	N/A	0	0	1.00	20	0.000
303	N/A	0	0	1.00	22	0.000
304	N/A	0	0	1.00	22	0.000
305	N/A	0	0	1.00	20	0.000
306	N/A	0	0	1.00	14	0.000
307	N/A	0	0	1.00	20	0.000
308	N/A	0	0	1.00	22	0.000
309	N/A	0	0	1.00	22	0.000
310	A	4	3	1.00	18	0.167
311	A	5	3	1.00	18	0.167
312	A	4	3	1.00	18	0.167
313	A	3	3	1.00	16	0.188
314	A	2	2	1.00	14	0.143
315	A	4	4	1.00	18	0.222
316	A	5	5	1.00	18	0.278
317	A	6	5	1.00	18	0.278
318	A	4	3	1.00	20	0.150
319	A	4	4	1.00	20	0.200
320	A	5	5	1.00	20	0.250
321	A	3	3	1.00	18	0.167
322	A	4	4	1.00	16	0.250
323	A	4	4	1.00	20	0.200
324	A	5	5	1.00	20	0.250
325	A	6	6	1.00	20	0.300
326	A	4	3	1.00	20	0.150
327	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
328	A	4	3	1.00	20	0.150
329	A	4	3	1.00	18	0.167
330	A	4	3	1.00	16	0.188
331	A	4	4	1.00	20	0.200
332	A	4	3	1.00	20	0.150
333	A	4	3	1.00	20	0.150
334	A	6	4	1.00	18	0.222
335	A	5	4	1.00	18	0.222
336	A	5	5	1.00	18	0.278
337	A	3	3	1.00	16	0.188
338	A	3	3	1.00	14	0.214
339	A	6	5	1.00	18	0.278
340	A	6	6	1.00	18	0.333
341	A	8	7	1.00	18	0.389
342	A	6	4	1.00	20	0.200
343	A	4	4	1.00	20	0.200
344	A	7	6	1.00	20	0.300
345	A	4	4	1.00	18	0.222
346	A	6	5	1.00	16	0.312
347	A	6	5	1.00	20	0.250
348	A	7	7	1.00	20	0.350
349	A	8	7	1.00	20	0.350
350	A	6	4	1.00	20	0.200
351	A	6	4	1.00	20	0.200
352	A	6	4	1.00	20	0.200
353	A	6	4	1.00	18	0.222
354	A	6	4	1.00	16	0.250
355	A	6	5	1.00	20	0.250
356	A	6	4	1.00	20	0.200
357	A	6	4	1.00	20	0.200

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^5(a+b\sin(c+dx^2))\ dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)\ dx=\frac{ax^6}{6}+\frac{b\cos\left(c+dx^2\right)}{d^3}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}+\frac{bx^2\sin\left(c+dx^2\right)}{d^2}$$

```
[Out] 1/6*a*x^6+b*cos(d*x^2+c)/d^3-1/2*b*x^4*cos(d*x^2+c)/d+b*x^2*sin(d*x^2+c)/d^
2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3460, 3377, 2718}

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)\ dx=\frac{ax^6}{6}+\frac{b\cos\left(c+dx^2\right)}{d^3}+\frac{bx^2\sin\left(c+dx^2\right)}{d^2}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}$$

```
[In] Int[x^5*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^6)/6 + (b*Cos[c + d*x^2])/d^3 - (b*x^4*Cos[c + d*x^2])/(2*d) + (b*x^2*
Sin[c + d*x^2])/d^2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^5 + bx^5 \sin\left(c + dx^2\right)\right) dx \\
&= \frac{ax^6}{6} + b \int x^5 \sin\left(c + dx^2\right) dx \\
&= \frac{ax^6}{6} + \frac{1}{2} b \text{Subst}\left(\int x^2 \sin(c + dx)\, dx, x, x^2\right) \\
&= \frac{ax^6}{6} - \frac{bx^4 \cos\left(c + dx^2\right)}{2d} + \frac{b \text{Subst}\left(\int x \cos(c + dx)\, dx, x, x^2\right)}{d} \\
&= \frac{ax^6}{6} - \frac{bx^4 \cos\left(c + dx^2\right)}{2d} + \frac{bx^2 \sin\left(c + dx^2\right)}{d^2} - \frac{b \text{Subst}\left(\int \sin(c + dx)\, dx, x, x^2\right)}{d^2} \\
&= \frac{ax^6}{6} + \frac{b \cos\left(c + dx^2\right)}{d^3} - \frac{bx^4 \cos\left(c + dx^2\right)}{2d} + \frac{bx^2 \sin\left(c + dx^2\right)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int x^5 \left(a + b \sin\left(c + dx^2\right)\right) dx = \frac{ad^3x^6 - 3b(-2 + d^2x^4) \cos\left(c + dx^2\right) + 6bdx^2 \sin\left(c + dx^2\right)}{6d^3}$$

```
[In] Integrate[x^5*(a + b*Sin[c + d*x^2]),x]
[Out] (a*d^3*x^6 - 3*b*(-2 + d^2*x^4)*Cos[c + d*x^2] + 6*b*d*x^2*Sin[c + d*x^2])/
(6*d^3)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{a\,x^{6}}{6}-\frac{b\left(x^{4} d^{2}-2\right) \cos \left(d\,x^{2}+c \right)}{2 d^{3}}+\frac{b\,x^{2} \sin \left(d\,x^{2}+c \right)}{d^{2}}$	47
parallelrisch	$\frac{\left(-3 x^{4} d^{2}+6\right) b \cos \left(d\,x^{2}+c \right)+a\,x^{6} d^{3}+6 b\,x^{2} \sin \left(d\,x^{2}+c \right) d -6 b}{6 d^{3}}$	53
default	$\frac{a\,x^{6}}{6}+b \left(-\frac{x^{4} \cos \left(d\,x^{2}+c \right)}{2 d}+\frac{\frac{x^{2} \sin \left(d\,x^{2}+c \right)}{d}+\frac{\cos \left(d\,x^{2}+c \right)}{d^{2}}}{d}\right)$	62
parts	$\frac{a\,x^{6}}{6}+b \left(-\frac{x^{4} \cos \left(d\,x^{2}+c \right)}{2 d}+\frac{\frac{x^{2} \sin \left(d\,x^{2}+c \right)}{d}+\frac{\cos \left(d\,x^{2}+c \right)}{d^{2}}}{d}\right)$	62
norman	$\frac{\frac{2 b}{d^{3}}+\frac{a\,x^{6}}{6}+\frac{a\,x^{6} \left(\tan^{2}\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{6}-\frac{b\,x^{4}}{2 d}+\frac{2 b\,x^{2} \tan \left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)}{d^{2}}+\frac{b\,x^{4} \left(\tan^{2}\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{2 d}}{1+\tan^{2}\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)}$	102

```
[In] int(x^5*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*x^6-1/2*b*(d^2*x^4-2)/d^3*cos(d*x^2+c)+b*x^2*sin(d*x^2+c)/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ad^3x^6+6\,bdx^2\sin\left(dx^2+c\right)-3\left(bd^2x^4-2\,b\right)\cos\left(dx^2+c\right)}{6\,d^3}$$

```
[In] integrate(x^5*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*d^3*x^6 + 6*b*d*x^2*sin(d*x^2 + c) - 3*(b*d^2*x^4 - 2*b)*cos(d*x^2 +
 c))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\begin{cases}\frac{ax^6}{6}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}+\frac{bx^2\sin\left(c+dx^2\right)}{d^2}+\frac{b\cos\left(c+dx^2\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{x^6\left(a+b\sin\left(c\right)\right)}{6} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**5*(a+b*sin(d*x**2+c)),x)
```

```
[Out] Piecewise((a*x**6/6 - b*x**4*cos(c + d*x**2)/(2*d) + b*x**2*sin(c + d*x**2)
/d**2 + b*cos(c + d*x**2)/d**3, Ne(d, 0)), (x**6*(a + b*sin(c))/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int x^5 \left(a + b\sin\left(c + dx^2\right)\right)\, dx = \frac{1}{6}\,ax^6 + \frac{(2\,dx^2\sin\left(dx^2 + c\right) - (d^2x^4 - 2)\cos\left(dx^2 + c\right))b}{2\,d^3}$$

```
[In] integrate(x^5*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/2*(2*d*x^2*sin(d*x^2 + c) - (d^2*x^4 - 2)*cos(d*x^2 + c))*b/d
^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(53) = 106.

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.25

$$\int x^5 \left(a + b\sin\left(c + dx^2\right)\right)\, dx = -\frac{\left((dx^2 + c)^2b - 2\,(dx^2 + c)bc - 2\,b\right)\cos\left(dx^2 + c\right)}{2\,d^3} + \frac{((dx^2 + c)b - bc)\sin\left(dx^2 + c\right)}{d^3} + \frac{(dx^2 + c)^3a - 3\,(dx^2 + c)^2ac}{6\,d^3} + \frac{(dx^2 + c)ac^2 - bc^2\cos\left(dx^2 + c\right)}{2\,d^3}$$

```
[In] integrate(x^5*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] -1/2*((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c - 2*b)*cos(d*x^2 + c)/d^3 + ((d*x
^2 + c)*b - b*c)*sin(d*x^2 + c)/d^3 + 1/6*((d*x^2 + c)^3*a - 3*(d*x^2 + c)^
2*a*c)/d^3 + 1/2*((d*x^2 + c)*a*c^2 - b*c^2*cos(d*x^2 + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int x^5 \left(a+b\sin\left(c+dx^2\right)\right)\, dx = \frac{a\,x^6}{6} + \frac{b\,\cos\left(d\,x^2 + c\right) - \frac{b\,d^2\,x^4\,\cos\left(d\,x^2+c\right)}{2} + b\,d\,x^2\,\sin\left(d\,x^2 + c\right)}{d^3}$$

```
[In] int(x^5*(a + b*sin(c + d*x^2)),x)
```

```
[Out] (a*x^6)/6 + (b*cos(c + d*x^2) - (b*d^2*x^4*cos(c + d*x^2))/2 + b*d*x^2*sin(
c + d*x^2))/d^3
```

3.2 $\int x^3(a+b\sin(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^4}{4}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}+\frac{b\sin\left(c+dx^2\right)}{2d^2}$$

```
[Out] 1/4*a*x^4-1/2*b*x^2*cos(d*x^2+c)/d+1/2*b*sin(d*x^2+c)/d^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3460, 3377, 2717}

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^4}{4}+\frac{b\sin\left(c+dx^2\right)}{2d^2}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}$$

```
[In] Int[x^3*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^4)/4 - (b*x^2*Cos[c + d*x^2])/(2*d) + (b*Sin[c + d*x^2])/(2*d^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^3 + bx^3 \sin\left(c + dx^2\right)\right) dx \\
&= \frac{ax^4}{4} + b \int x^3 \sin\left(c + dx^2\right) dx \\
&= \frac{ax^4}{4} + \frac{1}{2} b \text{Subst}\left(\int x \sin(c + dx)\, dx, x, x^2\right) \\
&= \frac{ax^4}{4} - \frac{bx^2 \cos\left(c + dx^2\right)}{2d} + \frac{b \text{Subst}\left(\int \cos(c + dx)\, dx, x, x^2\right)}{2d} \\
&= \frac{ax^4}{4} - \frac{bx^2 \cos\left(c + dx^2\right)}{2d} + \frac{b \sin\left(c + dx^2\right)}{2d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int x^3 \left(a + b \sin\left(c + dx^2\right)\right) dx = \frac{ax^4}{4} - \frac{bx^2 \cos\left(c + dx^2\right)}{2d} + \frac{b \sin\left(c + dx^2\right)}{2d^2}$$

```
[In] Integrate[x^3*(a + b*Sin[c + d*x^2]),x]
[Out] (a*x^4)/4 - (b*x^2*Cos[c + d*x^2])/(2*d) + (b*Sin[c + d*x^2])/(2*d^2)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{a\,x^{4}}{4}-\frac{b\,x^{2}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{b\sin\left(d\,x^{2}+c\right)}{2d^{2}}$	39
default	$\frac{a\,x^{4}}{4}+b\left(-\frac{x^{2}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{\sin\left(d\,x^{2}+c\right)}{2d^{2}}\right)$	40
parts	$\frac{a\,x^{4}}{4}+b\left(-\frac{x^{2}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{\sin\left(d\,x^{2}+c\right)}{2d^{2}}\right)$	40
parallelrisch	$\frac{a\,x^{4}d^{2}-2x^{2}bd\cos\left(d\,x^{2}+c\right)+2b\sin\left(d\,x^{2}+c\right)}{4d^{2}}$	41
norman	$\frac{\frac{b\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)}{d^{2}}+\frac{a\,x^{4}}{4}+\frac{a\,x^{4}\left(\tan^{2}\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{4}-\frac{b\,x^{2}}{2d}+\frac{b\,x^{2}\left(\tan^{2}\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{2d}}{1+\tan^{2}\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)}$	92

```
[In] int(x^3*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
[Out] 1/4*a*x^4-1/2*b*x^2*cos(d*x^2+c)/d+1/2*b*sin(d*x^2+c)/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ad^2x^4-2\,bdx^2\cos\left(dx^2+c\right)+2\,b\sin\left(dx^2+c\right)}{4\,d^2}$$

```
[In] integrate(x^3*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] 1/4*(a*d^2*x^4 - 2*b*d*x^2*cos(d*x^2 + c) + 2*b*sin(d*x^2 + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\begin{cases}\frac{ax^4}{4}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}+\frac{b\sin\left(c+dx^2\right)}{2d^2} & \text{for } d\neq 0\\ \frac{x^4(a+b\sin\left(c\right))}{4} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**3*(a+b*sin(d*x**2+c)),x)
[Out] Piecewise((a*x**4/4 - b*x**2*cos(c + d*x**2)/(2*d) + b*sin(c + d*x**2)/(2*d
**2), Ne(d, 0)), (x**4*(a + b*sin(c))/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int x^3 \left(a + b \sin \left(c + dx^2\right)\right) \, dx = \frac{1}{4} \, ax^4 - \frac{(dx^2 \cos \left(dx^2 + c\right) - \sin \left(dx^2 + c\right))b}{2 \, d^2}$$

```
[In] integrate(x^3*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] 1/4*a*x^4 - 1/2*(d*x^2*cos(d*x^2 + c) - sin(d*x^2 + c))*b/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int x^3 \left(a + b \sin \left(c + dx^2\right)\right) \, dx = \frac{(dx^2 + c)^2 a - 2 \, (dx^2 + c)b \cos \left(dx^2 + c\right) + 2 \, b \sin \left(dx^2 + c\right)}{4 \, d^2} - \frac{(dx^2 + c)ac - bc \cos \left(dx^2 + c\right)}{2 \, d^2}$$

```
[In] integrate(x^3*(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] 1/4*((d*x^2 + c)^2*a - 2*(d*x^2 + c)*b*cos(d*x^2 + c) + 2*b*sin(d*x^2 + c))
/d^2 - 1/2*((d*x^2 + c)*a*c - b*c*cos(d*x^2 + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int x^3 \left(a + b \sin \left(c + dx^2\right)\right) \, dx = \frac{a\,x^4}{4} + \frac{\frac{b\,\sin\left(d\,x^2+c\right)}{2} - \frac{b\,d\,x^2\,\cos\left(d\,x^2+c\right)}{2}}{d^2}$$

```
[In] int(x^3*(a + b*sin(c + d*x^2)),x)
[Out] (a*x^4)/4 + ((b*sin(c + d*x^2))/2 - (b*d*x^2*cos(c + d*x^2))/2)/d^2
```

3.3 $\int x(a+b\sin(c+dx^2))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}-\frac{b\cos\left(c+dx^2\right)}{2d}$$

```
[Out] 1/2*a*x^2-1/2*b*cos(d*x^2+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {14, 3460, 2718}

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}-\frac{b\cos\left(c+dx^2\right)}{2d}$$

```
[In] Int[x*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^2)/2 - (b*Cos[c + d*x^2])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ax + bx\sin\left(c + dx^2\right)\right)\,dx\\ &= \frac{ax^2}{2} + b\int x\sin\left(c + dx^2\right)\,dx\\ &= \frac{ax^2}{2} + \frac{1}{2}b\text{Subst}\left(\int \sin(c + dx)\,dx, x, x^2\right)\\ &= \frac{ax^2}{2} - \frac{b\cos\left(c + dx^2\right)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int x\left(a + b\sin\left(c + dx^2\right)\right)\,dx = \frac{ax^2}{2} - \frac{b\cos(c)\cos\left(dx^2\right)}{2d} + \frac{b\sin(c)\sin\left(dx^2\right)}{2d}$$

```
[In] Integrate[x*(a + b*Sin[c + d*x^2]),x]
[Out] (a*x^2)/2 - (b*Cos[c]*Cos[d*x^2])/(2*d) + (b*Sin[c]*Sin[d*x^2])/(2*d)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{x^2a}{2}-\frac{b\cos(d\,x^2+c)}{2d}$	22
parts	$\frac{x^2a}{2}-\frac{b\cos(d\,x^2+c)}{2d}$	22
parallelrisch	$\frac{x^2ad-b\cos(d\,x^2+c)+b}{2d}$	25
derivativedivides	$\frac{(d\,x^2+c)a-b\cos(d\,x^2+c)}{2d}$	27
default	$\frac{(d\,x^2+c)a-b\cos(d\,x^2+c)}{2d}$	27
norman	$\frac{\frac{b\left(\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{d}+\frac{x^2a}{2}+\frac{x^2a\left(\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}$	63

```
[In] int(x*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*a-1/2*b*cos(d*x^2+c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{adx^2-b\cos\left(dx^2+c\right)}{2\,d}$$

```
[In] integrate(x*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] 1/2*(a*d*x^2 - b*cos(d*x^2 + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\begin{cases}\frac{ax^2}{2}-\frac{b\cos\left(c+dx^2\right)}{2d} & \text{for } d\neq 0\\ \frac{x^2(a+b\sin(c))}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(a+b*sin(d*x**2+c)),x)
[Out] Piecewise((a*x**2/2 - b*cos(c + d*x**2)/(2*d), Ne(d, 0)), (x**2*(a + b*sin(
c))/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{1}{2}\,ax^2-\frac{b\cos\left(dx^2+c\right)}{2\,d}$$

```
[In] integrate(x*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] 1/2*a*x^2 - 1/2*b*cos(d*x^2 + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{(dx^2+c)a-b\cos\left(dx^2+c\right)}{2\,d}$$

```
[In] integrate(x*(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] 1/2*((d*x^2 + c)*a - b*cos(d*x^2 + c))/d
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,\cos\left(d\,x^2+c\right)}{2\,d}$$

```
[In] int(x*(a + b*sin(c + d*x^2)),x)
[Out] (a*x^2)/2 - (b*cos(c + d*x^2))/(2*d)
```

3.4 $\int \frac{a+b\sin(c+dx^2)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 31

$$\int \frac{a+b\sin(c+dx^2)}{x}\,dx = a\log(x)+\frac{1}{2}b\,\text{CosIntegral}\left(dx^2\right)\sin(c)+\frac{1}{2}b\cos(c)\text{Si}\left(dx^2\right)$$

```
[Out] a*ln(x)+1/2*b*cos(c)*Si(d*x^2)+1/2*b*Ci(d*x^2)*sin(c)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3458, 3457, 3456}

$$\int \frac{a+b\sin(c+dx^2)}{x}\,dx = a\log(x)+\frac{1}{2}b\sin(c)\,\text{CosIntegral}\left(dx^2\right)+\frac{1}{2}b\cos(c)\text{Si}\left(dx^2\right)$$

```
[In] Int[(a + b*Sin[c + d*x^2])/x,x]
```

```
[Out] a*Log[x] + (b*CosIntegral[d*x^2]*Sin[c])/2 + (b*Cos[c]*SinIntegral[d*x^2])/
2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a}{x}+\frac{b\sin(c+dx^2)}{x}\right)\,dx\\
&= a\log(x)+b\int \frac{\sin(c+dx^2)}{x}\,dx\\
&= a\log(x)+(b\cos(c))\int \frac{\sin(dx^2)}{x}\,dx+(b\sin(c))\int \frac{\cos(dx^2)}{x}\,dx\\
&= a\log(x)+\frac{1}{2}b\,\text{CosIntegral}\left(dx^2\right)\sin(c)+\frac{1}{2}b\cos(c)\text{Si}\left(dx^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{a+b\sin(c+dx^2)}{x}\,dx = a\log(x)+\frac{1}{2}b\left(\text{CosIntegral}\left(dx^2\right)\sin(c)+\cos(c)\text{Si}\left(dx^2\right)\right)$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x,x]
[Out] a*Log[x] + (b*(CosIntegral[d*x^2]*Sin[c] + Cos[c]*SinIntegral[d*x^2]))/2
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$a\ln(x)+b\left(\frac{\cos(c)\operatorname{Si}(d\,x^2)}{2}+\frac{\sin(c)\operatorname{Ci}(d\,x^2)}{2}\right)$	29
parts	$a\ln(x)+b\left(\frac{\cos(c)\operatorname{Si}(d\,x^2)}{2}+\frac{\sin(c)\operatorname{Ci}(d\,x^2)}{2}\right)$	29
risch	$a\ln(x)-\frac{e^{-ic}\pi\ \operatorname{csgn}(d\,x^2)b}{4}+\frac{e^{-ic}\operatorname{Si}(d\,x^2)b}{2}-\frac{ie^{-ic}\operatorname{Ei}_1(-id\,x^2)b}{4}+\frac{ib\,e^{ic}\operatorname{Ei}_1(-id\,x^2)}{4}$	71

```
[In] int((a+b*sin(d*x^2+c))/x,x,method=_RETURNVERBOSE)
[Out] a*ln(x)+b*(1/2*cos(c)*Si(d*x^2)+1/2*sin(c)*Ci(d*x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sin(c+dx^2)}{x}\,dx=\frac{1}{2}b\operatorname{Ci}\left(dx^2\right)\sin(c)+\frac{1}{2}b\cos(c)\operatorname{Si}\left(dx^2\right)+a\log(x)$$

```
[In] integrate((a+b*sin(d*x^2+c))/x,x, algorithm="fricas")
[Out] 1/2*b*cos_integral(d*x^2)*sin(c) + 1/2*b*cos(c)*sin_integral(d*x^2) + a*log
(x)
```

Sympy [F]

$$\int \frac{a+b\sin(c+dx^2)}{x}\,dx=\int \frac{a+b\sin(c+dx^2)}{x}\,dx$$

```
[In] integrate((a+b*sin(d*x**2+c))/x,x)
[Out] Integral((a + b*sin(c + d*x**2))/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{a+b\sin(c+dx^2)}{x}\,dx$$
$$=-\frac{1}{4}\left(\left(i\operatorname{Ei}\left(i\,dx^2\right)-i\operatorname{Ei}\left(-i\,dx^2\right)\right)\cos(c)-\left(\operatorname{Ei}\left(i\,dx^2\right)+\operatorname{Ei}\left(-i\,dx^2\right)\right)\sin(c)\right)b+a\log(x)$$

```
[In] integrate((a+b*sin(d*x^2+c))/x,x, algorithm="maxima")
[Out] -1/4*((I*Ei(I*d*x^2) - I*Ei(-I*d*x^2))*cos(c) - (Ei(I*d*x^2) + Ei(-I*d*x^2)
)*sin(c))*b + a*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sin(c+dx^2)}{x}\,dx = \frac{1}{2}\,b\,\mathrm{Ci}\left(dx^2\right)\sin(c)+\frac{1}{2}\,b\cos(c)\,\mathrm{Si}\left(dx^2\right)+\frac{1}{2}\,a\log\left(dx^2\right)$$

```
[In] integrate((a+b*sin(d*x^2+c))/x,x, algorithm="giac")
```

```
[Out] 1/2*b*cos_integral(d*x^2)*sin(c) + 1/2*b*cos(c)*sin_integral(d*x^2) + 1/2*a
*log(d*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(c+dx^2)}{x}\,dx = a\,\ln(x)+\frac{b\,\sin(c)\,\mathrm{cosint}(d\,x^2)}{2}+\frac{b\,\cos(c)\,\mathrm{sinint}(d\,x^2)}{2}$$

```
[In] int((a + b*sin(c + d*x^2))/x,x)
```

```
[Out] a*log(x) + (b*sin(c)*cosint(d*x^2))/2 + (b*cos(c)*sinint(d*x^2))/2
```

3.5 $\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 53

$$\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx = -\frac{a}{2x^2}+\frac{1}{2}bd\cos(c)\,\text{CosIntegral}\left(dx^2\right) - \frac{b\sin(c+dx^2)}{2x^2}-\frac{1}{2}bd\sin(c)\text{Si}\left(dx^2\right)$$

```
[Out] -1/2*a/x^2+1/2*b*d*Ci(d*x^2)*cos(c)-1/2*b*d*Si(d*x^2)*sin(c)-1/2*b*sin(d*x^
2+c)/x^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 3460, 3378, 3384, 3380, 3383}

$$\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx = -\frac{a}{2x^2}+\frac{1}{2}bd\cos(c)\,\text{CosIntegral}\left(dx^2\right) -\frac{1}{2}bd\sin(c)\text{Si}\left(dx^2\right) - \frac{b\sin(c+dx^2)}{2x^2}$$

```
[In] Int[(a + b*Sin[c + d*x^2])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*d*Cos[c]*CosIntegral[d*x^2])/2 - (b*Sin[c + d*x^2])/(2*x^2)
 - (b*d*Sin[c]*SinIntegral[d*x^2])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{x^3}+\frac{b\sin\left(c+dx^2\right)}{x^3}\right)dx\\
&= -\frac{a}{2x^2}+b\int \frac{\sin\left(c+dx^2\right)}{x^3}\,dx\\
&= -\frac{a}{2x^2}+\frac{1}{2}b\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,x^2\right)
\end{aligned}$$

$$= -\frac{a}{2x^2} - \frac{b\sin(c+dx^2)}{2x^2} + \frac{1}{2}(bd)\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx, x, x^2\right)$$
$$= -\frac{a}{2x^2} - \frac{b\sin(c+dx^2)}{2x^2} + \frac{1}{2}(bd\cos(c))\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx, x, x^2\right) - \frac{1}{2}(bd\sin(c))\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx, x, x^2\right)$$
$$= -\frac{a}{2x^2} + \frac{1}{2}bd\cos(c)\,\text{CosIntegral}\left(dx^2\right) - \frac{b\sin(c+dx^2)}{2x^2} - \frac{1}{2}bd\sin(c)\text{Si}\left(dx^2\right)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx = -\frac{a - bdx^2\cos(c)\,\text{CosIntegral}(dx^2) + b\sin(c+dx^2) + bdx^2\sin(c)\text{Si}(dx^2)}{2x^2}$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x^3,x]
[Out] -1/2*(a - b*d*x^2*Cos[c]*CosIntegral[d*x^2] + b*Sin[c + d*x^2] + b*d*x^2*Si
n[c]*SinIntegral[d*x^2])/x^2
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a}{2x^2} + b\left(-\frac{\sin(dx^2+c)}{2x^2} + d\left(\frac{\cos(c)\,\text{Ci}(dx^2)}{2} - \frac{\sin(c)\,\text{Si}(dx^2)}{2}\right)\right)$	47
parts	$-\frac{a}{2x^2} + b\left(-\frac{\sin(dx^2+c)}{2x^2} + d\left(\frac{\cos(c)\,\text{Ci}(dx^2)}{2} - \frac{\sin(c)\,\text{Si}(dx^2)}{2}\right)\right)$	47
risch	$-\frac{-i\,\text{csgn}(dx^2)\pi\,\mathrm{e}^{-ic}bd\,x^2+2i\,\text{Si}(dx^2)\mathrm{e}^{-ic}bd\,x^2+bd\,\text{Ei}_1(-id\,x^2)\mathrm{e}^{ic}x^2+\text{Ei}_1(-id\,x^2)\mathrm{e}^{-ic}bd\,x^2+2b\sin(dx^2+c)+2a}{4x^2}$	100

```
[In] int((a+b*sin(d*x^2+c))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a/x^2+b*(-1/2/x^2*sin(d*x^2+c)+d*(1/2*cos(c)*Ci(d*x^2)-1/2*sin(c)*Si(d
*x^2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{a + b\sin\left(c + dx^{2}\right)}{x^{3}}\, dx = \frac{bdx^{2}\cos\left(c\right)\operatorname{Ci}\left(dx^{2}\right) - bdx^{2}\sin\left(c\right)\operatorname{Si}\left(dx^{2}\right) - b\sin\left(dx^{2} + c\right) - a}{2\,x^{2}}$$

```
[In] integrate((a+b*sin(d*x^2+c))/x^3,x, algorithm="fricas")
[Out] 1/2*(b*d*x^2*cos(c)*cos_integral(d*x^2) - b*d*x^2*sin(c)*sin_integral(d*x^2
) - b*sin(d*x^2 + c) - a)/x^2
```

Sympy [F]

$$\int \frac{a + b\sin\left(c + dx^{2}\right)}{x^{3}}\, dx = \int \frac{a + b\sin\left(c + dx^{2}\right)}{x^{3}}\, dx$$

```
[In] integrate((a+b*sin(d*x**2+c))/x**3,x)
[Out] Integral((a + b*sin(c + d*x**2))/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \frac{a + b\sin\left(c + dx^{2}\right)}{x^{3}}\, dx = \frac{1}{4}\left(\left(\Gamma\left(-1, i\,dx^{2}\right) + \Gamma\left(-1, -i\,dx^{2}\right)\right)\cos\left(c\right) - \left(i\,\Gamma\left(-1, i\,dx^{2}\right) - i\,\Gamma\left(-1, -i\,dx^{2}\right)\right)\sin\left(c\right)\right)bd - \frac{a}{2\,x^{2}}$$

```
[In] integrate((a+b*sin(d*x^2+c))/x^3,x, algorithm="maxima")
[Out] 1/4*((gamma(-1, I*d*x^2) + gamma(-1, -I*d*x^2))*cos(c) - (I*gamma(-1, I*d*x
^2) - I*gamma(-1, -I*d*x^2))*sin(c))*b*d - 1/2*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(45) = 90$.

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.87

$$\int \frac{a + b\sin(c + dx^2)}{x^3}\,dx = \frac{(dx^2 + c)bd^2\cos(c)\operatorname{Ci}(dx^2) - bcd^2\cos(c)\operatorname{Ci}(dx^2) - (dx^2 + c)bd^2\sin(c)\operatorname{Si}(dx^2) + bcd^2\sin(c)\operatorname{Si}(dx^2) -}{2\,d^2x^2}$$

```
[In] integrate((a+b*sin(d*x^2+c))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*((d*x^2 + c)*b*d^2*cos(c)*cos_integral(d*x^2) - b*c*d^2*cos(c)*cos_inte
gral(d*x^2) - (d*x^2 + c)*b*d^2*sin(c)*sin_integral(d*x^2) + b*c*d^2*sin(c)
*sin_integral(d*x^2) - b*d^2*sin(d*x^2 + c) - a*d^2)/(d^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sin(c + dx^2)}{x^3}\,dx = \int \frac{a + b\,\sin(d\,x^2 + c)}{x^3}\,dx$$

```
[In] int((a + b*sin(c + d*x^2))/x^3,x)
```

```
[Out] int((a + b*sin(c + d*x^2))/x^3, x)
```

3.6 $\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 74

$$\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx = -\frac{a}{4x^4} - \frac{bd\cos(c+dx^2)}{4x^2} - \frac{1}{4}bd^2\,\text{CosIntegral}\left(dx^2\right)\sin(c) - \frac{b\sin(c+dx^2)}{4x^4} - \frac{1}{4}bd^2\cos(c)\text{Si}\left(dx^2\right)$$

```
[Out] -1/4*a/x^4-1/4*b*d*cos(d*x^2+c)/x^2-1/4*b*d^2*cos(c)*Si(d*x^2)-1/4*b*d^2*Ci
(d*x^2)*sin(c)-1/4*b*sin(d*x^2+c)/x^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 3460, 3378, 3384, 3380, 3383}

$$\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx = -\frac{a}{4x^4} - \frac{1}{4}bd^2\sin(c)\,\text{CosIntegral}\left(dx^2\right) - \frac{1}{4}bd^2\cos(c)\text{Si}\left(dx^2\right) - \frac{bd\cos(c+dx^2)}{4x^2} - \frac{b\sin(c+dx^2)}{4x^4}$$

```
[In] Int[(a + b*Sin[c + d*x^2])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (b*d*Cos[c + d*x^2])/(4*x^2) - (b*d^2*CosIntegral[d*x^2]*Sin[c
])/4 - (b*Sin[c + d*x^2])/(4*x^4) - (b*d^2*Cos[c]*SinIntegral[d*x^2])/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a}{x^5}+\frac{b\sin\left(c+dx^2\right)}{x^5}\right)dx\\ &= -\frac{a}{4x^4}+b\int \frac{\sin\left(c+dx^2\right)}{x^5}\,dx\\ &= -\frac{a}{4x^4}+\frac{1}{2}b\text{Subst}\left(\int \frac{\sin(c+dx)}{x^3}\,dx,x,x^2\right)\end{aligned}$$

$$= -\frac{a}{4x^4} - \frac{b\sin(c+dx^2)}{4x^4} + \frac{1}{4}(bd)\text{Subst}\left(\int \frac{\cos(c+dx)}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{a}{4x^4} - \frac{bd\cos(c+dx^2)}{4x^2} - \frac{b\sin(c+dx^2)}{4x^4} - \frac{1}{4}(bd^2)\,\text{Subst}\left(\int \frac{\sin(c+dx)}{x}\,dx, x, x^2\right)$$

$$= -\frac{a}{4x^4} - \frac{bd\cos(c+dx^2)}{4x^2} - \frac{b\sin(c+dx^2)}{4x^4} - \frac{1}{4}(bd^2\cos(c))\,\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx, x, x^2\right) - \frac{1}{4}(bd^2\sin(c))\,\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx, x, x^2\right)$$

$$= -\frac{a}{4x^4} - \frac{bd\cos(c+dx^2)}{4x^2} - \frac{1}{4}bd^2\,\text{CosIntegral}\left(dx^2\right)\sin(c) - \frac{b\sin(c+dx^2)}{4x^4} - \frac{1}{4}bd^2\cos(c)\text{Si}\left(dx^2\right)$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx = -\frac{a}{4x^4} - \frac{b\cos(dx^2)(dx^2\cos(c)+\sin(c))}{4x^4} + \frac{b(-\cos(c)+dx^2\sin(c))\sin(dx^2)}{4x^4} - \frac{1}{4}bd^2\left(\text{CosIntegral}\left(dx^2\right)\sin(c)+\cos(c)\text{Si}\left(dx^2\right)\right)$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (b*Cos[d*x^2]*(d*x^2*Cos[c] + Sin[c]))/(4*x^4) + (b*(-Cos[c] +
 d*x^2*Sin[c])*Sin[d*x^2])/(4*x^4) - (b*d^2*(CosIntegral[d*x^2]*Sin[c] + Co
s[c]*SinIntegral[d*x^2]))/4
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a}{4x^4}+b\left(-\frac{\sin\left(d\,x^2+c\right)}{4x^4}+\frac{d\left(-\frac{\cos\left(d\,x^2+c\right)}{2x^2}-d\left(\frac{\cos(c)\,\mathrm{Si}\left(d\,x^2\right)}{2}+\frac{\sin(c)\,\mathrm{Ci}\left(d\,x^2\right)}{2}\right)\right)}{2}\right)$
parts	$-\frac{a}{4x^4}+b\left(-\frac{\sin\left(d\,x^2+c\right)}{4x^4}+\frac{d\left(-\frac{\cos\left(d\,x^2+c\right)}{2x^2}-d\left(\frac{\cos(c)\,\mathrm{Si}\left(d\,x^2\right)}{2}+\frac{\sin(c)\,\mathrm{Ci}\left(d\,x^2\right)}{2}\right)\right)}{2}\right)$
risch	$-\frac{-\pi\ \mathrm{csgn}\left(d\,x^2\right)\mathrm{e}^{-ic}b\,d^2x^4-i\mathrm{e}^{-ic}\ \mathrm{Ei}_1\left(-id\,x^2\right)b\,d^2x^4+ib\,d^2\ \mathrm{Ei}_1\left(-id\,x^2\right)\mathrm{e}^{ic}x^4+2\ \mathrm{Si}\left(d\,x^2\right)\mathrm{e}^{-ic}b\,d^2x^4+2x^2bd\cos\left(d\,x^2+c\right)+2b\sin\left(d\,\right.}{8x^4}$

```
[In] int((a+b*sin(d*x^2+c))/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a/x^4+b*(-1/4/x^4*sin(d*x^2+c)+1/2*d*(-1/2/x^2*cos(d*x^2+c)-d*(1/2*cos
(c)*Si(d*x^2)+1/2*sin(c)*Ci(d*x^2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^5}\,dx = -\frac{bd^2x^4\,\mathrm{Ci}\left(dx^2\right)\sin\left(c\right)+bd^2x^4\cos\left(c\right)\mathrm{Si}\left(dx^2\right)+bdx^2\cos\left(dx^2+c\right)+b\sin\left(dx^2+c\right)+a}{4\,x^4}$$

```
[In] integrate((a+b*sin(d*x^2+c))/x^5,x, algorithm="fricas")
[Out] -1/4*(b*d^2*x^4*cos_integral(d*x^2)*sin(c) + b*d^2*x^4*cos(c)*sin_integral(
d*x^2) + b*d*x^2*cos(d*x^2 + c) + b*sin(d*x^2 + c) + a)/x^4
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^5}\,dx = \int \frac{a+b\sin\left(c+dx^2\right)}{x^5}\,dx$$

```
[In] integrate((a+b*sin(d*x**2+c))/x**5,x)
[Out] Integral((a + b*sin(c + d*x**2))/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx = \frac{1}{4}\left(\left(i\,\Gamma(-2,i\,dx^2)-i\,\Gamma(-2,-i\,dx^2)\right)\cos(c)+\left(\Gamma(-2,i\,dx^2)+\Gamma(-2,-i\,dx^2)\right)\sin(c)\right)bd^2 - \frac{a}{4\,x^4}$$

```
[In] integrate((a+b*sin(d*x^2+c))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*((I*gamma(-2, I*d*x^2) - I*gamma(-2, -I*d*x^2))*cos(c) + (gamma(-2, I*d
*x^2) + gamma(-2, -I*d*x^2))*sin(c))*b*d^2 - 1/4*a/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(64) = 128.

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.76

$$\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx = -\frac{(dx^2+c)^2bd^3\operatorname{Ci}(dx^2)\sin(c)-2(dx^2+c)bcd^3\operatorname{Ci}(dx^2)\sin(c)+bc^2d^3\operatorname{Ci}(dx^2)\sin(c)+(dx^2+c)^2bd^3\cos$$

```
[In] integrate((a+b*sin(d*x^2+c))/x^5,x, algorithm="giac")
```

```
[Out] -1/4*((d*x^2 + c)^2*b*d^3*cos_integral(d*x^2)*sin(c) - 2*(d*x^2 + c)*b*c*d^
3*cos_integral(d*x^2)*sin(c) + b*c^2*d^3*cos_integral(d*x^2)*sin(c) + (d*x^
2 + c)^2*b*d^3*cos(c)*sin_integral(d*x^2) - 2*(d*x^2 + c)*b*c*d^3*cos(c)*si
n_integral(d*x^2) + b*c^2*d^3*cos(c)*sin_integral(d*x^2) + (d*x^2 + c)*b*d^
3*cos(d*x^2 + c) - b*c*d^3*cos(d*x^2 + c) + b*d^3*sin(d*x^2 + c) + a*d^3)/(
((d*x^2 + c)^2 - 2*(d*x^2 + c)*c + c^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx = \int \frac{a+b\,\sin(d\,x^2+c)}{x^5}\,dx$$

```
[In] int((a + b*sin(c + d*x^2))/x^5,x)
[Out] int((a + b*sin(c + d*x^2))/x^5, x)
```

3.7 $\int x^4(a+b\sin(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^5}{5}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}-\frac{3b\sqrt{\frac{\pi}{2}}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}}-\frac{3b\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{4d^{5/2}}+\frac{3bx\sin\left(c+dx^2\right)}{4d^2}$$

```
[Out] 1/5*a*x^5-1/2*b*x^3*cos(d*x^2+c)/d+3/4*b*x*sin(d*x^2+c)/d^2-3/8*b*cos(c)*Fr
esnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)-3/8*b*FresnelC(
x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)/d^(5/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 3466, 3467, 3434, 3433, 3432}

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^5}{5}-\frac{3\sqrt{\frac{\pi}{2}}b\sin(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}b\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}}+\frac{3bx\sin\left(c+dx^2\right)}{4d^2}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}$$

```
[In] Int[x^4*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^5)/5 - (b*x^3*Cos[c + d*x^2])/(2*d) - (3*b*Sqrt[Pi/2]*Cos[c]*FresnelS[
Sqrt[d]*Sqrt[2/Pi]*x])/(4*d^(5/2)) - (3*b*Sqrt[Pi/2]*FresnelC[Sqrt[d]*Sqrt[
2/Pi]*x]*Sin[c])/(4*d^(5/2)) + (3*b*x*Sin[c + d*x^2])/(4*d^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\text{integral} = \int \left(ax^4 + bx^4 \sin\left(c + dx^2\right)\right)\, dx$$
$$= \frac{ax^5}{5} + b\int x^4 \sin\left(c + dx^2\right)\, dx$$

$$= \frac{ax^5}{5} - \frac{bx^3 \cos(c+dx^2)}{2d} + \frac{(3b)\int x^2 \cos(c+dx^2)\,dx}{2d}$$

$$= \frac{ax^5}{5} - \frac{bx^3 \cos(c+dx^2)}{2d} + \frac{3bx \sin(c+dx^2)}{4d^2} - \frac{(3b)\int \sin(c+dx^2)\,dx}{4d^2}$$

$$= \frac{ax^5}{5} - \frac{bx^3 \cos(c+dx^2)}{2d} + \frac{3bx \sin(c+dx^2)}{4d^2} - \frac{(3b\cos(c))\int \sin(dx^2)\,dx}{4d^2} - \frac{(3b\sin(c))\int \cos(dx^2)\,dx}{4d^2}$$

$$= \frac{ax^5}{5} - \frac{bx^3 \cos(c+dx^2)}{2d} - \frac{3b\sqrt{\frac{\pi}{2}}\cos(c)\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}} - \frac{3b\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{4d^{5/2}} + \frac{3bx \sin(c+dx^2)}{4d^2}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

$$\int x^4 (a + b\sin(c+dx^2))\,dx = \frac{ax^5}{5} - \frac{bx\cos(dx^2)(2dx^2\cos(c) - 3\sin(c))}{4d^2} - \frac{3b\sqrt{\frac{\pi}{2}}\left(\cos(c)\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) + \operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right)}{4d^{5/2}} + \frac{bx(3\cos(c) + 2dx^2\sin(c))\sin(dx^2)}{4d^2}$$

```
[In] Integrate[x^4*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^5)/5 - (b*x*Cos[d*x^2]*(2*d*x^2*Cos[c] - 3*Sin[c]))/(4*d^2) - (3*b*Sqr
t[Pi/2]*(Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + FresnelC[Sqrt[d]*Sqrt[2/Pi
]*x]*Sin[c]))/(4*d^(5/2)) + (b*x*(3*Cos[c] + 2*d*x^2*Sin[c])*Sin[d*x^2])/(4
*d^2)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{a\,x^{5}}{5}+b\left(-\frac{x^{3}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{\frac{3x\sin\left(d\,x^{2}+c\right)}{4d}-\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(c\right)\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin\left(c\right)\mathrm{C}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{8d^{\frac{3}{2}}}}{d}\right)$	89
parts	$\frac{a\,x^{5}}{5}+b\left(-\frac{x^{3}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{\frac{3x\sin\left(d\,x^{2}+c\right)}{4d}-\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(c\right)\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin\left(c\right)\mathrm{C}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{8d^{\frac{3}{2}}}}{d}\right)$	89
risch	$\frac{a\,x^{5}}{5}-\frac{3ib\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{i d}\,x\right)\mathrm{e}^{-ic}}{16d^{2}\sqrt{i d}}+\frac{3ib\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-i d}\,x\right)\mathrm{e}^{ic}}{16d^{2}\sqrt{-i d}}-\frac{b\,x^{3}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{3bx\sin\left(d\,x^{2}+c\right)}{4d^{2}}$	100

```
[In] int(x^4*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+b*(-1/2/d*x^3*cos(d*x^2+c)+3/2/d*(1/2/d*x*sin(d*x^2+c)-1/4/d^(3/2
)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))+sin(c)*Fres
nelC(x*d^(1/2)*2^(1/2)/Pi^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx$$
$$=\frac{8\,ad^3x^5-20\,bd^2x^3\cos\left(dx^2+c\right)-15\,\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)-15\,\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)+}{40\,d^3}$$

```
[In] integrate(x^4*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*d^3*x^5 - 20*b*d^2*x^3*cos(d*x^2 + c) - 15*sqrt(2)*pi*b*sqrt(d/pi
)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/pi)) - 15*sqrt(2)*pi*b*sqrt(d/pi)*fre
snel_cos(sqrt(2)*x*sqrt(d/pi))*sin(c) + 30*b*d*x*sin(d*x^2 + c))/d^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(126) = 252.

Time = 2.37 (sec) , antiderivative size = 488, normalized size of antiderivative = 4.03

$$\int x^4(a+b\sin(c+dx^2))\,dx = \frac{ax^5}{5} - \frac{5\sqrt{2}\sqrt{\pi}bx^4\sqrt{\frac{1}{d}}\sin(c)C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{32\Gamma\left(\frac{9}{4}\right)} + \frac{\sqrt{2}\sqrt{\pi}bx^4\sqrt{\frac{1}{d}}\sin(c)C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)}{2} - \frac{21\sqrt{2}\sqrt{\pi}bx^4\sqrt{\frac{1}{d}}\cos(c)S\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{32\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{2}\sqrt{\pi}bx^4\sqrt{\frac{1}{d}}\cos(c)S\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)}{2} - \frac{15\sqrt{2}\sqrt{\pi}b\sqrt{\frac{1}{d}}\sin(c)C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{128d^2\Gamma\left(\frac{9}{4}\right)} - \frac{63\sqrt{2}\sqrt{\pi}b\sqrt{\frac{1}{d}}\cos(c)S\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{128d^2\Gamma\left(\frac{11}{4}\right)} + \frac{5bx^3\sqrt{\frac{1}{d}}\sin(c)\sin(dx^2)\Gamma\left(\frac{1}{4}\right)}{32\sqrt{d}\Gamma\left(\frac{9}{4}\right)} - \frac{21bx^3\sqrt{\frac{1}{d}}\cos(c)\cos(dx^2)\Gamma\left(\frac{3}{4}\right)}{32\sqrt{d}\Gamma\left(\frac{11}{4}\right)} + \frac{15bx\sqrt{\frac{1}{d}}\sin(c)\cos(dx^2)\Gamma\left(\frac{1}{4}\right)}{64d^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{63bx\sqrt{\frac{1}{d}}\sin(dx^2)\cos(c)\Gamma\left(\frac{3}{4}\right)}{64d^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate(x**4*(a+b*sin(d*x**2+c)),x)
```

```
[Out] a*x**5/5 - 5*sqrt(2)*sqrt(pi)*b*x**4*sqrt(1/d)*sin(c)*fresnelc(sqrt(2)*sqrt
(d)*x/sqrt(pi))*gamma(1/4)/(32*gamma(9/4)) + sqrt(2)*sqrt(pi)*b*x**4*sqrt(1
/d)*sin(c)*fresnelc(sqrt(2)*sqrt(d)*x/sqrt(pi))/2 - 21*sqrt(2)*sqrt(pi)*b*x
**4*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi))*gamma(3/4)/(32*ga
mma(11/4)) + sqrt(2)*sqrt(pi)*b*x**4*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt
(d)*x/sqrt(pi))/2 - 15*sqrt(2)*sqrt(pi)*b*sqrt(1/d)*sin(c)*fresnelc(sqrt(2)
*sqrt(d)*x/sqrt(pi))*gamma(1/4)/(128*d**2*gamma(9/4)) - 63*sqrt(2)*sqrt(pi)
*b*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi))*gamma(3/4)/(128*d*
```

```
*2*gamma(11/4)) + 5*b*x**3*sqrt(1/d)*sin(c)*sin(d*x**2)*gamma(1/4)/(32*sqrt
(d)*gamma(9/4)) - 21*b*x**3*sqrt(1/d)*cos(c)*cos(d*x**2)*gamma(3/4)/(32*sqr
t(d)*gamma(11/4)) + 15*b*x*sqrt(1/d)*sin(c)*cos(d*x**2)*gamma(1/4)/(64*d**(
3/2)*gamma(9/4)) + 63*b*x*sqrt(1/d)*sin(d*x**2)*cos(c)*gamma(3/4)/(64*d**(3
/2)*gamma(11/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{1}{5}\,ax^5-\frac{\left(16\,d^3x^3\cos\left(dx^2+c\right)-24\,d^2x\sin\left(dx^2+c\right)+3\,\sqrt{2}\sqrt{\pi}\left(\left(\left(i+1\right)\,\cos\left(c\right)-\left(i-1\right)\,\sin\left(c\right)\right)\operatorname{erf}\left(\sqrt{i\,d}x\right)\right.\right.}{32\,d^4}$$

```
[In] integrate(x^4*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 - 1/32*(16*d^3*x^3*cos(d*x^2 + c) - 24*d^2*x*sin(d*x^2 + c) + 3*s
qrt(2)*sqrt(pi)*(((I + 1)*cos(c) - (I - 1)*sin(c))*erf(sqrt(I*d)*x) + (-(I
- 1)*cos(c) + (I + 1)*sin(c))*erf(sqrt(-I*d)*x))*d^(3/2))*b/d^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.36

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{1}{5}\,ax^5-\frac{3\,\sqrt{2}\sqrt{\pi}b\operatorname{erf}\left(-\frac{1}{2}i\,\sqrt{2}x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(i\,c)}}{16\,d^2\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}-\frac{3\,\sqrt{2}\sqrt{\pi}b\operatorname{erf}\left(\frac{1}{2}i\,\sqrt{2}x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-i\,c)}}{16\,d^2\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}+\frac{i\,(2i\,bdx^3-3\,bx)e^{(i\,dx^2+i\,c)}}{8\,d^2}+\frac{i\,(2i\,bdx^3+3\,bx)e^{(-i\,dx^2-i\,c)}}{8\,d^2}$$

```
[In] integrate(x^4*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/5*a*x^5 - 3/16*sqrt(2)*sqrt(pi)*b*erf(-1/2*I*sqrt(2)*x*(I*d/abs(d) + 1)*s
qrt(abs(d)))*e^(I*c)/(d^2*(I*d/abs(d) + 1)*sqrt(abs(d))) - 3/16*sqrt(2)*sqr
t(pi)*b*erf(1/2*I*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/(d^2*(
-I*d/abs(d) + 1)*sqrt(abs(d))) + 1/8*I*(2*I*b*d*x^3 - 3*b*x)*e^(I*d*x^2 + I
*c)/d^2 + 1/8*I*(2*I*b*d*x^3 + 3*b*x)*e^(-I*d*x^2 - I*c)/d^2
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\int x^4\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)\,dx$$

```
[In] int(x^4*(a + b*sin(c + d*x^2)),x)
[Out] int(x^4*(a + b*sin(c + d*x^2)), x)
```

3.8 $\int x^2(a+b\sin(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 102

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^3}{3}-\frac{bx\cos\left(c+dx^2\right)}{2d}+\frac{b\sqrt{\frac{\pi}{2}}\cos(c)\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{3/2}}-\frac{b\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{2d^{3/2}}$$

```
[Out] 1/3*a*x^3-1/2*b*x*cos(d*x^2+c)/d+1/4*b*cos(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi
^(1/2))*2^(1/2)*Pi^(1/2)/d^(3/2)-1/4*b*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))
*sin(c)*2^(1/2)*Pi^(1/2)/d^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {14, 3466, 3435, 3433, 3432}

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^3}{3}+\frac{\sqrt{\frac{\pi}{2}}b\cos(c)\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}b\sin(c)\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{3/2}}-\frac{bx\cos\left(c+dx^2\right)}{2d}$$

```
[In] Int[x^2*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^3)/3 - (b*x*Cos[c + d*x^2])/(2*d) + (b*Sqrt[Pi/2]*Cos[c]*FresnelC[Sqrt
[d]*Sqrt[2/Pi]*x])/(2*d^(3/2)) - (b*Sqrt[Pi/2]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*
x]*Sin[c])/(2*d^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^2 + bx^2 \sin\left(c + dx^2\right)\right) \, dx \\
&= \frac{ax^3}{3} + b \int x^2 \sin\left(c + dx^2\right) \, dx \\
&= \frac{ax^3}{3} - \frac{bx \cos\left(c + dx^2\right)}{2d} + \frac{b \int \cos\left(c + dx^2\right) \, dx}{2d} \\
&= \frac{ax^3}{3} - \frac{bx \cos\left(c + dx^2\right)}{2d} + \frac{(b \cos(c)) \int \cos\left(dx^2\right) \, dx}{2d} - \frac{(b \sin(c)) \int \sin\left(dx^2\right) \, dx}{2d} \\
&= \frac{ax^3}{3} - \frac{bx \cos\left(c + dx^2\right)}{2d} + \frac{b\sqrt{\frac{\pi}{2}} \cos(c) \, \text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{3/2}} - \frac{b\sqrt{\frac{\pi}{2}} \, \text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) \sin(c)}{2d^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx = \frac{ax^3}{3}-\frac{bx\cos(c)\cos\left(dx^2\right)}{2d} + \frac{b\sqrt{\frac{\pi}{2}}\left(\cos(c)\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right)}{2d^{3/2}}+\frac{bx\sin(c)\sin\left(dx^2\right)}{2d}$$

```
[In] Integrate[x^2*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^3)/3 - (b*x*Cos[c]*Cos[d*x^2])/(2*d) + (b*Sqrt[Pi/2]*(Cos[c]*FresnelC[
Sqrt[d]*Sqrt[2/Pi]*x] - FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c]))/(2*d^(3/2))
 + (b*x*Sin[c]*Sin[d*x^2])/(2*d)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{a\,x^3}{3}+b\left(-\frac{x\cos\left(d\,x^2+c\right)}{2d}+\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\,\mathrm{C}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(c)\,\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4d^{\frac{3}{2}}}\right)$	68
parts	$\frac{a\,x^3}{3}+b\left(-\frac{x\cos\left(d\,x^2+c\right)}{2d}+\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\,\mathrm{C}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(c)\,\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4d^{\frac{3}{2}}}\right)$	68
risch	$\frac{b\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right)\mathrm{e}^{ic}}{8d\sqrt{-id}}+\frac{b\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{8d\sqrt{id}}+\frac{a\,x^3}{3}-\frac{bx\cos\left(d\,x^2+c\right)}{2d}$	81

```
[In] int(x^2*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3+b*(-1/2/d*x*cos(d*x^2+c)+1/4/d^(3/2)*2^(1/2)*Pi^(1/2)*(cos(c)*Fre
snelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))-sin(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2
))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.84

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx = \frac{4\,ad^2x^3+3\,\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)-3\,\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)-6\,bdx\cos\left(dx^2+c\right)}{12\,d^2}$$

```
[In] integrate(x^2*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*d^2*x^3 + 3*sqrt(2)*pi*b*sqrt(d/pi)*cos(c)*fresnel_cos(sqrt(2)*x*
sqrt(d/pi)) - 3*sqrt(2)*pi*b*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*s
in(c) - 6*b*d*x*cos(d*x^2 + c))/d^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(102) = 204.

Time = 1.87 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.19

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx = \frac{ax^3}{3} - \frac{bd^{\frac{3}{2}}x^5\sqrt{\frac{1}{d}}\cos\left(c\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_{2}F_{3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{d^2x^4}{4}\right)}{8\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)} - \frac{b\sqrt{d}x^3\sqrt{\frac{1}{d}}\sin\left(c\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_{2}F_{3}\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{d^2x^4}{4}\right)}{8\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{2}\sqrt{\pi}bx^2\sqrt{\frac{1}{d}}\sin\left(c\right)C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)}{2} + \frac{\sqrt{2}\sqrt{\pi}bx^2\sqrt{\frac{1}{d}}\cos\left(c\right)S\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)}{2}$$

```
[In] integrate(x**2*(a+b*sin(d*x**2+c)),x)
```

```
[Out] a*x**3/3 - b*d**(3/2)*x**5*sqrt(1/d)*cos(c)*gamma(3/4)*gamma(5/4)*hyper((3/
4, 5/4), (3/2, 7/4, 9/4), -d**2*x**4/4)/(8*gamma(7/4)*gamma(9/4)) - b*sqrt(
d)*x**3*sqrt(1/d)*sin(c)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4,
 7/4), -d**2*x**4/4)/(8*gamma(5/4)*gamma(7/4)) + sqrt(2)*sqrt(pi)*b*x**2*sq
rt(1/d)*sin(c)*fresnelc(sqrt(2)*sqrt(d)*x/sqrt(pi))/2 + sqrt(2)*sqrt(pi)*b*
x**2*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi))/2
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{1}{3}\,ax^3-\frac{\left(8\,d^2x\cos\left(dx^2+c\right)+\sqrt{2}\sqrt{\pi}\Big(\left(\left(i-1\right)\,\cos\left(c\right)+\left(i+1\right)\,\sin\left(c\right)\right)\mathrm{erf}\left(\sqrt{i\,d}x\right)+\left(-\left(i+1\right)\,\cos\left(c\right)-\left(i-\right.\right.}{16\,d^3}$$

```
[In] integrate(x^2*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 - 1/16*(8*d^2*x*cos(d*x^2 + c) + sqrt(2)*sqrt(pi)*(((I - 1)*cos(c
) + (I + 1)*sin(c))*erf(sqrt(I*d)*x) + (-(I + 1)*cos(c) - (I - 1)*sin(c))*e
rf(sqrt(-I*d)*x))*d^(3/2))*b/d^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.42

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{1}{3}\,ax^3-\frac{bxe^{\left(i\,dx^2+i\,c\right)}}{4\,d}-\frac{bxe^{\left(-i\,dx^2-i\,c\right)}}{4\,d}+\frac{i\,\sqrt{2}\sqrt{\pi}b\,\mathrm{erf}\left(-\frac{1}{2}i\,\sqrt{2}x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{\left(i\,c\right)}}{8\,d\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}-\frac{i\,\sqrt{2}\sqrt{\pi}b\,\mathrm{erf}\left(\frac{1}{2}i\,\sqrt{2}x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{\left(-i\,c\right)}}{8\,d\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}$$

```
[In] integrate(x^2*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 - 1/4*b*x*e^(I*d*x^2 + I*c)/d - 1/4*b*x*e^(-I*d*x^2 - I*c)/d + 1/
8*I*sqrt(2)*sqrt(pi)*b*erf(-1/2*I*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*
e^(I*c)/(d*(I*d/abs(d) + 1)*sqrt(abs(d))) - 1/8*I*sqrt(2)*sqrt(pi)*b*erf(1/
2*I*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/(d*(-I*d/abs(d) + 1)
*sqrt(abs(d)))
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\sin(c+dx^2))\,dx=\int x^2\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)\,dx$$

```
[In] int(x^2*(a + b*sin(c + d*x^2)),x)
[Out] int(x^2*(a + b*sin(c + d*x^2)), x)
```

3.9 $\int (a+b\sin(c+dx^2))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 74

$$\int (a+b\sin(c+dx^2))\,dx = ax+\frac{b\sqrt{\frac{\pi}{2}}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}+\frac{b\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{\sqrt{d}}$$

```
[Out] a*x+1/2*b*cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^(1
/2)+1/2*b*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)/d^(1
/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3434, 3433, 3432}

$$\int (a+b\sin(c+dx^2))\,dx = ax+\frac{\sqrt{\frac{\pi}{2}}b\sin(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}b\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}$$

```
[In] Int[a + b*Sin[c + d*x^2],x]
```

```
[Out] a*x + (b*Sqrt[Pi/2]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/Sqrt[d] + (b*Sqr
t[Pi/2]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/Sqrt[d]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \sin\left(c + dx^2\right)\,dx \\
&= ax + (b\cos(c))\int \sin\left(dx^2\right)\,dx + (b\sin(c))\int \cos\left(dx^2\right)\,dx \\
&= ax + \frac{b\sqrt{\frac{\pi}{2}}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}} + \frac{b\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{\sqrt{d}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \left(a + b\sin\left(c + dx^2\right)\right)\,dx = ax + \frac{b\sqrt{\frac{\pi}{2}}\left(\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) + \text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right)}{\sqrt{d}}$$

```
[In] Integrate[a + b*Sin[c + d*x^2],x]
[Out] a*x + (b*Sqrt[Pi/2]*(Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + FresnelC[Sqrt[
d]*Sqrt[2/Pi]*x]*Sin[c]))/Sqrt[d]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

method	result	size
default	$ax+\frac{b\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\,\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(c)\,\mathrm{C}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{2\sqrt{d}}$	48
parts	$ax+\frac{b\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\,\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(c)\,\mathrm{C}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{2\sqrt{d}}$	48
risch	$ax+\frac{ib\,\mathrm{e}^{-ic}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{id}\,x\right)}{4\sqrt{id}}-\frac{ib\,\mathrm{e}^{ic}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-id}\,x\right)}{4\sqrt{-id}}$	59

```
[In] int(a+b*sin(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/2*b*2^(1/2)*Pi^(1/2)/d^(1/2)*(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1
/2))+sin(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \left(a+b\sin\left(c+dx^2\right)\right)\,dx = \frac{\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)+\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)+2\,adx}{2\,d}$$

```
[In] integrate(a+b*sin(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*b*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/pi)) + sqr
t(2)*pi*b*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))*sin(c) + 2*a*d*x)/d
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \left(a+b\sin\left(c+dx^2\right)\right)\,dx = ax+\frac{\sqrt{2}\sqrt{\pi}b\left(\sin\left(c\right)C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)+\cos\left(c\right)S\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{d}}}{2}$$

```
[In] integrate(a+b*sin(d*x**2+c),x)
```

```
[Out] a*x + sqrt(2)*sqrt(pi)*b*(sin(c)*fresnelc(sqrt(2)*sqrt(d)*x/sqrt(pi)) + cos
(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi)))*sqrt(1/d)/2
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

$$\int \left(a + b\sin\left(c + dx^2\right)\right)\, dx =$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\cos(c) + (i-1)\sin(c)\right)\operatorname{erf}\left(\sqrt{i\,d}x\right) + \left((i-1)\cos(c) - (i+1)\sin(c)\right)\operatorname{erf}\left(\sqrt{-i\,d}x\right)\right.}{8\sqrt{d}}$$
$$+ ax$$

```
[In] integrate(a+b*sin(d*x^2+c),x, algorithm="maxima")
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(c) + (I - 1)*sin(c))*erf(sqrt(I*d)*x)
+ ((I - 1)*cos(c) - (I + 1)*sin(c))*erf(sqrt(-I*d)*x))*b/sqrt(d) + a*x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \left(a + b\sin\left(c + dx^2\right)\right)\, dx$$
$$= \frac{1}{4}\left(\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}i\sqrt{2}x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(i\,c)}}{\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} + \frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}i\sqrt{2}x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-i\,c)}}{\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}\right)b$$
$$+ ax$$

```
[In] integrate(a+b*sin(d*x^2+c),x, algorithm="giac")
[Out] 1/4*(sqrt(2)*sqrt(pi)*erf(-1/2*I*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e
^(I*c)/((I*d/abs(d) + 1)*sqrt(abs(d))) + sqrt(2)*sqrt(pi)*erf(1/2*I*sqrt(2)
*x*(-I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/((-I*d/abs(d) + 1)*sqrt(abs(d))
))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \left(a+b\sin \left(c+dx^2\right)\right)\,dx=a\,x+\frac{\sqrt{2}\,b\,\sqrt{\pi }\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{d}\,x}{\sqrt{\pi }}\right)\,\cos \left(c\right)}{2\,\sqrt{d}}+\frac{\sqrt{2}\,b\,\sqrt{\pi }\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{d}\,x}{\sqrt{\pi }}\right)\,\sin \left(c\right)}{2\,\sqrt{d}}$$

```
[In] int(a + b*sin(c + d*x^2),x)
```

```
[Out] a*x + (2^(1/2)*b*pi^(1/2)*fresnels((2^(1/2)*d^(1/2)*x)/pi^(1/2))*cos(c))/(2
*d^(1/2)) + (2^(1/2)*b*pi^(1/2)*fresnelc((2^(1/2)*d^(1/2)*x)/pi^(1/2))*sin(
c))/(2*d^(1/2))
```

3.10 $\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx = -\frac{a}{x}+b\sqrt{d}\sqrt{2\pi}\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) - b\sqrt{d}\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)-\frac{b\sin(c+dx^2)}{x}$$

```
[Out] -a/x-b*sin(d*x^2+c)/x+b*cos(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))*d^(1/2)
*2^(1/2)*Pi^(1/2)-b*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*d^(1/2)*2^(
1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {14, 3468, 3435, 3433, 3432}

$$\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx = -\frac{a}{x}+\sqrt{2\pi}b\sqrt{d}\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) - \sqrt{2\pi}b\sqrt{d}\sin(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{b\sin(c+dx^2)}{x}$$

```
[In] Int[(a + b*Sin[c + d*x^2])/x^2,x]
```

```
[Out] -(a/x) + b*Sqrt[d]*Sqrt[2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] - b*Sqr
t[d]*Sqrt[2*Pi]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] - (b*Sin[c + d*x^2])/
x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a}{x^2}+\frac{b\sin\left(c+dx^2\right)}{x^2}\right)\,dx\\
&= -\frac{a}{x}+b\int \frac{\sin\left(c+dx^2\right)}{x^2}\,dx\\
&= -\frac{a}{x}-\frac{b\sin\left(c+dx^2\right)}{x}+(2bd)\int \cos\left(c+dx^2\right)\,dx\\
&= -\frac{a}{x}-\frac{b\sin\left(c+dx^2\right)}{x}+(2bd\cos(c))\int \cos\left(dx^2\right)\,dx-(2bd\sin(c))\int \sin\left(dx^2\right)\,dx\\
&= -\frac{a}{x}+b\sqrt{d}\sqrt{2\pi}\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\\
&\quad -b\sqrt{d}\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)-\frac{b\sin\left(c+dx^2\right)}{x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx = -\frac{a}{x}-\frac{b\cos(dx^2)\sin(c)}{x}+b\sqrt{d}\sqrt{2\pi}\left(\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right)-\frac{b\cos(c)\sin(dx^2)}{x}$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x^2,x]
```

```
[Out] -(a/x) - (b*Cos[d*x^2]*Sin[c])/x + b*Sqrt[d]*Sqrt[2*Pi]*(Cos[c]*FresnelC[Sq
rt[d]*Sqrt[2/Pi]*x] - FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c]) - (b*Cos[c]*Si
n[d*x^2])/x
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{a}{x}+b\left(-\frac{\sin(d\,x^2+c)}{x}+\sqrt{d}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\,\mathrm{C}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(c)\,\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)\right)$	66
parts	$-\frac{a}{x}+b\left(-\frac{\sin(d\,x^2+c)}{x}+\sqrt{d}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\,\mathrm{C}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(c)\,\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)\right)$	66
risch	$\frac{bd\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right)\mathrm{e}^{ic}}{2\sqrt{-id}}+\frac{bd\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{2\sqrt{id}}-\frac{a}{x}-\frac{b\sin(d\,x^2+c)}{x}$	76

```
[In] int((a+b*sin(d*x^2+c))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x+b*(-sin(d*x^2+c)/x+d^(1/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelC(x*d^(1/2)
*2^(1/2)/Pi^(1/2))-sin(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx = \frac{\sqrt{2}\pi bx\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)-\sqrt{2}\pi bx\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin(c)-b\sin(dx^2+c)-a}{x}$$

```
[In] integrate((a+b*sin(d*x^2+c))/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(2)*pi*b*x*sqrt(d/pi)*cos(c)*fresnel_cos(sqrt(2)*x*sqrt(d/pi)) - sqrt(
2)*pi*b*x*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*sin(c) - b*sin(d*x^2
 + c) - a)/x
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^2}\,dx=\int \frac{a+b\sin\left(c+dx^2\right)}{x^2}\,dx$$

```
[In] integrate((a+b*sin(d*x**2+c))/x**2,x)
[Out] Integral((a + b*sin(c + d*x**2))/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^2}\,dx=-\frac{\sqrt{dx^2}\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{1}{2},i\,dx^2\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},-i\,dx^2\right)\right)\cos\left(c\right)+\left((i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},i\,dx^2\right)-(i-\right.}{8\,x}-\frac{a}{x}$$

```
[In] integrate((a+b*sin(d*x^2+c))/x^2,x, algorithm="maxima")
[Out] -1/8*sqrt(d*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, I*d*x^2) - (I + 1)*sqrt(2)*g
amma(-1/2, -I*d*x^2))*cos(c) + ((I + 1)*sqrt(2)*gamma(-1/2, I*d*x^2) - (I -
 1)*sqrt(2)*gamma(-1/2, -I*d*x^2))*sin(c))*b/x - a/x
```

Giac [F]

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^2}\,dx=\int \frac{b\sin\left(dx^2+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sin(d*x^2+c))/x^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x^2 + c) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx = \int \frac{a+b\,\sin(d\,x^2+c)}{x^2}\,dx$$

```
[In] int((a + b*sin(c + d*x^2))/x^2,x)
[Out] int((a + b*sin(c + d*x^2))/x^2, x)
```

3.11 $\int \frac{a+b\sin(c+dx^2)}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 114

$$\int \frac{a+b\sin(c+dx^2)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{2bd\cos(c+dx^2)}{3x} - \frac{2}{3}bd^{3/2}\sqrt{2\pi}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) - \frac{2}{3}bd^{3/2}\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)-\frac{b\sin(c+dx^2)}{3x^3}$$

```
[Out] -1/3*a/x^3-2/3*b*d*cos(d*x^2+c)/x-1/3*b*sin(d*x^2+c)/x^3-2/3*b*d^(3/2)*cos(
c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)-2/3*b*d^(3/2)*Fres
nelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 3468, 3469, 3434, 3433, 3432}

$$\int \frac{a+b\sin(c+dx^2)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{2}{3}\sqrt{2\pi}bd^{3/2}\sin(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) - \frac{2}{3}\sqrt{2\pi}bd^{3/2}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{2bd\cos(c+dx^2)}{3x} - \frac{b\sin(c+dx^2)}{3x^3}$$

```
[In] Int[(a + b*Sin[c + d*x^2])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (2*b*d*Cos[c + d*x^2])/(3*x) - (2*b*d^(3/2)*Sqrt[2*Pi]*Cos[c]*
FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/3 - (2*b*d^(3/2)*Sqrt[2*Pi]*FresnelC[Sqrt[d
]*Sqrt[2/Pi]*x]*Sin[c])/3 - (b*Sin[c + d*x^2])/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a}{x^4} + \frac{b \sin \left(c + d x^2\right)}{x^4} \right) dx$$

$$
\begin{aligned}
&= -\frac{a}{3x^3} + b\int \frac{\sin(c+dx^2)}{x^4}\,dx \\
&= -\frac{a}{3x^3} - \frac{b\sin(c+dx^2)}{3x^3} + \frac{1}{3}(2bd)\int \frac{\cos(c+dx^2)}{x^2}\,dx \\
&= -\frac{a}{3x^3} - \frac{2bd\cos(c+dx^2)}{3x} - \frac{b\sin(c+dx^2)}{3x^3} - \frac{1}{3}(4bd^2)\int \sin(c+dx^2)\,dx \\
&= -\frac{a}{3x^3} - \frac{2bd\cos(c+dx^2)}{3x} - \frac{b\sin(c+dx^2)}{3x^3} \\
&\quad - \frac{1}{3}(4bd^2\cos(c))\int \sin(dx^2)\,dx - \frac{1}{3}(4bd^2\sin(c))\int \cos(dx^2)\,dx \\
&= -\frac{a}{3x^3} - \frac{2bd\cos(c+dx^2)}{3x} - \frac{2}{3}bd^{3/2}\sqrt{2\pi}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) \\
&\quad - \frac{2}{3}bd^{3/2}\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c) - \frac{b\sin(c+dx^2)}{3x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$
\begin{aligned}
\int \frac{a+b\sin(c+dx^2)}{x^4}\,dx = &-\frac{a}{3x^3} - \frac{b\cos(dx^2)(2dx^2\cos(c)+\sin(c))}{3x^3} \\
&- \frac{2}{3}bd^{3/2}\sqrt{2\pi}\left(\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\right. \\
&\left.+ \text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right) \\
&+ \frac{b(-\cos(c)+2dx^2\sin(c))\sin(dx^2)}{3x^3}
\end{aligned}
$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*Cos[d*x^2]*(2*d*x^2*Cos[c] + Sin[c]))/(3*x^3) - (2*b*d^(3/2
)*Sqrt[2*Pi]*(Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + FresnelC[Sqrt[d]*Sqrt
[2/Pi]*x]*Sin[c]))/3 + (b*(-Cos[c] + 2*d*x^2*Sin[c])*Sin[d*x^2])/(3*x^3)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{a}{3x^3}+b\left(-\frac{\sin(dx^2+c)}{3x^3}+\frac{2d\left(-\frac{\cos(dx^2+c)}{x}-\sqrt{d}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\,\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(c)\,\mathrm{C}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)\right)}{3}\right)$	83
parts	$-\frac{a}{3x^3}+b\left(-\frac{\sin(dx^2+c)}{3x^3}+\frac{2d\left(-\frac{\cos(dx^2+c)}{x}-\sqrt{d}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\,\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(c)\,\mathrm{C}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)\right)}{3}\right)$	83
risch	$-\frac{a}{3x^3}-\frac{ib\,d^2\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{3\sqrt{id}}+\frac{ib\,d^2\sqrt{\pi}\,\mathrm{erf}(\sqrt{-id}\,x)\mathrm{e}^{ic}}{3\sqrt{-id}}-\frac{2bd\cos(dx^2+c)}{3x}-\frac{b\sin(dx^2+c)}{3x^3}$	97

```
[In] int((a+b*sin(d*x^2+c))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3+b*(-1/3*sin(d*x^2+c)/x^3+2/3*d*(-1/x*cos(d*x^2+c)-d^(1/2)*2^(1/2
)*Pi^(1/2)*(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))+sin(c)*FresnelC(x*d
^(1/2)*2^(1/2)/Pi^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.86

$$\int \frac{a+b\sin(c+dx^2)}{x^4}\,dx = -\frac{2\sqrt{2}\pi bdx^3\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)+2\sqrt{2}\pi bdx^3\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin(c)+2\,bdx^2\cos(dx^2+c)+b\sin(}{3\,x^3}$$

```
[In] integrate((a+b*sin(d*x^2+c))/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(2)*pi*b*d*x^3*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/p
i)) + 2*sqrt(2)*pi*b*d*x^3*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))*sin
(c) + 2*b*d*x^2*cos(d*x^2 + c) + b*sin(d*x^2 + c) + a)/x^3
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx = \int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx$$

```
[In] integrate((a+b*sin(d*x**2+c))/x**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.72

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx = -\frac{\sqrt{dx^2}\left(\left(-(i+1)\,\sqrt{2}\Gamma\left(-\frac{3}{2},i\,dx^2\right)+(i-1)\,\sqrt{2}\Gamma\left(-\frac{3}{2},-i\,dx^2\right)\right)\cos\left(c\right)+\left((i-1)\,\sqrt{2}\Gamma\left(-\frac{3}{2},i\,dx^2\right)-(i\cdots\right.\right.}{8\,x} - \frac{a}{3\,x^3}$$

```
[In] integrate((a+b*sin(d*x^2+c))/x^4,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(d*x^2)*((-(I + 1)*sqrt(2)*gamma(-3/2, I*d*x^2) + (I - 1)*sqrt(2)*
gamma(-3/2, -I*d*x^2))*cos(c) + ((I - 1)*sqrt(2)*gamma(-3/2, I*d*x^2) - (I
+ 1)*sqrt(2)*gamma(-3/2, -I*d*x^2))*sin(c))*b*d/x - 1/3*a/x^3
```

Giac [F]

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx = \int \frac{b\sin\left(dx^2+c\right)+a}{x^4}\,dx$$

```
[In] integrate((a+b*sin(d*x^2+c))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(c+dx^2)}{x^4}\,dx = \int \frac{a+b\,\sin(d\,x^2+c)}{x^4}\,dx$$

```
[In] int((a + b*sin(c + d*x^2))/x^4,x)
[Out] int((a + b*sin(c + d*x^2))/x^4, x)
```

3.12 $\int x^5(a+b\sin(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 163

$$\int x^5(a+b\sin(c+dx^2))^2\,dx=-\frac{b^2x^2}{8d^2}+\frac{a^2x^6}{6}+\frac{b^2x^6}{12}+\frac{2ab\cos(c+dx^2)}{d^3}-\frac{abx^4\cos(c+dx^2)}{d}+\frac{2abx^2\sin(c+dx^2)}{d^2}+\frac{b^2\cos(c+dx^2)\sin(c+dx^2)}{8d^3}-\frac{b^2x^4\cos(c+dx^2)\sin(c+dx^2)}{4d}+\frac{b^2x^2\sin^2(c+dx^2)}{4d^2}$$

```
[Out] -1/8*b^2*x^2/d^2+1/6*a^2*x^6+1/12*b^2*x^6+2*a*b*cos(d*x^2+c)/d^3-a*b*x^4*co
s(d*x^2+c)/d+2*a*b*x^2*sin(d*x^2+c)/d^2+1/8*b^2*cos(d*x^2+c)*sin(d*x^2+c)/d
^3-1/4*b^2*x^4*cos(d*x^2+c)*sin(d*x^2+c)/d+1/4*b^2*x^2*sin(d*x^2+c)^2/d^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3460, 3398, 3377, 2718, 3392, 30, 2715, 8}

$$\int x^5(a+b\sin(c+dx^2))^2\,dx=\frac{a^2x^6}{6}+\frac{2ab\cos(c+dx^2)}{d^3}+\frac{2abx^2\sin(c+dx^2)}{d^2}-\frac{abx^4\cos(c+dx^2)}{d}+\frac{b^2\sin(c+dx^2)\cos(c+dx^2)}{8d^3}+\frac{b^2x^2\sin^2(c+dx^2)}{4d^2}-\frac{b^2x^4\sin(c+dx^2)\cos(c+dx^2)}{4d}-\frac{b^2x^2}{8d^2}+\frac{b^2x^6}{12}$$

```
[In] Int[x^5*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] -1/8*(b^2*x^2)/d^2 + (a^2*x^6)/6 + (b^2*x^6)/12 + (2*a*b*Cos[c + d*x^2])/d^
3 - (a*b*x^4*Cos[c + d*x^2])/d + (2*a*b*x^2*Sin[c + d*x^2])/d^2 + (b^2*Cos[
c + d*x^2]*Sin[c + d*x^2])/(8*d^3) - (b^2*x^4*Cos[c + d*x^2]*Sin[c + d*x^2]
)/(4*d) + (b^2*x^2*Sin[c + d*x^2]^2)/(4*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2(a+b\sin(c+dx))^2\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x^2+2abx^2\sin(c+dx)+b^2x^2\sin^2(c+dx)\right)\,dx, x, x^2\right)\\
&= \frac{a^2x^6}{6}+(ab)\text{Subst}\left(\int x^2\sin(c+dx)\,dx, x, x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int x^2\sin^2(c+dx)\,dx, x, x^2\right)\\
&= \frac{a^2x^6}{6}-\frac{abx^4\cos\left(c+dx^2\right)}{d}-\frac{b^2x^4\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}\\
&\quad+\frac{b^2x^2\sin^2\left(c+dx^2\right)}{4d^2}+\frac{1}{4}b^2\text{Subst}\left(\int x^2\,dx, x, x^2\right)\\
&\quad-\frac{b^2\text{Subst}\left(\int\sin^2(c+dx)\,dx, x, x^2\right)}{4d^2}+\frac{(2ab)\text{Subst}\left(\int x\cos(c+dx)\,dx, x, x^2\right)}{d}\\
&= \frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{abx^4\cos\left(c+dx^2\right)}{d}\\
&\quad+\frac{2abx^2\sin\left(c+dx^2\right)}{d^2}+\frac{b^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{8d^3}\\
&\quad-\frac{b^2x^4\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}+\frac{b^2x^2\sin^2\left(c+dx^2\right)}{4d^2}\\
&\quad-\frac{(2ab)\text{Subst}\left(\int\sin(c+dx)\,dx, x, x^2\right)}{d^2}-\frac{b^2\text{Subst}\left(\int 1\,dx, x, x^2\right)}{8d^2}\\
&= -\frac{b^2x^2}{8d^2}+\frac{a^2x^6}{6}+\frac{b^2x^6}{12}+\frac{2ab\cos\left(c+dx^2\right)}{d^3}-\frac{abx^4\cos\left(c+dx^2\right)}{d}+\frac{2abx^2\sin\left(c+dx^2\right)}{d^2}\\
&\quad+\frac{b^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{8d^3}-\frac{b^2x^4\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}+\frac{b^2x^2\sin^2\left(c+dx^2\right)}{4d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{8a^2d^3x^6+4b^2d^3x^6-48ab(-2+d^2x^4)\cos\left(c+dx^2\right)-6b^2dx^2\cos\left(2(c+dx^2)\right)+96abdx^2\sin\left(c+dx^2\right)+3b}{48d^3}$$

[In] Integrate[x^5*(a + b*Sin[c + d*x^2])^2,x]

[Out] (8*a^2*d^3*x^6 + 4*b^2*d^3*x^6 - 48*a*b*(-2 + d^2*x^4)*Cos[c + d*x^2] - 6*b ^2*d*x^2*Cos[2*(c + d*x^2)] + 96*a*b*d*x^2*Sin[c + d*x^2] + 3*b^2*Sin[2*(c + d*x^2)] - 6*b^2*d^2*x^4*Sin[2*(c + d*x^2)])/(48*d^3)

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.68

method	result
risch	$\frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{ab(x^4d^2-2)\cos(dx^2+c)}{d^3}+\frac{2ab\,x^2\sin(dx^2+c)}{d^2}-\frac{b^2x^2\cos(2dx^2+2c)}{8d^2}-\frac{b^2(2x^4d^2-1)\sin(2dx^2+2c)}{16d^3}$
parallelrisch	$\frac{(-6x^4d^2+3)b^2\sin(2dx^2+2c)-6b^2x^2\cos(2dx^2+2c)d-48ab(x^4d^2-2)\cos(dx^2+c)+8a^2d^3x^6+4b^2d^3x^6+96ab\,x^2\sin(dx^2+c)d+}{48d^3}$
parts	$\frac{a^2x^6}{6}+b^2\left(\frac{x^6}{12}-\frac{x^4\sin(2dx^2+2c)}{8d}+\frac{-\frac{x^2\cos(2dx^2+2c)}{4d}+\frac{\sin(2dx^2+2c)}{8d^2}}{2d}\right)+2ab\left(-\frac{x^4\cos(dx^2+c)}{2d}+\frac{\frac{x^2\sin(dx^2+c}{d}}{}\right.$
default	$\frac{\left(a^2+\frac{b^2}{2}\right)x^6}{6}-\frac{b^2\left(\frac{x^4\sin(2dx^2+2c)}{4d}-\frac{-\frac{x^2\cos(2dx^2+2c)}{4d}+\frac{\sin(2dx^2+2c)}{8d^2}}{d}\right)}{2}+2ab\left(-\frac{x^4\cos(dx^2+c)}{2d}+\frac{\frac{x^2\sin(dx^2+c)}{d}+\frac{c}{}}{d}\right.$
norman	$\frac{\left(\frac{a^2}{6}+\frac{b^2}{12}\right)x^6+\left(\frac{a^2}{3}+\frac{b^2}{6}\right)x^6\left(\tan^2\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{6}+\frac{b^2}{12}\right)x^6\left(\tan^4\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)+\frac{ab\,x^4\left(\tan^4\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b^2\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{4d^3}-\frac{b^2(t}{}}{}$

[In] int(x^5*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/6*a^2*x^6+1/12*b^2*x^6-a*b*(d^2*x^4-2)/d^3*cos(d*x^2+c)+2*a*b*x^2*sin(d*x ^2+c)/d^2-1/8*b^2/d^2*x^2*cos(2*d*x^2+2*c)-1/16*b^2*(2*d^2*x^4-1)/d^3*sin(2 *d*x^2+2*c)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{2\,(2\,a^2+b^2)d^3x^6-6\,b^2dx^2\cos\left(dx^2+c\right)^2+3\,b^2dx^2-24\,(abd^2x^4-2\,ab)\cos\left(dx^2+c\right)+3\,(16\,abdx^2-(2\,b}{24\,d^3}$$

```
[In] integrate(x^5*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(2*(2*a^2 + b^2)*d^3*x^6 - 6*b^2*d*x^2*cos(d*x^2 + c)^2 + 3*b^2*d*x^2
- 24*(a*b*d^2*x^4 - 2*a*b)*cos(d*x^2 + c) + 3*(16*a*b*d*x^2 - (2*b^2*d^2*x^
4 - b^2)*cos(d*x^2 + c))*sin(d*x^2 + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.28

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \begin{cases} \frac{a^2x^6}{6}-\frac{abx^4\cos\left(c+dx^2\right)}{d}+\frac{2abx^2\sin\left(c+dx^2\right)}{d^2}+\frac{2ab\cos\left(c+dx^2\right)}{d^3}+\frac{b^2x^6\sin^2\left(c+dx^2\right)}{12}+\frac{b^2x^6\cos^2\left(c+dx^2\right)}{12}-\frac{b^2x^4\sin\left(c+dx^2\right)\cos}{4d} \\ \frac{x^6(a+b\sin(c))^2}{6} \end{cases}$$

```
[In] integrate(x**5*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 - a*b*x**4*cos(c + d*x**2)/d + 2*a*b*x**2*sin(c + d*
x**2)/d**2 + 2*a*b*cos(c + d*x**2)/d**3 + b**2*x**6*sin(c + d*x**2)**2/12 +
 b**2*x**6*cos(c + d*x**2)**2/12 - b**2*x**4*sin(c + d*x**2)*cos(c + d*x**2
)/(4*d) + b**2*x**2*sin(c + d*x**2)**2/(8*d**2) - b**2*x**2*cos(c + d*x**2)
**2/(8*d**2) + b**2*sin(c + d*x**2)*cos(c + d*x**2)/(8*d**3), Ne(d, 0)), (x
**6*(a + b*sin(c))**2/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.65

$$\begin{aligned}\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx &= \frac{1}{6}\,a^2x^6+\frac{(2\,dx^2\sin\left(dx^2+c\right)-(d^2x^4-2)\cos\left(dx^2+c\right))ab}{d^3} \\ &\quad+\frac{(4\,d^3x^6-6\,dx^2\cos\left(2\,dx^2+2\,c\right)-3\,(2\,d^2x^4-1)\sin\left(2\,dx^2+2\,c\right))b^2}{48\,d^3}\end{aligned}$$

```
[In] integrate(x^5*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*x^6 + (2*d*x^2*sin(d*x^2 + c) - (d^2*x^4 - 2)*cos(d*x^2 + c))*a*b/d
^3 + 1/48*(4*d^3*x^6 - 6*d*x^2*cos(2*d*x^2 + 2*c) - 3*(2*d^2*x^4 - 1)*sin(2
*d*x^2 + 2*c))*b^2/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.74

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$= -\frac{((dx^2+c)b^2-b^2c)\cos\left(2\,dx^2+2\,c\right)}{8\,d^3} - \frac{\left((dx^2+c)^2ab-2\,(dx^2+c)abc-2\,ab\right)\cos\left(dx^2+c\right)}{d^3} - \frac{\left(2\,(dx^2+c)^2b^2-4\,(dx^2+c)b^2c-b^2\right)\sin\left(2\,dx^2+2\,c\right)}{16\,d^3} + \frac{2\,((dx^2+c)ab-abc)\sin\left(dx^2+c\right)}{d^3} + \frac{2\,(dx^2+c)^3a^2+(dx^2+c)^3b^2-6\,(dx^2+c)^2a^2c-3\,(dx^2+c)^2b^2c}{12\,d^3} + \frac{4\,(dx^2+c)a^2c^2+(2\,dx^2+2\,c-\sin\left(2\,dx^2+2\,c\right))b^2c^2-8\,abc^2\cos\left(dx^2+c\right)}{8\,d^3}$$

```
[In] integrate(x^5*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*((d*x^2 + c)*b^2 - b^2*c)*cos(2*d*x^2 + 2*c)/d^3 - ((d*x^2 + c)^2*a*b
- 2*(d*x^2 + c)*a*b*c - 2*a*b)*cos(d*x^2 + c)/d^3 - 1/16*(2*(d*x^2 + c)^2*b
^2 - 4*(d*x^2 + c)*b^2*c - b^2)*sin(2*d*x^2 + 2*c)/d^3 + 2*((d*x^2 + c)*a*b
 - a*b*c)*sin(d*x^2 + c)/d^3 + 1/12*(2*(d*x^2 + c)^3*a^2 + (d*x^2 + c)^3*b^
2 - 6*(d*x^2 + c)^2*a^2*c - 3*(d*x^2 + c)^2*b^2*c)/d^3 + 1/8*(4*(d*x^2 + c)
*a^2*c^2 + (2*d*x^2 + 2*c - sin(2*d*x^2 + 2*c))*b^2*c^2 - 8*a*b*c^2*cos(d*x
^2 + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.91

$$\int x^5 \left(a + b \sin \left(c + dx^2\right)\right)^2 \, dx$$

$$= \frac{\frac{3\,b^2\,\sin\left(2\,d\,x^2+2\,c\right)}{2} - 96\,a\,b\sin\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2 + 4\,a^2\,d^3\,x^6 + 2\,b^2\,d^3\,x^6 + 3\,b^2\,d\,x^2\left(2\sin\left(d\,x^2+c\right)^2-1\right) - 3\,b^2\,d^2}{24\,d^3}$$

```
[In] int(x^5*(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] ((3*b^2*sin(2*c + 2*d*x^2))/2 - 96*a*b*sin(c/2 + (d*x^2)/2)^2 + 4*a^2*d^3*x
^6 + 2*b^2*d^3*x^6 + 3*b^2*d*x^2*(2*sin(c + d*x^2)^2 - 1) - 3*b^2*d^2*x^4*s
in(2*c + 2*d*x^2) + 24*a*b*d^2*x^4*(2*sin(c/2 + (d*x^2)/2)^2 - 1) + 48*a*b*
d*x^2*sin(c + d*x^2))/(24*d^3)
```

3.13 $\int x^3(a+b\sin(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 102

$$\int x^3(a+b\sin(c+dx^2))^2\,dx=\frac{a^2x^4}{4}+\frac{b^2x^4}{8}-\frac{abx^2\cos(c+dx^2)}{d}+\frac{ab\sin(c+dx^2)}{d^2}-\frac{b^2x^2\cos(c+dx^2)\sin(c+dx^2)}{4d}+\frac{b^2\sin^2(c+dx^2)}{8d^2}$$

```
[Out] 1/4*a^2*x^4+1/8*b^2*x^4-a*b*x^2*cos(d*x^2+c)/d+a*b*sin(d*x^2+c)/d^2-1/4*b^2
*x^2*cos(d*x^2+c)*sin(d*x^2+c)/d+1/8*b^2*sin(d*x^2+c)^2/d^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3460, 3398, 3377, 2717, 3391, 30}

$$\int x^3(a+b\sin(c+dx^2))^2\,dx=\frac{a^2x^4}{4}+\frac{ab\sin(c+dx^2)}{d^2}-\frac{abx^2\cos(c+dx^2)}{d}+\frac{b^2\sin^2(c+dx^2)}{8d^2}-\frac{b^2x^2\sin(c+dx^2)\cos(c+dx^2)}{4d}+\frac{b^2x^4}{8}$$

```
[In] Int[x^3*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (a^2*x^4)/4 + (b^2*x^4)/8 - (a*b*x^2*Cos[c + d*x^2])/d + (a*b*Sin[c + d*x^2
])/d^2 - (b^2*x^2*Cos[c + d*x^2]*Sin[c + d*x^2])/(4*d) + (b^2*Sin[c + d*x^2
]^2)/(8*d^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(a+b\sin(c+dx))^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x+2abx\sin(c+dx)+b^2x\sin^2(c+dx)\right)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}+(ab)\text{Subst}\left(\int x\sin(c+dx)\,dx,x,x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int x\sin^2(c+dx)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}-\frac{abx^2\cos\left(c+dx^2\right)}{d}-\frac{b^2x^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}+\frac{b^2\sin^2\left(c+dx^2\right)}{8d^2}\\
&\quad+\frac{1}{4}b^2\text{Subst}\left(\int x\,dx,x,x^2\right)+\frac{(ab)\text{Subst}\left(\int\cos(c+dx)\,dx,x,x^2\right)}{d}
\end{aligned}$$

$$= \frac{a^2x^4}{4} + \frac{b^2x^4}{8} - \frac{abx^2\cos(c+dx^2)}{d} + \frac{ab\sin(c+dx^2)}{d^2} - \frac{b^2x^2\cos(c+dx^2)\sin(c+dx^2)}{4d} + \frac{b^2\sin^2(c+dx^2)}{8d^2}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{4a^2d^2x^4 + 2b^2d^2x^4 - 16abdx^2\cos(c+dx^2) - b^2\cos(2(c+dx^2)) + 16ab\sin(c+dx^2) - 2b^2dx^2\sin(2(c+dx}{16d^2}$$

```
[In] Integrate[x^3*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (4*a^2*d^2*x^4 + 2*b^2*d^2*x^4 - 16*a*b*d*x^2*Cos[c + d*x^2] - b^2*Cos[2*(c
 + d*x^2)] + 16*a*b*Sin[c + d*x^2] - 2*b^2*d*x^2*Sin[2*(c + d*x^2)])/(16*d^
2)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

method	result
parts	$\frac{a^2x^4}{4}+b^2\left(\frac{x^4}{8}-\frac{x^2\sin(2dx^2+2c)}{8d}-\frac{\cos(2dx^2+2c)}{16d^2}\right)+2ab\left(-\frac{x^2\cos(dx^2+c)}{2d}+\frac{\sin(dx^2+c)}{2d^2}\right)$
default	$\frac{\left(a^2+\frac{b^2}{2}\right)x^4}{4}-\frac{b^2\left(\frac{x^2\sin(2dx^2+2c)}{4d}+\frac{\cos(2dx^2+2c)}{8d^2}\right)}{2}+2ab\left(-\frac{x^2\cos(dx^2+c)}{2d}+\frac{\sin(dx^2+c)}{2d^2}\right)$
risch	$\frac{a^2x^4}{4}+\frac{b^2x^4}{8}-\frac{ab\,x^2\cos(dx^2+c)}{d}+\frac{ab\sin(dx^2+c)}{d^2}-\frac{b^2\cos(2dx^2+2c)}{16d^2}-\frac{b^2x^2\sin(2dx^2+2c)}{8d}$
parallelrisch	$\frac{4a^2d^2x^4+2b^2d^2x^4-16ab\,x^2\cos(dx^2+c)d-2b^2x^2\sin(2dx^2+2c)d+16\sin(dx^2+c)ab-b^2\cos(2dx^2+2c)+b^2}{16d^2}$
norman	$\frac{\left(\frac{a^2}{4}+\frac{b^2}{8}\right)x^4+\left(\frac{a^2}{2}+\frac{b^2}{4}\right)x^4\left(\tan^2\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{4}+\frac{b^2}{8}\right)x^4\left(\tan^4\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)+\frac{ab\,x^2\left(\tan^4\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b^2\left(\tan^2\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)}{2d^2}-}{\left(1+\tan^2\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)^2}$

```
[In] int(x^3*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4+b^2*(1/8*x^4-1/8/d*x^2*sin(2*d*x^2+2*c)-1/16/d^2*cos(2*d*x^2+2*
c))+2*a*b*(-1/2/d*x^2*cos(d*x^2+c)+1/2/d^2*sin(d*x^2+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{(2\,a^2+b^2)d^2x^4-8\,abdx^2\cos\left(dx^2+c\right)-b^2\cos\left(dx^2+c\right)^2-2\left(b^2dx^2\cos\left(dx^2+c\right)-4\,ab\right)\sin\left(dx^2+c\right)}{8\,d^2}$$

```
[In] integrate(x^3*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
[Out] 1/8*((2*a^2 + b^2)*d^2*x^4 - 8*a*b*d*x^2*cos(d*x^2 + c) - b^2*cos(d*x^2 + c
)^2 - 2*(b^2*d*x^2*cos(d*x^2 + c) - 4*a*b)*sin(d*x^2 + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \begin{cases} \frac{a^2x^4}{4}-\frac{abx^2\cos\left(c+dx^2\right)}{d}+\frac{ab\sin\left(c+dx^2\right)}{d^2}+\frac{b^2x^4\sin^2\left(c+dx^2\right)}{8}+\frac{b^2x^4\cos^2\left(c+dx^2\right)}{8}-\frac{b^2x^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d}-\frac{b^2\cos^2\left(c+\right.}{8d^2} \\ \frac{x^4(a+b\sin\left(c\right))^2}{4} \end{cases}$$

```
[In] integrate(x**3*(a+b*sin(d*x**2+c))**2,x)
[Out] Piecewise((a**2*x**4/4 - a*b*x**2*cos(c + d*x**2)/d + a*b*sin(c + d*x**2)/d
**2 + b**2*x**4*sin(c + d*x**2)**2/8 + b**2*x**4*cos(c + d*x**2)**2/8 - b**
2*x**2*sin(c + d*x**2)*cos(c + d*x**2)/(4*d) - b**2*cos(c + d*x**2)**2/(8*d
**2), Ne(d, 0)), (x**4*(a + b*sin(c))**2/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.85

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{1}{4}\,a^2x^4-\frac{\left(dx^2\cos\left(dx^2+c\right)-\sin\left(dx^2+c\right)\right)ab}{d^2}+\frac{\left(2\,d^2x^4-2\,dx^2\sin\left(2\,dx^2+2\,c\right)-\cos\left(2\,dx^2+2\,c\right)\right)b^2}{16\,d^2}$$

```
[In] integrate(x^3*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
[Out] 1/4*a^2*x^4 - (d*x^2*cos(d*x^2 + c) - sin(d*x^2 + c))*a*b/d^2 + 1/16*(2*d^2
*x^4 - 2*d*x^2*sin(2*d*x^2 + 2*c) - cos(2*d*x^2 + 2*c))*b^2/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int x^3 \left(a + b\sin\left(c + dx^2\right)\right)^2 \, dx$$
$$= \frac{4\,(dx^2 + c)^2 a^2 + 2\,(dx^2 + c)^2 b^2 - 16\,(dx^2 + c)ab\cos\left(dx^2 + c\right) - 2\,(dx^2 + c)b^2\sin\left(2\,dx^2 + 2\,c\right) - b^2\cos\left(2\,dx\right.}{16\,d^2}$$
$$- \frac{4\,(dx^2 + c)a^2c + \left(2\,dx^2 + 2\,c - \sin\left(2\,dx^2 + 2\,c\right)\right)b^2c - 8\,abc\cos\left(dx^2 + c\right)}{8\,d^2}$$

```
[In] integrate(x^3*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*(d*x^2 + c)^2*a^2 + 2*(d*x^2 + c)^2*b^2 - 16*(d*x^2 + c)*a*b*cos(d*
x^2 + c) - 2*(d*x^2 + c)*b^2*sin(2*d*x^2 + 2*c) - b^2*cos(2*d*x^2 + 2*c) +
16*a*b*sin(d*x^2 + c))/d^2 - 1/8*(4*(d*x^2 + c)*a^2*c + (2*d*x^2 + 2*c - si
n(2*d*x^2 + 2*c))*b^2*c - 8*a*b*c*cos(d*x^2 + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

$$\int x^3 \left(a + b\sin\left(c + dx^2\right)\right)^2 \, dx =$$
$$-\frac{b^2\cos\left(d\,x^2 + c\right)^2 - 2\,a^2\,d^2\,x^4 - b^2\,d^2\,x^4 - 8\,a\,b\,\sin\left(d\,x^2 + c\right) + 8\,a\,b\,d\,x^2\,\cos\left(d\,x^2 + c\right) + 2\,b^2\,d\,x^2\,\cos\left(d\,x\right.}{8\,d^2}$$

```
[In] int(x^3*(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] -(b^2*cos(c + d*x^2)^2 - 2*a^2*d^2*x^4 - b^2*d^2*x^4 - 8*a*b*sin(c + d*x^2)
 + 8*a*b*d*x^2*cos(c + d*x^2) + 2*b^2*d*x^2*cos(c + d*x^2)*sin(c + d*x^2))/
(8*d^2)
```

3.14 $\int x(a+b\sin(c+dx^2))^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 58

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{1}{4}\left(2a^2+b^2\right)x^2-\frac{ab\cos\left(c+dx^2\right)}{d}-\frac{b^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}$$

```
[Out] 1/4*(2*a^2+b^2)*x^2-a*b*cos(d*x^2+c)/d-1/4*b^2*cos(d*x^2+c)*sin(d*x^2+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3460, 2723}

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{1}{4}x^2\left(2a^2+b^2\right)-\frac{ab\cos\left(c+dx^2\right)}{d}-\frac{b^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d}$$

```
[In] Int[x*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^2)/4 - (a*b*Cos[c + d*x^2])/d - (b^2*Cos[c + d*x^2]*Sin[c
+ d*x^2])/(4*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (a+b\sin(c+dx))^2\,dx, x, x^2\right)\\ &= \frac{1}{4}\left(2a^2+b^2\right)x^2 - \frac{ab\cos\left(c+dx^2\right)}{d} - \frac{b^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = -\frac{-2(2a^2+b^2)\left(c+dx^2\right)+8ab\cos\left(c+dx^2\right)+b^2\sin\left(2(c+dx^2)\right)}{8d}$$

```
[In] Integrate[x*(a + b*Sin[c + d*x^2])^2,x]
[Out] -1/8*(-2*(2*a^2 + b^2)*(c + d*x^2) + 8*a*b*Cos[c + d*x^2] + b^2*Sin[2*(c +
d*x^2)])/d
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result
risch	$\frac{x^2 a^2}{2}+\frac{x^2 b^2}{4}-\frac{ab\cos(d\,x^2+c)}{d}-\frac{b^2\sin(2d\,x^2+2c)}{8d}$
parallelrisch	$\frac{4a^2 d\,x^2+2x^2 b^2 d-8ab\cos(d\,x^2+c)-b^2\sin(2d\,x^2+2c)+8ab}{8d}$
parts	$\frac{x^2 a^2}{2}+\frac{b^2\left(-\frac{\cos\left(d\,x^2+c\right)\sin\left(d\,x^2+c\right)}{2}+\frac{d\,x^2}{2}+\frac{c}{2}\right)}{2d}-\frac{ab\cos(d\,x^2+c)}{d}$
derivativedivides	$\frac{b^2\left(-\frac{\cos\left(d\,x^2+c\right)\sin\left(d\,x^2+c\right)}{2}+\frac{d\,x^2}{2}+\frac{c}{2}\right)-2ab\cos(d\,x^2+c)+a^2(d\,x^2+c)}{2d}$
default	$\frac{b^2\left(-\frac{\cos\left(d\,x^2+c\right)\sin\left(d\,x^2+c\right)}{2}+\frac{d\,x^2}{2}+\frac{c}{2}\right)-2ab\cos(d\,x^2+c)+a^2(d\,x^2+c)}{2d}$
norman	$\frac{\left(\frac{a^2}{2}+\frac{b^2}{4}\right)x^2+\left(a^2+\frac{b^2}{2}\right)x^2\left(\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{2}+\frac{b^2}{4}\right)x^2\left(\tan^4\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)+\frac{2ab\left(\tan^4\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{2d}}{\left(1+\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)^2}$

```
[In] int(x*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*a^2+1/4*x^2*b^2-a*b*cos(d*x^2+c)/d-1/8*b^2/d*sin(2*d*x^2+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int x\big(a+b\sin\left(c+dx^2\right)\big)^2\,dx = \frac{(2\,a^2+b^2)dx^2-b^2\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)-4\,ab\cos\left(dx^2+c\right)}{4\,d}$$

```
[In] integrate(x*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
[Out] 1/4*((2*a^2 + b^2)*d*x^2 - b^2*cos(d*x^2 + c)*sin(d*x^2 + c) - 4*a*b*cos(d*
x^2 + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \begin{cases} \frac{a^2x^2}{2} - \frac{ab\cos\left(c+dx^2\right)}{d} + \frac{b^2x^2\sin^2\left(c+dx^2\right)}{4} + \frac{b^2x^2\cos^2\left(c+dx^2\right)}{4} - \frac{b^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d} & \text{for } d\neq 0 \\ \frac{x^2(a+b\sin(c))^2}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 - a*b*cos(c + d*x**2)/d + b**2*x**2*sin(c + d*x**2)*
*2/4 + b**2*x**2*cos(c + d*x**2)**2/4 - b**2*sin(c + d*x**2)*cos(c + d*x**2
)/(4*d), Ne(d, 0)), (x**2*(a + b*sin(c))**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{1}{2}\,a^2x^2 + \frac{\left(2\,dx^2 - \sin\left(2\,dx^2+2\,c\right)\right)b^2}{8\,d} - \frac{ab\cos\left(dx^2+c\right)}{d}$$

```
[In] integrate(x*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/8*(2*d*x^2 - sin(2*d*x^2 + 2*c))*b^2/d - a*b*cos(d*x^2 + c)
/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{4\left(dx^2+c\right)a^2 + \left(2\,dx^2+2\,c-\sin\left(2\,dx^2+2\,c\right)\right)b^2 - 8\,ab\cos\left(dx^2+c\right)}{8\,d}$$

```
[In] integrate(x*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*(d*x^2 + c)*a^2 + (2*d*x^2 + 2*c - sin(2*d*x^2 + 2*c))*b^2 - 8*a*b*c
os(d*x^2 + c))/d
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{a^2\,x^2}{2}+\frac{b^2\,x^2}{4}-\frac{b^2\,\sin\left(2\,d\,x^2+2\,c\right)}{8\,d}-\frac{a\,b\,\cos\left(d\,x^2+c\right)}{d}$$

```
[In] int(x*(a + b*sin(c + d*x^2))^2,x)
[Out] (a^2*x^2)/2 + (b^2*x^2)/4 - (b^2*sin(2*c + 2*d*x^2))/(8*d) - (a*b*cos(c + d
*x^2))/d
```

3.15 $\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 74

$$\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx = -\frac{1}{4}b^2\cos(2c)\,\text{CosIntegral}\left(2dx^2\right) + \frac{1}{2}\left(2a^2+b^2\right)\log(x) + ab\,\text{CosIntegral}\left(dx^2\right)\sin(c) + ab\cos(c)\text{Si}\left(dx^2\right) + \frac{1}{4}b^2\sin(2c)\text{Si}\left(2dx^2\right)$$

```
[Out] -1/4*b^2*Ci(2*d*x^2)*cos(2*c)+1/2*(2*a^2+b^2)*ln(x)+a*b*cos(c)*Si(d*x^2)+a*
b*Ci(d*x^2)*sin(c)+1/4*b^2*Si(2*d*x^2)*sin(2*c)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3484, 6, 3459, 3457, 3456, 3458}

$$\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx = \frac{1}{2}\left(2a^2+b^2\right)\log(x) + ab\sin(c)\,\text{CosIntegral}\left(dx^2\right) + ab\cos(c)\text{Si}\left(dx^2\right) - \frac{1}{4}b^2\cos(2c)\,\text{CosIntegral}\left(2dx^2\right) + \frac{1}{4}b^2\sin(2c)\text{Si}\left(2dx^2\right)$$

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x,x]
```

```
[Out] -1/4*(b^2*Cos[2*c]*CosIntegral[2*d*x^2]) + ((2*a^2 + b^2)*Log[x])/2 + a*b*C
osIntegral[d*x^2]*Sin[c] + a*b*Cos[c]*SinIntegral[d*x^2] + (b^2*Sin[2*c]*Si
nIntegral[2*d*x^2])/4
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{x}+\frac{b^2}{2x}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x}+\frac{2ab\sin\left(c+dx^2\right)}{x}\right)dx\\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x}+\frac{2ab\sin\left(c+dx^2\right)}{x}\right)dx\\
&= \frac{1}{2}\left(2a^2+b^2\right)\log(x)+(2ab)\int \frac{\sin\left(c+dx^2\right)}{x}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^2\right)}{x}\,dx\\
&= \frac{1}{2}\left(2a^2+b^2\right)\log(x)+(2ab\cos(c))\int \frac{\sin\left(dx^2\right)}{x}\,dx-\frac{1}{2}\left(b^2\cos(2c)\right)\int \frac{\cos\left(2dx^2\right)}{x}\,dx\\
&\quad+(2ab\sin(c))\int \frac{\cos\left(dx^2\right)}{x}\,dx+\frac{1}{2}\left(b^2\sin(2c)\right)\int \frac{\sin\left(2dx^2\right)}{x}\,dx
\end{aligned}$$

$$= -\frac{1}{4}b^2\cos(2c)\,\text{CosIntegral}\left(2dx^2\right) + \frac{1}{2}\left(2a^2+b^2\right)\log(x) + ab\,\text{CosIntegral}\left(dx^2\right)\sin(c) + ab\cos(c)\text{Si}\left(dx^2\right) + \frac{1}{4}b^2\sin(2c)\text{Si}\left(2dx^2\right)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx = \frac{1}{2}\left(2a^2+b^2\right)\log(x) - \frac{1}{4}b\left(b\cos(2c)\,\text{CosIntegral}\left(2dx^2\right) - 4a\,\text{CosIntegral}\left(dx^2\right)\sin(c) - 4a\cos(c)\text{Si}\left(dx^2\right) - b\sin(2c)\text{Si}\left(2dx^2\right)\right)$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x,x]
```

```
[Out] ((2*a^2 + b^2)*Log[x])/2 - (b*(b*Cos[2*c]*CosIntegral[2*d*x^2] - 4*a*CosInt
egral[d*x^2]*Sin[c] - 4*a*Cos[c]*SinIntegral[d*x^2] - b*Sin[2*c]*SinIntegra
l[2*d*x^2]))/4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

method	result
risch	$-\frac{e^{-ic}\pi\ \text{csgn}(d\,x^2)ab}{2} + e^{-ic}\,\text{Si}\left(d\,x^2\right)ab - \frac{i e^{-ic}\,\text{Ei}_1\left(-i d\,x^2\right)ab}{2} + \ln(x)\,a^2 + \frac{\ln(x)b^2}{2} - \frac{i e^{-2ic}\pi\ \text{csgn}(d\,x^2)b^2}{8} + \frac{i e^{-2}$

```
[In] int((a+b*sin(d*x^2+c))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(-I*c)*Pi*csgn(d*x^2)*a*b+exp(-I*c)*Si(d*x^2)*a*b-1/2*I*exp(-I*c)*E
i(1,-I*d*x^2)*a*b+ln(x)*a^2+1/2*ln(x)*b^2-1/8*I*exp(-2*I*c)*Pi*csgn(d*x^2)*
b^2+1/4*I*exp(-2*I*c)*Si(2*d*x^2)*b^2+1/8*exp(-2*I*c)*Ei(1,-2*I*d*x^2)*b^2+
1/8*b^2*exp(2*I*c)*Ei(1,-2*I*d*x^2)+1/2*I*a*b*exp(I*c)*Ei(1,-I*d*x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx = -\frac{1}{4}b^2\cos(2c)\operatorname{Ci}(2dx^2) + ab\operatorname{Ci}(dx^2)\sin(c) + \frac{1}{4}b^2\sin(2c)\operatorname{Si}(2dx^2) + ab\cos(c)\operatorname{Si}(dx^2) + \frac{1}{2}(2a^2+b^2)\log(x)$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x,x, algorithm="fricas")
```

```
[Out] -1/4*b^2*cos(2*c)*cos_integral(2*d*x^2) + a*b*cos_integral(d*x^2)*sin(c) +
1/4*b^2*sin(2*c)*sin_integral(2*d*x^2) + a*b*cos(c)*sin_integral(d*x^2) + 1
/2*(2*a^2 + b^2)*log(x)
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx = \int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx$$

```
[In] integrate((a+b*sin(d*x**2+c))**2/x,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx = -\frac{1}{2}\left(\left(i\operatorname{Ei}(i\,dx^2) - i\operatorname{Ei}(-i\,dx^2)\right)\cos(c) - \left(\operatorname{Ei}(i\,dx^2) + \operatorname{Ei}(-i\,dx^2)\right)\sin(c)\right)ab - \frac{1}{8}\left(\left(\operatorname{Ei}(2i\,dx^2) + \operatorname{Ei}(-2i\,dx^2)\right)\cos(2c) - \left(-i\operatorname{Ei}(2i\,dx^2) + i\operatorname{Ei}(-2i\,dx^2)\right)\sin(2c) - 4\log(x)\right)b^2 + a^2\log(x)$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*((I*Ei(I*d*x^2) - I*Ei(-I*d*x^2))*cos(c) - (Ei(I*d*x^2) + Ei(-I*d*x^2)
)*sin(c))*a*b - 1/8*((Ei(2*I*d*x^2) + Ei(-2*I*d*x^2))*cos(2*c) - (-I*Ei(2*I
*d*x^2) + I*Ei(-2*I*d*x^2))*sin(2*c) - 4*log(x))*b^2 + a^2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx = -\frac{1}{4}\,b^2\cos(2\,c)\,\mathrm{Ci}\left(2\,dx^2\right)+ab\,\mathrm{Ci}\left(dx^2\right)\sin(c)+ab\cos(c)\,\mathrm{Si}\left(dx^2\right)-\frac{1}{4}\,b^2\sin(2\,c)\,\mathrm{Si}\left(-2\,dx^2\right)+\frac{1}{2}\,a^2\log\left(dx^2\right)+\frac{1}{4}\,b^2\log\left(dx^2\right)$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x,x, algorithm="giac")
```

```
[Out] -1/4*b^2*cos(2*c)*cos_integral(2*d*x^2) + a*b*cos_integral(d*x^2)*sin(c) +
a*b*cos(c)*sin_integral(d*x^2) - 1/4*b^2*sin(2*c)*sin_integral(-2*d*x^2) +
1/2*a^2*log(d*x^2) + 1/4*b^2*log(d*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx = \int \frac{\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}{x}\,dx$$

```
[In] int((a + b*sin(c + d*x^2))^2/x,x)
```

```
[Out] int((a + b*sin(c + d*x^2))^2/x, x)
```

3.16 $\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 115

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^3} \, dx = -\frac{2a^2+b^2}{4x^2} + \frac{b^2\cos(2(c+dx^2))}{4x^2} + abd\cos(c)\operatorname{CosIntegral}\left(dx^2\right) + \frac{1}{2}b^2d\operatorname{CosIntegral}\left(2dx^2\right)\sin(2c) - \frac{ab\sin(c+dx^2)}{x^2} - abd\sin(c)\operatorname{Si}\left(dx^2\right) + \frac{1}{2}b^2d\cos(2c)\operatorname{Si}\left(2dx^2\right)$$

```
[Out] 1/4*(-2*a^2-b^2)/x^2+a*b*d*Ci(d*x^2)*cos(c)+1/4*b^2*cos(2*d*x^2+2*c)/x^2+1/
2*b^2*d*cos(2*c)*Si(2*d*x^2)-a*b*d*Si(d*x^2)*sin(c)+1/2*b^2*d*Ci(2*d*x^2)*s
in(2*c)-a*b*sin(d*x^2+c)/x^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3484, 6, 3461, 3378, 3384, 3380, 3383, 3460}

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^3} \, dx = -\frac{2a^2+b^2}{4x^2} + abd\cos(c)\operatorname{CosIntegral}\left(dx^2\right) - abd\sin(c)\operatorname{Si}\left(dx^2\right) - \frac{ab\sin(c+dx^2)}{x^2} + \frac{1}{2}b^2d\sin(2c)\operatorname{CosIntegral}\left(2dx^2\right) + \frac{1}{2}b^2d\cos(2c)\operatorname{Si}\left(2dx^2\right) + \frac{b^2\cos(2(c+dx^2))}{4x^2}$$

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x^3,x]
```

```
[Out] -1/4*(2*a^2 + b^2)/x^2 + (b^2*Cos[2*(c + d*x^2)])/(4*x^2) + a*b*d*Cos[c]*Co
sIntegral[d*x^2] + (b^2*d*CosIntegral[2*d*x^2]*Sin[2*c])/2 - (a*b*Sin[c + d
*x^2])/x^2 - a*b*d*Sin[c]*SinIntegral[d*x^2] + (b^2*d*Cos[2*c]*SinIntegral[
2*d*x^2])/2
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{x^3}+\frac{b^2}{2x^3}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^3}+\frac{2ab\sin\left(c+dx^2\right)}{x^3}\right)\,dx\\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^3}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^3}+\frac{2ab\sin\left(c+dx^2\right)}{x^3}\right)\,dx\\
&= -\frac{2a^2+b^2}{4x^2}+(2ab)\int \frac{\sin\left(c+dx^2\right)}{x^3}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^2\right)}{x^3}\,dx\\
&= -\frac{2a^2+b^2}{4x^2}+(ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,x^2\right)-\frac{1}{4}b^2\text{Subst}\left(\int \frac{\cos(2c+2dx)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{4x^2}-\frac{ab\sin\left(c+dx^2\right)}{x^2}\\
&\quad+(abd)\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx,x,x^2\right)\\
&\quad+\frac{1}{2}\left(b^2d\right)\text{Subst}\left(\int \frac{\sin(2c+2dx)}{x}\,dx,x,x^2\right)\\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{4x^2}-\frac{ab\sin\left(c+dx^2\right)}{x^2}\\
&\quad+(abd\cos(c))\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx,x,x^2\right)\\
&\quad+\frac{1}{2}\left(b^2d\cos(2c)\right)\text{Subst}\left(\int \frac{\sin(2dx)}{x}\,dx,x,x^2\right)\\
&\quad-(abd\sin(c))\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx,x,x^2\right)\\
&\quad+\frac{1}{2}\left(b^2d\sin(2c)\right)\text{Subst}\left(\int \frac{\cos(2dx)}{x}\,dx,x,x^2\right)\\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{4x^2}+abd\cos(c)\,\text{CosIntegral}\left(dx^2\right)\\
&\quad+\frac{1}{2}b^2d\,\text{CosIntegral}\left(2dx^2\right)\sin(2c)-\frac{ab\sin\left(c+dx^2\right)}{x^2}\\
&\quad-abd\sin(c)\text{Si}\left(dx^2\right)+\frac{1}{2}b^2d\cos(2c)\text{Si}\left(2dx^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

$$\int \frac{(a + b\sin(c + dx^2))^2}{x^3} dx$$
$$= \frac{-2a^2 - b^2 + b^2\cos(2(c + dx^2)) + 4abdx^2\cos(c)\,\text{CosIntegral}(dx^2) + 2b^2dx^2\,\text{CosIntegral}(2dx^2)\sin(2c) - 4a}{4x^2}$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x^3,x]
```

```
[Out] (-2*a^2 - b^2 + b^2*Cos[2*(c + d*x^2)] + 4*a*b*d*x^2*Cos[c]*CosIntegral[d*x
^2] + 2*b^2*d*x^2*CosIntegral[2*d*x^2]*Sin[2*c] - 4*a*b*Sin[c + d*x^2] - 4*
a*b*d*x^2*Sin[c]*SinIntegral[d*x^2] + 2*b^2*d*x^2*Cos[2*c]*SinIntegral[2*d*
x^2])/(4*x^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.90

method	result
risch	$\frac{2i\pi\,\text{csgn}(d\,x^2)\text{e}^{-ic}abd\,x^2 - 4i\,\text{Si}(d\,x^2)\text{e}^{-ic}abd\,x^2 - \pi\,\text{csgn}(d\,x^2)\text{e}^{-2ic}b^2d\,x^2 - i\,\text{Ei}_1(-2id\,x^2)\text{e}^{-2ic}b^2d\,x^2 + ib^2d\,\text{Ei}_1(-2id\,x^2)\text{e}^{2ic}x^2 + 2}{4x^2}$

```
[In] int((a+b*sin(d*x^2+c))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*I*Pi*csgn(d*x^2)*exp(-I*c)*a*b*d*x^2-4*I*Si(d*x^2)*exp(-I*c)*a*b*d*x
^2-Pi*csgn(d*x^2)*exp(-2*I*c)*b^2*d*x^2-I*Ei(1,-2*I*d*x^2)*exp(-2*I*c)*b^2*
d*x^2+I*b^2*d*Ei(1,-2*I*d*x^2)*exp(2*I*c)*x^2+2*Si(2*d*x^2)*exp(-2*I*c)*b^2
*d*x^2-2*a*b*d*Ei(1,-I*d*x^2)*exp(I*c)*x^2-2*Ei(1,-I*d*x^2)*exp(-I*c)*a*b*d
*x^2-4*sin(d*x^2+c)*a*b+b^2*cos(2*d*x^2+2*c)-2*a^2-b^2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{(a + b\sin(c + dx^2))^2}{x^3} dx$$
$$= \frac{2\,abdx^2\cos(c)\,\text{Ci}(dx^2) + b^2dx^2\,\text{Ci}(2\,dx^2)\sin(2\,c) + b^2dx^2\cos(2\,c)\,\text{Si}(2\,dx^2) - 2\,abdx^2\sin(c)\,\text{Si}(dx^2) + b^2}{2\,x^2}$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*d*x^2*cos(c)*cos_integral(d*x^2) + b^2*d*x^2*cos_integral(2*d*x^
2)*sin(2*c) + b^2*d*x^2*cos(2*c)*sin_integral(2*d*x^2) - 2*a*b*d*x^2*sin(c)
*sin_integral(d*x^2) + b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 -
b^2)/x^2
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx = \int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx$$

```
[In] integrate((a+b*sin(d*x**2+c))**2/x**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.08

$$\begin{aligned}
&\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx \\
&= \frac{1}{2}\left(\left(\Gamma\left(-1, i\,dx^2\right) + \Gamma\left(-1, -i\,dx^2\right)\right)\cos(c) - \left(i\,\Gamma\left(-1, i\,dx^2\right) - i\,\Gamma\left(-1, -i\,dx^2\right)\right)\sin(c)\right)abd \\
&\quad + \frac{(((i\,\Gamma(-1, 2i\,dx^2) - i\,\Gamma(-1, -2i\,dx^2))\cos(2\,c) + (\Gamma(-1, 2i\,dx^2) + \Gamma(-1, -2i\,dx^2))\sin(2\,c))dx^2 - 1)b^2}{4\,x^2} \\
&\quad - \frac{a^2}{2\,x^2}
\end{aligned}$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((gamma(-1, I*d*x^2) + gamma(-1, -I*d*x^2))*cos(c) - (I*gamma(-1, I*d*x
^2) - I*gamma(-1, -I*d*x^2))*sin(c))*a*b*d + 1/4*(((I*gamma(-1, 2*I*d*x^2)
- I*gamma(-1, -2*I*d*x^2))*cos(2*c) + (gamma(-1, 2*I*d*x^2) + gamma(-1, -2*
I*d*x^2))*sin(2*c))*d*x^2 - 1)*b^2/x^2 - 1/2*a^2/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(108) = 216.

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.97

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx$$
$$= \frac{4(dx^2+c)abd^2\cos(c)\operatorname{Ci}(dx^2) - 4abcd^2\cos(c)\operatorname{Ci}(dx^2) + 2(dx^2+c)b^2d^2\operatorname{Ci}(2dx^2)\sin(2c) - 2b^2cd^2\operatorname{Ci}(2}{}$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*(d*x^2 + c)*a*b*d^2*cos(c)*cos_integral(d*x^2) - 4*a*b*c*d^2*cos(c)*
cos_integral(d*x^2) + 2*(d*x^2 + c)*b^2*d^2*cos_integral(2*d*x^2)*sin(2*c)
- 2*b^2*c*d^2*cos_integral(2*d*x^2)*sin(2*c) - 4*(d*x^2 + c)*a*b*d^2*sin(c)
*sin_integral(d*x^2) + 4*a*b*c*d^2*sin(c)*sin_integral(d*x^2) - 2*(d*x^2 +
c)*b^2*d^2*cos(2*c)*sin_integral(-2*d*x^2) + 2*b^2*c*d^2*cos(2*c)*sin_integ
ral(-2*d*x^2) + b^2*d^2*cos(2*d*x^2 + 2*c) - 4*a*b*d^2*sin(d*x^2 + c) - 2*a
^2*d^2 - b^2*d^2)/(d^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx = \int \frac{(a+b\,\sin(d\,x^2+c))^2}{x^3}\,dx$$

```
[In] int((a + b*sin(c + d*x^2))^2/x^3,x)
```

```
[Out] int((a + b*sin(c + d*x^2))^2/x^3, x)
```

3.17 $\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 169

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx = &-\frac{2a^2+b^2}{8x^4}-\frac{abd\cos(c+dx^2)}{2x^2}+\frac{b^2\cos(2(c+dx^2))}{8x^4}\\&+\frac{1}{2}b^2d^2\cos(2c)\operatorname{CosIntegral}(2dx^2)\\&-\frac{1}{2}abd^2\operatorname{CosIntegral}(dx^2)\sin(c)\\&-\frac{ab\sin(c+dx^2)}{2x^4}-\frac{b^2d\sin(2(c+dx^2))}{4x^2}\\&-\frac{1}{2}abd^2\cos(c)\operatorname{Si}(dx^2)-\frac{1}{2}b^2d^2\sin(2c)\operatorname{Si}(2dx^2)\end{aligned}$$

```
[Out] 1/8*(-2*a^2-b^2)/x^4+1/2*b^2*d^2*Ci(2*d*x^2)*cos(2*c)-1/2*a*b*d*cos(d*x^2+c
)/x^2+1/8*b^2*cos(2*d*x^2+2*c)/x^4-1/2*a*b*d^2*cos(c)*Si(d*x^2)-1/2*a*b*d^2
*Ci(d*x^2)*sin(c)-1/2*b^2*d^2*Si(2*d*x^2)*sin(2*c)-1/2*a*b*sin(d*x^2+c)/x^4
-1/4*b^2*d*sin(2*d*x^2+2*c)/x^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used

$= \{3484, 6, 3461, 3378, 3384, 3380, 3383, 3460\}$

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx = -\frac{2a^2+b^2}{8x^4} - \frac{1}{2}abd^2\sin(c)\,\mathrm{CosIntegral}\left(dx^2\right) - \frac{1}{2}abd^2\cos(c)\mathrm{Si}\left(dx^2\right) - \frac{abd\cos(c+dx^2)}{2x^2} - \frac{ab\sin(c+dx^2)}{2x^4} + \frac{1}{2}b^2d^2\cos(2c)\,\mathrm{CosIntegral}\left(2dx^2\right) - \frac{1}{2}b^2d^2\sin(2c)\mathrm{Si}\left(2dx^2\right) - \frac{b^2d\sin(2(c+dx^2))}{4x^2} + \frac{b^2\cos(2(c+dx^2))}{8x^4}$$

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x^5,x]
```

```
[Out] -1/8*(2*a^2 + b^2)/x^4 - (a*b*d*Cos[c + d*x^2])/(2*x^2) + (b^2*Cos[2*(c + d
*x^2)])/(8*x^4) + (b^2*d^2*Cos[2*c]*CosIntegral[2*d*x^2])/2 - (a*b*d^2*CosI
ntegral[d*x^2]*Sin[c])/2 - (a*b*Sin[c + d*x^2])/(2*x^4) - (b^2*d*Sin[2*(c +
 d*x^2)])/(4*x^2) - (a*b*d^2*Cos[c]*SinIntegral[d*x^2])/2 - (b^2*d^2*Sin[2*
c]*SinIntegral[2*d*x^2])/2
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{x^5}+\frac{b^2}{2x^5}-\frac{b^2\cos(2c+2dx^2)}{2x^5}+\frac{2ab\sin(c+dx^2)}{x^5}\right)dx\\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^5}-\frac{b^2\cos(2c+2dx^2)}{2x^5}+\frac{2ab\sin(c+dx^2)}{x^5}\right)dx\\
&= -\frac{2a^2+b^2}{8x^4}+(2ab)\int \frac{\sin(c+dx^2)}{x^5}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^2)}{x^5}\,dx\\
&= -\frac{2a^2+b^2}{8x^4}+(ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^3}\,dx,x,x^2\right)-\frac{1}{4}b^2\text{Subst}\left(\int \frac{\cos(2c+2dx)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{2a^2+b^2}{8x^4}+\frac{b^2\cos(2(c+dx^2))}{8x^4}-\frac{ab\sin(c+dx^2)}{2x^4}\\
&\quad+\frac{1}{2}(abd)\text{Subst}\left(\int \frac{\cos(c+dx)}{x^2}\,dx,x,x^2\right)\\
&\quad+\frac{1}{4}(b^2d)\,\text{Subst}\left(\int \frac{\sin(2c+2dx)}{x^2}\,dx,x,x^2\right)
\end{aligned}
$$

$$= -\frac{2a^2+b^2}{8x^4} - \frac{abd\cos(c+dx^2)}{2x^2} + \frac{b^2\cos(2(c+dx^2))}{8x^4} - \frac{ab\sin(c+dx^2)}{2x^4} - \frac{b^2d\sin(2(c+dx^2))}{4x^2} - \frac{1}{2}(abd^2)\,\text{Subst}\left(\int \frac{\sin(c+dx)}{x}\,dx, x, x^2\right) + \frac{1}{2}(b^2d^2)\,\text{Subst}\left(\int \frac{\cos(2c+2dx)}{x}\,dx, x, x^2\right)$$

$$= -\frac{2a^2+b^2}{8x^4} - \frac{abd\cos(c+dx^2)}{2x^2} + \frac{b^2\cos(2(c+dx^2))}{8x^4} - \frac{ab\sin(c+dx^2)}{2x^4} - \frac{b^2d\sin(2(c+dx^2))}{4x^2} - \frac{1}{2}(abd^2\cos(c))\,\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx, x, x^2\right) + \frac{1}{2}(b^2d^2\cos(2c))\,\text{Subst}\left(\int \frac{\cos(2dx)}{x}\,dx, x, x^2\right) - \frac{1}{2}(abd^2\sin(c))\,\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx, x, x^2\right) - \frac{1}{2}(b^2d^2\sin(2c))\,\text{Subst}\left(\int \frac{\sin(2dx)}{x}\,dx, x, x^2\right)$$

$$= -\frac{2a^2+b^2}{8x^4} - \frac{abd\cos(c+dx^2)}{2x^2} + \frac{b^2\cos(2(c+dx^2))}{8x^4} + \frac{1}{2}b^2d^2\cos(2c)\,\text{CosIntegral}(2dx^2) - \frac{1}{2}abd^2\,\text{CosIntegral}(dx^2)\sin(c) - \frac{ab\sin(c+dx^2)}{2x^4} - \frac{b^2d\sin(2(c+dx^2))}{4x^2} - \frac{1}{2}abd^2\cos(c)\text{Si}(dx^2) - \frac{1}{2}b^2d^2\sin(2c)\text{Si}(2dx^2)$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx = -\frac{2a^2+b^2+4abdx^2\cos(c+dx^2)-b^2\cos(2(c+dx^2))-4b^2d^2x^4\cos(2c)\,\text{CosIntegral}(2dx^2)+4abd^2x^4\,\text{Co}}{}$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x^5,x]
```

```
[Out] -1/8*(2*a^2 + b^2 + 4*a*b*d*x^2*Cos[c + d*x^2] - b^2*Cos[2*(c + d*x^2)] - 4
*b^2*d^2*x^4*Cos[2*c]*CosIntegral[2*d*x^2] + 4*a*b*d^2*x^4*CosIntegral[d*x^
2]*Sin[c] + 4*a*b*Sin[c + d*x^2] + 2*b^2*d*x^2*Sin[2*(c + d*x^2)] + 4*a*b*d
^2*x^4*Cos[c]*SinIntegral[d*x^2] + 4*b^2*d^2*x^4*Sin[2*c]*SinIntegral[2*d*x
^2])/x^4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.60

method	result
risch	$\frac{2i e^{-2ic}\pi\ \mathrm{csgn}(d x^2) b^2 d^2 x^4+2\pi\ \mathrm{csgn}(d x^2) e^{-ic} ab\, d^2 x^4-4i e^{-2ic}\ \mathrm{Si}(2d x^2) b^2 d^2 x^4-2iab\, d^2\ \mathrm{Ei}_1(-id x^2) e^{ic} x^4+2i\ \mathrm{Ei}_1(-id x^2) e^{-ic} ab}{}$

```
[In] int((a+b*sin(d*x^2+c))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*I*exp(-2*I*c)*Pi*csgn(d*x^2)*b^2*d^2*x^4+2*Pi*csgn(d*x^2)*exp(-I*c)*
a*b*d^2*x^4-4*I*exp(-2*I*c)*Si(2*d*x^2)*b^2*d^2*x^4-2*I*a*b*d^2*Ei(1,-I*d*x
^2)*exp(I*c)*x^4+2*I*Ei(1,-I*d*x^2)*exp(-I*c)*a*b*d^2*x^4-4*Si(d*x^2)*exp(-
I*c)*a*b*d^2*x^4-2*exp(-2*I*c)*Ei(1,-2*I*d*x^2)*b^2*d^2*x^4-2*b^2*d^2*Ei(1,
-2*I*d*x^2)*exp(2*I*c)*x^4-4*a*b*x^2*cos(d*x^2+c)*d-2*b^2*x^2*sin(2*d*x^2+2
*c)*d-4*sin(d*x^2+c)*a*b+b^2*cos(2*d*x^2+2*c)-2*a^2-b^2)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx = \frac{2\,b^2d^2x^4\cos(2\,c)\,\mathrm{Ci}(2\,dx^2)-2\,abd^2x^4\,\mathrm{Ci}(dx^2)\sin(c)-2\,b^2d^2x^4\sin(2\,c)\,\mathrm{Si}(2\,dx^2)-2\,abd^2x^4\cos(c)\,\mathrm{Si}(d}{4\,x^4}$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^2*x^4*cos(2*c)*cos_integral(2*d*x^2) - 2*a*b*d^2*x^4*cos_integ
ral(d*x^2)*sin(c) - 2*b^2*d^2*x^4*sin(2*c)*sin_integral(2*d*x^2) - 2*a*b*d^
2*x^4*cos(c)*sin_integral(d*x^2) - 2*a*b*d*x^2*cos(d*x^2 + c) + b^2*cos(d*x
^2 + c)^2 - a^2 - b^2 - 2*(b^2*d*x^2*cos(d*x^2 + c) + a*b)*sin(d*x^2 + c))/
x^4
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx = \int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx$$

```
[In] integrate((a+b*sin(d*x**2+c))**2/x**5,x)
[Out] Integral((a + b*sin(c + d*x**2))**2/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx$$
$$= \frac{1}{2}\left(\left(i\,\Gamma\left(-2,i\,dx^2\right)-i\,\Gamma\left(-2,-i\,dx^2\right)\right)\cos(c)+\left(\Gamma\left(-2,i\,dx^2\right)+\Gamma\left(-2,-i\,dx^2\right)\right)\sin(c)\right)abd^2$$
$$-\frac{(4\left((\Gamma(-2,2i\,dx^2)+\Gamma(-2,-2i\,dx^2))\cos(2\,c)+(-i\,\Gamma(-2,2i\,dx^2)+i\,\Gamma(-2,-2i\,dx^2))\sin(2\,c)\right)d^2x^4+1)}{8\,x^4}$$
$$-\frac{a^2}{4\,x^4}$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^5,x, algorithm="maxima")
[Out] 1/2*((I*gamma(-2, I*d*x^2) - I*gamma(-2, -I*d*x^2))*cos(c) + (gamma(-2, I*d
*x^2) + gamma(-2, -I*d*x^2))*sin(c))*a*b*d^2 - 1/8*(4*((gamma(-2, 2*I*d*x^2
) + gamma(-2, -2*I*d*x^2))*cos(2*c) + (-I*gamma(-2, 2*I*d*x^2) + I*gamma(-2
, -2*I*d*x^2))*sin(2*c))*d^2*x^4 + 1)*b^2/x^4 - 1/4*a^2/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(153) = 306.

Time = 0.30 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.65

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx$$
$$= \frac{4\,(dx^2+c)^2b^2d^3\cos(2\,c)\,\mathrm{Ci}\,(2\,dx^2)-8\,(dx^2+c)b^2cd^3\cos(2\,c)\,\mathrm{Ci}\,(2\,dx^2)+4\,b^2c^2d^3\cos(2\,c)\,\mathrm{Ci}\,(2\,dx^2)-4\,(}{}$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(4*(d*x^2 + c)^2*b^2*d^3*cos(2*c)*cos_integral(2*d*x^2) - 8*(d*x^2 + c)
*b^2*c*d^3*cos(2*c)*cos_integral(2*d*x^2) + 4*b^2*c^2*d^3*cos(2*c)*cos_inte
gral(2*d*x^2) - 4*(d*x^2 + c)^2*a*b*d^3*cos_integral(d*x^2)*sin(c) + 8*(d*x
^2 + c)*a*b*c*d^3*cos_integral(d*x^2)*sin(c) - 4*a*b*c^2*d^3*cos_integral(d
*x^2)*sin(c) - 4*(d*x^2 + c)^2*a*b*d^3*cos(c)*sin_integral(d*x^2) + 8*(d*x^
2 + c)*a*b*c*d^3*cos(c)*sin_integral(d*x^2) - 4*a*b*c^2*d^3*cos(c)*sin_inte
gral(d*x^2) + 4*(d*x^2 + c)^2*b^2*d^3*sin(2*c)*sin_integral(-2*d*x^2) - 8*(
d*x^2 + c)*b^2*c*d^3*sin(2*c)*sin_integral(-2*d*x^2) + 4*b^2*c^2*d^3*sin(2*
c)*sin_integral(-2*d*x^2) - 4*(d*x^2 + c)*a*b*d^3*cos(d*x^2 + c) + 4*a*b*c*
d^3*cos(d*x^2 + c) - 2*(d*x^2 + c)*b^2*d^3*sin(2*d*x^2 + 2*c) + 2*b^2*c*d^3
*sin(2*d*x^2 + 2*c) + b^2*d^3*cos(2*d*x^2 + 2*c) - 4*a*b*d^3*sin(d*x^2 + c)
 - 2*a^2*d^3 - b^2*d^3)/(((d*x^2 + c)^2 - 2*(d*x^2 + c)*c + c^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^5}\,dx=\int \frac{{\left(a+b\,\sin\left(d\,x^2+c\right)\right)}^2}{x^5}\,dx$$

```
[In] int((a + b*sin(c + d*x^2))^2/x^5,x)
```

```
[Out] int((a + b*sin(c + d*x^2))^2/x^5, x)
```

3.18 $\int x^4(a+b\sin(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 247

$$\int x^4(a+b\sin(c+dx^2))^2\,dx = \frac{1}{10}(2a^2+b^2)x^5 - \frac{abx^3\cos(c+dx^2)}{d} - \frac{3b^2x\cos(2c+2dx^2)}{32d^2} + \frac{3b^2\sqrt{\pi}\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{64d^{5/2}} - \frac{3ab\sqrt{\frac{\pi}{2}}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{5/2}} - \frac{3ab\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{2d^{5/2}} - \frac{3b^2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)}{64d^{5/2}} + \frac{3abx\sin(c+dx^2)}{2d^2} - \frac{b^2x^3\sin(2c+2dx^2)}{8d}$$

```
[Out] 1/10*(2*a^2+b^2)*x^5-a*b*x^3*cos(d*x^2+c)/d-3/32*b^2*x*cos(2*d*x^2+2*c)/d^2
+3/2*a*b*x*sin(d*x^2+c)/d^2-1/8*b^2*x^3*sin(2*d*x^2+2*c)/d-3/4*a*b*cos(c)*F
resnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)-3/4*a*b*Fresne
lC(x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)/d^(5/2)+3/64*b^2*cos
(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))*Pi^(1/2)/d^(5/2)-3/64*b^2*FresnelS(2*x
*d^(1/2)/Pi^(1/2))*sin(2*c)*Pi^(1/2)/d^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3484, 6, 3467, 3466, 3435, 3433, 3432, 3434}

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{1}{10}x^5\left(2a^2+b^2\right)-\frac{3\sqrt{\frac{\pi}{2}}ab\sin(c)\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}ab\cos(c)\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{5/2}}+\frac{3abx\sin\left(c+dx^2\right)}{2d^2}-\frac{abx^3\cos\left(c+dx^2\right)}{d}+\frac{3\sqrt{\pi}b^2\cos(2c)\operatorname{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{64d^{5/2}}-\frac{3\sqrt{\pi}b^2\sin(2c)\operatorname{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{64d^{5/2}}-\frac{3b^2x\cos\left(2c+2dx^2\right)}{32d^2}-\frac{b^2x^3\sin\left(2c+2dx^2\right)}{8d}$$

```
[In] Int[x^4*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^5)/10 - (a*b*x^3*Cos[c + d*x^2])/d - (3*b^2*x*Cos[2*c + 2*
d*x^2])/(32*d^2) + (3*b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]
])/(64*d^(5/2)) - (3*a*b*Sqrt[Pi/2]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/
(2*d^(5/2)) - (3*a*b*Sqrt[Pi/2]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/(2*d
^(5/2)) - (3*b^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(64*d^
(5/2)) + (3*a*b*x*Sin[c + d*x^2])/(2*d^2) - (b^2*x^3*Sin[2*c + 2*d*x^2])/(8
*d)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2x^4+\frac{b^2x^4}{2}-\frac{1}{2}b^2x^4\cos\left(2c+2dx^2\right)+2abx^4\sin\left(c+dx^2\right)\right)\,dx \\
&= \int \left(\left(a^2+\frac{b^2}{2}\right)x^4-\frac{1}{2}b^2x^4\cos\left(2c+2dx^2\right)+2abx^4\sin\left(c+dx^2\right)\right)\,dx \\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5+(2ab)\int x^4\sin\left(c+dx^2\right)\,dx-\frac{1}{2}b^2\int x^4\cos\left(2c+2dx^2\right)\,dx \\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{abx^3\cos\left(c+dx^2\right)}{d}-\frac{b^2x^3\sin\left(2c+2dx^2\right)}{8d} \\
&\quad+\frac{(3ab)\int x^2\cos\left(c+dx^2\right)\,dx}{d}+\frac{\left(3b^2\right)\int x^2\sin\left(2c+2dx^2\right)\,dx}{8d}
\end{aligned}$$

$$= \frac{1}{10}(2a^2+b^2)x^5 - \frac{abx^3\cos(c+dx^2)}{d} - \frac{3b^2x\cos(2c+2dx^2)}{32d^2} + \frac{3abx\sin(c+dx^2)}{2d^2} - \frac{b^2x^3\sin(2c+2dx^2)}{8d} - \frac{(3ab)\int\sin(c+dx^2)\,dx}{2d^2} + \frac{(3b^2)\int\cos(2c+2dx^2)\,dx}{32d^2}$$

$$= \frac{1}{10}(2a^2+b^2)x^5 - \frac{abx^3\cos(c+dx^2)}{d} - \frac{3b^2x\cos(2c+2dx^2)}{32d^2} + \frac{3abx\sin(c+dx^2)}{2d^2} - \frac{b^2x^3\sin(2c+2dx^2)}{8d} - \frac{(3ab\cos(c))\int\sin(dx^2)\,dx}{2d^2} + \frac{(3b^2\cos(2c))\int\cos(2dx^2)\,dx}{32d^2} - \frac{(3ab\sin(c))\int\cos(dx^2)\,dx}{2d^2} - \frac{(3b^2\sin(2c))\int\sin(2dx^2)\,dx}{32d^2}$$

$$= \frac{1}{10}(2a^2+b^2)x^5 - \frac{abx^3\cos(c+dx^2)}{d} - \frac{3b^2x\cos(2c+2dx^2)}{32d^2} + \frac{3b^2\sqrt{\pi}\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{64d^{5/2}} - \frac{3ab\sqrt{\frac{\pi}{2}}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{5/2}} - \frac{3ab\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{2d^{5/2}} - \frac{3b^2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)}{64d^{5/2}} + \frac{3abx\sin(c+dx^2)}{2d^2} - \frac{b^2x^3\sin(2c+2dx^2)}{8d}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.95

$$\int x^4(a+b\sin(c+dx^2))^2\,dx = \frac{64a^2d^{5/2}x^5 + 32b^2d^{5/2}x^5 - 320abd^{3/2}x^3\cos(c+dx^2) - 30b^2\sqrt{d}x\cos(2(c+dx^2)) + 15b^2\sqrt{\pi}\cos(2c)\,\text{Fresne}}{}$$

```
[In] Integrate[x^4*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (64*a^2*d^(5/2)*x^5 + 32*b^2*d^(5/2)*x^5 - 320*a*b*d^(3/2)*x^3*Cos[c + d*x^
2] - 30*b^2*Sqrt[d]*x*Cos[2*(c + d*x^2)] + 15*b^2*Sqrt[Pi]*Cos[2*c]*Fresnel
C[(2*Sqrt[d]*x)/Sqrt[Pi]] - 240*a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt
[2/Pi]*x] - 240*a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] - 15*b
^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c] + 480*a*b*Sqrt[d]*x*S
in[c + d*x^2] - 40*b^2*d^(3/2)*x^3*Sin[2*(c + d*x^2)])/(320*d^(5/2))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.75

method	result
parts	$\frac{x^5a^2}{5}+b^2\left(\frac{x^5}{10}-\frac{x^3\sin\left(2d\,x^2+2c\right)}{8d}+\frac{-\frac{3x\cos\left(2d\,x^2+2c\right)}{32d}+\frac{3\sqrt{\pi}\left(\cos(2c)\,\mathrm{C}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)-\sin(2c)\,\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)}{64d^{\frac{3}{2}}}}{d}\right)+2ab\left(-\frac{x^3\cos(d\,x\right.$
default	$\frac{\left(a^2+\frac{b^2}{2}\right)x^5}{5}-\frac{b^2\left(\frac{x^3\sin\left(2d\,x^2+2c\right)}{4d}-\frac{3\left(-\frac{x\cos\left(2d\,x^2+2c\right)}{4d}+\frac{\sqrt{\pi}\left(\cos(2c)\,\mathrm{C}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)-\sin(2c)\,\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)}{8d^{\frac{3}{2}}}\right)}{4d}\right)}{2}+2ab\left(-\frac{x^3\cos(d\,x^2+\right.$
risch	$\frac{x^5a^2}{5}-\frac{3iab\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{8d^2\sqrt{id}}+\frac{x^5b^2}{10}+\frac{3b^2\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{id}\,x\right)\mathrm{e}^{-2ic}}{256d^2\sqrt{id}}+\frac{3b^2\sqrt{\pi}\,\mathrm{erf}(\sqrt{-2id}\,x)\mathrm{e}^{2ic}}{128d^2\sqrt{-2id}}+\frac{3iab\sqrt{\pi}\,\mathrm{erf}(\sqrt{-id}\,x}{8d^2\sqrt{-id}}$

```
[In] int(x^4*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*a^2+b^2*(1/10*x^5-1/8/d*x^3*sin(2*d*x^2+2*c)+3/8/d*(-1/4/d*x*cos(2*
d*x^2+2*c)+1/8/d^(3/2)*Pi^(1/2)*(cos(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))-si
n(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2)))))+2*a*b*(-1/2/d*x^3*cos(d*x^2+c)+3/2
/d*(1/2/d*x*sin(d*x^2+c)-1/4/d^(3/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelS(x*d^
(1/2)*2^(1/2)/Pi^(1/2))+sin(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.87

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{32\left(2a^2+b^2\right)d^3x^5-320\,abd^2x^3\cos\left(dx^2+c\right)-60\,b^2dx\cos\left(dx^2+c\right)^2-240\,\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)}{}$$

```
[In] integrate(x^4*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/320*(32*(2*a^2 + b^2)*d^3*x^5 - 320*a*b*d^2*x^3*cos(d*x^2 + c) - 60*b^2*d
*x*cos(d*x^2 + c)^2 - 240*sqrt(2)*pi*a*b*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt
(2)*x*sqrt(d/pi)) - 240*sqrt(2)*pi*a*b*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqr
t(d/pi))*sin(c) + 15*pi*b^2*sqrt(d/pi)*cos(2*c)*fresnel_cos(2*x*sqrt(d/pi))
 - 15*pi*b^2*sqrt(d/pi)*fresnel_sin(2*x*sqrt(d/pi))*sin(2*c) + 30*b^2*d*x -
 80*(b^2*d^2*x^3*cos(d*x^2 + c) - 6*a*b*d*x)*sin(d*x^2 + c))/d^3
```

Sympy [F]

$$\int x^4 \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\int x^4 \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x**4*(a+b*sin(d*x**2+c))**2,x)
[Out] Integral(x**4*(a + b*sin(c + d*x**2))**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.84

$$\int x^4 \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{1}{5}\,a^2x^5 - \frac{\left(16\,d^3x^3\cos\left(dx^2+c\right)-24\,d^2x\sin\left(dx^2+c\right)+3\,\sqrt{2}\sqrt{\pi}\left(\left(\left(i+1\right)\cos\left(c\right)-\left(i-1\right)\sin\left(c\right)\right)\operatorname{erf}\left(\sqrt{i\,d}x\right)\right.\right.}{16\,d^4} + \frac{\left(256\,d^4x^5-320\,d^3x^3\sin\left(2\,dx^2+2\,c\right)-240\,d^2x\cos\left(2\,dx^2+2\,c\right)+15\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-\left(i-1\right)\cos\left(2\,c\right)-\right.\right.\right.}{2560\,d^4}$$

```
[In] integrate(x^4*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
[Out] 1/5*a^2*x^5 - 1/16*(16*d^3*x^3*cos(d*x^2 + c) - 24*d^2*x*sin(d*x^2 + c) + 3
*sqrt(2)*sqrt(pi)*(((I + 1)*cos(c) - (I - 1)*sin(c))*erf(sqrt(I*d)*x) + (-(
I - 1)*cos(c) + (I + 1)*sin(c))*erf(sqrt(-I*d)*x))*d^(3/2))*a*b/d^4 + 1/256
0*(256*d^4*x^5 - 320*d^3*x^3*sin(2*d*x^2 + 2*c) - 240*d^2*x*cos(2*d*x^2 + 2
*c) + 15*4^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*cos(2*c) - (I + 1)*sin(2*c))*e
rf(sqrt(2*I*d)*x) + ((I + 1)*cos(2*c) + (I - 1)*sin(2*c))*erf(sqrt(-2*I*d)*
x))*d^(3/2))*b^2/d^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.33

$$\int x^4 \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{1}{5}\,a^2x^5+\frac{1}{10}\,b^2x^5 - \frac{3\,\sqrt{2}\sqrt{\pi}ab\operatorname{erf}\left(-\frac{1}{2}i\,\sqrt{2}x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(i\,c)}}{8\,d^2\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} - \frac{3\,\sqrt{2}\sqrt{\pi}ab\operatorname{erf}\left(\frac{1}{2}i\,\sqrt{2}x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-i\,c)}}{8\,d^2\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} + \frac{3i\,\sqrt{\pi}b^2\operatorname{erf}\left(-i\,\sqrt{d}x\left(\frac{i\,d}{|d|}+1\right)\right)e^{(2i\,c)}}{128\,d^{\frac{5}{2}}\left(\frac{i\,d}{|d|}+1\right)} - \frac{3i\,\sqrt{\pi}b^2\operatorname{erf}\left(i\,\sqrt{d}x\left(-\frac{i\,d}{|d|}+1\right)\right)e^{(-2i\,c)}}{128\,d^{\frac{5}{2}}\left(-\frac{i\,d}{|d|}+1\right)} - \frac{(-4i\,b^2dx^3+3\,b^2x)e^{(2i\,dx^2+2i\,c)}}{64\,d^2} + \frac{i\,(2i\,abdx^3-3\,abx)e^{(i\,dx^2+i\,c)}}{4\,d^2} + \frac{i\,(2i\,abdx^3+3\,abx)e^{(-i\,dx^2-i\,c)}}{4\,d^2} - \frac{(4i\,b^2dx^3+3\,b^2x)e^{(-2i\,dx^2-2i\,c)}}{64\,d^2}$$

```
[In] integrate(x^4*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/5*a^2*x^5 + 1/10*b^2*x^5 - 3/8*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*I*sqrt(2)*x*
(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(I*c)/(d^2*(I*d/abs(d) + 1)*sqrt(abs(d)))
- 3/8*sqrt(2)*sqrt(pi)*a*b*erf(1/2*I*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d
)))*e^(-I*c)/(d^2*(-I*d/abs(d) + 1)*sqrt(abs(d))) + 3/128*I*sqrt(pi)*b^2*er
f(-I*sqrt(d)*x*(I*d/abs(d) + 1))*e^(2*I*c)/(d^(5/2)*(I*d/abs(d) + 1)) - 3/1
28*I*sqrt(pi)*b^2*erf(I*sqrt(d)*x*(-I*d/abs(d) + 1))*e^(-2*I*c)/(d^(5/2)*(-
I*d/abs(d) + 1)) - 1/64*(-4*I*b^2*d*x^3 + 3*b^2*x)*e^(2*I*d*x^2 + 2*I*c)/d^
2 + 1/4*I*(2*I*a*b*d*x^3 - 3*a*b*x)*e^(I*d*x^2 + I*c)/d^2 + 1/4*I*(2*I*a*b*
d*x^3 + 3*a*b*x)*e^(-I*d*x^2 - I*c)/d^2 - 1/64*(4*I*b^2*d*x^3 + 3*b^2*x)*e^
(-2*I*d*x^2 - 2*I*c)/d^2
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\int x^4\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2\,dx$$

```
[In] int(x^4*(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] int(x^4*(a + b*sin(c + d*x^2))^2, x)
```

3.19 $\int x^2(a+b\sin(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 198

$$\begin{aligned}\int x^2\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=&\ \frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{abx\cos\left(c+dx^2\right)}{d}\\&+\frac{ab\sqrt{\frac{\pi}{2}}\cos(c)\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}}\\&+\frac{b^2\sqrt{\pi}\cos(2c)\operatorname{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{16d^{3/2}}\\&-\frac{ab\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{d^{3/2}}\\&+\frac{b^2\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)}{16d^{3/2}}-\frac{b^2x\sin\left(2c+2dx^2\right)}{8d}\end{aligned}$$

```
[Out] 1/6*(2*a^2+b^2)*x^3-a*b*x*cos(d*x^2+c)/d-1/8*b^2*x*sin(2*d*x^2+2*c)/d+1/2*a
*b*cos(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^(3/2)-1/2
*a*b*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)/d^(3/2)+1
/16*b^2*cos(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2))*Pi^(1/2)/d^(3/2)+1/16*b^2*F
resnelC(2*x*d^(1/2)/Pi^(1/2))*sin(2*c)*Pi^(1/2)/d^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3484, 6, 3467, 3434, 3433, 3432, 3466, 3435}

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{1}{6}x^3\left(2a^2+b^2\right)+\frac{\sqrt{\frac{\pi}{2}}ab\cos(c)\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}ab\sin(c)\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}}-\frac{abx\cos\left(c+dx^2\right)}{d}+\frac{\sqrt{\pi}b^2\sin(2c)\operatorname{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{16d^{3/2}}+\frac{\sqrt{\pi}b^2\cos(2c)\operatorname{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{16d^{3/2}}-\frac{b^2x\sin\left(2c+2dx^2\right)}{8d}$$

```
[In] Int[x^2*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^3)/6 - (a*b*x*Cos[c + d*x^2])/d + (a*b*Sqrt[Pi/2]*Cos[c]*F
resnelC[Sqrt[d]*Sqrt[2/Pi]*x])/d^(3/2) + (b^2*Sqrt[Pi]*Cos[2*c]*FresnelS[(2
*Sqrt[d]*x)/Sqrt[Pi]])/(16*d^(3/2)) - (a*b*Sqrt[Pi/2]*FresnelS[Sqrt[d]*Sqrt
[2/Pi]*x]*Sin[c])/d^(3/2) + (b^2*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]]*
Sin[2*c])/(16*d^(3/2)) - (b^2*x*Sin[2*c + 2*d*x^2])/(8*d)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2x^2+\frac{b^2x^2}{2}-\frac{1}{2}b^2x^2\cos\left(2c+2dx^2\right)+2abx^2\sin\left(c+dx^2\right)\right)\,dx\\
&= \int \left(\left(a^2+\frac{b^2}{2}\right)x^2-\frac{1}{2}b^2x^2\cos\left(2c+2dx^2\right)+2abx^2\sin\left(c+dx^2\right)\right)\,dx\\
&= \frac{1}{6}\left(2a^2+b^2\right)x^3+(2ab)\int x^2\sin\left(c+dx^2\right)\,dx-\frac{1}{2}b^2\int x^2\cos\left(2c+2dx^2\right)\,dx\\
&= \frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{abx\cos\left(c+dx^2\right)}{d}-\frac{b^2x\sin\left(2c+2dx^2\right)}{8d}\\
&\quad+\frac{(ab)\int\cos\left(c+dx^2\right)\,dx}{d}+\frac{b^2\int\sin\left(2c+2dx^2\right)\,dx}{8d}\\
&= \frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{abx\cos\left(c+dx^2\right)}{d}-\frac{b^2x\sin\left(2c+2dx^2\right)}{8d}+\frac{(ab\cos(c))\int\cos\left(dx^2\right)\,dx}{d}\\
&\quad+\frac{\left(b^2\cos(2c)\right)\int\sin\left(2dx^2\right)\,dx}{8d}-\frac{(ab\sin(c))\int\sin\left(dx^2\right)\,dx}{d}+\frac{\left(b^2\sin(2c)\right)\int\cos\left(2dx^2\right)\,dx}{8d}
\end{aligned}
$$

$$= \frac{1}{6}(2a^2+b^2)\,x^3 - \frac{abx\cos(c+dx^2)}{d} + \frac{ab\sqrt{\frac{\pi}{2}}\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}} + \frac{b^2\sqrt{\pi}\cos(2c)\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{16d^{3/2}} - \frac{ab\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{d^{3/2}} + \frac{b^2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)}{16d^{3/2}} - \frac{b^2x\sin(2c+2dx^2)}{8d}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.96

$$\int x^2(a+b\sin(c+dx^2))^2\,dx$$

$$= \frac{16a^2d^{3/2}x^3 + 8b^2d^{3/2}x^3 - 48ab\sqrt{d}x\cos(c+dx^2) + 24ab\sqrt{2\pi}\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) + 3b^2\sqrt{\pi}\cos(2c)}{}$$

```
[In] Integrate[x^2*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (16*a^2*d^(3/2)*x^3 + 8*b^2*d^(3/2)*x^3 - 48*a*b*Sqrt[d]*x*Cos[c + d*x^2] +
 24*a*b*Sqrt[2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + 3*b^2*Sqrt[Pi]*C
os[2*c]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]] - 24*a*b*Sqrt[2*Pi]*FresnelS[Sqrt[
d]*Sqrt[2/Pi]*x]*Sin[c] + 3*b^2*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]]*S
in[2*c] - 6*b^2*Sqrt[d]*x*Sin[2*(c + d*x^2)])/(48*d^(3/2))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.70

method	result
parts	$\frac{x^3a^2}{3} + b^2\left(\frac{x^3}{6} - \frac{x\sin(2dx^2+2c)}{8d} + \frac{\sqrt{\pi}\left(\cos(2c)\,S\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)+\sin(2c)\,C\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)}{16d^{\frac{3}{2}}}\right) + 2ab\left(-\frac{x\cos(dx^2+c)}{2d} + \frac{\sqrt{2}\sqrt{\pi}\left(\text{co}\right.}{}\right.$
default	$\frac{\left(a^2+\frac{b^2}{2}\right)x^3}{3} - \frac{b^2\left(\frac{x\sin(2dx^2+2c)}{4d} - \frac{\sqrt{\pi}\left(\cos(2c)\,S\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)+\sin(2c)\,C\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)}{8d^{\frac{3}{2}}}\right)}{2} + 2ab\left(-\frac{x\cos(dx^2+c)}{2d} + \frac{\sqrt{2}\sqrt{\pi}\left(\cos(c)\,C\right.}{}\right.$
risch	$\frac{ib^2\sqrt{\pi}\sqrt{2}\,\text{erf}\left(\sqrt{2}\sqrt{id}\,x\right)e^{-2ic}}{64d\sqrt{id}} - \frac{ib^2\sqrt{\pi}\,\text{erf}\left(\sqrt{-2id}\,x\right)e^{2ic}}{32d\sqrt{-2id}} + \frac{ab\sqrt{\pi}\,\text{erf}\left(\sqrt{-id}\,x\right)e^{ic}}{4d\sqrt{-id}} + \frac{ab\sqrt{\pi}\,\text{erf}\left(\sqrt{id}\,x\right)e^{-ic}}{4d\sqrt{id}} + \frac{x^3a^2}{3} + \frac{x^3b}{6}$

```
[In] int(x^2*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*a^2+b^2*(1/6*x^3-1/8/d*x*sin(2*d*x^2+2*c)+1/16/d^(3/2)*Pi^(1/2)*(co
s(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2))+sin(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2
))))+2*a*b*(-1/2/d*x*cos(d*x^2+c)+1/4/d^(3/2)*2^(1/2)*Pi^(1/2)*(cos(c)*Fres
```

```
nelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))-sin(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2)
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.89

$$\int x^2(a+b\sin(c+dx^2))^2\,dx = \frac{8(2a^2+b^2)d^2x^3+24\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)-12b^2dx\cos(dx^2+c)\sin(dx^2+c)-24\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}}{48}$$

```
[In] integrate(x^2*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(8*(2*a^2 + b^2)*d^2*x^3 + 24*sqrt(2)*pi*a*b*sqrt(d/pi)*cos(c)*fresnel
_cos(sqrt(2)*x*sqrt(d/pi)) - 12*b^2*d*x*cos(d*x^2 + c)*sin(d*x^2 + c) - 24*
sqrt(2)*pi*a*b*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*sin(c) + 3*pi*b
^2*sqrt(d/pi)*cos(2*c)*fresnel_sin(2*x*sqrt(d/pi)) + 3*pi*b^2*sqrt(d/pi)*fr
esnel_cos(2*x*sqrt(d/pi))*sin(2*c) - 48*a*b*d*x*cos(d*x^2 + c))/d^2
```

Sympy [F]

$$\int x^2(a+b\sin(c+dx^2))^2\,dx = \int x^2(a+b\sin(c+dx^2))^2\,dx$$

```
[In] integrate(x**2*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(x**2*(a + b*sin(c + d*x**2))**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

$$\int x^2(a+b\sin(c+dx^2))^2\,dx = \frac{1}{3}a^2x^3 - \frac{\left(8d^2x\cos(dx^2+c)+\sqrt{2}\sqrt{\pi}\left(((i-1)\cos(c)+(i+1)\sin(c))\operatorname{erf}\left(\sqrt{i\,d}x\right)+(-(i+1)\cos(c)-(i-1)\right.\right.}{8d^3} + \frac{\left(64d^3x^3-48d^2x\sin(2dx^2+2c)+3\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(((i+1)\cos(2c)-(i-1)\sin(2c))\operatorname{erf}\left(\sqrt{2i\,d}x\right)+(\right.\right.}{384d^3}$$

```
[In] integrate(x^2*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 - 1/8*(8*d^2*x*cos(d*x^2 + c) + sqrt(2)*sqrt(pi)*(((I - 1)*cos(
c) + (I + 1)*sin(c))*erf(sqrt(I*d)*x) + (-(I + 1)*cos(c) - (I - 1)*sin(c))*
erf(sqrt(-I*d)*x))*d^(3/2))*a*b/d^3 + 1/384*(64*d^3*x^3 - 48*d^2*x*sin(2*d*
x^2 + 2*c) + 3*4^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*cos(2*c) - (I - 1)*sin(2*
c))*erf(sqrt(2*I*d)*x) + (-(I - 1)*cos(2*c) + (I + 1)*sin(2*c))*erf(sqrt(-2
*I*d)*x))*d^(3/2))*b^2/d^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.43

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{1}{3}\,a^2x^3+\frac{1}{6}\,b^2x^3+\frac{i\,b^2xe^{\left(2i\,dx^2+2i\,c\right)}}{16\,d}-\frac{abxe^{\left(i\,dx^2+i\,c\right)}}{2\,d}-\frac{abxe^{\left(-i\,dx^2-i\,c\right)}}{2\,d}-\frac{i\,b^2xe^{\left(-2i\,dx^2-2i\,c\right)}}{16\,d}+\frac{i\,\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}i\,\sqrt{2}x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{\left(i\,c\right)}}{4\,d\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}-\frac{i\,\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(\frac{1}{2}i\,\sqrt{2}x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{\left(-i\,c\right)}}{4\,d\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}+\frac{\sqrt{\pi}b^2\,\mathrm{erf}\left(-i\,\sqrt{d}x\left(\frac{i\,d}{|d|}+1\right)\right)e^{\left(2i\,c\right)}}{32\,d^{\frac{3}{2}}\left(\frac{i\,d}{|d|}+1\right)}+\frac{\sqrt{\pi}b^2\,\mathrm{erf}\left(i\,\sqrt{d}x\left(-\frac{i\,d}{|d|}+1\right)\right)e^{\left(-2i\,c\right)}}{32\,d^{\frac{3}{2}}\left(-\frac{i\,d}{|d|}+1\right)}$$

```
[In] integrate(x^2*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*x^3 + 1/6*b^2*x^3 + 1/16*I*b^2*x*e^(2*I*d*x^2 + 2*I*c)/d - 1/2*a*b*
x*e^(I*d*x^2 + I*c)/d - 1/2*a*b*x*e^(-I*d*x^2 - I*c)/d - 1/16*I*b^2*x*e^(-2
*I*d*x^2 - 2*I*c)/d + 1/4*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*I*sqrt(2)*x*(I*d/
abs(d) + 1)*sqrt(abs(d)))*e^(I*c)/(d*(I*d/abs(d) + 1)*sqrt(abs(d))) - 1/4*I
*sqrt(2)*sqrt(pi)*a*b*erf(1/2*I*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d)))*e
^(-I*c)/(d*(-I*d/abs(d) + 1)*sqrt(abs(d))) + 1/32*sqrt(pi)*b^2*erf(-I*sqrt(
d)*x*(I*d/abs(d) + 1))*e^(2*I*c)/(d^(3/2)*(I*d/abs(d) + 1)) + 1/32*sqrt(pi)
*b^2*erf(I*sqrt(d)*x*(-I*d/abs(d) + 1))*e^(-2*I*c)/(d^(3/2)*(-I*d/abs(d) +
1))
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a + b \sin(c + dx^2))^2 \, dx = \int x^2 \left(a + b \sin\left(d x^2 + c\right)\right)^2 dx$$

```
[In] int(x^2*(a + b*sin(c + d*x^2))^2,x)
[Out] int(x^2*(a + b*sin(c + d*x^2))^2, x)
```

3.20 $\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 153

$$\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x-\frac{b^2\sqrt{\pi}\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{4\sqrt{d}}+\frac{ab\sqrt{2\pi}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}+\frac{ab\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{\sqrt{d}}+\frac{b^2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)}{4\sqrt{d}}$$

```
[Out] 1/2*(2*a^2+b^2)*x-1/4*b^2*cos(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))*Pi^(1/2)/
d^(1/2)+1/4*b^2*FresnelS(2*x*d^(1/2)/Pi^(1/2))*sin(2*c)*Pi^(1/2)/d^(1/2)+a*
b*cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^(1/2)+a*b*
FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used

$= \{3438, 3435, 3433, 3432, 3434\}$

$$\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{1}{2}x\left(2a^2+b^2\right)+\frac{\sqrt{2\pi}ab\sin(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}} + \frac{\sqrt{2\pi}ab\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}} - \frac{\sqrt{\pi}b^2\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{4\sqrt{d}} + \frac{\sqrt{\pi}b^2\sin(2c)\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{4\sqrt{d}}$$

```
[In] Int[(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 - (b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi
]])/(4*Sqrt[d]) + (a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/Sq
rt[d] + (a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/Sqrt[d] + (b
^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(4*Sqrt[d])
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2+\frac{b^2}{2}-\frac{1}{2}b^2\cos\left(2c+2dx^2\right)+2ab\sin\left(c+dx^2\right)\right)\,dx \\
&= \frac{1}{2}\left(2a^2+b^2\right)x+(2ab)\int \sin\left(c+dx^2\right)\,dx-\frac{1}{2}b^2\int \cos\left(2c+2dx^2\right)\,dx \\
&= \frac{1}{2}\left(2a^2+b^2\right)x+(2ab\cos(c))\int \sin\left(dx^2\right)\,dx-\frac{1}{2}\left(b^2\cos(2c)\right)\int \cos\left(2dx^2\right)\,dx \\
&\quad +(2ab\sin(c))\int \cos\left(dx^2\right)\,dx+\frac{1}{2}\left(b^2\sin(2c)\right)\int \sin\left(2dx^2\right)\,dx \\
&= \frac{1}{2}\left(2a^2+b^2\right)x-\frac{b^2\sqrt{\pi}\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{4\sqrt{d}}+\frac{ab\sqrt{2\pi}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}} \\
&\quad +\frac{ab\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{\sqrt{d}}+\frac{b^2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)}{4\sqrt{d}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

$$
\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx
$$

$$
=\frac{4a^2\sqrt{d}x+2b^2\sqrt{d}x-b^2\sqrt{\pi}\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)+4ab\sqrt{2\pi}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+4ab\sqrt{2\pi}\,\text{Fres}}{4\sqrt{d}}
$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (4*a^2*Sqrt[d]*x + 2*b^2*Sqrt[d]*x - b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt
[d]*x)/Sqrt[Pi]] + 4*a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] +
 4*a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] + b^2*Sqrt[Pi]*Fres
nelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(4*Sqrt[d])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

method	result
parts	$a^{2}x+b^{2}\left(\frac{x}{2}-\frac{\sqrt{\pi}\left(\cos(2c)\,\mathrm{C}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)-\sin(2c)\,\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)}{4\sqrt{d}}\right)+\frac{ab\sqrt{2}\,\sqrt{\pi}\left(\cos(c)\,\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(c)\,\mathrm{C}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{\sqrt{d}}$
default	$a^{2}x+\frac{b^{2}x}{2}-\frac{b^{2}\sqrt{\pi}\left(\cos(2c)\,\mathrm{C}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)-\sin(2c)\,\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)}{4\sqrt{d}}+\frac{ab\sqrt{2}\,\sqrt{\pi}\left(\cos(c)\,\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(c)\,\mathrm{C}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{\sqrt{d}}$
risch	$a^{2}x+\frac{iab\,\mathrm{e}^{-ic}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)}{2\sqrt{id}}+\frac{b^{2}x}{2}-\frac{b^{2}\mathrm{e}^{-2ic}\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{id}\,x\right)}{16\sqrt{id}}-\frac{b^{2}\mathrm{e}^{2ic}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-2id}\,x\right)}{8\sqrt{-2id}}-\frac{iab\,\mathrm{e}^{ic}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right)}{2\sqrt{-id}}$

```
[In] int((a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+b^2*(1/2*x-1/4*Pi^(1/2)/d^(1/2)*(cos(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/
2))-sin(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2))))+a*b*2^(1/2)*Pi^(1/2)/d^(1/2)*
(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))+sin(c)*FresnelC(x*d^(1/2)*2^(1
/2)/Pi^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.88

$$\int \left(a+b\sin\left(c+dx^{2}\right)\right)^{2}\,dx$$
$$=\frac{4\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)+4\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)-\pi b^{2}\sqrt{\frac{d}{\pi}}\cos\left(2\,c\right)\mathrm{C}\left(2\,x\sqrt{\frac{d}{\pi}}\right)+\pi b^{2}\sqrt{}}{4\,d}$$

```
[In] integrate((a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*pi*a*b*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/pi)) +
 4*sqrt(2)*pi*a*b*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))*sin(c) - pi*
b^2*sqrt(d/pi)*cos(2*c)*fresnel_cos(2*x*sqrt(d/pi)) + pi*b^2*sqrt(d/pi)*fre
snel_sin(2*x*sqrt(d/pi))*sin(2*c) + 2*(2*a^2 + b^2)*d*x)/d
```

Sympy [F]

$$\int \left(a+b\sin \left(c+dx^2\right)\right)^2\,dx=\int \left(a+b\sin \left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate((a+b*sin(d*x**2+c))**2,x)
[Out] Integral((a + b*sin(c + d*x**2))**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int \left(a+b\sin \left(c+dx^2\right)\right)^2\,dx=$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\,\cos \left(c\right)+(i-1)\,\sin \left(c\right)\right)\mathrm{erf}\left(\sqrt{i\,d}x\right)+\left((i-1)\,\cos \left(c\right)-(i+1)\,\sin \left(c\right)\right)\mathrm{erf}\left(\sqrt{-i\,d}x\right.}{4\,\sqrt{d}}$$
$$+a^2x$$
$$+\frac{\left(4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left((i-1)\,\cos \left(2\,c\right)+(i+1)\,\sin \left(2\,c\right)\right)\mathrm{erf}\left(\sqrt{2i\,d}x\right)+\left(-(i+1)\,\cos \left(2\,c\right)-(i-1)\,\sin \left(2\,c\right)\right)\mathrm{e}\right.}{32\,d^2}$$

```
[In] integrate((a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
[Out] -1/4*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(c) + (I - 1)*sin(c))*erf(sqrt(I*d)*x)
+ ((I - 1)*cos(c) - (I + 1)*sin(c))*erf(sqrt(-I*d)*x))*a*b/sqrt(d) + a^2*x
+ 1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*cos(2*c) + (I + 1)*sin(2*c))*erf
(sqrt(2*I*d)*x) + (-(I + 1)*cos(2*c) - (I - 1)*sin(2*c))*erf(sqrt(-2*I*d)*x
))*d^(3/2) + 16*d^2*x)*b^2/d^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.27

$$\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}i\,\sqrt{2}x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(i\,c)}}{2\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} + \frac{\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(\frac{1}{2}i\,\sqrt{2}x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-i\,c)}}{2\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} - \frac{i\,\sqrt{\pi}b^2\,\mathrm{erf}\left(-i\,\sqrt{d}x\left(\frac{i\,d}{|d|}+1\right)\right)e^{(2i\,c)}}{8\,\sqrt{d}\left(\frac{i\,d}{|d|}+1\right)} + \frac{i\,\sqrt{\pi}b^2\,\mathrm{erf}\left(i\,\sqrt{d}x\left(-\frac{i\,d}{|d|}+1\right)\right)e^{(-2i\,c)}}{8\,\sqrt{d}\left(-\frac{i\,d}{|d|}+1\right)} + \frac{1}{2}\left(2\,a^2+b^2\right)x$$

```
[In] integrate((a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*I*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d))
)*e^(I*c)/((I*d/abs(d) + 1)*sqrt(abs(d))) + 1/2*sqrt(2)*sqrt(pi)*a*b*erf(1/
2*I*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/((-I*d/abs(d) + 1)*s
qrt(abs(d))) - 1/8*I*sqrt(pi)*b^2*erf(-I*sqrt(d)*x*(I*d/abs(d) + 1))*e^(2*I
*c)/(sqrt(d)*(I*d/abs(d) + 1)) + 1/8*I*sqrt(pi)*b^2*erf(I*sqrt(d)*x*(-I*d/a
bs(d) + 1))*e^(-2*I*c)/(sqrt(d)*(-I*d/abs(d) + 1)) + 1/2*(2*a^2 + b^2)*x
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \int \left(a+b\,\sin\left(d\,x^2+c\right)\right)^2\,dx$$

```
[In] int((a + b*sin(c + d*x^2))^2,x)
```

```
[Out] int((a + b*sin(c + d*x^2))^2, x)
```

3.21 $\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 187

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx = &-\frac{2a^2+b^2}{2x}+\frac{b^2\cos(2c+2dx^2)}{2x}\\ &+2ab\sqrt{d}\sqrt{2\pi}\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\\ &+b^2\sqrt{d}\sqrt{\pi}\cos(2c)\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\\ &-2ab\sqrt{d}\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\\ &+b^2\sqrt{d}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)-\frac{2ab\sin(c+dx^2)}{x}\end{aligned}$$

```
[Out] 1/2*(-2*a^2-b^2)/x+1/2*b^2*cos(2*d*x^2+2*c)/x-2*a*b*sin(d*x^2+c)/x+b^2*cos(
2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2))*d^(1/2)*Pi^(1/2)+b^2*FresnelC(2*x*d^(1/
2)/Pi^(1/2))*sin(2*c)*d^(1/2)*Pi^(1/2)+2*a*b*cos(c)*FresnelC(x*d^(1/2)*2^(1
/2)/Pi^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)-2*a*b*FresnelS(x*d^(1/2)*2^(1/2)/Pi^
(1/2))*sin(c)*d^(1/2)*2^(1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3484, 6, 3469, 3434, 3433, 3432, 3468, 3435}

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx = -\frac{2a^2+b^2}{2x}+2\sqrt{2\pi}ab\sqrt{d}\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) - 2\sqrt{2\pi}ab\sqrt{d}\sin(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) - \frac{2ab\sin(c+dx^2)}{x}+\sqrt{\pi}b^2\sqrt{d}\sin(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right) + \sqrt{\pi}b^2\sqrt{d}\cos(2c)\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)+\frac{b^2\cos(2c+2dx^2)}{2x}$$

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x^2,x]
```

```
[Out] -1/2*(2*a^2 + b^2)/x + (b^2*Cos[2*c + 2*d*x^2])/(2*x) + 2*a*b*Sqrt[d]*Sqrt[
2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + b^2*Sqrt[d]*Sqrt[Pi]*Cos[2*c]
*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]] - 2*a*b*Sqrt[d]*Sqrt[2*Pi]*FresnelS[Sqrt[
d]*Sqrt[2/Pi]*x]*Sin[c] + b^2*Sqrt[d]*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[
Pi]]*Sin[2*c] - (2*a*b*Sin[c + d*x^2])/x
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{x^2}+\frac{b^2}{2x^2}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^2}+\frac{2ab\sin\left(c+dx^2\right)}{x^2}\right)dx \\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^2}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^2}+\frac{2ab\sin\left(c+dx^2\right)}{x^2}\right)dx \\
&= -\frac{2a^2+b^2}{2x}+(2ab)\int \frac{\sin\left(c+dx^2\right)}{x^2}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^2\right)}{x^2}\,dx \\
&= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos\left(2c+2dx^2\right)}{2x}-\frac{2ab\sin\left(c+dx^2\right)}{x} \\
&\quad+(4abd)\int \cos\left(c+dx^2\right)\,dx+\left(2b^2d\right)\int \sin\left(2c+2dx^2\right)\,dx \\
&= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos\left(2c+2dx^2\right)}{2x}-\frac{2ab\sin\left(c+dx^2\right)}{x} \\
&\quad+(4abd\cos(c))\int \cos\left(dx^2\right)\,dx+\left(2b^2d\cos(2c)\right)\int \sin\left(2dx^2\right)\,dx \\
&\quad-(4abd\sin(c))\int \sin\left(dx^2\right)\,dx+\left(2b^2d\sin(2c)\right)\int \cos\left(2dx^2\right)\,dx
\end{aligned}
$$

$$= -\frac{2a^2+b^2}{2x} + \frac{b^2\cos(2c+2dx^2)}{2x} + 2ab\sqrt{d}\sqrt{2\pi}\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)$$
$$+ b^2\sqrt{d}\sqrt{\pi}\cos(2c)\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right) - 2ab\sqrt{d}\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)$$
$$+ b^2\sqrt{d}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c) - \frac{2ab\sin(c+dx^2)}{x}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx$$
$$= \frac{-2a^2-b^2+b^2\cos(2(c+dx^2))+4ab\sqrt{d}\sqrt{2\pi}x\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+2b^2\sqrt{d}\sqrt{\pi}x\cos(2c)\,\text{FresnelS}\left(\right.}{2}$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x^2,x]
```

```
[Out] (-2*a^2 - b^2 + b^2*Cos[2*(c + d*x^2)] + 4*a*b*Sqrt[d]*Sqrt[2*Pi]*x*Cos[c]*
FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + 2*b^2*Sqrt[d]*Sqrt[Pi]*x*Cos[2*c]*FresnelS
[(2*Sqrt[d]*x)/Sqrt[Pi]] - 4*a*b*Sqrt[d]*Sqrt[2*Pi]*x*FresnelS[Sqrt[d]*Sqrt
[2/Pi]*x]*Sin[c] + 2*b^2*Sqrt[d]*Sqrt[Pi]*x*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]
]*Sin[2*c] - 4*a*b*Sin[c + d*x^2])/(2*x)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.72

method	result
parts	$-\frac{a^2}{x}+b^2\left(-\frac{1}{2x}+\frac{\cos(2d\,x^2+2c)}{2x}+\sqrt{d}\,\sqrt{\pi}\left(\cos(2c)\,\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)+\sin(2c)\,\mathrm{C}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)\right)+2ab\left(-\frac{\sin(d\,x^2+}{x}\right.$
default	$-\frac{a^2+\frac{b^2}{2}}{x}-\frac{b^2\left(-\frac{\cos(2d\,x^2+2c)}{x}-2\sqrt{d}\,\sqrt{\pi}\left(\cos(2c)\,\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)+\sin(2c)\,\mathrm{C}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)\right)}{2}+2ab\left(-\frac{\sin(d\,x^2+c)}{x}+\sqrt{d}\,\sqrt{2}\,\sqrt{\pi}\left(\right.\right.$
risch	$\frac{ib^2d\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{id}\,x\right)\mathrm{e}^{-2ic}}{4\sqrt{id}}-\frac{ib^2d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-2id}\,x\right)\mathrm{e}^{2ic}}{2\sqrt{-2id}}+\frac{abd\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-id}\,x\right)\mathrm{e}^{ic}}{\sqrt{-id}}+\frac{abd\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{\sqrt{id}}-\frac{a^2}{x}-\frac{b^2}{2x}$

```
[In] int((a+b*sin(d*x^2+c))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x*a^2+b^2*(-1/2/x+1/2/x*cos(2*d*x^2+2*c)+d^(1/2)*Pi^(1/2)*(cos(2*c)*Fres
nelS(2*x*d^(1/2)/Pi^(1/2))+sin(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))))+2*a*b*
(-sin(d*x^2+c)/x+d^(1/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelC(x*d^(1/2)*2^(1/2
)/Pi^(1/2))-sin(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx = \frac{2\sqrt{2}\pi abx\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right) - 2\sqrt{2}\pi abx\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin(c) + \pi b^2x\sqrt{\frac{d}{\pi}}\cos(2c)\,\mathrm{S}\left(2x\sqrt{\frac{d}{\pi}}\right) +}{x}$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^2,x, algorithm="fricas")
```

```
[Out] (2*sqrt(2)*pi*a*b*x*sqrt(d/pi)*cos(c)*fresnel_cos(sqrt(2)*x*sqrt(d/pi)) - 2
*sqrt(2)*pi*a*b*x*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*sin(c) + pi*
b^2*x*sqrt(d/pi)*cos(2*c)*fresnel_sin(2*x*sqrt(d/pi)) + pi*b^2*x*sqrt(d/pi)
*fresnel_cos(2*x*sqrt(d/pi))*sin(2*c) + b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*
x^2 + c) - a^2 - b^2)/x
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx = \int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx$$

```
[In] integrate((a+b*sin(d*x**2+c))**2/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx = -\frac{\sqrt{dx^2}\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{1}{2}, i\,dx^2\right) - (i+1)\sqrt{2}\Gamma\left(-\frac{1}{2}, -i\,dx^2\right)\right)\cos(c) + \left((i+1)\sqrt{2}\Gamma\left(-\frac{1}{2}, i\,dx^2\right) - (i-\right.}{4x} - \frac{\left(\sqrt{2}\sqrt{dx^2}\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{1}{2}, 2i\,dx^2\right) + (i-1)\sqrt{2}\Gamma\left(-\frac{1}{2}, -2i\,dx^2\right)\right)\cos(2c) + \left((i-1)\sqrt{2}\Gamma\left(-\frac{1}{2}, 2i\,d\right.\right.}{16x} - \frac{a^2}{x}$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(d*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, I*d*x^2) - (I + 1)*sqrt(2)*g
amma(-1/2, -I*d*x^2))*cos(c) + ((I + 1)*sqrt(2)*gamma(-1/2, I*d*x^2) - (I -
 1)*sqrt(2)*gamma(-1/2, -I*d*x^2))*sin(c))*a*b/x - 1/16*(sqrt(2)*sqrt(d*x^2
)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*d*x^2) + (I - 1)*sqrt(2)*gamma(-1/2, -
2*I*d*x^2))*cos(2*c) + ((I - 1)*sqrt(2)*gamma(-1/2, 2*I*d*x^2) - (I + 1)*sq
rt(2)*gamma(-1/2, -2*I*d*x^2))*sin(2*c)) + 8)*b^2/x - a^2/x
```

Giac [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx=\int \frac{(b\sin(dx^2+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x^2 + c) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx=\int \frac{(a+b\,\sin(d\,x^2+c))^2}{x^2}\,dx$$

```
[In] int((a + b*sin(c + d*x^2))^2/x^2,x)
[Out] int((a + b*sin(c + d*x^2))^2/x^2, x)
```

3.22 $\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx = -\frac{2a^2+b^2}{6x^3}-\frac{4abd\cos(c+dx^2)}{3x}+\frac{b^2\cos(2c+2dx^2)}{6x^3} + \frac{4}{3}b^2d^{3/2}\sqrt{\pi}\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)-\frac{4}{3}abd^{3/2}\sqrt{2\pi}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) -\frac{4}{3}abd^{3/2}\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)-\frac{4}{3}b^2d^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)-\frac{2ab\sin(c+dx^2)}{3x^3}-$$

```
[Out] 1/6*(-2*a^2-b^2)/x^3-4/3*a*b*d*cos(d*x^2+c)/x+1/6*b^2*cos(2*d*x^2+2*c)/x^3-
2/3*a*b*sin(d*x^2+c)/x^3-2/3*b^2*d*sin(2*d*x^2+2*c)/x+4/3*b^2*d^(3/2)*cos(2
*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))*Pi^(1/2)-4/3*b^2*d^(3/2)*FresnelS(2*x*d^
(1/2)/Pi^(1/2))*sin(2*c)*Pi^(1/2)-4/3*a*b*d^(3/2)*cos(c)*FresnelS(x*d^(1/2)
*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)-4/3*a*b*d^(3/2)*FresnelC(x*d^(1/2)*2^(1
/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used

= {3484, 6, 3469, 3468, 3435, 3433, 3432, 3434}

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx = -\frac{2a^2+b^2}{6x^3} - \frac{4}{3}\sqrt{2\pi}abd^{3/2}\sin(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) - \frac{4}{3}\sqrt{2\pi}abd^{3/2}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) - \frac{4abd\cos(c+dx^2)}{3x} - \frac{2ab\sin(c+dx^2)}{3x^3} + \frac{4}{3}\sqrt{\pi}b^2d^{3/2}\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right) - \frac{4}{3}\sqrt{\pi}b^2d^{3/2}\sin(2c)\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right) - \frac{2b^2d\sin(2c+2dx^2)}{3x} + \frac{b}{}$$

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x^4,x]
```

```
[Out] -1/6*(2*a^2 + b^2)/x^3 - (4*a*b*d*Cos[c + d*x^2])/(3*x) + (b^2*Cos[2*c + 2*
d*x^2])/(6*x^3) + (4*b^2*d^(3/2)*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/S
qrt[Pi]])/3 - (4*a*b*d^(3/2)*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*
x])/3 - (4*a*b*d^(3/2)*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/3
- (4*b^2*d^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/3 - (2
*a*b*Sin[c + d*x^2])/(3*x^3) - (2*b^2*d*Sin[2*c + 2*d*x^2])/(3*x)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{x^4}+\frac{b^2}{2x^4}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^4}+\frac{2ab\sin\left(c+dx^2\right)}{x^4}\right)\,dx\\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^4}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^4}+\frac{2ab\sin\left(c+dx^2\right)}{x^4}\right)\,dx\\
&= -\frac{2a^2+b^2}{6x^3}+(2ab)\int \frac{\sin\left(c+dx^2\right)}{x^4}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^2\right)}{x^4}\,dx\\
&= -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos\left(2c+2dx^2\right)}{6x^3}-\frac{2ab\sin\left(c+dx^2\right)}{3x^3}\\
&\quad+\frac{1}{3}(4abd)\int \frac{\cos\left(c+dx^2\right)}{x^2}\,dx+\frac{1}{3}\left(2b^2d\right)\int \frac{\sin\left(2c+2dx^2\right)}{x^2}\,dx\\
&= -\frac{2a^2+b^2}{6x^3}-\frac{4abd\cos\left(c+dx^2\right)}{3x}+\frac{b^2\cos\left(2c+2dx^2\right)}{6x^3}-\frac{2ab\sin\left(c+dx^2\right)}{3x^3}\\
&\quad-\frac{2b^2d\sin\left(2c+2dx^2\right)}{3x}-\frac{1}{3}\left(8abd^2\right)\int \sin\left(c+dx^2\right)\,dx+\frac{1}{3}\left(8b^2d^2\right)\int \cos\left(2c+2dx^2\right)\,dx
\end{aligned}$$

$$= -\frac{2a^2+b^2}{6x^3} - \frac{4abd\cos(c+dx^2)}{3x} + \frac{b^2\cos(2c+2dx^2)}{6x^3} - \frac{2ab\sin(c+dx^2)}{3x^3} - \frac{2b^2d\sin(2c+2dx^2)}{3x} - \frac{1}{3}\left(8abd^2\cos(c)\right)\int\sin\left(dx^2\right)\,dx + \frac{1}{3}\left(8b^2d^2\cos(2c)\right)\int\cos\left(2dx^2\right)\,dx - \frac{1}{3}\left(8abd^2\sin(c)\right)\int\cos\left(dx^2\right)\,dx - \frac{1}{3}\left(8b^2d^2\sin(2c)\right)\int\sin\left(2dx^2\right)\,dx$$

$$= -\frac{2a^2+b^2}{6x^3} - \frac{4abd\cos(c+dx^2)}{3x} + \frac{b^2\cos(2c+2dx^2)}{6x^3} + \frac{4}{3}b^2d^{3/2}\sqrt{\pi}\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right) - \frac{4}{3}abd^{3/2}\sqrt{2\pi}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) - \frac{4}{3}abd^{3/2}\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c) - \frac{4}{3}b^2d^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c) - \frac{2ab\sin(c+d}{3x^3}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.95

$$\int\frac{\left(a+b\sin(c+dx^2)\right)^2}{x^4}\,dx = -\frac{2a^2+b^2+8abdx^2\cos(c+dx^2)-b^2\cos(2(c+dx^2))-8b^2d^{3/2}\sqrt{\pi}x^3\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)+8abd^{3/2}}{}$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x^4,x]
```

```
[Out] -1/6*(2*a^2 + b^2 + 8*a*b*d*x^2*Cos[c + d*x^2] - b^2*Cos[2*(c + d*x^2)] - 8
*b^2*d^(3/2)*Sqrt[Pi]*x^3*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]] + 8*a*b
*d^(3/2)*Sqrt[2*Pi]*x^3*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + 8*a*b*d^(3/
2)*Sqrt[2*Pi]*x^3*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] + 8*b^2*d^(3/2)*Sqr
t[Pi]*x^3*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c] + 4*a*b*Sin[c + d*x^2]
+ 4*b^2*d*x^2*Sin[2*(c + d*x^2)])/x^3
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.72

method	result
parts	$-\frac{a^2}{3x^3}+b^2\left(-\frac{1}{6x^3}+\frac{\cos(2d\,x^2+2c)}{6x^3}+\frac{2d\left(-\frac{\sin(2d\,x^2+2c)}{x}+2\sqrt{d}\,\sqrt{\pi}\,\left(\cos(2c)\,\mathrm{C}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)-\sin(2c)\,\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)\right)}{3}\right)+2ab\Bigg($
default	$-\frac{a^2+\frac{b^2}{2}}{3x^3}-\frac{b^2\left(-\frac{\cos(2d\,x^2+2c)}{3x^3}-\frac{4d\left(-\frac{\sin(2d\,x^2+2c)}{x}+2\sqrt{d}\,\sqrt{\pi}\,\left(\cos(2c)\,\mathrm{C}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)-\sin(2c)\,\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)\right)}{3}\right)}{2}+2ab\left(-\frac{\sin(d\,x^2+c)}{3x^3}\right.$
risch	$-\frac{2iab\,d^2\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{3\sqrt{id}}-\frac{a^2}{3x^3}-\frac{b^2}{6x^3}+\frac{b^2d^2\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{id}\,x\right)\mathrm{e}^{-2ic}}{3\sqrt{id}}+\frac{2b^2d^2\sqrt{\pi}\,\mathrm{erf}(\sqrt{-2id}\,x)\mathrm{e}^{2ic}}{3\sqrt{-2id}}+\frac{2iab\,d^2\sqrt{\pi}}{3}$

```
[In] int((a+b*sin(d*x^2+c))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3*a^2+b^2*(-1/6/x^3+1/6/x^3*cos(2*d*x^2+2*c)+2/3*d*(-1/x*sin(2*d*x^2
+2*c)+2*d^(1/2)*Pi^(1/2)*(cos(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))-sin(2*c)*
FresnelS(2*x*d^(1/2)/Pi^(1/2)))))+2*a*b*(-1/3*sin(d*x^2+c)/x^3+2/3*d*(-1/x*
cos(d*x^2+c)-d^(1/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi
^(1/2))+sin(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx =$$
$$-\frac{4\sqrt{2}\pi abdx^3\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)+4\sqrt{2}\pi abdx^3\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin(c)-4\pi b^2dx^3\sqrt{\frac{d}{\pi}}\cos(2c)\,\mathrm{C}\left(\right.}{}$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(2)*pi*a*b*d*x^3*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d
/pi)) + 4*sqrt(2)*pi*a*b*d*x^3*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))
*sin(c) - 4*pi*b^2*d*x^3*sqrt(d/pi)*cos(2*c)*fresnel_cos(2*x*sqrt(d/pi)) +
4*pi*b^2*d*x^3*sqrt(d/pi)*fresnel_sin(2*x*sqrt(d/pi))*sin(2*c) + 4*a*b*d*x^
2*cos(d*x^2 + c) - b^2*cos(d*x^2 + c)^2 + a^2 + b^2 + 2*(2*b^2*d*x^2*cos(d*
x^2 + c) + a*b)*sin(d*x^2 + c))/x^3
```

Sympy [F]

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^4}\,dx$$

```
[In] integrate((a+b*sin(d*x**2+c))**2/x**4,x)
[Out] Integral((a + b*sin(c + d*x**2))**2/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^4}\,dx=$$
$$-\frac{\sqrt{dx^2}\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},i\,dx^2\right)+(i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},-i\,dx^2\right)\right)\cos(c)+\left((i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},i\,dx^2\right)-(i+\right.}{4\,x}$$
$$-\frac{\left(3\sqrt{2}\sqrt{dx^2}\left(\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},2i\,dx^2\right)+(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},-2i\,dx^2\right)\right)\cos(2\,c)+\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},2\right.\right.\right.}{24\,x^3}$$
$$-\frac{a^2}{3\,x^3}$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^4,x, algorithm="maxima")
[Out] -1/4*sqrt(d*x^2)*((-(I + 1)*sqrt(2)*gamma(-3/2, I*d*x^2) + (I - 1)*sqrt(2)*
gamma(-3/2, -I*d*x^2))*cos(c) + ((I - 1)*sqrt(2)*gamma(-3/2, I*d*x^2) - (I
+ 1)*sqrt(2)*gamma(-3/2, -I*d*x^2))*sin(c))*a*b*d/x - 1/24*(3*sqrt(2)*sqrt(
d*x^2)*((-(I - 1)*sqrt(2)*gamma(-3/2, 2*I*d*x^2) + (I + 1)*sqrt(2)*gamma(-3
/2, -2*I*d*x^2))*cos(2*c) + (-(I + 1)*sqrt(2)*gamma(-3/2, 2*I*d*x^2) + (I -
 1)*sqrt(2)*gamma(-3/2, -2*I*d*x^2))*sin(2*c))*d*x^2 + 4)*b^2/x^3 - 1/3*a^2
/x^3
```

Giac [F]

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^4}\,dx=\int \frac{\left(b\sin\left(dx^2+c\right)+a\right)^2}{x^4}\,dx$$

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^4,x, algorithm="giac")
[Out] integrate((b*sin(d*x^2 + c) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\sin\left(c + dx^2\right)\right)^2}{x^4}\,dx = \int \frac{\left(a + b\,\sin\left(d\,x^2 + c\right)\right)^2}{x^4}\,dx$$

```
[In] int((a + b*sin(c + d*x^2))^2/x^4,x)
[Out] int((a + b*sin(c + d*x^2))^2/x^4, x)
```

3.23 $\int x^5 \sin^3(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 117

$$\int x^5 \sin^3(a+bx^2)\,dx = \frac{7\cos(a+bx^2)}{9b^3} - \frac{x^4\cos(a+bx^2)}{3b} - \frac{\cos^3(a+bx^2)}{27b^3} + \frac{2x^2\sin(a+bx^2)}{3b^2} - \frac{x^4\cos(a+bx^2)\sin^2(a+bx^2)}{6b} + \frac{x^2\sin^3(a+bx^2)}{9b^2}$$

```
[Out] 7/9*cos(b*x^2+a)/b^3-1/3*x^4*cos(b*x^2+a)/b-1/27*cos(b*x^2+a)^3/b^3+2/3*x^2
*sin(b*x^2+a)/b^2-1/6*x^4*cos(b*x^2+a)*sin(b*x^2+a)^2/b+1/9*x^2*sin(b*x^2+a
)^3/b^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3460, 3392, 3377, 2718, 2713}

$$\int x^5 \sin^3(a+bx^2)\,dx = -\frac{\cos^3(a+bx^2)}{27b^3} + \frac{7\cos(a+bx^2)}{9b^3} + \frac{x^2\sin^3(a+bx^2)}{9b^2} + \frac{2x^2\sin(a+bx^2)}{3b^2} - \frac{x^4\cos(a+bx^2)}{3b} - \frac{x^4\sin^2(a+bx^2)\cos(a+bx^2)}{6b}$$

```
[In] Int[x^5*Sin[a + b*x^2]^3,x]
```

```
[Out] (7*Cos[a + b*x^2])/(9*b^3) - (x^4*Cos[a + b*x^2])/(3*b) - Cos[a + b*x^2]^3/
(27*b^3) + (2*x^2*Sin[a + b*x^2])/(3*b^2) - (x^4*Cos[a + b*x^2]*Sin[a + b*x
^2]^2)/(6*b) + (x^2*Sin[a + b*x^2]^3)/(9*b^2)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2\sin^3(a+bx)\,dx,x,x^2\right)\\
&= -\frac{x^4\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b}+\frac{x^2\sin^3\left(a+bx^2\right)}{9b^2}\\
&\quad+\frac{1}{3}\text{Subst}\left(\int x^2\sin(a+bx)\,dx,x,x^2\right)-\frac{\text{Subst}\left(\int\sin^3(a+bx)\,dx,x,x^2\right)}{9b^2}\\
&= -\frac{x^4\cos\left(a+bx^2\right)}{3b}-\frac{x^4\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b}+\frac{x^2\sin^3\left(a+bx^2\right)}{9b^2}\\
&\quad+\frac{\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cos\left(a+bx^2\right)\right)}{9b^3}+\frac{2\text{Subst}\left(\int x\cos(a+bx)\,dx,x,x^2\right)}{3b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\cos(a+bx^2)}{9b^3} - \frac{x^4\cos(a+bx^2)}{3b} - \frac{\cos^3(a+bx^2)}{27b^3} \\
&\quad + \frac{2x^2\sin(a+bx^2)}{3b^2} - \frac{x^4\cos(a+bx^2)\sin^2(a+bx^2)}{6b} \\
&\quad + \frac{x^2\sin^3(a+bx^2)}{9b^2} - \frac{2\text{Subst}\left(\int \sin(a+bx)\,dx, x, x^2\right)}{3b^2} \\
&= \frac{7\cos(a+bx^2)}{9b^3} - \frac{x^4\cos(a+bx^2)}{3b} - \frac{\cos^3(a+bx^2)}{27b^3} + \frac{2x^2\sin(a+bx^2)}{3b^2} \\
&\quad - \frac{x^4\cos(a+bx^2)\sin^2(a+bx^2)}{6b} + \frac{x^2\sin^3(a+bx^2)}{9b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$
\int x^5\sin^3\left(a+bx^2\right)\,dx
= \frac{-81(-2+b^2x^4)\cos(a+bx^2)+(-2+9b^2x^4)\cos(3(a+bx^2))-6bx^2(-27\sin(a+bx^2)+\sin(3(a+bx^2)))}{216b^3}
$$

```
[In] Integrate[x^5*Sin[a + b*x^2]^3,x]
[Out] (-81*(-2 + b^2*x^4)*Cos[a + b*x^2] + (-2 + 9*b^2*x^4)*Cos[3*(a + b*x^2)] -
6*b*x^2*(-27*Sin[a + b*x^2] + Sin[3*(a + b*x^2)]))/(216*b^3)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{3(b^2x^4-2)\cos(bx^2+a)}{8b^3}+\frac{3x^2\sin(bx^2+a)}{4b^2}+\frac{(9b^2x^4-2)\cos(3bx^2+3a)}{216b^3}-\frac{x^2\sin(3bx^2+3a)}{36b^2}$	85
default	$-\frac{3x^4\cos(bx^2+a)}{8b}+\frac{\frac{3x^2\sin(bx^2+a)}{4b}+\frac{3\cos(bx^2+a)}{4b^2}}{b}+\frac{x^4\cos(3bx^2+3a)}{24b}-\frac{\frac{x^2\sin(3bx^2+3a)}{6b}+\frac{\cos(3bx^2+3a)}{18b^2}}{6b}$	113

```
[In] int(x^5*sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] -3/8*(b^2*x^4-2)/b^3*cos(b*x^2+a)+3/4*x^2*sin(b*x^2+a)/b^2+1/216*(9*b^2*x^4
-2)/b^3*cos(3*b*x^2+3*a)-1/36*x^2/b^2*sin(3*b*x^2+3*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int x^5 \sin^3\left(a+bx^2\right)\, dx$$
$$= \frac{\left(9\, b^2 x^4 - 2\right) \cos\left(bx^2+a\right)^3 - 3\left(9\, b^2 x^4 - 14\right) \cos\left(bx^2+a\right) - 6\left(bx^2 \cos\left(bx^2+a\right)^2 - 7\, bx^2\right) \sin\left(bx^2+a\right)}{54\, b^3}$$

```
[In] integrate(x^5*sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/54*((9*b^2*x^4 - 2)*cos(b*x^2 + a)^3 - 3*(9*b^2*x^4 - 14)*cos(b*x^2 + a)
- 6*(b*x^2*cos(b*x^2 + a)^2 - 7*b*x^2)*sin(b*x^2 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int x^5 \sin^3\left(a+bx^2\right)\, dx$$
$$= \begin{cases} -\frac{x^4 \sin^2\left(a+bx^2\right) \cos\left(a+bx^2\right)}{2b} - \frac{x^4 \cos^3\left(a+bx^2\right)}{3b} + \frac{7x^2 \sin^3\left(a+bx^2\right)}{9b^2} + \frac{2x^2 \sin\left(a+bx^2\right) \cos^2\left(a+bx^2\right)}{3b^2} + \frac{7 \sin^2\left(a+bx^2\right) \cos\left(a+bx^2\right)}{9b^3} \\ \frac{x^6 \sin^3(a)}{6} \end{cases}$$

```
[In] integrate(x**5*sin(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-x**4*sin(a + b*x**2)**2*cos(a + b*x**2)/(2*b) - x**4*cos(a + b*
x**2)**3/(3*b) + 7*x**2*sin(a + b*x**2)**3/(9*b**2) + 2*x**2*sin(a + b*x**2
)*cos(a + b*x**2)**2/(3*b**2) + 7*sin(a + b*x**2)**2*cos(a + b*x**2)/(9*b**
3) + 20*cos(a + b*x**2)**3/(27*b**3), Ne(b, 0)), (x**6*sin(a)**3/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int x^5 \sin^3\left(a+bx^2\right)\, dx =$$
$$-\frac{6\, bx^2 \sin\left(3\, bx^2 + 3\, a\right) - 162\, bx^2 \sin\left(bx^2+a\right) - \left(9\, b^2 x^4 - 2\right) \cos\left(3\, bx^2 + 3\, a\right) + 81\left(b^2 x^4 - 2\right) \cos\left(bx^2 + \right.}{216\, b^3}$$

```
[In] integrate(x^5*sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/216*(6*b*x^2*sin(3*b*x^2 + 3*a) - 162*b*x^2*sin(b*x^2 + a) - (9*b^2*x^4
- 2)*cos(3*b*x^2 + 3*a) + 81*(b^2*x^4 - 2)*cos(b*x^2 + a))/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int x^5 \sin^3\left(a+bx^2\right)\,dx = -\frac{x^2\sin\left(3\,bx^2+3\,a\right)}{36\,b^2} + \frac{3\,x^2\sin\left(bx^2+a\right)}{4\,b^2} + \frac{\left(\cos\left(bx^2+a\right)^3 - 3\,\cos\left(bx^2+a\right)\right)a^2}{6\,b^3} + \frac{\left(9\left(bx^2+a\right)^2 - 18\left(bx^2+a\right)a - 2\right)\cos\left(3\,bx^2+3\,a\right)}{216\,b^3} - \frac{3\left(\left(bx^2+a\right)^2 - 2\left(bx^2+a\right)a - 2\right)\cos\left(bx^2+a\right)}{8\,b^3}$$

```
[In] integrate(x^5*sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/36*x^2*sin(3*b*x^2 + 3*a)/b^2 + 3/4*x^2*sin(b*x^2 + a)/b^2 + 1/6*(cos(b*
x^2 + a)^3 - 3*cos(b*x^2 + a))*a^2/b^3 + 1/216*(9*(b*x^2 + a)^2 - 18*(b*x^2
 + a)*a - 2)*cos(3*b*x^2 + 3*a)/b^3 - 3/8*((b*x^2 + a)^2 - 2*(b*x^2 + a)*a
- 2)*cos(b*x^2 + a)/b^3
```

Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int x^5 \sin^3\left(a+bx^2\right)\,dx = \frac{\frac{3\,\cos\left(b\,x^2+a\right)}{4} - \frac{\cos\left(3\,b\,x^2+3\,a\right)}{108} + b\left(\frac{3\,x^2\,\sin\left(b\,x^2+a\right)}{4} - \frac{x^2\,\sin\left(3\,b\,x^2+3\,a\right)}{36}\right) + b^2\left(\frac{x^4\,\cos\left(3\,b\,x^2+3\,a\right)}{24} - \frac{3\,x^4\,\cos\left(b\,x^2+a\right)}{8}\right)}{b^3}$$

```
[In] int(x^5*sin(a + b*x^2)^3,x)
```

```
[Out] ((3*cos(a + b*x^2))/4 - cos(3*a + 3*b*x^2)/108 + b*((3*x^2*sin(a + b*x^2))/
4 - (x^2*sin(3*a + 3*b*x^2))/36) + b^2*((x^4*cos(3*a + 3*b*x^2))/24 - (3*x^
4*cos(a + b*x^2))/8))/b^3
```

3.24 $\int x^3 \sin^3(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int x^3 \sin^3(a+bx^2)\,dx = -\frac{x^2\cos(a+bx^2)}{3b} + \frac{\sin(a+bx^2)}{3b^2} - \frac{x^2\cos(a+bx^2)\sin^2(a+bx^2)}{6b} + \frac{\sin^3(a+bx^2)}{18b^2}$$

```
[Out] -1/3*x^2*cos(b*x^2+a)/b+1/3*sin(b*x^2+a)/b^2-1/6*x^2*cos(b*x^2+a)*sin(b*x^2
+a)^2/b+1/18*sin(b*x^2+a)^3/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3460, 3391, 3377, 2717}

$$\int x^3 \sin^3(a+bx^2)\,dx = \frac{\sin^3(a+bx^2)}{18b^2} + \frac{\sin(a+bx^2)}{3b^2} - \frac{x^2\cos(a+bx^2)}{3b} - \frac{x^2\sin^2(a+bx^2)\cos(a+bx^2)}{6b}$$

```
[In] Int[x^3*Sin[a + b*x^2]^3,x]
```

```
[Out] -1/3*(x^2*Cos[a + b*x^2])/b + Sin[a + b*x^2]/(3*b^2) - (x^2*Cos[a + b*x^2]*
Sin[a + b*x^2]^2)/(6*b) + Sin[a + b*x^2]^3/(18*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\sin^3(a+bx)\,dx, x, x^2\right)\\
&= -\frac{x^2\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b}+\frac{\sin^3\left(a+bx^2\right)}{18b^2}+\frac{1}{3}\text{Subst}\left(\int x\sin(a+bx)\,dx, x, x^2\right)\\
&= -\frac{x^2\cos\left(a+bx^2\right)}{3b}-\frac{x^2\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b}\\
&\quad+\frac{\sin^3\left(a+bx^2\right)}{18b^2}+\frac{\text{Subst}\left(\int\cos(a+bx)\,dx, x, x^2\right)}{3b}\\
&= -\frac{x^2\cos\left(a+bx^2\right)}{3b}+\frac{\sin\left(a+bx^2\right)}{3b^2}-\frac{x^2\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b}+\frac{\sin^3\left(a+bx^2\right)}{18b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int x^3 \sin^3\left(a+bx^2\right) dx$$
$$= -\frac{27bx^2\cos\left(a+bx^2\right)-3bx^2\cos\left(3\left(a+bx^2\right)\right)-27\sin\left(a+bx^2\right)+\sin\left(3\left(a+bx^2\right)\right)}{72b^2}$$

```
[In] Integrate[x^3*Sin[a + b*x^2]^3,x]
```

```
[Out] -1/72*(27*b*x^2*Cos[a + b*x^2] - 3*b*x^2*Cos[3*(a + b*x^2)] - 27*Sin[a + b*
x^2] + Sin[3*(a + b*x^2)])/b^2
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

method	result
default	$-\frac{3x^2\cos\left(b\,x^2+a\right)}{8b}+\frac{3\sin\left(b\,x^2+a\right)}{8b^2}+\frac{x^2\cos\left(3b\,x^2+3a\right)}{24b}-\frac{\sin\left(3b\,x^2+3a\right)}{72b^2}$
risch	$-\frac{3x^2\cos\left(b\,x^2+a\right)}{8b}+\frac{3\sin\left(b\,x^2+a\right)}{8b^2}+\frac{x^2\cos\left(3b\,x^2+3a\right)}{24b}-\frac{\sin\left(3b\,x^2+3a\right)}{72b^2}$
parallelrisch	$\frac{-27\cos\left(b\,x^2+a\right)b\,x^2+3x^2\cos\left(3b\,x^2+3a\right)b-24b\,x^2-24i\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)-i\right)+24i\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)+i\right)+27\sin\left(b\,x^2+a\right)-\sin}{72b^2}$
norman	$\frac{\frac{x^2\left(\tan^4\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{b}+\frac{2\tan\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)}{3b^2}+\frac{16\left(\tan^3\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{9b^2}+\frac{2\left(\tan^5\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{3b^2}-\frac{x^2}{3b}-\frac{x^2\left(\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{b}+\frac{x^2\left(\tan^6\left(\frac{a}{2}+\frac{b\,x^2}{2}\right.\right.}{3b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)^3}$

```
[In] int(x^3*sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*x^2*cos(b*x^2+a)/b+3/8*sin(b*x^2+a)/b^2+1/24/b*x^2*cos(3*b*x^2+3*a)-1/
72/b^2*sin(3*b*x^2+3*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int x^3 \sin^3\left(a+bx^2\right) dx$$
$$= \frac{3\,bx^2\cos\left(bx^2+a\right)^3-9\,bx^2\cos\left(bx^2+a\right)-\left(\cos\left(bx^2+a\right)^2-7\right)\sin\left(bx^2+a\right)}{18\,b^2}$$

```
[In] integrate(x^3*sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(3*b*x^2*cos(b*x^2 + a)^3 - 9*b*x^2*cos(b*x^2 + a) - (cos(b*x^2 + a)^2
 - 7)*sin(b*x^2 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int x^3 \sin^3\left(a+bx^2\right)\,dx = \begin{cases} -\frac{x^2\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b} - \frac{x^2\cos^3\left(a+bx^2\right)}{3b} + \frac{7\sin^3\left(a+bx^2\right)}{18b^2} + \frac{\sin\left(a+bx^2\right)\cos^2\left(a+bx^2\right)}{3b^2} & \text{for } b \neq 0 \\ \frac{x^4\sin^3(a)}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*sin(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-x**2*sin(a + b*x**2)**2*cos(a + b*x**2)/(2*b) - x**2*cos(a + b*
x**2)**3/(3*b) + 7*sin(a + b*x**2)**3/(18*b**2) + sin(a + b*x**2)*cos(a + b
*x**2)**2/(3*b**2), Ne(b, 0)), (x**4*sin(a)**3/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

$$\int x^3 \sin^3\left(a+bx^2\right)\,dx = \frac{3\,bx^2\cos\left(3\,bx^2+3\,a\right) - 27\,bx^2\cos\left(bx^2+a\right) - \sin\left(3\,bx^2+3\,a\right) + 27\,\sin\left(bx^2+a\right)}{72\,b^2}$$

```
[In] integrate(x^3*sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/72*(3*b*x^2*cos(3*b*x^2 + 3*a) - 27*b*x^2*cos(b*x^2 + a) - sin(3*b*x^2 +
3*a) + 27*sin(b*x^2 + a))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int x^3 \sin^3\left(a+bx^2\right)\,dx = -\frac{\left(\cos\left(bx^2+a\right)^3 - 3\,\cos\left(bx^2+a\right)\right)a}{6\,b^2} + \frac{3\left(bx^2+a\right)\cos\left(3\,bx^2+3\,a\right) - 27\left(bx^2+a\right)\cos\left(bx^2+a\right) - \sin\left(3\,bx^2+3\,a\right) + 27\,\sin\left(bx^2+a\right)}{72\,b^2}$$

```
[In] integrate(x^3*sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/6*(cos(b*x^2 + a)^3 - 3*cos(b*x^2 + a))*a/b^2 + 1/72*(3*(b*x^2 + a)*cos(
3*b*x^2 + 3*a) - 27*(b*x^2 + a)*cos(b*x^2 + a) - sin(3*b*x^2 + 3*a) + 27*si
n(b*x^2 + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int x^3 \sin^3\left(a+bx^2\right)\,dx = \frac{\frac{7\,\sin\left(b\,x^2+a\right)}{18}-\frac{\cos\left(b\,x^2+a\right)^2\,\sin\left(b\,x^2+a\right)}{18}+b\,\left(\frac{x^2\,\cos\left(b\,x^2+a\right)^3}{6}-\frac{x^2\,\cos\left(b\,x^2+a\right)}{2}\right)}{b^2}$$

```
[In] int(x^3*sin(a + b*x^2)^3,x)
```

```
[Out] ((7*sin(a + b*x^2))/18 - (cos(a + b*x^2)^2*sin(a + b*x^2))/18 + b*((x^2*cos
(a + b*x^2)^3)/6 - (x^2*cos(a + b*x^2))/2))/b^2
```

3.25 $\int x \sin^3(a+bx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int x \sin^3(a+bx^2)\,dx = -\frac{\cos(a+bx^2)}{2b} + \frac{\cos^3(a+bx^2)}{6b}$$

```
[Out] -1/2*cos(b*x^2+a)/b+1/6*cos(b*x^2+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3460, 2713}

$$\int x \sin^3(a+bx^2)\,dx = \frac{\cos^3(a+bx^2)}{6b} - \frac{\cos(a+bx^2)}{2b}$$

```
[In] Int[x*Sin[a + b*x^2]^3,x]
```

```
[Out] -1/2*Cos[a + b*x^2]/b + Cos[a + b*x^2]^3/(6*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sin^3(a+bx)\,dx, x, x^2\right)\\
&= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos\left(a+bx^2\right)\right)}{2b}\\
&= -\frac{\cos\left(a+bx^2\right)}{2b}+\frac{\cos^3\left(a+bx^2\right)}{6b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x\sin^3\left(a+bx^2\right)\,dx = -\frac{3\cos\left(a+bx^2\right)}{8b}+\frac{\cos\left(3\left(a+bx^2\right)\right)}{24b}$$

```
[In] Integrate[x*Sin[a + b*x^2]^3,x]
[Out] (-3*Cos[a + b*x^2])/(8*b) + Cos[3*(a + b*x^2)]/(24*b)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{\left(2+\sin^2\left(b\,x^2+a\right)\right)\cos\left(b\,x^2+a\right)}{6b}$	26
default	$-\frac{\left(2+\sin^2\left(b\,x^2+a\right)\right)\cos\left(b\,x^2+a\right)}{6b}$	26
parallelrisch	$\frac{-8-9\cos\left(b\,x^2+a\right)+\cos\left(3b\,x^2+3a\right)}{24b}$	29
risch	$-\frac{3\cos\left(b\,x^2+a\right)}{8b}+\frac{\cos\left(3b\,x^2+3a\right)}{24b}$	31
norman	$\frac{-\frac{2\left(\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{b}-\frac{2}{3b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)^3}$	43

```
[In] int(x*sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/6/b*(2+sin(b*x^2+a)^2)*cos(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int x\sin^3\left(a+bx^2\right)\,dx=\frac{\cos\left(bx^2+a\right)^3-3\,\cos\left(bx^2+a\right)}{6\,b}$$

```
[In] integrate(x*sin(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/6*(cos(b*x^2 + a)^3 - 3*cos(b*x^2 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int x\sin^3\left(a+bx^2\right)\,dx=\begin{cases}-\frac{\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}-\frac{\cos^3\left(a+bx^2\right)}{3b} & \text{for } b\neq 0\\ \frac{x^2\sin^3\left(a\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*sin(b*x**2+a)**3,x)
[Out] Piecewise((-sin(a + b*x**2)**2*cos(a + b*x**2)/(2*b) - cos(a + b*x**2)**3/(
3*b), Ne(b, 0)), (x**2*sin(a)**3/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x\sin^3\left(a+bx^2\right)\,dx=\frac{\cos\left(3\,bx^2+3\,a\right)-9\,\cos\left(bx^2+a\right)}{24\,b}$$

```
[In] integrate(x*sin(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/24*(cos(3*b*x^2 + 3*a) - 9*cos(b*x^2 + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int x \sin^3\left(a+bx^2\right)\,dx = \frac{\cos\left(bx^2+a\right)^3 - 3\,\cos\left(bx^2+a\right)}{6\,b}$$

```
[In] integrate(x*sin(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/6*(cos(b*x^2 + a)^3 - 3*cos(b*x^2 + a))/b
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x \sin^3\left(a+bx^2\right)\,dx = -\frac{3\,\cos\left(b\,x^2+a\right) - \cos\left(b\,x^2+a\right)^3}{6\,b}$$

```
[In] int(x*sin(a + b*x^2)^3,x)
[Out] -(3*cos(a + b*x^2) - cos(a + b*x^2)^3)/(6*b)
```

3.26 $\int \frac{\sin^3(a+bx^2)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = \frac{3}{8}\operatorname{CosIntegral}(bx^2)\sin(a) - \frac{1}{8}\operatorname{CosIntegral}(3bx^2)\sin(3a) + \frac{3}{8}\cos(a)\operatorname{Si}(bx^2) - \frac{1}{8}\cos(3a)\operatorname{Si}(3bx^2)$$

```
[Out] 3/8*cos(a)*Si(b*x^2)-1/8*cos(3*a)*Si(3*b*x^2)+3/8*Ci(b*x^2)*sin(a)-1/8*Ci(3
*b*x^2)*sin(3*a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3484, 3458, 3457, 3456}

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = \frac{3}{8}\sin(a)\operatorname{CosIntegral}(bx^2) - \frac{1}{8}\sin(3a)\operatorname{CosIntegral}(3bx^2) + \frac{3}{8}\cos(a)\operatorname{Si}(bx^2) - \frac{1}{8}\cos(3a)\operatorname{Si}(3bx^2)$$

```
[In] Int[Sin[a + b*x^2]^3/x,x]
```

```
[Out] (3*CosIntegral[b*x^2]*Sin[a])/8 - (CosIntegral[3*b*x^2]*Sin[3*a])/8 + (3*Co
s[a]*SinIntegral[b*x^2])/8 - (Cos[3*a]*SinIntegral[3*b*x^2])/8
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3\sin\left(a+bx^2\right)}{4x}-\frac{\sin\left(3a+3bx^2\right)}{4x}\right)dx \\
&= -\left(\frac{1}{4}\int \frac{\sin\left(3a+3bx^2\right)}{x}\,dx\right)+\frac{3}{4}\int \frac{\sin\left(a+bx^2\right)}{x}\,dx \\
&= \frac{1}{4}(3\cos(a))\int \frac{\sin\left(bx^2\right)}{x}\,dx-\frac{1}{4}\cos(3a)\int \frac{\sin\left(3bx^2\right)}{x}\,dx \\
&\quad +\frac{1}{4}(3\sin(a))\int \frac{\cos\left(bx^2\right)}{x}\,dx-\frac{1}{4}\sin(3a)\int \frac{\cos\left(3bx^2\right)}{x}\,dx \\
&= \frac{3}{8}\,\text{CosIntegral}\left(bx^2\right)\sin(a)-\frac{1}{8}\,\text{CosIntegral}\left(3bx^2\right)\sin(3a) \\
&\quad +\frac{3}{8}\cos(a)\text{Si}\left(bx^2\right)-\frac{1}{8}\cos(3a)\text{Si}\left(3bx^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\sin^3\left(a+bx^2\right)}{x}\,dx = \frac{1}{8}\Big(3\,\text{CosIntegral}\left(bx^2\right)\sin(a)-\text{CosIntegral}\left(3bx^2\right)\sin(3a) +3\cos(a)\text{Si}\left(bx^2\right)-\cos(3a)\text{Si}\left(3bx^2\right)\Big)$$

```
[In] Integrate[Sin[a + b*x^2]^3/x,x]
[Out] (3*CosIntegral[b*x^2]*Sin[a] - CosIntegral[3*b*x^2]*Sin[3*a] + 3*Cos[a]*Sin
Integral[b*x^2] - Cos[3*a]*SinIntegral[3*b*x^2])/8
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.62 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.27

method	result
risch	$-\frac{i e^{3 i a} \operatorname{Ei}_{1}\left(-3 i x^{2} b\right)}{16}+\frac{\operatorname{csgn}\left(b x^{2}\right) \pi e^{-3 i a}}{16}-\frac{\operatorname{Si}\left(3 b x^{2}\right) e^{-3 i a}}{8}+\frac{i \operatorname{Ei}_{1}\left(-3 i x^{2} b\right) e^{-3 i a}}{16}-\frac{3 \operatorname{csgn}\left(b x^{2}\right) \pi e^{-i a}}{16}+\frac{3 \operatorname{Si}\left(b x^{2}\right) e^{-i a}}{8}-$

```
[In] int(sin(b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*exp(3*I*a)*Ei(1,-3*I*x^2*b)+1/16*csgn(b*x^2)*Pi*exp(-3*I*a)-1/8*Si(
3*b*x^2)*exp(-3*I*a)+1/16*I*Ei(1,-3*I*x^2*b)*exp(-3*I*a)-3/16*csgn(b*x^2)*P
i*exp(-I*a)+3/8*Si(b*x^2)*exp(-I*a)-3/16*I*Ei(1,-I*b*x^2)*exp(-I*a)+3/16*I*
exp(I*a)*Ei(1,-I*b*x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = -\frac{1}{8}\,\operatorname{Ci}\left(3\,bx^2\right)\sin\left(3\,a\right)+\frac{3}{8}\,\operatorname{Ci}\left(bx^2\right)\sin\left(a\right) - \frac{1}{8}\,\cos\left(3\,a\right)\operatorname{Si}\left(3\,bx^2\right)+\frac{3}{8}\,\cos\left(a\right)\operatorname{Si}\left(bx^2\right)$$

```
[In] integrate(sin(b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] -1/8*cos_integral(3*b*x^2)*sin(3*a) + 3/8*cos_integral(b*x^2)*sin(a) - 1/8*
cos(3*a)*sin_integral(3*b*x^2) + 3/8*cos(a)*sin_integral(b*x^2)
```

Sympy [F]

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = \int \frac{\sin^3(a+bx^2)}{x}\,dx$$

```
[In] integrate(sin(b*x**2+a)**3/x,x)
```

```
[Out] Integral(sin(a + b*x**2)**3/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = \frac{1}{16}\left(i\,\mathrm{Ei}\left(3i\,bx^2\right) - i\,\mathrm{Ei}\left(-3i\,bx^2\right)\right)\cos(3a) - \frac{3}{16}\left(i\,\mathrm{Ei}\left(i\,bx^2\right) - i\,\mathrm{Ei}\left(-i\,bx^2\right)\right)\cos(a) - \frac{1}{16}\left(\mathrm{Ei}\left(3i\,bx^2\right) + \mathrm{Ei}\left(-3i\,bx^2\right)\right)\sin(3a) + \frac{3}{16}\left(\mathrm{Ei}\left(i\,bx^2\right) + \mathrm{Ei}\left(-i\,bx^2\right)\right)\sin(a)$$

```
[In] integrate(sin(b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*(I*Ei(3*I*b*x^2) - I*Ei(-3*I*b*x^2))*cos(3*a) - 3/16*(I*Ei(I*b*x^2) -
I*Ei(-I*b*x^2))*cos(a) - 1/16*(Ei(3*I*b*x^2) + Ei(-3*I*b*x^2))*sin(3*a) + 3
/16*(Ei(I*b*x^2) + Ei(-I*b*x^2))*sin(a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = -\frac{1}{8}\,\mathrm{Ci}\left(3\,bx^2\right)\sin(3\,a) + \frac{3}{8}\,\mathrm{Ci}\left(bx^2\right)\sin(a) + \frac{3}{8}\,\cos(a)\,\mathrm{Si}\left(bx^2\right) + \frac{1}{8}\,\cos(3\,a)\,\mathrm{Si}\left(-3\,bx^2\right)$$

```
[In] integrate(sin(b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] -1/8*cos_integral(3*b*x^2)*sin(3*a) + 3/8*cos_integral(b*x^2)*sin(a) + 3/8*
cos(a)*sin_integral(b*x^2) + 1/8*cos(3*a)*sin_integral(-3*b*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = \int \frac{\sin\left(b\,x^2+a\right)^3}{x}\,dx$$

```
[In] int(sin(a + b*x^2)^3/x,x)
```

```
[Out] int(sin(a + b*x^2)^3/x, x)
```

3.27 $\int \frac{\sin^3(a+bx^2)}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx = \frac{3}{8}b\cos(a)\,\text{CosIntegral}\left(bx^2\right) - \frac{3}{8}b\cos(3a)\,\text{CosIntegral}\left(3bx^2\right) - \frac{3\sin(a+bx^2)}{8x^2} + \frac{\sin(3(a+bx^2))}{8x^2} - \frac{3}{8}b\sin(a)\text{Si}\left(bx^2\right) + \frac{3}{8}b\sin(3a)\text{Si}\left(3bx^2\right)$$

```
[Out] 3/8*b*Ci(b*x^2)*cos(a)-3/8*b*Ci(3*b*x^2)*cos(3*a)-3/8*b*Si(b*x^2)*sin(a)+3/
8*b*Si(3*b*x^2)*sin(3*a)-3/8*sin(b*x^2+a)/x^2+1/8*sin(3*b*x^2+3*a)/x^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3484, 3460, 3378, 3384, 3380, 3383}

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx = \frac{3}{8}b\cos(a)\,\text{CosIntegral}\left(bx^2\right) - \frac{3}{8}b\cos(3a)\,\text{CosIntegral}\left(3bx^2\right) - \frac{3}{8}b\sin(a)\text{Si}\left(bx^2\right) + \frac{3}{8}b\sin(3a)\text{Si}\left(3bx^2\right) - \frac{3\sin(a+bx^2)}{8x^2} + \frac{\sin(3(a+bx^2))}{8x^2}$$

```
[In] Int[Sin[a + b*x^2]^3/x^3,x]
```

```
[Out] (3*b*Cos[a]*CosIntegral[b*x^2])/8 - (3*b*Cos[3*a]*CosIntegral[3*b*x^2])/8 -
 (3*Sin[a + b*x^2])/(8*x^2) + Sin[3*(a + b*x^2)]/(8*x^2) - (3*b*Sin[a]*SinI
ntegral[b*x^2])/8 + (3*b*Sin[3*a]*SinIntegral[3*b*x^2])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{3\sin(a+bx^2)}{4x^3} - \frac{\sin(3a+3bx^2)}{4x^3}\right) dx$$

$$= -\left(\frac{1}{4}\int \frac{\sin(3a+3bx^2)}{x^3}\,dx\right) + \frac{3}{4}\int \frac{\sin(a+bx^2)}{x^3}\,dx$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{\sin(3a+3bx)}{x^2}\,dx, x, x^2\right)\right) + \frac{3}{8}\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{3\sin(a+bx^2)}{8x^2} + \frac{\sin(3(a+bx^2))}{8x^2} + \frac{1}{8}(3b)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, x^2\right) - \frac{1}{8}(3b)\text{Subst}\left(\int \frac{\cos(3a+3bx)}{x}\,dx, x, x^2\right)$$

$$= -\frac{3\sin(a+bx^2)}{8x^2} + \frac{\sin(3(a+bx^2))}{8x^2} + \frac{1}{8}(3b\cos(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, x^2\right) - \frac{1}{8}(3b\cos(3a))\text{Subst}\left(\int \frac{\cos(3bx)}{x}\,dx, x, x^2\right) - \frac{1}{8}(3b\sin(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, x^2\right) + \frac{1}{8}(3b\sin(3a))\text{Subst}\left(\int \frac{\sin(3bx)}{x}\,dx, x, x^2\right)$$

$$= \frac{3}{8}b\cos(a)\,\text{CosIntegral}\left(bx^2\right) - \frac{3}{8}b\cos(3a)\,\text{CosIntegral}\left(3bx^2\right) - \frac{3\sin(a+bx^2)}{8x^2} + \frac{\sin(3(a+bx^2))}{8x^2} - \frac{3}{8}b\sin(a)\text{Si}\left(bx^2\right) + \frac{3}{8}b\sin(3a)\text{Si}\left(3bx^2\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx = \frac{3bx^2\cos(a)\,\text{CosIntegral}(bx^2) - 3bx^2\cos(3a)\,\text{CosIntegral}(3bx^2) - 3\sin(a+bx^2) + \sin(3(a+bx^2)) - 3bx^2}{8x^2}$$

```
[In] Integrate[Sin[a + b*x^2]^3/x^3,x]
```

```
[Out] (3*b*x^2*Cos[a]*CosIntegral[b*x^2] - 3*b*x^2*Cos[3*a]*CosIntegral[3*b*x^2]
- 3*Sin[a + b*x^2] + Sin[3*(a + b*x^2)] - 3*b*x^2*Sin[a]*SinIntegral[b*x^2]
 + 3*b*x^2*Sin[3*a]*SinIntegral[3*b*x^2])/(8*x^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.70 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{3 i \mathrm{e}^{-3 i a} \operatorname{csgn}\left(b\,x^{2}\right) \pi b\,x^{2}-3 i \mathrm{e}^{-i a} \operatorname{csgn}\left(b\,x^{2}\right) \pi b\,x^{2}-6 i \mathrm{e}^{-3 i a} \operatorname{Si}\left(3 b\,x^{2}\right) b\,x^{2}+6 i \mathrm{e}^{-i a} \operatorname{Si}\left(b\,x^{2}\right) b\,x^{2}-3\,\mathrm{e}^{-3 i a} \operatorname{Ei}_{1}\left(-3 i x^{2} b\right) b\,x^{2}-3\,\mathrm{e}^{3 i a} b\ \mathrm{E}}{16 x^{2}}$

```
[In] int(sin(b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(3*I*exp(-3*I*a)*csgn(b*x^2)*Pi*b*x^2-3*I*exp(-I*a)*csgn(b*x^2)*Pi*b*
x^2-6*I*exp(-3*I*a)*Si(3*b*x^2)*b*x^2+6*I*exp(-I*a)*Si(b*x^2)*b*x^2-3*exp(-
3*I*a)*Ei(1,-3*I*x^2*b)*b*x^2-3*exp(3*I*a)*b*Ei(1,-3*I*x^2*b)*x^2+3*exp(I*a
)*b*Ei(1,-I*b*x^2)*x^2+3*exp(-I*a)*Ei(1,-I*b*x^2)*b*x^2+6*sin(b*x^2+a)-2*si
n(3*b*x^2+3*a))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{\sin^3\left(a+bx^2\right)}{x^3}\,dx =$$
$$-\frac{3\,bx^{2}\cos\left(3\,a\right)\operatorname{Ci}\left(3\,bx^{2}\right)-3\,bx^{2}\cos\left(a\right)\operatorname{Ci}\left(bx^{2}\right)-3\,bx^{2}\sin\left(3\,a\right)\operatorname{Si}\left(3\,bx^{2}\right)+3\,bx^{2}\sin\left(a\right)\operatorname{Si}\left(bx^{2}\right)-4\,\Big(\mathrm{co}}{8\,x^{2}}$$

```
[In] integrate(sin(b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*b*x^2*cos(3*a)*cos_integral(3*b*x^2) - 3*b*x^2*cos(a)*cos_integral(
b*x^2) - 3*b*x^2*sin(3*a)*sin_integral(3*b*x^2) + 3*b*x^2*sin(a)*sin_integr
al(b*x^2) - 4*(cos(b*x^2 + a)^2 - 1)*sin(b*x^2 + a))/x^2
```

Sympy [F]

$$\int \frac{\sin^3\left(a+bx^2\right)}{x^3}\,dx = \int \frac{\sin^3\left(a+bx^2\right)}{x^3}\,dx$$

```
[In] integrate(sin(b*x**2+a)**3/x**3,x)
```

```
[Out] Integral(sin(a + b*x**2)**3/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx =$$
$$-\frac{3}{16}\left(\left(\Gamma\left(-1,3i\,bx^2\right)+\Gamma\left(-1,-3i\,bx^2\right)\right)\cos\left(3\,a\right)-\left(\Gamma\left(-1,i\,bx^2\right)+\Gamma\left(-1,-i\,bx^2\right)\right)\cos\left(a\right)+\left(-i\,\Gamma\left(-1,3i\right.\right.$$

```
[In] integrate(sin(b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/16*((gamma(-1, 3*I*b*x^2) + gamma(-1, -3*I*b*x^2))*cos(3*a) - (gamma(-1,
 I*b*x^2) + gamma(-1, -I*b*x^2))*cos(a) + (-I*gamma(-1, 3*I*b*x^2) + I*gamm
a(-1, -3*I*b*x^2))*sin(3*a) + (I*gamma(-1, I*b*x^2) - I*gamma(-1, -I*b*x^2)
)*sin(a))*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(80) = 160.

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.04

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx =$$
$$-\frac{3\,(bx^2+a)b^2\cos\left(3\,a\right)\mathrm{Ci}\left(3\,bx^2\right)-3\,ab^2\cos\left(3\,a\right)\mathrm{Ci}\left(3\,bx^2\right)-3\,(bx^2+a)b^2\cos\left(a\right)\mathrm{Ci}\left(bx^2\right)+3\,ab^2\cos\left(a\right)}{}$$

```
[In] integrate(sin(b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(b*x^2 + a)*b^2*cos(3*a)*cos_integral(3*b*x^2) - 3*a*b^2*cos(3*a)*c
os_integral(3*b*x^2) - 3*(b*x^2 + a)*b^2*cos(a)*cos_integral(b*x^2) + 3*a*b
^2*cos(a)*cos_integral(b*x^2) + 3*(b*x^2 + a)*b^2*sin(a)*sin_integral(b*x^2
) - 3*a*b^2*sin(a)*sin_integral(b*x^2) + 3*(b*x^2 + a)*b^2*sin(3*a)*sin_int
egral(-3*b*x^2) - 3*a*b^2*sin(3*a)*sin_integral(-3*b*x^2) - b^2*sin(3*b*x^2
 + 3*a) + 3*b^2*sin(b*x^2 + a))/(b^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx = \int \frac{\sin\left(b\,x^2+a\right)^3}{x^3}\,dx$$

```
[In] int(sin(a + b*x^2)^3/x^3,x)
```

```
[Out] int(sin(a + b*x^2)^3/x^3, x)
```

3.28 $\int x^2 \sin^3(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 188

$$\int x^2 \sin^3\left(a+bx^2\right)\,dx = -\frac{3x\cos\left(a+bx^2\right)}{8b}+\frac{x\cos\left(3a+3bx^2\right)}{24b}+\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)}{24b^{3/2}}$$

```
[Out] -3/8*x*cos(b*x^2+a)/b+1/24*x*cos(3*b*x^2+3*a)/b-1/144*cos(3*a)*FresnelC(x*b
^(1/2)*6^(1/2)/Pi^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)+1/144*FresnelS(x*b^(1/2)*
6^(1/2)/Pi^(1/2))*sin(3*a)*6^(1/2)*Pi^(1/2)/b^(3/2)+3/16*cos(a)*FresnelC(x*
b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)-3/16*FresnelS(x*b^(1/2)*
2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3484, 3466, 3435, 3433, 3432}

$$\int x^2 \sin^3\left(a+bx^2\right)\,dx = \frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}}\sin(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}} - \frac{3x\cos\left(a+bx^2\right)}{8b} + \frac{x\cos\left(3a+3bx^2\right)}{24b}$$

```
[In] Int[x^2*Sin[a + b*x^2]^3,x]
```

```
[Out] (-3*x*Cos[a + b*x^2])/(8*b) + (x*Cos[3*a + 3*b*x^2])/(24*b) + (3*Sqrt[Pi/2]
*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/(8*b^(3/2)) - (Sqrt[Pi/6]*Cos[3*a]*
FresnelC[Sqrt[b]*Sqrt[6/Pi]*x])/(24*b^(3/2)) - (3*Sqrt[Pi/2]*FresnelS[Sqrt[
b]*Sqrt[2/Pi]*x]*Sin[a])/(8*b^(3/2)) + (Sqrt[Pi/6]*FresnelS[Sqrt[b]*Sqrt[6/
Pi]*x]*Sin[3*a])/(24*b^(3/2))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
```

```
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{4}x^2\sin\left(a+bx^2\right)-\frac{1}{4}x^2\sin\left(3a+3bx^2\right)\right)\,dx\\
&= -\left(\frac{1}{4}\int x^2\sin\left(3a+3bx^2\right)\,dx\right)+\frac{3}{4}\int x^2\sin\left(a+bx^2\right)\,dx\\
&= -\frac{3x\cos\left(a+bx^2\right)}{8b}+\frac{x\cos\left(3a+3bx^2\right)}{24b}-\frac{\int\cos\left(3a+3bx^2\right)\,dx}{24b}+\frac{3\int\cos\left(a+bx^2\right)\,dx}{8b}\\
&= -\frac{3x\cos\left(a+bx^2\right)}{8b}+\frac{x\cos\left(3a+3bx^2\right)}{24b}+\frac{(3\cos(a))\int\cos\left(bx^2\right)\,dx}{8b}\\
&\quad-\frac{\cos(3a)\int\cos\left(3bx^2\right)\,dx}{24b}-\frac{(3\sin(a))\int\sin\left(bx^2\right)\,dx}{8b}+\frac{\sin(3a)\int\sin\left(3bx^2\right)\,dx}{24b}\\
&= -\frac{3x\cos\left(a+bx^2\right)}{8b}+\frac{x\cos\left(3a+3bx^2\right)}{24b}\\
&\quad+\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}}\\
&\quad-\frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)}{24b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

$$\int x^2\sin^3\left(a+bx^2\right)\,dx$$
$$=\frac{-54\sqrt{b}x\cos\left(a+bx^2\right)+6\sqrt{b}x\cos\left(3\left(a+bx^2\right)\right)+27\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{6\pi}\cos(3a)\,\text{Fresn}}{144b^{3/2}}$$

```
[In] Integrate[x^2*Sin[a + b*x^2]^3,x]
```

```
[Out] (-54*Sqrt[b]*x*Cos[a + b*x^2] + 6*Sqrt[b]*x*Cos[3*(a + b*x^2)] + 27*Sqrt[2*
Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] - Sqrt[6*Pi]*Cos[3*a]*FresnelC[Sq
rt[b]*Sqrt[6/Pi]*x] - 27*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a] +
 Sqrt[6*Pi]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(144*b^(3/2))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.70

method	result
default	$-\frac{3x\cos\left(b\,x^2+a\right)}{8b}+\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\,\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{16b^{\frac{3}{2}}}+\frac{x\cos\left(3b\,x^2+3a\right)}{24b}-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\left(\cos(3a)\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)-\right.}{144b^{\frac{3}{2}}}$
risch	$-\frac{\mathrm{e}^{-3ia}\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{ib}\,x\right)}{288b\sqrt{ib}}-\frac{\mathrm{e}^{3ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3ib}\,x\right)}{96b\sqrt{-3ib}}+\frac{3\,\mathrm{e}^{ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)}{32b\sqrt{-ib}}+\frac{3\,\mathrm{e}^{-ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ib}\,x\right)}{32b\sqrt{ib}}-\frac{3x\cos\left(b\,x^2+a\right)}{8b}+$

```
[In] int(x^2*sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*x*cos(b*x^2+a)/b+3/16/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(x*b^(1
/2)*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2)))+1/24*x*c
os(3*b*x^2+3*a)/b-1/144/b^(3/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*FresnelC
(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)-sin(3*a)*FresnelS(2^(1/2)/Pi^(1/2)*3^(
1/2)*b^(1/2)*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.78

$$\int x^2\sin^3\left(a+bx^2\right)\,dx$$
$$=\frac{24\,bx\cos\left(bx^2+a\right)^3-\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\cos\left(3\,a\right)\mathrm{C}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)+27\,\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{6}\right.}{144\,b^2}$$

```
[In] integrate(x^2*sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/144*(24*b*x*cos(b*x^2 + a)^3 - sqrt(6)*pi*sqrt(b/pi)*cos(3*a)*fresnel_cos
(sqrt(6)*x*sqrt(b/pi)) + 27*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2
)*x*sqrt(b/pi)) + sqrt(6)*pi*sqrt(b/pi)*fresnel_sin(sqrt(6)*x*sqrt(b/pi))*s
in(3*a) - 27*sqrt(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi))*sin(a)
 - 72*b*x*cos(b*x^2 + a))/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(194) = 388$.

Time = 1.89 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.34

$$\int x^2 \sin^3\left(a+bx^2\right)\,dx = -\frac{3b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\cos(a)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{b^2x^4}{4}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)} + \frac{3b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\cos(3a)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{9b^2x^4}{4}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)} - \frac{3\sqrt{b}x^3\sqrt{\frac{1}{b}}\sin(a)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{b^2x^4}{4}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{b}x^3\sqrt{\frac{1}{b}}\sin(3a)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{9b^2x^4}{4}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)} + \frac{3\sqrt{2}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\sin(a)C\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)}{8} - \frac{\sqrt{6}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\sin(3a)C\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)}{24} + \frac{3\sqrt{2}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\cos(a)S\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)}{8} - \frac{\sqrt{6}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\cos(3a)S\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)}{24}$$

```
[In] integrate(x**2*sin(b*x**2+a)**3,x)
```

```
[Out] -3*b**(3/2)*x**5*sqrt(1/b)*cos(a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (
3/2, 7/4, 9/4), -b**2*x**4/4)/(32*gamma(7/4)*gamma(9/4)) + 3*b**(3/2)*x**5*
sqrt(1/b)*cos(3*a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4),
 -9*b**2*x**4/4)/(32*gamma(7/4)*gamma(9/4)) - 3*sqrt(b)*x**3*sqrt(1/b)*sin(
a)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*x**4/4)/(
32*gamma(5/4)*gamma(7/4)) + sqrt(b)*x**3*sqrt(1/b)*sin(3*a)*gamma(1/4)*gamm
a(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -9*b**2*x**4/4)/(32*gamma(5/4)*ga
mma(7/4)) + 3*sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*sin(a)*fresnelc(sqrt(2)*sqrt(
b)*x/sqrt(pi))/8 - sqrt(6)*sqrt(pi)*x**2*sqrt(1/b)*sin(3*a)*fresnelc(sqrt(6
```

```
)*sqrt(b)*x/sqrt(pi))/24 + 3*sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*cos(a)*fresnel
s(sqrt(2)*sqrt(b)*x/sqrt(pi))/8 - sqrt(6)*sqrt(pi)*x**2*sqrt(1/b)*cos(3*a)*
fresnels(sqrt(6)*sqrt(b)*x/sqrt(pi))/24
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

$$\int x^2 \sin^3\left(a+bx^2\right)\,dx$$
$$= \frac{24\,b^2x\cos\left(3\,bx^2+3\,a\right)-216\,b^2x\cos\left(bx^2+a\right)+9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\Big(\left(\left(i-1\right)\,\cos\left(3\,a\right)+\left(i+1\right)\,\sin\left(3\,a\right)\right)\mathrm{erf}\left(\sqrt{3i\,b}x\right.}{}$$

```
[In] integrate(x^2*sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576*(24*b^2*x*cos(3*b*x^2 + 3*a) - 216*b^2*x*cos(b*x^2 + a) + 9^(1/4)*sqr
t(2)*sqrt(pi)*(((I - 1)*cos(3*a) + (I + 1)*sin(3*a))*erf(sqrt(3*I*b)*x) + (
-(I + 1)*cos(3*a) - (I - 1)*sin(3*a))*erf(sqrt(-3*I*b)*x))*b^(3/2) - 27*sqr
t(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf(sqrt(I*b)*x) + (-(I +
1)*cos(a) - (I - 1)*sin(a))*erf(sqrt(-I*b)*x))*b^(3/2))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.38

$$\begin{aligned}\int x^2 \sin^3\left(a+bx^2\right)\,dx &= \frac{xe^{\left(3i\,bx^2+3i\,a\right)}}{48\,b} - \frac{3\,xe^{\left(i\,bx^2+i\,a\right)}}{16\,b} - \frac{3\,xe^{\left(-i\,bx^2-i\,a\right)}}{16\,b} \\ &\quad + \frac{xe^{\left(-3i\,bx^2-3i\,a\right)}}{48\,b} - \frac{i\,\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,\sqrt{6}\sqrt{b}x\left(\frac{i\,b}{|b|}+1\right)\right)e^{\left(3i\,a\right)}}{288\,b^{\frac{3}{2}}\left(\frac{i\,b}{|b|}+1\right)} \\ &\quad + \frac{3i\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,\sqrt{2}x\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{\left(i\,a\right)}}{32\,b\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}} \\ &\quad - \frac{3i\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,\sqrt{2}x\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{\left(-i\,a\right)}}{32\,b\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}} \\ &\quad + \frac{i\,\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,\sqrt{6}\sqrt{b}x\left(-\frac{i\,b}{|b|}+1\right)\right)e^{\left(-3i\,a\right)}}{288\,b^{\frac{3}{2}}\left(-\frac{i\,b}{|b|}+1\right)}\end{aligned}$$

[In] integrate(x^2*sin(b*x^2+a)^3,x, algorithm="giac")

[Out] 1/48*x*e^(3*I*b*x^2 + 3*I*a)/b - 3/16*x*e^(I*b*x^2 + I*a)/b - 3/16*x*e^(-I*b*x^2 - I*a)/b + 1/48*x*e^(-3*I*b*x^2 - 3*I*a)/b - 1/288*I*sqrt(6)*sqrt(pi)*erf(-1/2*I*sqrt(6)*sqrt(b)*x*(I*b/abs(b) + 1))*e^(3*I*a)/(b^(3/2)*(I*b/abs(b) + 1)) + 3/32*I*sqrt(2)*sqrt(pi)*erf(-1/2*I*sqrt(2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) - 3/32*I*sqrt(2)*sqrt(pi)*erf(1/2*I*sqrt(2)*x*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + 1/288*I*sqrt(6)*sqrt(pi)*erf(1/2*I*sqrt(6)*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(-3*I*a)/(b^(3/2)*(-I*b/abs(b) + 1))

Mupad [F(-1)]

Timed out.

$$\int x^2 \sin^3(a+bx^2)\, dx = \int x^2 \sin\left(b\,x^2+a\right)^3 dx$$

[In] int(x^2*sin(a + b*x^2)^3,x)

[Out] int(x^2*sin(a + b*x^2)^3, x)

3.29 $\int \sin^3(a+bx^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 153

$$\int \sin^3(a+bx^2)\,dx = \frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}} - \frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{4\sqrt{b}} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)}{4\sqrt{b}}$$

```
[Out] -1/24*cos(3*a)*FresnelS(x*b^(1/2)*6^(1/2)/Pi^(1/2))*6^(1/2)*Pi^(1/2)/b^(1/2
)-1/24*FresnelC(x*b^(1/2)*6^(1/2)/Pi^(1/2))*sin(3*a)*6^(1/2)*Pi^(1/2)/b^(1/
2)+3/8*cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)
+3/8*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3438, 3434, 3433, 3432}

$$\int \sin^3(a+bx^2)\,dx = \frac{3\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}} - \frac{\sqrt{\frac{\pi}{6}}\sin(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}} + \frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}} - \frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}$$

```
[In] Int[Sin[a + b*x^2]^3,x]
```

```
[Out] (3*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x])/(4*Sqrt[b]) - (Sqrt[Pi
/6]*Cos[3*a]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x])/(4*Sqrt[b]) + (3*Sqrt[Pi/2]*Fr
```

```
esnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/(4*Sqrt[b]) - (Sqrt[Pi/6]*FresnelC[Sqr
t[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(4*Sqrt[b])
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{3}{4}\sin\left(a+bx^2\right)-\frac{1}{4}\sin\left(3a+3bx^2\right)\right)\,dx\\
&= -\left(\frac{1}{4}\int \sin\left(3a+3bx^2\right)\,dx\right)+\frac{3}{4}\int \sin\left(a+bx^2\right)\,dx\\
&= \frac{1}{4}(3\cos(a))\int \sin\left(bx^2\right)\,dx-\frac{1}{4}\cos(3a)\int \sin\left(3bx^2\right)\,dx\\
&\quad+\frac{1}{4}(3\sin(a))\int \cos\left(bx^2\right)\,dx-\frac{1}{4}\sin(3a)\int \cos\left(3bx^2\right)\,dx\\
&= \frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}\\
&\quad+\frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)}{4\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.76

$$\int \sin^3(a+bx^2)\,dx = \frac{\sqrt{\frac{\pi}{6}}\left(3\sqrt{3}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)+3\sqrt{3}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)-\text{F}\right.}{4\sqrt{b}}$$

[In] Integrate[Sin[a + b*x^2]^3,x]

[Out] (Sqrt[Pi/6]*(3*Sqrt[3]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] - Cos[3*a]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x] + 3*Sqrt[3]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a] - FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a]))/(4*Sqrt[b])

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{3\sqrt{2}\sqrt{\pi}\left(\cos(a)\,\mathrm{S}\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right)+\sin(a)\,\mathrm{C}\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right)\right)}{8\sqrt{b}}-\frac{\sqrt{2}\sqrt{\pi}\sqrt{3}\left(\cos(3a)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{3}\sqrt{b}\,x}{\sqrt{\pi}}\right)+\sin(3a)\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{3}\sqrt{b}\,x}{\sqrt{\pi}}\right)\right)}{24\sqrt{b}}$	99
risch	$\frac{i e^{3ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3ib}\,x\right)}{16\sqrt{-3ib}}-\frac{i e^{-3ia}\sqrt{\pi}\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\sqrt{ib}\,x\right)}{48\sqrt{ib}}+\frac{3i e^{-ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ib}\,x\right)}{16\sqrt{ib}}-\frac{3i e^{ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)}{16\sqrt{-ib}}$	112

[In] int(sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] 3/8*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2)))-1/24*2^(1/2)*Pi^(1/2)*3^(1/2)/b^(1/2)*(cos(3*a)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)+sin(3*a)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.78

$$\int \sin^3(a+bx^2)\,dx = -\frac{\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\cos(3a)\,\mathrm{S}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)-9\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)\sin(3a)-9\sqrt{}}{24\,b}$$

[In] integrate(sin(b*x^2+a)^3,x, algorithm="fricas")

```
[Out] -1/24*(sqrt(6)*pi*sqrt(b/pi)*cos(3*a)*fresnel_sin(sqrt(6)*x*sqrt(b/pi)) - 9
*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt(6)*p
i*sqrt(b/pi)*fresnel_cos(sqrt(6)*x*sqrt(b/pi))*sin(3*a) - 9*sqrt(2)*pi*sqrt
(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a))/b
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int \sin^3\left(a+bx^2\right)\,dx = \frac{3\sqrt{2}\sqrt{\pi}\left(\sin\left(a\right)C\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)+\cos\left(a\right)S\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{8} - \frac{\sqrt{6}\sqrt{\pi}\left(\sin\left(3a\right)C\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)+\cos\left(3a\right)S\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{24}$$

```
[In] integrate(sin(b*x**2+a)**3,x)
```

```
[Out] 3*sqrt(2)*sqrt(pi)*(sin(a)*fresnelc(sqrt(2)*sqrt(b)*x/sqrt(pi)) + cos(a)*fr
esnels(sqrt(2)*sqrt(b)*x/sqrt(pi)))*sqrt(1/b)/8 - sqrt(6)*sqrt(pi)*(sin(3*a
)*fresnelc(sqrt(6)*sqrt(b)*x/sqrt(pi)) + cos(3*a)*fresnels(sqrt(6)*sqrt(b)*
x/sqrt(pi)))*sqrt(1/b)/24
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73

$$\int \sin^3\left(a+bx^2\right)\,dx$$
$$= \frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\,\cos\left(3\,a\right)+(i-1)\,\sin\left(3\,a\right)\right)\mathrm{erf}\left(\sqrt{3i\,b}x\right)+\left((i-1)\,\cos\left(3\,a\right)-(i+1)\,\sin\left(3\,a\right)\right)\mathrm{erf}\left(\right.}{}$$

```
[In] integrate(sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(3*a) + (I - 1)*sin(3*a))*erf(
sqrt(3*I*b)*x) + ((I - 1)*cos(3*a) - (I + 1)*sin(3*a))*erf(sqrt(-3*I*b)*x))
*b^(3/2) - 9*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf(sqrt(
I*b)*x) + ((I - 1)*cos(a) - (I + 1)*sin(a))*erf(sqrt(-I*b)*x))*b^(3/2))/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.21

$$\int \sin^3\left(a+bx^2\right)\,dx = -\frac{\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\sqrt{6}\sqrt{b}x\left(\frac{ib}{|b|}+1\right)\right)e^{(3ia)}}{48\sqrt{b}\left(\frac{ib}{|b|}+1\right)} + \frac{3\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\sqrt{2}x\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(ia)}}{16\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}} + \frac{3\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\sqrt{2}x\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(-ia)}}{16\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}} - \frac{\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\sqrt{6}\sqrt{b}x\left(-\frac{ib}{|b|}+1\right)\right)e^{(-3ia)}}{48\sqrt{b}\left(-\frac{ib}{|b|}+1\right)}$$

```
[In] integrate(sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(6)*sqrt(pi)*erf(-1/2*I*sqrt(6)*sqrt(b)*x*(I*b/abs(b) + 1))*e^(3*
I*a)/(sqrt(b)*(I*b/abs(b) + 1)) + 3/16*sqrt(2)*sqrt(pi)*erf(-1/2*I*sqrt(2)*
x*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/((I*b/abs(b) + 1)*sqrt(abs(b))) +
3/16*sqrt(2)*sqrt(pi)*erf(1/2*I*sqrt(2)*x*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e
^(-I*a)/((-I*b/abs(b) + 1)*sqrt(abs(b))) - 1/48*sqrt(6)*sqrt(pi)*erf(1/2*I*
sqrt(6)*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(-3*I*a)/(sqrt(b)*(-I*b/abs(b) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3\left(a+bx^2\right)\,dx = \int \sin\left(b\,x^2+a\right)^3\,dx$$

```
[In] int(sin(a + b*x^2)^3,x)
```

```
[Out] int(sin(a + b*x^2)^3, x)
```

3.30 $\int \frac{\sin^3(a+bx^2)}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 168

$$\int \frac{\sin^3(a+bx^2)}{x^2} dx = \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right) - \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a) + \frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a) - \frac{\sin^3(a+bx^2)}{x}$$

```
[Out] -sin(b*x^2+a)^3/x+3/4*cos(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*b^(1/2)*2
^(1/2)*Pi^(1/2)-3/4*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*b^(1/2)*2^(
1/2)*Pi^(1/2)-1/4*cos(3*a)*FresnelC(x*b^(1/2)*6^(1/2)/Pi^(1/2))*b^(1/2)*6^(
1/2)*Pi^(1/2)+1/4*FresnelS(x*b^(1/2)*6^(1/2)/Pi^(1/2))*sin(3*a)*b^(1/2)*6^(
1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used

= {3474, 4670, 3435, 3433, 3432}

$$\int \frac{\sin^3(a+bx^2)}{x^2}\,dx = \frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) + \frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right) - \frac{\sin^3(a+bx^2)}{x}$$

```
[In] Int[Sin[a + b*x^2]^3/x^2,x]
```

```
[Out] (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/2 - (Sqrt[b]*S
qrt[(3*Pi)/2]*Cos[3*a]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x])/2 - (3*Sqrt[b]*Sqrt[
Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/2 + (Sqrt[b]*Sqrt[(3*Pi)/2]*Fr
esnelS[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/2 - Sin[a + b*x^2]^3/x
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3474

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[x^(m +
1)*(Sin[a + b*x^n]^p/(m + 1)), x] - Dist[b*n*(p/(m + 1)), Int[Sin[a + b*x^n
]^(p - 1)*Cos[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
```

```
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin^3\left(a+bx^2\right)}{x}+(6b)\int \cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)\,dx\\
&= -\frac{\sin^3\left(a+bx^2\right)}{x}+(6b)\int \left(\frac{1}{4}\cos\left(a+bx^2\right)-\frac{1}{4}\cos\left(3a+3bx^2\right)\right)dx\\
&= -\frac{\sin^3\left(a+bx^2\right)}{x}+\frac{1}{2}(3b)\int \cos\left(a+bx^2\right)\,dx-\frac{1}{2}(3b)\int \cos\left(3a+3bx^2\right)\,dx\\
&= -\frac{\sin^3\left(a+bx^2\right)}{x}+\frac{1}{2}(3b\cos(a))\int \cos\left(bx^2\right)\,dx-\frac{1}{2}(3b\cos(3a))\int \cos\left(3bx^2\right)\,dx\\
&\quad -\frac{1}{2}(3b\sin(a))\int \sin\left(bx^2\right)\,dx+\frac{1}{2}(3b\sin(3a))\int \sin\left(3bx^2\right)\,dx\\
&= \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\\
&\quad -\frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\\
&\quad +\frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)-\frac{\sin^3\left(a+bx^2\right)}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int \frac{\sin^3\left(a+bx^2\right)}{x^2}\,dx$$

$$=\frac{3\sqrt{b}\sqrt{2\pi}x\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{b}\sqrt{6\pi}x\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)-3\sqrt{b}\sqrt{2\pi}x\,\text{FresnelS}\left(\sqrt{b}\right.}{4x}$$

```
[In] Integrate[Sin[a + b*x^2]^3/x^2,x]

[Out] (3*Sqrt[b]*Sqrt[2*Pi]*x*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] - Sqrt[b]*Sqr
t[6*Pi]*x*Cos[3*a]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x] - 3*Sqrt[b]*Sqrt[2*Pi]*x*
FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a] + Sqrt[b]*Sqrt[6*Pi]*x*FresnelS[Sqrt[
b]*Sqrt[6/Pi]*x]*Sin[3*a] - 3*Sin[a + b*x^2] + Sin[3*(a + b*x^2)])/(4*x)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

method	result
default	$-\frac{3\sin(b\,x^2+a)}{4x}+\frac{3\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\,\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4}+\frac{\sin(3b\,x^2+3a)}{4x}-\frac{\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\left(\cos(3a)\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{b}}{\sqrt{\pi}}\right.\right.}{4}$
risch	$-\frac{\mathrm{e}^{-3ia}b\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{ib}\,x\right)}{8\sqrt{ib}}-\frac{3\,\mathrm{e}^{3ia}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3ib}\,x\right)}{8\sqrt{-3ib}}+\frac{3\,\mathrm{e}^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)}{8\sqrt{-ib}}+\frac{3\,\mathrm{e}^{-ia}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ib}\,x\right)}{8\sqrt{ib}}-\frac{3\sin(b\,x^2+a)}{4x}$

```
[In] int(sin(b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4/x*sin(b*x^2+a)+3/4*b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(x*b^(1/2)
*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2)))+1/4*sin(3*b
*x^2+3*a)/x-1/4*b^(1/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*FresnelC(2^(1/2)
/Pi^(1/2)*3^(1/2)*b^(1/2)*x)-sin(3*a)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(
1/2)*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(a+bx^2)}{x^2}\,dx =$$
$$-\frac{\sqrt{6}\pi x\sqrt{\frac{b}{\pi}}\cos(3\,a)\,\mathrm{C}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)-3\,\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)-\sqrt{6}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)\sin(3\,a)+}{4\,x}$$

```
[In] integrate(sin(b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(6)*pi*x*sqrt(b/pi)*cos(3*a)*fresnel_cos(sqrt(6)*x*sqrt(b/pi)) -
3*sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x*sqrt(b/pi)) - sqrt(6
)*pi*x*sqrt(b/pi)*fresnel_sin(sqrt(6)*x*sqrt(b/pi))*sin(3*a) + 3*sqrt(2)*pi
*x*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi))*sin(a) - 4*(cos(b*x^2 + a)^
2 - 1)*sin(b*x^2 + a))/x
```

Sympy [F]

$$\int \frac{\sin^3(a+bx^2)}{x^2} dx = \int \frac{\sin^3(a+bx^2)}{x^2} dx$$

```
[In] integrate(sin(b*x**2+a)**3/x**2,x)
[Out] Integral(sin(a + b*x**2)**3/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.90

$$\int \frac{\sin^3(a+bx^2)}{x^2} dx = \frac{\sqrt{3}\sqrt{bx^2}\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{1}{2},3i\,bx^2\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},-3i\,bx^2\right)\right)\cos(3\,a)+\left((i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},3i\,bx^2\right)-\right.}{}$$

```
[In] integrate(sin(b*x^2+a)^3/x^2,x, algorithm="maxima")
[Out] 1/32*(sqrt(3)*sqrt(b*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, 3*I*b*x^2) - (I + 1
)*sqrt(2)*gamma(-1/2, -3*I*b*x^2))*cos(3*a) + ((I + 1)*sqrt(2)*gamma(-1/2,
3*I*b*x^2) - (I - 1)*sqrt(2)*gamma(-1/2, -3*I*b*x^2))*sin(3*a)) - 3*sqrt(b*
x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I + 1)*sqrt(2)*gamma(-1/2, -
I*b*x^2))*cos(a) + ((I + 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I - 1)*sqrt(2)*
gamma(-1/2, -I*b*x^2))*sin(a)))/x
```

Giac [F]

$$\int \frac{\sin^3(a+bx^2)}{x^2} dx = \int \frac{\sin(bx^2+a)^3}{x^2} dx$$

```
[In] integrate(sin(b*x^2+a)^3/x^2,x, algorithm="giac")
[Out] integrate(sin(b*x^2 + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx^2)}{x^2}\,dx = \int \frac{\sin\left(b\,x^2+a\right)^3}{x^2}\,dx$$

```
[In] int(sin(a + b*x^2)^3/x^2,x)
[Out] int(sin(a + b*x^2)^3/x^2, x)
```

3.31 $\int x^2 \sin^3(x^2)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 71

$$\int x^2 \sin^3(x^2)\, dx = -\frac{1}{2}x\cos(x^2) + \frac{1}{6}x\cos^3(x^2) + \frac{3}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{24}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right)$$

```
[Out] -1/2*x*cos(x^2)+1/6*x*cos(x^2)^3-1/144*FresnelC(x*6^(1/2)/Pi^(1/2))*6^(1/2)
*Pi^(1/2)+3/16*FresnelC(x*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3484, 3466, 3433}

$$\int x^2 \sin^3(x^2)\, dx = \frac{3}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{24}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right) - \frac{3}{8}x\cos(x^2) + \frac{1}{24}x\cos(3x^2)$$

```
[In] Int[x^2*Sin[x^2]^3,x]
```

```
[Out] (-3*x*Cos[x^2])/8 + (x*Cos[3*x^2])/24 + (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*x
])/8 - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*x])/24
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{3}{4}x^2\sin\left(x^2\right)-\frac{1}{4}x^2\sin\left(3x^2\right)\right)\,dx\\
&= -\left(\frac{1}{4}\int x^2\sin\left(3x^2\right)\,dx\right)+\frac{3}{4}\int x^2\sin\left(x^2\right)\,dx\\
&= -\frac{3}{8}x\cos\left(x^2\right)+\frac{1}{24}x\cos\left(3x^2\right)-\frac{1}{24}\int\cos\left(3x^2\right)\,dx+\frac{3}{8}\int\cos\left(x^2\right)\,dx\\
&= -\frac{3}{8}x\cos\left(x^2\right)+\frac{1}{24}x\cos\left(3x^2\right)+\frac{3}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right)-\frac{1}{24}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$
\int x^2\sin^3\left(x^2\right)\,dx = \frac{1}{144}\left(6x\left(-9\cos\left(x^2\right)+\cos\left(3x^2\right)\right)+27\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right)-\sqrt{6\pi}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right)\right)
$$

```
[In] Integrate[x^2*Sin[x^2]^3,x]
```

```
[Out] (6*x*(-9*Cos[x^2] + Cos[3*x^2]) + 27*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*x] - Sq
rt[6*Pi]*FresnelC[Sqrt[6/Pi]*x])/144
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

method	result
default	$-\frac{3x\cos\left(x^{2}\right)}{8}+\frac{3\,\mathrm{C}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{16}+\frac{x\cos\left(3x^{2}\right)}{24}-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{\sqrt{\pi}}\right)}{144}$
risch	$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3i}\,x\right)}{96\sqrt{-3i}}+\frac{(-1)^{\frac{3}{4}}\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\,(-1)^{\frac{1}{4}}x\right)}{288}-\frac{3(-1)^{\frac{3}{4}}\sqrt{\pi}\,\mathrm{erf}\left((-1)^{\frac{1}{4}}x\right)}{32}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-i}\,x\right)}{32\sqrt{-i}}-\frac{3x\cos\left(x^{2}\right)}{8}+\frac{x\cos}{}$

```
[In] int(x^2*sin(x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*x*cos(x^2)+3/16*FresnelC(x*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)+1/24*x*c
os(3*x^2)-1/144*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*
x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int x^2\sin^3\left(x^2\right)\,dx=\frac{1}{6}\,x\cos\left(x^2\right)^3-\frac{1}{2}\,x\cos\left(x^2\right)-\frac{1}{144}\,\sqrt{6}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{6}x}{\sqrt{\pi}}\right)+\frac{3}{16}\,\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}x}{\sqrt{\pi}}\right)$$

```
[In] integrate(x^2*sin(x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/6*x*cos(x^2)^3 - 1/2*x*cos(x^2) - 1/144*sqrt(6)*sqrt(pi)*fresnel_cos(sqrt
(6)*x/sqrt(pi)) + 3/16*sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*x/sqrt(pi))
```

Sympy [A] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.63

$$\int x^2\sin^3\left(x^2\right)\,dx=-\frac{15x\cos\left(x^2\right)\Gamma\left(\frac{5}{4}\right)}{32\Gamma\left(\frac{9}{4}\right)}+\frac{5x\cos\left(3x^2\right)\Gamma\left(\frac{5}{4}\right)}{96\Gamma\left(\frac{9}{4}\right)}+\frac{15\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{64\Gamma\left(\frac{9}{4}\right)}-\frac{5\sqrt{6}\sqrt{\pi}C\left(\frac{\sqrt{6}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{576\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate(x**2*sin(x**2)**3,x)
```

```
[Out] -15*x*cos(x**2)*gamma(5/4)/(32*gamma(9/4)) + 5*x*cos(3*x**2)*gamma(5/4)/(96
*gamma(9/4)) + 15*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*x/sqrt(pi))*gamma(5/4)/
(64*gamma(9/4)) - 5*sqrt(6)*sqrt(pi)*fresnelc(sqrt(6)*x/sqrt(pi))*gamma(5/4
)/(576*gamma(9/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int x^2 \sin^3(x^2)\, dx = \frac{1}{24}x\cos(3x^2) - \frac{3}{8}x\cos(x^2) + \frac{1}{1152}\sqrt{\pi}\left((2i-2)\sqrt{3}\sqrt{2}\,\mathrm{erf}\left(\sqrt{3i}x\right) - (2i+2)\sqrt{3}\sqrt{2}\,\mathrm{erf}\left(\sqrt{-3i}x\right) - (27i-27)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\right.\right.$$

```
[In] integrate(x^2*sin(x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*x*cos(3*x^2) - 3/8*x*cos(x^2) + 1/1152*sqrt(pi)*((2*I - 2)*sqrt(3)*sqr
t(2)*erf(sqrt(3*I)*x) - (2*I + 2)*sqrt(3)*sqrt(2)*erf(sqrt(-3*I)*x) - (27*I
 - 27)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*x) - (27*I + 27)*sqrt(2)*erf((1/2*
I - 1/2)*sqrt(2)*x) + (27*I + 27)*sqrt(2)*erf(sqrt(-I)*x) - (27*I - 27)*sqr
t(2)*erf((-1)^(1/4)*x))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\begin{aligned}\int x^2 \sin^3(x^2)\, dx = &\left(\frac{1}{576}i + \frac{1}{576}\right)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{6}x\right) \\ &- \left(\frac{1}{576}i - \frac{1}{576}\right)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{6}x\right) \\ &- \left(\frac{3}{64}i + \frac{3}{64}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}x\right) \\ &+ \left(\frac{3}{64}i - \frac{3}{64}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{2}x\right) \\ &+ \frac{1}{48}xe^{(3i x^2)} - \frac{3}{16}xe^{(i x^2)} - \frac{3}{16}xe^{(-i x^2)} + \frac{1}{48}xe^{(-3i x^2)}\end{aligned}$$

```
[In] integrate(x^2*sin(x^2)^3,x, algorithm="giac")
```

```
[Out] (1/576*I + 1/576)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*x) - (1/576*I
- 1/576)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*x) - (3/64*I + 3/64)*s
qrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*x) + (3/64*I - 3/64)*sqrt(2)*sqrt
(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*x) + 1/48*x*e^(3*I*x^2) - 3/16*x*e^(I*x^2)
- 3/16*x*e^(-I*x^2) + 1/48*x*e^(-3*I*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sin^3\left(x^2\right)\,dx = \int x^2\,\sin\left(x^2\right)^3\,dx$$

```
[In] int(x^2*sin(x^2)^3,x)
```

```
[Out] int(x^2*sin(x^2)^3, x)
```

3.32 $\int x^4 \cos(x^2) \sin^2(x^2)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 84

$$\int x^4 \cos(x^2) \sin^2(x^2)\, dx = \frac{1}{4}x\cos(x^2) - \frac{1}{12}x\cos^3(x^2) - \frac{3}{16}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right) + \frac{1}{48}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right) + \frac{1}{6}x^3\sin^3(x^2)$$

```
[Out] 1/4*x*cos(x^2)-1/12*x*cos(x^2)^3+1/6*x^3*sin(x^2)^3+1/288*FresnelC(x*6^(1/2
)/Pi^(1/2))*6^(1/2)*Pi^(1/2)-3/32*FresnelC(x*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(
1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3524, 3484, 3466, 3433}

$$\int x^4 \cos(x^2) \sin^2(x^2)\, dx = -\frac{3}{16}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right) + \frac{1}{48}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right) + \frac{3}{16}x\cos(x^2) - \frac{1}{48}x\cos(3x^2) + \frac{1}{6}x^3\sin^3(x^2)$$

```
[In] Int[x^4*Cos[x^2]*Sin[x^2]^2,x]
```

```
[Out] (3*x*Cos[x^2])/16 - (x*Cos[3*x^2])/48 - (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*x
])/16 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*x])/48 + (x^3*Sin[x^2]^3)/6
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{6}x^3\sin^3\left(x^2\right)-\frac{1}{2}\int x^2\sin^3\left(x^2\right)\,dx\\
&= \frac{1}{6}x^3\sin^3\left(x^2\right)-\frac{1}{2}\int\left(\frac{3}{4}x^2\sin\left(x^2\right)-\frac{1}{4}x^2\sin\left(3x^2\right)\right)\,dx\\
&= \frac{1}{6}x^3\sin^3\left(x^2\right)+\frac{1}{8}\int x^2\sin\left(3x^2\right)\,dx-\frac{3}{8}\int x^2\sin\left(x^2\right)\,dx\\
&= \frac{3}{16}x\cos\left(x^2\right)-\frac{1}{48}x\cos\left(3x^2\right)+\frac{1}{6}x^3\sin^3\left(x^2\right)+\frac{1}{48}\int\cos\left(3x^2\right)\,dx-\frac{3}{16}\int\cos\left(x^2\right)\,dx\\
&= \frac{3}{16}x\cos\left(x^2\right)-\frac{1}{48}x\cos\left(3x^2\right)-\frac{3}{16}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right)\\
&\quad+\frac{1}{48}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right)+\frac{1}{6}x^3\sin^3\left(x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int x^4 \cos \left(x^2\right) \sin ^2\left(x^2\right) \, dx=\frac{1}{288} \left(-27 \sqrt{2 \pi } \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi }} x\right)+\sqrt{6 \pi } \operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi }} x\right)+6 x \left(9 \cos \left(x^2\right)-\cos \left(3 x^2\right)+8 x^2 \sin ^3\left(x^2\right)\right)\right)$$

```
[In] Integrate[x^4*Cos[x^2]*Sin[x^2]^2,x]
```

```
[Out] (-27*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*x] + Sqrt[6*Pi]*FresnelC[Sqrt[6/Pi]*x]
+ 6*x*(9*Cos[x^2] - Cos[3*x^2] + 8*x^2*Sin[x^2]^3))/288
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

method	result
default	$\frac{x^{3} \sin\left(x^{2}\right)}{8}+\frac{3 x \cos\left(x^{2}\right)}{16}-\frac{3 \operatorname{C}\left(\frac{x \sqrt{2}}{\sqrt{\pi}}\right) \sqrt{2}\, \sqrt{\pi}}{32}-\frac{x^{3} \sin\left(3 x^{2}\right)}{24}-\frac{x \cos\left(3 x^{2}\right)}{48}+\frac{\sqrt{2}\, \sqrt{\pi}\, \sqrt{3}\, \operatorname{C}\left(\frac{\sqrt{2}\, \sqrt{3}\, x}{\sqrt{\pi}}\right)}{288}$
risch	$-\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{\pi}\, \sqrt{3}\, \operatorname{erf}\left(\sqrt{3}\, \left(-1\right)^{\frac{1}{4}} x\right)}{576}+\frac{3 \left(-1\right)^{\frac{3}{4}} \sqrt{\pi}\, \operatorname{erf}\left(\left(-1\right)^{\frac{1}{4}} x\right)}{64}-\frac{3 \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-i}\, x\right)}{64 \sqrt{-i}}+\frac{\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-3 i}\, x\right)}{192 \sqrt{-3 i}}+\frac{3 x \cos\left(x^{2}\right)}{16}+\frac{x^{3} \sin(}{8}$

```
[In] int(x^4*cos(x^2)*sin(x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^3*sin(x^2)+3/16*x*cos(x^2)-3/32*FresnelC(x*2^(1/2)/Pi^(1/2))*2^(1/2)*
Pi^(1/2)-1/24*x^3*sin(3*x^2)-1/48*x*cos(3*x^2)+1/288*2^(1/2)*Pi^(1/2)*3^(1/
2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int x^4 \cos \left(x^2\right) \sin ^2\left(x^2\right) \, dx=-\frac{1}{12} \, x \cos \left(x^{2}\right)^{3} + \frac{1}{4} \, x \cos \left(x^{2}\right) + \frac{1}{288} \, \sqrt{6} \sqrt{\pi} \operatorname{C}\left(\frac{\sqrt{6} x}{\sqrt{\pi}}\right) - \frac{3}{32} \, \sqrt{2} \sqrt{\pi} \operatorname{C}\left(\frac{\sqrt{2} x}{\sqrt{\pi}}\right) - \frac{1}{6} \, {\left(x^{3} \cos \left(x^{2}\right)^{2} - x^{3}\right)} \sin \left(x^{2}\right)$$

```
[In] integrate(x^4*cos(x^2)*sin(x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/12*x*cos(x^2)^3 + 1/4*x*cos(x^2) + 1/288*sqrt(6)*sqrt(pi)*fresnel_cos(sq
rt(6)*x/sqrt(pi)) - 3/32*sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*x/sqrt(pi)) -
 1/6*(x^3*cos(x^2)^2 - x^3)*sin(x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(82) = 164$.

Time = 2.11 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.46

$$\int x^4 \cos(x^2) \sin^2(x^2)\, dx = -\frac{9x^5\Gamma\left(-\frac{9}{4}\right)}{40\Gamma\left(-\frac{5}{4}\right)} + \frac{9x^3\sin(x^2)\Gamma\left(-\frac{9}{4}\right)}{32\Gamma\left(-\frac{5}{4}\right)} - \frac{5x^3\sin(x^2)\Gamma\left(-\frac{5}{4}\right)}{16\Gamma\left(-\frac{1}{4}\right)} + \frac{3x^3\sin(3x^2)\Gamma\left(-\frac{9}{4}\right)}{32\Gamma\left(-\frac{5}{4}\right)} + \frac{27x\cos(x^2)\Gamma\left(-\frac{9}{4}\right)}{64\Gamma\left(-\frac{5}{4}\right)} - \frac{15x\cos(x^2)\Gamma\left(-\frac{5}{4}\right)}{32\Gamma\left(-\frac{1}{4}\right)} + \frac{3x\cos(3x^2)\Gamma\left(-\frac{9}{4}\right)}{64\Gamma\left(-\frac{5}{4}\right)} + \frac{15\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}x}{\sqrt{\pi}}\right)\Gamma\left(-\frac{5}{4}\right)}{64\Gamma\left(-\frac{1}{4}\right)} - \frac{27\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}x}{\sqrt{\pi}}\right)\Gamma\left(-\frac{9}{4}\right)}{128\Gamma\left(-\frac{5}{4}\right)} - \frac{\sqrt{6}\sqrt{\pi}C\left(\frac{\sqrt{6}x}{\sqrt{\pi}}\right)\Gamma\left(-\frac{9}{4}\right)}{128\Gamma\left(-\frac{5}{4}\right)}$$

```
[In] integrate(x**4*cos(x**2)*sin(x**2)**2,x)
```

```
[Out] -9*x**5*gamma(-9/4)/(40*gamma(-5/4)) + 9*x**3*sin(x**2)*gamma(-9/4)/(32*gam
ma(-5/4)) - 5*x**3*sin(x**2)*gamma(-5/4)/(16*gamma(-1/4)) + 3*x**3*sin(3*x*
*2)*gamma(-9/4)/(32*gamma(-5/4)) + 27*x*cos(x**2)*gamma(-9/4)/(64*gamma(-5/
4)) - 15*x*cos(x**2)*gamma(-5/4)/(32*gamma(-1/4)) + 3*x*cos(3*x**2)*gamma(-
9/4)/(64*gamma(-5/4)) + 15*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*x/sqrt(pi))*ga
mma(-5/4)/(64*gamma(-1/4)) - 27*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*x/sqrt(pi
))*gamma(-9/4)/(128*gamma(-5/4)) - sqrt(6)*sqrt(pi)*fresnelc(sqrt(6)*x/sqrt
(pi))*gamma(-9/4)/(128*gamma(-5/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int x^4 \cos(x^2) \sin^2(x^2)\, dx = -\frac{1}{24}x^3\sin(3x^2) + \frac{1}{8}x^3\sin(x^2) - \frac{1}{48}x\cos(3x^2) + \frac{3}{16}x\cos(x^2) - \frac{1}{2304}\sqrt{\pi}\left((2i-2)\sqrt{3}\sqrt{2}\operatorname{erf}\left(\sqrt{3i}x\right) - (2i+2)\sqrt{3}\sqrt{2}\operatorname{erf}\left(\sqrt{-3i}x\right) - (27i-27)\sqrt{2}\operatorname{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\right.\right.$$

```
[In] integrate(x^4*cos(x^2)*sin(x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*x^3*sin(3*x^2) + 1/8*x^3*sin(x^2) - 1/48*x*cos(3*x^2) + 3/16*x*cos(x^
2) - 1/2304*sqrt(pi)*((2*I - 2)*sqrt(3)*sqrt(2)*erf(sqrt(3*I)*x) - (2*I + 2
)*sqrt(3)*sqrt(2)*erf(sqrt(-3*I)*x) - (27*I - 27)*sqrt(2)*erf((1/2*I + 1/2)
*sqrt(2)*x) - (27*I + 27)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*x) + (27*I + 27
)*sqrt(2)*erf(sqrt(-I)*x) - (27*I - 27)*sqrt(2)*erf((-1)^(1/4)*x))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.49

$$\begin{aligned}\int x^4 \cos\left(x^2\right) \sin^2\left(x^2\right)\, dx = & -\left(\frac{1}{1152}i + \frac{1}{1152}\right)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{6}x\right) \\ & + \left(\frac{1}{1152}i - \frac{1}{1152}\right)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{6}x\right) \\ & + \left(\frac{3}{128}i + \frac{3}{128}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}x\right) \\ & - \left(\frac{3}{128}i - \frac{3}{128}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{2}x\right) \\ & - \frac{1}{96}\left(-2i\,x^3 + x\right)e^{\left(3i\,x^2\right)} - \frac{1}{32}\left(2i\,x^3 - 3\,x\right)e^{\left(i\,x^2\right)} \\ & - \frac{1}{32}\left(-2i\,x^3 - 3\,x\right)e^{\left(-i\,x^2\right)} - \frac{1}{96}\left(2i\,x^3 + x\right)e^{\left(-3i\,x^2\right)}\end{aligned}$$

```
[In] integrate(x^4*cos(x^2)*sin(x^2)^2,x, algorithm="giac")
```

```
[Out] -(1/1152*I + 1/1152)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*x) + (1/115
2*I - 1/1152)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*x) + (3/128*I + 3
/128)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*x) - (3/128*I - 3/128)*sqr
t(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*x) - 1/96*(-2*I*x^3 + x)*e^(3*I*x^
2) - 1/32*(2*I*x^3 - 3*x)*e^(I*x^2) - 1/32*(-2*I*x^3 - 3*x)*e^(-I*x^2) - 1/
96*(2*I*x^3 + x)*e^(-3*I*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \cos\left(x^2\right) \sin^2\left(x^2\right)\, dx = \int x^4\,\cos\left(x^2\right)\,{\sin\left(x^2\right)}^2\,dx$$

```
[In] int(x^4*cos(x^2)*sin(x^2)^2,x)
```

```
[Out] int(x^4*cos(x^2)*sin(x^2)^2, x)
```

3.33 $\int x \sin^7(a+bx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 67

$$\int x\sin^7\left(a+bx^2\right)\,dx=-\frac{\cos\left(a+bx^2\right)}{2b}+\frac{\cos^3\left(a+bx^2\right)}{2b}-\frac{3\cos^5\left(a+bx^2\right)}{10b}+\frac{\cos^7\left(a+bx^2\right)}{14b}$$

```
[Out] -1/2*cos(b*x^2+a)/b+1/2*cos(b*x^2+a)^3/b-3/10*cos(b*x^2+a)^5/b+1/14*cos(b*x
^2+a)^7/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3460, 2713}

$$\int x\sin^7\left(a+bx^2\right)\,dx=\frac{\cos^7\left(a+bx^2\right)}{14b}-\frac{3\cos^5\left(a+bx^2\right)}{10b}+\frac{\cos^3\left(a+bx^2\right)}{2b}-\frac{\cos\left(a+bx^2\right)}{2b}$$

```
[In] Int[x*Sin[a + b*x^2]^7,x]
```

```
[Out] -1/2*Cos[a + b*x^2]/b + Cos[a + b*x^2]^3/(2*b) - (3*Cos[a + b*x^2]^5)/(10*b
) + Cos[a + b*x^2]^7/(14*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sin^7(a+bx)\,dx, x, x^2\right)\\
&= -\frac{\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx, x, \cos\left(a+bx^2\right)\right)}{2b}\\
&= -\frac{\cos\left(a+bx^2\right)}{2b}+\frac{\cos^3\left(a+bx^2\right)}{2b}-\frac{3\cos^5\left(a+bx^2\right)}{10b}+\frac{\cos^7\left(a+bx^2\right)}{14b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int x\sin^7\left(a+bx^2\right)\,dx = -\frac{35\cos\left(a+bx^2\right)}{128b}+\frac{7\cos\left(3\left(a+bx^2\right)\right)}{128b}-\frac{7\cos\left(5\left(a+bx^2\right)\right)}{640b}+\frac{\cos\left(7\left(a+bx^2\right)\right)}{896b}$$

```
[In] Integrate[x*Sin[a + b*x^2]^7,x]
[Out] (-35*Cos[a + b*x^2])/(128*b) + (7*Cos[3*(a + b*x^2)])/(128*b) - (7*Cos[5*(a
 + b*x^2)])/(640*b) + Cos[7*(a + b*x^2)]/(896*b)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{\left(\frac{16}{5}+\sin^6\left(b\,x^2+a\right)+\frac{6\left(\sin^4\left(b\,x^2+a\right)\right)}{5}+\frac{8\left(\sin^2\left(b\,x^2+a\right)\right)}{5}\right)\cos\left(b\,x^2+a\right)}{14b}$	50
default	$-\frac{\left(\frac{16}{5}+\sin^6\left(b\,x^2+a\right)+\frac{6\left(\sin^4\left(b\,x^2+a\right)\right)}{5}+\frac{8\left(\sin^2\left(b\,x^2+a\right)\right)}{5}\right)\cos\left(b\,x^2+a\right)}{14b}$	50
parallelrisch	$\frac{-1024+245\cos\left(3b\,x^2+3a\right)-49\cos\left(5b\,x^2+5a\right)-1225\cos\left(b\,x^2+a\right)+5\cos\left(7b\,x^2+7a\right)}{4480b}$	57
risch	$-\frac{35\cos\left(b\,x^2+a\right)}{128b}+\frac{\cos\left(7b\,x^2+7a\right)}{896b}-\frac{7\cos\left(5b\,x^2+5a\right)}{640b}+\frac{7\cos\left(3b\,x^2+3a\right)}{128b}$	63

```
[In] int(x*sin(b*x^2+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/14/b*(16/5+sin(b*x^2+a)^6+6/5*sin(b*x^2+a)^4+8/5*sin(b*x^2+a)^2)*cos(b*x
^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int x\sin^7\left(a+bx^2\right)\,dx = \frac{5\cos\left(bx^2+a\right)^7-21\cos\left(bx^2+a\right)^5+35\cos\left(bx^2+a\right)^3-35\cos\left(bx^2+a\right)}{70\,b}$$

```
[In] integrate(x*sin(b*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/70*(5*cos(b*x^2 + a)^7 - 21*cos(b*x^2 + a)^5 + 35*cos(b*x^2 + a)^3 - 35*c
os(b*x^2 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.42

$$\int x\sin^7\left(a+bx^2\right)\,dx = \begin{cases} -\frac{\sin^6\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}-\frac{\sin^4\left(a+bx^2\right)\cos^3\left(a+bx^2\right)}{b}-\frac{4\sin^2\left(a+bx^2\right)\cos^5\left(a+bx^2\right)}{5b}-\frac{8\cos^7\left(a+bx^2\right)}{35b} & \text{for } b\neq 0 \\ \frac{x^2\sin^7\left(a\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*sin(b*x**2+a)**7,x)
```

```
[Out] Piecewise((-sin(a + b*x**2)**6*cos(a + b*x**2)/(2*b) - sin(a + b*x**2)**4*c
os(a + b*x**2)**3/b - 4*sin(a + b*x**2)**2*cos(a + b*x**2)**5/(5*b) - 8*cos
(a + b*x**2)**7/(35*b), Ne(b, 0)), (x**2*sin(a)**7/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int x\sin^7\left(a+bx^2\right)\,dx = \frac{5\cos\left(7\,bx^2+7\,a\right)-49\cos\left(5\,bx^2+5\,a\right)+245\cos\left(3\,bx^2+3\,a\right)-1225\cos\left(bx^2+a\right)}{4480\,b}$$

```
[In] integrate(x*sin(b*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/4480*(5*cos(7*b*x^2 + 7*a) - 49*cos(5*b*x^2 + 5*a) + 245*cos(3*b*x^2 + 3*
a) - 1225*cos(b*x^2 + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int x\sin^7\left(a+bx^2\right)\,dx = \frac{5\cos\left(bx^2+a\right)^7-21\cos\left(bx^2+a\right)^5+35\cos\left(bx^2+a\right)^3-35\cos\left(bx^2+a\right)}{70\,b}$$

```
[In] integrate(x*sin(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 1/70*(5*cos(b*x^2 + a)^7 - 21*cos(b*x^2 + a)^5 + 35*cos(b*x^2 + a)^3 - 35*c
os(b*x^2 + a))/b
```

Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int x\sin^7\left(a+bx^2\right)\,dx = \frac{245\cos\left(3\,b\,x^2+3\,a\right)-49\cos\left(5\,b\,x^2+5\,a\right)+5\cos\left(7\,b\,x^2+7\,a\right)-1225\cos\left(b\,x^2+a\right)}{4480\,b}$$

```
[In] int(x*sin(a + b*x^2)^7,x)
```

```
[Out] (245*cos(3*a + 3*b*x^2) - 49*cos(5*a + 5*b*x^2) + 5*cos(7*a + 7*b*x^2) - 12
25*cos(a + b*x^2))/(4480*b)
```

3.34 $\int \frac{(1+\sin(x^2))^2}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 44

$$\int \frac{(1+\sin(x^2))^2}{x^3}\,dx = -\frac{3}{4x^2} + \frac{\cos(2x^2)}{4x^2} + \text{CosIntegral}(x^2) - \frac{\sin(x^2)}{x^2} + \frac{\text{Si}(2x^2)}{2}$$

```
[Out] -3/4/x^2+Ci(x^2)+1/4*cos(2*x^2)/x^2+1/2*Si(2*x^2)-sin(x^2)/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3484, 3461, 3378, 3380, 3460, 3383}

$$\int \frac{(1+\sin(x^2))^2}{x^3}\,dx = \text{CosIntegral}(x^2) + \frac{\text{Si}(2x^2)}{2} - \frac{3}{4x^2} - \frac{\sin(x^2)}{x^2} + \frac{\cos(2x^2)}{4x^2}$$

```
[In] Int[(1 + Sin[x^2])^2/x^3,x]
```

```
[Out] -3/(4*x^2) + Cos[2*x^2]/(4*x^2) + CosIntegral[x^2] - Sin[x^2]/x^2 + SinInte
gral[2*x^2]/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{3}{2x^3} - \frac{\cos(2x^2)}{2x^3} + \frac{2\sin(x^2)}{x^3}\right) dx \\
&= -\frac{3}{4x^2} - \frac{1}{2}\int \frac{\cos(2x^2)}{x^3}\, dx + 2\int \frac{\sin(x^2)}{x^3}\, dx \\
&= -\frac{3}{4x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{\cos(2x)}{x^2}\, dx, x, x^2\right) + \text{Subst}\left(\int \frac{\sin(x)}{x^2}\, dx, x, x^2\right) \\
&= -\frac{3}{4x^2} + \frac{\cos(2x^2)}{4x^2} - \frac{\sin(x^2)}{x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{\sin(2x)}{x}\, dx, x, x^2\right) + \text{Subst}\left(\int \frac{\cos(x)}{x}\, dx, x, x^2\right) \\
&= -\frac{3}{4x^2} + \frac{\cos(2x^2)}{4x^2} + \text{CosIntegral}\left(x^2\right) - \frac{\sin(x^2)}{x^2} + \frac{\text{Si}(2x^2)}{2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{(1+\sin(x^2))^2}{x^3}\,dx = \frac{-3+\cos(2x^2)+4x^2\,\text{CosIntegral}(x^2)-4\sin(x^2)+2x^2\text{Si}(2x^2)}{4x^2}$$

```
[In] Integrate[(1 + Sin[x^2])^2/x^3,x]
```

```
[Out] (-3 + Cos[2*x^2] + 4*x^2*CosIntegral[x^2] - 4*Sin[x^2] + 2*x^2*SinIntegral[
2*x^2])/(4*x^2)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{3}{4x^2}+\text{Ci}(x^2)+\frac{\cos(2x^2)}{4x^2}+\frac{\text{Si}(2x^2)}{2}-\frac{\sin(x^2)}{x^2}$	39
parts	$-\frac{3}{4x^2}+\text{Ci}(x^2)+\frac{\cos(2x^2)}{4x^2}+\frac{\text{Si}(2x^2)}{2}-\frac{\sin(x^2)}{x^2}$	39
risch	$\text{Ci}(x^2)-\frac{i\pi\,\text{csgn}(ix^2)\,\text{csgn}(x^2)}{2}+\frac{i\pi\,\text{csgn}(ix^2)}{2}-\frac{3}{4x^2}-\frac{\pi\,\text{csgn}(x^2)}{4}+\frac{\text{Si}(2x^2)}{2}-\frac{\sin(x^2)}{x^2}+\frac{\cos(2x^2)}{4x^2}$	72

```
[In] int((1+sin(x^2))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4/x^2+Ci(x^2)+1/4*cos(2*x^2)/x^2+1/2*Si(2*x^2)-sin(x^2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(1+\sin(x^2))^2}{x^3}\,dx = \frac{2\,x^2\,\text{Ci}(x^2)+x^2\,\text{Si}(2\,x^2)+\cos(x^2)^2-2\,\sin(x^2)-2}{2\,x^2}$$

```
[In] integrate((1+sin(x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^2*cos_integral(x^2) + x^2*sin_integral(2*x^2) + cos(x^2)^2 - 2*sin
(x^2) - 2)/x^2
```

Sympy [A] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{(1+\sin(x^2))^2}{x^3}\,dx = -\log(x^2) + \frac{\log(x^4)}{2} + \operatorname{Ci}(x^2) + \frac{\operatorname{Si}(2x^2)}{2} - \frac{\sin(x^2)}{x^2} + \frac{\cos(2x^2)}{4x^2} - \frac{3}{4x^2}$$

```
[In] integrate((1+sin(x**2))**2/x**3,x)
[Out] -log(x**2) + log(x**4)/2 + Ci(x**2) + Si(2*x**2)/2 - sin(x**2)/x**2 + cos(2
*x**2)/(4*x**2) - 3/(4*x**2)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{(1+\sin(x^2))^2}{x^3}\,dx = \frac{x^2(i\,\Gamma(-1,2i\,x^2) - i\,\Gamma(-1,-2i\,x^2)) - 1}{4\,x^2} - \frac{1}{2\,x^2} + \frac{1}{2}\Gamma(-1,i\,x^2) + \frac{1}{2}\Gamma(-1,-i\,x^2)$$

```
[In] integrate((1+sin(x^2))^2/x^3,x, algorithm="maxima")
[Out] 1/4*(x^2*(I*gamma(-1, 2*I*x^2) - I*gamma(-1, -2*I*x^2)) - 1)/x^2 - 1/2/x^2
+ 1/2*gamma(-1, I*x^2) + 1/2*gamma(-1, -I*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{(1+\sin(x^2))^2}{x^3}\,dx = \frac{4\,x^2\operatorname{Ci}(x^2) + 2\,x^2\operatorname{Si}(2\,x^2) + \cos(2\,x^2) - 4\,\sin(x^2) - 3}{4\,x^2}$$

```
[In] integrate((1+sin(x^2))^2/x^3,x, algorithm="giac")
[Out] 1/4*(4*x^2*cos_integral(x^2) + 2*x^2*sin_integral(2*x^2) + cos(2*x^2) - 4*s
in(x^2) - 3)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+\sin(x^2))^2}{x^3}\,dx = \operatorname{cosint}\left(x^2\right) + \frac{\operatorname{sinint}(2\,x^2)}{2} - \frac{\sin(x^2)}{x^2} + \frac{\cos(x^2)^2}{2\,x^2} - \frac{1}{x^2}$$

```
[In] int((sin(x^2) + 1)^2/x^3,x)
[Out] cosint(x^2) + sinint(2*x^2)/2 - sin(x^2)/x^2 + cos(x^2)^2/(2*x^2) - 1/x^2
```

3.35 $\int \frac{x^5}{a+b\sin(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 362

$$\int \frac{x^5}{a+b\sin(c+dx^2)}\,dx = -\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}+\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} - \frac{x^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2}+\frac{x^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} - \frac{i\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^3}+\frac{i\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^3}$$

```
[Out] -1/2*I*x^4*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)
+1/2*I*x^4*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)
-x^2*polylog(2,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2
)+x^2*polylog(2,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/
2)-I*polylog(3,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/d^3/(a^2-b^2)^(1/2
)+I*polylog(3,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/d^3/(a^2-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3460, 3404, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{x^5}{a+b\sin(c+dx^2)}\,dx = -\frac{i\,\text{PolyLog}\left(3,\frac{ibe^{i(dx^2+c)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}}+\frac{i\,\text{PolyLog}\left(3,\frac{ibe^{i(dx^2+c)}}{a+\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}}-\frac{x^2\,\text{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}+\frac{x^2\,\text{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}-\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2d\sqrt{a^2-b^2}}+\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{2d\sqrt{a^2-b^2}}$$

```
[In] Int[x^5/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ((-1/2*I)*x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(Sqrt
[a^2 - b^2]*d) + ((I/2)*x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 -
 b^2])])/(Sqrt[a^2 - b^2]*d) - (x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a -
 Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^2) + (x^2*PolyLog[2, (I*b*E^(I*(c +
d*x^2)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^2) - (I*PolyLog[3, (I*b
*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^3) + (I*Poly
Log[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^3
)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+b\sin(c+dx)}\,dx, x, x^2\right)\\
&= \text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^2\right)\\
&= -\frac{(ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{a^2-b^2}}+\frac{(ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{a^2-b^2}}
\end{aligned}$$

$$= -\frac{ix^4 \log\left(1 - \frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} + \frac{ix^4 \log\left(1 - \frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} + \frac{i\text{Subst}\left(\int x \log\left(1 - \frac{2ibe^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right) dx, x, x^2\right)}{\sqrt{a^2-b^2}d} - \frac{i\text{Subst}\left(\int x \log\left(1 - \frac{2ibe^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right) dx, x, x^2\right)}{\sqrt{a^2-b^2}d}$$

$$= -\frac{ix^4 \log\left(1 - \frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} + \frac{ix^4 \log\left(1 - \frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} - \frac{x^2 \text{PolyLog}\left(2, \frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{x^2 \text{PolyLog}\left(2, \frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{\text{Subst}\left(\int \text{PolyLog}\left(2, \frac{2ibe^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right) dx, x, x^2\right)}{\sqrt{a^2-b^2}d^2} - \frac{\text{Subst}\left(\int \text{PolyLog}\left(2, \frac{2ibe^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right) dx, x, x^2\right)}{\sqrt{a^2-b^2}d^2}$$

$$= -\frac{ix^4 \log\left(1 - \frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} + \frac{ix^4 \log\left(1 - \frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} - \frac{x^2 \text{PolyLog}\left(2, \frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{x^2 \text{PolyLog}\left(2, \frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} - \frac{i\text{Subst}\left(\int \frac{\text{PolyLog}\left(2, \frac{ibx}{a-\sqrt{a^2-b^2}}\right)}{x} dx, x, e^{i\left(c+dx^2\right)}\right)}{\sqrt{a^2-b^2}d^3} + \frac{i\text{Subst}\left(\int \frac{\text{PolyLog}\left(2, \frac{ibx}{a+\sqrt{a^2-b^2}}\right)}{x} dx, x, e^{i\left(c+dx^2\right)}\right)}{\sqrt{a^2-b^2}d^3}$$

$$= -\frac{ix^4 \log\left(1 - \frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} + \frac{ix^4 \log\left(1 - \frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} - \frac{x^2 \text{PolyLog}\left(2, \frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{x^2 \text{PolyLog}\left(2, \frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} - \frac{i \text{PolyLog}\left(3, \frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^3} + \frac{i \text{PolyLog}\left(3, \frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^3}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{a+b\sin(c+dx^2)}\,dx = \frac{-2dx^2\operatorname{PolyLog}\left(2,-\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)-i\left(d^2x^4\log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)-d^2x^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)+2idx^2\operatorname{Poly}}{2\sqrt{a^2-b^2}d^3}$$

```
[In] Integrate[x^5/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (-2*d*x^2*PolyLog[2, ((-I)*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] - I
*(d^2*x^4*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] - d^2*x^4
*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])] + (2*I)*d*x^2*PolyL
og[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])] + 2*PolyLog[3, (I*b*E^
(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])] - 2*PolyLog[3, (I*b*E^(I*(c + d*x^2
)))/(a + Sqrt[a^2 - b^2])]))/(2*Sqrt[a^2 - b^2]*d^3)
```

Maple [F]

$$\int \frac{x^5}{a+b\sin(d\,x^2+c)}dx$$

```
[In] int(x^5/(a+b*sin(d*x^2+c)),x)
```

```
[Out] int(x^5/(a+b*sin(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1435 vs. 2(300) = 600.

Time = 0.46 (sec) , antiderivative size = 1435, normalized size of antiderivative = 3.96

$$\int \frac{x^5}{a+b\sin(c+dx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c) - a*sin(d
*x^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) - 2*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c)
- a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b
^2)/b^2) - b)/b + 1) - 2*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d
*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt
```

```
(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog((
-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + b*c^2*sqrt(-(a^2 - b^2)/b^2)*log
(2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2
*I*a) + b*c^2*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x
^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - b*c^2*sqrt(-(a^2 - b^2)/b^2
)*log(-2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^
2) + 2*I*a) - b*c^2*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x^2 + c) - 2*I*b*
sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b*d^2*x^4 - b*c^2)*
sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos
(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d^2*x
^4 - b*c^2)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x^2 +
 c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b
) - (b*d^2*x^4 - b*c^2)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x^2 + c) -
a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2
)/b^2) - b)/b) + (b*d^2*x^4 - b*c^2)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(
d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b) + 2*b*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*c
os(d*x^2 + c) + a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*
sqrt(-(a^2 - b^2)/b^2))/b) - 2*b*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*co
s(d*x^2 + c) + a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*s
qrt(-(a^2 - b^2)/b^2))/b) + 2*b*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*co
s(d*x^2 + c) + a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*s
qrt(-(a^2 - b^2)/b^2))/b) - 2*b*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*co
s(d*x^2 + c) + a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*s
qrt(-(a^2 - b^2)/b^2))/b))/((a^2 - b^2)*d^3)
```

Sympy [F]

$$\int \frac{x^5}{a + b\sin(c + dx^2)}\,dx = \int \frac{x^5}{a + b\sin(c + dx^2)}\,dx$$

```
[In] integrate(x**5/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral(x**5/(a + b*sin(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^5}{a + b\sin(c + dx^2)}\,dx = \int \frac{x^5}{b\sin(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^5/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate(x^5/(b*sin(d*x^2 + c) + a), x)
```

Giac [F]

$$\int \frac{x^5}{a + b\sin(c + dx^2)}\,dx = \int \frac{x^5}{b\sin(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^5/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^5/(b*sin(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{a + b\sin(c + dx^2)}\,dx = \int \frac{x^5}{a + b\,\sin(d\,x^2 + c)}\,dx$$

```
[In] int(x^5/(a + b*sin(c + d*x^2)),x)
[Out] int(x^5/(a + b*sin(c + d*x^2)), x)
```

3.36 $\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 245

$$\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx = -\frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d}+\frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} - \frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d^2}+\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d^2}$$

```
[Out] -1/2*I*x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)
+1/2*I*x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)
-1/2*polylog(2,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2
)+1/2*polylog(2,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3460, 3404, 2296, 2221, 2317, 2438}

$$\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx = -\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a-\sqrt{a^2-b^2}}\right)}{2d^2\sqrt{a^2-b^2}}+\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a+\sqrt{a^2-b^2}}\right)}{2d^2\sqrt{a^2-b^2}} - \frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2d\sqrt{a^2-b^2}}+\frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{2d\sqrt{a^2-b^2}}$$

```
[In] Int[x^3/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ((-1/2*I)*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(Sqrt
[a^2 - b^2]*d) + ((I/2)*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 -
 b^2])])/(Sqrt[a^2 - b^2]*d) - PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt
[a^2 - b^2])]/(2*Sqrt[a^2 - b^2]*d^2) + PolyLog[2, (I*b*E^(I*(c + d*x^2)))/
(a + Sqrt[a^2 - b^2])]/(2*Sqrt[a^2 - b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+b\sin(c+dx)}\,dx, x, x^2\right)\\
&= \text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^2\right)\\
&= -\frac{(ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{a^2-b^2}}+\frac{(ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{a^2-b^2}}\\
&= -\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}+\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}\\
&\quad+\frac{i\text{Subst}\left(\int \log\left(1-\frac{2ibe^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)\,dx, x, x^2\right)}{2\sqrt{a^2-b^2}d}\\
&\quad-\frac{i\text{Subst}\left(\int \log\left(1-\frac{2ibe^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right)\,dx, x, x^2\right)}{2\sqrt{a^2-b^2}d}\\
&= -\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}+\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}\\
&\quad+\frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{2ibx}{2a-2\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i\left(c+dx^2\right)}\right)}{2\sqrt{a^2-b^2}d^2}\\
&\quad-\frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{2ibx}{2a+2\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i\left(c+dx^2\right)}\right)}{2\sqrt{a^2-b^2}d^2}\\
&= -\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}+\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}\\
&\quad-\frac{\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d^2}+\frac{\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx = \frac{-idx^2\left(\log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)-\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)\right)-\text{PolyLog}\left(2,-\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)+\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d^2}$$

```
[In] Integrate[x^3/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ((-I)*d*x^2*(Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] - Log[
1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])]) - PolyLog[2, ((-I)*b*E^
(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] + PolyLog[2, (I*b*E^(I*(c + d*x^2)
))/(a + Sqrt[a^2 - b^2])])/(2*Sqrt[a^2 - b^2]*d^2)
```

Maple [F]

$$\int \frac{x^3}{a+b\sin(d\,x^2+c)}dx$$

```
[In] int(x^3/(a+b*sin(d*x^2+c)),x)
```

```
[Out] int(x^3/(a+b*sin(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(199) = 398.

Time = 0.42 (sec) , antiderivative size = 1041, normalized size of antiderivative = 4.25

$$\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + b*c*sqrt(-(a^2 - b^2)/b^2)*log(
2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*
I*a) - b*c*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - b*c*sqrt(-(a^2 - b^2)/b^2)*lo
g(-2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a) - I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c) - a*sin(d*x^
2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b
```

```
)/b + 1) + I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c) - a*sin(d*x
^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) + I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^2 + c) - a*sin(d
*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) - I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^2 + c) - a*sin
(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b + 1) + (b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2
+ c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^
2 - b^2)/b^2) - b)/b) - (b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*co
s(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(
-(-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^
2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/
b^2)*log(-(-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b
*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/((a^2 - b^2)*d^2)
```

Sympy [F]

$$\int \frac{x^3}{a + b\sin(c + dx^2)}\,dx = \int \frac{x^3}{a + b\sin(c + dx^2)}\,dx$$

```
[In] integrate(x**3/(a+b*sin(d*x**2+c)),x)
[Out] Integral(x**3/(a + b*sin(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{a + b\sin(c + dx^2)}\,dx = \int \frac{x^3}{b\sin(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^3/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate(x^3/(b*sin(d*x^2 + c) + a), x)
```

Giac [F]

$$\int \frac{x^3}{a + b\sin(c + dx^2)}\,dx = \int \frac{x^3}{b\sin(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^3/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^3/(b*sin(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a + b\sin(c + dx^2)}\,dx = \int \frac{x^3}{a + b\,\sin(d\,x^2 + c)}\,dx$$

```
[In] int(x^3/(a + b*sin(c + d*x^2)),x)
[Out] int(x^3/(a + b*sin(c + d*x^2)), x)
```

3.37 $\int \frac{x}{a+b\sin(c+dx^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{x}{a+b\sin(c+dx^2)}\,dx=\frac{\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}$$

```
[Out] arctan((b+a*tan(1/2*d*x^2+1/2*c))/(a^2-b^2)^(1/2))/d/(a^2-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3460, 2739, 632, 210}

$$\int \frac{x}{a+b\sin(c+dx^2)}\,dx=\frac{\arctan\left(\frac{a\tan\left(\frac{1}{2}(c+dx^2)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}$$

```
[In] Int[x/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]]/(Sqrt[a^2 - b^2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b\sin(c+dx)}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{d}\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{d}\\
&= \frac{\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\sin(c+dx^2)}\,dx = \frac{\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}$$

```
[In] Integrate[x/(a + b*Sin[c + d*x^2]),x]
[Out] ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]]/(Sqrt[a^2 - b^2]*d)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{2a\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}$	48
default	$\frac{\arctan\left(\frac{2a\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}$	48
risch	$-\frac{\ln\left(\mathrm{e}^{i\left(d\,x^2+c\right)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}\,d}+\frac{\ln\left(\mathrm{e}^{i\left(d\,x^2+c\right)}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{b\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}\,d}$	138

```
[In] int(x/(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x^2+1/2*c)+2*b)/(a^2-b^2)^(1/
2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 208, normalized size of antiderivative = 4.33

$$\int \frac{x}{a+b\sin(c+dx^2)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(2\,a^2-b^2)\cos(dx^2+c)^2-2\,ab\sin(dx^2+c)-a^2-b^2+2\,(a\cos(dx^2+c)\sin(dx^2+c)+b\cos(dx^2+c))\sqrt{-a^2+b^2}}{b^2\cos(dx^2+c)^2-2\,ab\sin(dx^2+c)-a^2-b^2}\right)}{4\,(a^2-b^2)d}, -\frac{\arctan\left(-\frac{a\sin(dx^2+c)+b}{\sqrt{a^2-b^2}\cos(dx^2+c)}\right)}{2\,\sqrt{a^2-b^2}d}\right]$$

```
[In] integrate(x/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^
2 + c) - a^2 - b^2 + 2*(a*cos(d*x^2 + c)*sin(d*x^2 + c) + b*cos(d*x^2 + c))
*sqrt(-a^2 + b^2))/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2
))/((a^2 - b^2)*d), -1/2*arctan(-(a*sin(d*x^2 + c) + b)/(sqrt(a^2 - b^2)*co
s(d*x^2 + c)))/(sqrt(a^2 - b^2)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(37) = 74$.

Time = 3.56 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.44

$$\int \frac{x}{a + b\sin(c + dx^2)}\,dx = \begin{cases} \frac{\tilde{\infty}x^2}{\sin(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx^2}{2}\right)\right)}{2bd} & \text{for } a = 0 \\ \frac{x^2}{2(a + b\sin(c))} & \text{for } d = 0 \\ \frac{1}{bd\tan\left(\frac{c}{2} + \frac{dx^2}{2}\right) - bd} & \text{for } a = -b \\ -\frac{1}{bd\tan\left(\frac{c}{2} + \frac{dx^2}{2}\right) + bd} & \text{for } a = b \\ \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx^2}{2}\right) + \frac{b}{a} - \frac{\sqrt{-a^2+b^2}}{a}\right)}{2d\sqrt{-a^2+b^2}} - \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx^2}{2}\right) + \frac{b}{a} + \frac{\sqrt{-a^2+b^2}}{a}\right)}{2d\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Piecewise((zoo*x**2/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c/2 +
 d*x**2/2))/(2*b*d), Eq(a, 0)), (x**2/(2*(a + b*sin(c))), Eq(d, 0)), (1/(b*
d*tan(c/2 + d*x**2/2) - b*d), Eq(a, -b)), (-1/(b*d*tan(c/2 + d*x**2/2) + b*
d), Eq(a, b)), (log(tan(c/2 + d*x**2/2) + b/a - sqrt(-a**2 + b**2)/a)/(2*d*
sqrt(-a**2 + b**2)) - log(tan(c/2 + d*x**2/2) + b/a + sqrt(-a**2 + b**2)/a)
/(2*d*sqrt(-a**2 + b**2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8078 vs. $2(43) = 86$.

Time = 24.83 (sec) , antiderivative size = 8078, normalized size of antiderivative = 168.29

$$\int \frac{x}{a + b\sin(c + dx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan2(-2*(4*(a^2*b^4 - b^6)*cos(d*x^2 + 2*c)^4*cos(c)*sin(c) - 4*(a^2
*b^4 - b^6)*cos(c)*sin(d*x^2 + 2*c)^4*sin(c) - 4*((a^3*b^3 - a*b^5)*cos(c)^
3 + 3*(a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 4*(3*(a^3*b^3
 - a*b^5)*cos(c)^2*sin(c) + (a^3*b^3 - a*b^5)*sin(c)^3 + ((a^2*b^4 - b^6)*c
os(c)^2 - (a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^3 +
4*((4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 +
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 b^6)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2 - 4*((4*a^4*b^2 - 5*a^2*b^4 + b^6
)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)*sin(c)^3 + 3*((a^3
*b^3 - a*b^5)*cos(c)^3 - (a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c
))*sin(d*x^2 + 2*c)^2 - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^5 + 2*(2*a^
5*b - 3*a^3*b^3 + a*b^5)*cos(c)^3*sin(c)^2 + (2*a^5*b - 3*a^3*b^3 + a*b^5)*
cos(c)*sin(c)^4)*cos(d*x^2 + 2*c) - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)
^4*sin(c) + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^2*sin(c)^3 + (2*a^5*b -
3*a^3*b^3 + a*b^5)*sin(c)^5 + ((a^2*b^4 - b^6)*cos(c)^2 - (a^2*b^4 - b^6)*s
in(c)^2)*cos(d*x^2 + 2*c)^3 - 3*((a^3*b^3 - a*b^5)*cos(c)^2*sin(c) - (a^3*b
^3 - a*b^5)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + ((4*a^4*b^2 - 5*a^2*b^4 + b^6)*c
os(c)^4 - (4*a^4*b^2 - 5*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^2 + 2*c))*sin(d*x
^2 + 2*c) + (b^5*cos(d*x^2 + 2*c)^5*cos(c) - 4*a*b^4*cos(d*x^2 + 2*c)^4*cos
(c)*sin(c) + b^5*sin(d*x^2 + 2*c)^5*sin(c) + (b^5*cos(d*x^2 + 2*c)*cos(c) +
 4*a*b^4*cos(c)*sin(c))*sin(d*x^2 + 2*c)^4 + 2*((2*a^2*b^3 - b^5)*cos(c)^3
+ 3*(2*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 + 2*(b^5*cos(d*x^
2 + 2*c)^2*sin(c) + 3*(2*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (2*a^2*b^3 - b^5)
*sin(c)^3 + 2*(a*b^4*cos(c)^2 - a*b^4*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2
 + 2*c)^3 - 4*((4*a^3*b^2 - 3*a*b^4)*cos(c)^3*sin(c) + (4*a^3*b^2 - 3*a*b^4
)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + 2*(b^5*cos(d*x^2 + 2*c)^3*cos(c) +
2*(4*a^3*b^2 - 3*a*b^4)*cos(c)^3*sin(c) + 2*(4*a^3*b^2 - 3*a*b^4)*cos(c)*si
n(c)^3 + 3*((2*a^2*b^3 - b^5)*cos(c)^3 - (2*a^2*b^3 - b^5)*cos(c)*sin(c)^2)
*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 + ((8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)
^5 + 2*(8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)^3*sin(c)^2 + (8*a^4*b - 8*a^2*b^3
 + b^5)*cos(c)*sin(c)^4)*cos(d*x^2 + 2*c) + (b^5*cos(d*x^2 + 2*c)^4*sin(c)
+ (8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)^4*sin(c) + 2*(8*a^4*b - 8*a^2*b^3 + b^
5)*cos(c)^2*sin(c)^3 + (8*a^4*b - 8*a^2*b^3 + b^5)*sin(c)^5 + 4*(a*b^4*cos(
c)^2 - a*b^4*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 6*((2*a^2*b^3 - b^5)*cos(c)^2*s
in(c) - (2*a^2*b^3 - b^5)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + 4*((4*a^3*b^2 - 3*
a*b^4)*cos(c)^4 - (4*a^3*b^2 - 3*a*b^4)*sin(c)^4)*cos(d*x^2 + 2*c))*sin(d*x
^2 + 2*c))*sqrt(a^2 - b^2))/(b^6*cos(d*x^2 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*
x^2 + 2*c)^5 + b^6*sin(d*x^2 + 2*c)^6 - 6*a*b^5*cos(d*x^2 + 2*c)^5*sin(c) +
 (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^6 + 3*(32*a^6 - 48*a^4*b^2
 + 18*a^2*b^4 - b^6)*cos(c)^4*sin(c)^2 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^
4 - b^6)*cos(c)^2*sin(c)^4 + (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*sin(c
)^6 + 3*((2*a^2*b^4 - b^6)*cos(c)^2 + 5*(2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x
^2 + 2*c)^4 + 3*(b^6*cos(d*x^2 + 2*c)^2 - 2*a*b^5*cos(d*x^2 + 2*c)*sin(c) +
 5*(2*a^2*b^4 - b^6)*cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^2 + 2*c
)^4 - 4*(3*(4*a^3*b^3 - 3*a*b^5)*cos(c)^2*sin(c) + 5*(4*a^3*b^3 - 3*a*b^5)*
sin(c)^3)*cos(d*x^2 + 2*c)^3 + 4*(3*a*b^5*cos(d*x^2 + 2*c)^2*cos(c) + 5*(4*
a^3*b^3 - 3*a*b^5)*cos(c)^3 - 6*(2*a^2*b^4 - b^6)*cos(d*x^2 + 2*c)*cos(c)*s
in(c) + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*sin(d*x^2 + 2*c)^3 + 3*((8
*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(
c)^2*sin(c)^2 + 5*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^2 + 2*c)^
2 + 3*(b^6*cos(d*x^2 + 2*c)^4 - 4*a*b^5*cos(d*x^2 + 2*c)^3*sin(c) + 5*(8*a^
4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^
```

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2*sin(c)^2 + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4 + 6*((2*a^2*b^4 - b^6)*
cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 4*(3*(4*a^3*b^3
 - 3*a*b^5)*cos(c)^2*sin(c) + (4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^2 + 2
*c))*sin(d*x^2 + 2*c)^2 - 6*((16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^4*sin
(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^2*sin(c)^3 + (16*a^5*b - 2
0*a^3*b^3 + 5*a*b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + 6*(a*b^5*cos(d*x^2 + 2*c)
^4*cos(c) + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^5 - 4*(2*a^2*b^4 - b^6
)*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*co
s(c)^3*sin(c)^2 + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)*sin(c)^4 + 2*((4
*a^3*b^3 - 3*a*b^5)*cos(c)^3 + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*cos
(d*x^2 + 2*c)^2 - 4*((8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (8*a^4
*b^2 - 8*a^2*b^4 + b^6)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)
 - 2*(3*b^5*cos(c)*sin(d*x^2 + 2*c)^5 - 3*b^5*cos(d*x^2 + 2*c)^5*sin(c) + (
16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^6 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)
*cos(c)^4*sin(c)^2 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^2*sin(c)^4 +
(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*sin(c)^6 + 3*(a*b^4*cos(c)^2 + 5*a*b^4*sin(
c)^2)*cos(d*x^2 + 2*c)^4 + 3*(5*a*b^4*cos(c)^2 - b^5*cos(d*x^2 + 2*c)*sin(c
) + a*b^4*sin(c)^2)*sin(d*x^2 + 2*c)^4 - 2*(3*(4*a^2*b^3 - b^5)*cos(c)^2*si
n(c) + 5*(4*a^2*b^3 - b^5)*sin(c)^3)*cos(d*x^2 + 2*c)^3 + 2*(3*b^5*cos(d*x^
2 + 2*c)^2*cos(c) - 12*a*b^4*cos(d*x^2 + 2*c)*cos(c)*sin(c) + 5*(4*a^2*b^3
- b^5)*cos(c)^3 + 3*(4*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*sin(d*x^2 + 2*c)^3 +
 6*((2*a^3*b^2 - a*b^4)*cos(c)^4 + 6*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^2
+ 5*(2*a^3*b^2 - a*b^4)*sin(c)^4)*cos(d*x^2 + 2*c)^2 - 6*(b^5*cos(d*x^2 + 2
*c)^3*sin(c) - 5*(2*a^3*b^2 - a*b^4)*cos(c)^4 - 6*(2*a^3*b^2 - a*b^4)*cos(c
)^2*sin(c)^2 - (2*a^3*b^2 - a*b^4)*sin(c)^4 - 3*(a*b^4*cos(c)^2 + a*b^4*sin
(c)^2)*cos(d*x^2 + 2*c)^2 + (3*(4*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (4*a^2*b
^3 - b^5)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - 3*((16*a^4*b - 1
2*a^2*b^3 + b^5)*cos(c)^4*sin(c) + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^2
*sin(c)^3 + (16*a^4*b - 12*a^2*b^3 + b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + 3*(b
^5*cos(d*x^2 + 2*c)^4*cos(c) - 8*a*b^4*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + (
16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^5 + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos
(c)^3*sin(c)^2 + (16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)*sin(c)^4 + 2*((4*a^2*
b^3 - b^5)*cos(c)^3 + 3*(4*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)
^2 - 16*((2*a^3*b^2 - a*b^4)*cos(c)^3*sin(c) + (2*a^3*b^2 - a*b^4)*cos(c)*s
in(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c))*sqrt(a^2 - b^2)), (b^6*cos(d*x
^2 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*x^2 + 2*c)^5 + b^6*sin(d*x^2 + 2*c)^6 -
6*a*b^5*cos(d*x^2 + 2*c)^5*sin(c) + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^6
+ 3*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4*sin(c)^2 + 3*(8*a^4*b^2 - 8*a^2*
b^4 + b^6)*cos(c)^2*sin(c)^4 + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^6 + ((4
*a^2*b^4 - b^6)*cos(c)^2 + 5*(4*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c)^4
 + (3*b^6*cos(d*x^2 + 2*c)^2 - 6*a*b^5*cos(d*x^2 + 2*c)*sin(c) + 5*(4*a^2*b
^4 - b^6)*cos(c)^2 + (4*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^2 + 2*c)^4 - 4*(3*
(2*a^3*b^3 - a*b^5)*cos(c)^2*sin(c) + 5*(2*a^3*b^3 - a*b^5)*sin(c)^3)*cos(d
*x^2 + 2*c)^3 + 4*(3*a*b^5*cos(d*x^2 + 2*c)^2*cos(c) + 5*(2*a^3*b^3 - a*b^5
)*cos(c)^3 - 2*(4*a^2*b^4 - b^6)*cos(d*x^2 + 2*c)*cos(c)*sin(c) + 3*(2*a^3*
```

b^3 - a*b^5)*cos(c)*sin(c)^2)*sin(d*x^2 + 2*c)^3 + ((8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^2*sin(c)^2 + 5*(8*a^4*b^2 - 4*a^2*b^4 - b^6)*sin(c)^4)*cos(d*x^2 + 2*c)^2 + (3*b^6*cos(d*x^2 + 2*c)^4 - 12*a*b^5*cos(d*x^2 + 2*c)^3*sin(c) + 5*(8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^2*sin(c)^2 + (8*a^4*b^2 - 4*a^2*b^4 - b^6)*sin(c)^4 + 6*((4*a^2*b^4 - b^6)*cos(c)^2 + (4*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 12*(3*(2*a^3*b^3 - a*b^5)*cos(c)^2*sin(c) + (2*a^3*b^3 - a*b^5)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - 2*((8*a^5*b - 5*a*b^5)*cos(c)^4*sin(c) + 2*(8*a^5*b - 5*a*b^5)*cos(c)^2*sin(c)^3 + (8*a^5*b - 5*a*b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + 2*(3*a*b^5*cos(d*x^2 + 2*c)^4*cos(c) + (8*a^5*b - 5*a*b^5)*cos(c)^5 - 4*(4*a^2*b^4 - b^6)*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + 2*(8*a^5*b - 5*a*b^5)*cos(c)^3*sin(c)^2 + (8*a^5*b - 5*a*b^5)*cos(c)*sin(c)^4 + 6*((2*a^3*b^3 - a*b^5)*cos(c)^3 + 3*(2*a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 4*((8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^3*sin(c) + (8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c) - 4*(b^5*cos(c)*sin(d*x^2 + 2*c)^5 - b^5*cos(d*x^2 + 2*c)^5*sin(c) + (2*a^3*b^2 - a*b^4)*cos(c)^6 + 3*(2*a^3*b^2 - a*b^4)*cos(c)^4*sin(c)^2 + 3*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^4 + (2*a^3*b^2 - a*b^4)*sin(c)^6 + (a*b^4*cos(c)^2 + 5*a*b^4*sin(c)^2)*cos(d*x^2 + 2*c)^4 + (5*a*b^4*cos(c)^2 - b^5*cos(d*x^2 + 2*c)*sin(c) + a*b^4*sin(c)^2)*sin(d*x^2 + 2*c)^4 - 2*(3*a^2*b^3*cos(c)^2*sin(c) + 5*a^2*b^3*sin(c)^3)*cos(d*x^2 + 2*c)^3 + 2*(b^5*cos(d*x^2 + 2*c)^2*cos(c) + 5*a^2*b^3*cos(c)^3 - 4*a*b^4*cos(d*x^2 + 2*c)*cos(c)*sin(c) + 3*a^2*b^3*cos(c)*sin(c)^2)*sin(d*x^2 + 2*c)^3 + 2*(a^3*b^2*cos(c)^4 + 6*a^3*b^2*cos(c)^2*sin(c)^2 + 5*a^3*b^2*sin(c)^4)*cos(d*x^2 + 2*c)^2 + 2*(5*a^3*b^2*cos(c)^4 - b^5*cos(d*x^2 + 2*c)^3*sin(c) + 6*a^3*b^2*cos(c)^2*sin(c)^2 + a^3*b^2*sin(c)^4 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 3*(3*a^2*b^3*cos(c)^2*sin(c) + a^2*b^3*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - ((4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)^4*sin(c) + 2*(4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)^2*sin(c)^3 + (4*a^4*b + 2*a^2*b^3 - b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + (b^5*cos(d*x^2 + 2*c)^4*cos(c) - 8*a*b^4*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + (4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)^5 + 2*(4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)^3*sin(c)^2 + (4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)*sin(c)^4 + 6*(a^2*b^3*cos(c)^3 + 3*a^2*b^3*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 16*(a^3*b^2*cos(c)^3*sin(c) + a^3*b^2*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c))*sqrt(a^2 - b^2))/(b^6*cos(d*x^2 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*x^2 + 2*c)^5 + b^6*sin(d*x^2 + 2*c)^6 - 6*a*b^5*cos(d*x^2 + 2*c)^5*sin(c) + (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^6 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^4*sin(c)^2 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^2*sin(c)^4 + (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*sin(c)^6 + 3*((2*a^2*b^4 - b^6)*cos(c)^2 + 5*(2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c)^4 + 3*(b^6*cos(d*x^2 + 2*c)^2 - 2*a*b^5*cos(d*x^2 + 2*c)*sin(c) + 5*(2*a^2*b^4 - b^6)*cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^2 + 2*c)^4 - 4*(3*(4*a^3*b^3 - 3*a*b^5)*cos(c)^2*sin(c) + 5*(4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^2 + 2*c)^3 + 4*(3*a*b^5*cos(d*x^2 + 2*c)^2*cos(c) + 5*(4*a^3*b^3 - 3*a*b^5)*cos(c

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)^3 - 6*(2*a^2*b^4 - b^6)*cos(d*x^2 + 2*c)*cos(c)*sin(c) + 3*(4*a^3*b^3 - 3
*a*b^5)*cos(c)*sin(c)^2)*sin(d*x^2 + 2*c)^3 + 3*((8*a^4*b^2 - 8*a^2*b^4 + b
^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^2*sin(c)^2 + 5*(8*a^4
*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^2 + 2*c)^2 + 3*(b^6*cos(d*x^2 + 2
*c)^4 - 4*a*b^5*cos(d*x^2 + 2*c)^3*sin(c) + 5*(8*a^4*b^2 - 8*a^2*b^4 + b^6)
*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^2*sin(c)^2 + (8*a^4*b^2
- 8*a^2*b^4 + b^6)*sin(c)^4 + 6*((2*a^2*b^4 - b^6)*cos(c)^2 + (2*a^2*b^4 -
b^6)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 4*(3*(4*a^3*b^3 - 3*a*b^5)*cos(c)^2*sin
(c) + (4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2
- 6*((16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^4*sin(c) + 2*(16*a^5*b - 20*a
^3*b^3 + 5*a*b^5)*cos(c)^2*sin(c)^3 + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*sin
(c)^5)*cos(d*x^2 + 2*c) + 6*(a*b^5*cos(d*x^2 + 2*c)^4*cos(c) + (16*a^5*b -
20*a^3*b^3 + 5*a*b^5)*cos(c)^5 - 4*(2*a^2*b^4 - b^6)*cos(d*x^2 + 2*c)^3*cos
(c)*sin(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^3*sin(c)^2 + (16*a^
5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)*sin(c)^4 + 2*((4*a^3*b^3 - 3*a*b^5)*cos(
c)^3 + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 4*((8*
a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*
cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c) - 2*(3*b^5*cos(c)*sin(d
*x^2 + 2*c)^5 - 3*b^5*cos(d*x^2 + 2*c)^5*sin(c) + (16*a^5 - 16*a^3*b^2 + 3*
a*b^4)*cos(c)^6 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^4*sin(c)^2 + 3*(
16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^2*sin(c)^4 + (16*a^5 - 16*a^3*b^2 + 3
*a*b^4)*sin(c)^6 + 3*(a*b^4*cos(c)^2 + 5*a*b^4*sin(c)^2)*cos(d*x^2 + 2*c)^4
 + 3*(5*a*b^4*cos(c)^2 - b^5*cos(d*x^2 + 2*c)*sin(c) + a*b^4*sin(c)^2)*sin(
d*x^2 + 2*c)^4 - 2*(3*(4*a^2*b^3 - b^5)*cos(c)^2*sin(c) + 5*(4*a^2*b^3 - b^
5)*sin(c)^3)*cos(d*x^2 + 2*c)^3 + 2*(3*b^5*cos(d*x^2 + 2*c)^2*cos(c) - 12*a
*b^4*cos(d*x^2 + 2*c)*cos(c)*sin(c) + 5*(4*a^2*b^3 - b^5)*cos(c)^3 + 3*(4*a
^2*b^3 - b^5)*cos(c)*sin(c)^2)*sin(d*x^2 + 2*c)^3 + 6*((2*a^3*b^2 - a*b^4)*
cos(c)^4 + 6*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^2 + 5*(2*a^3*b^2 - a*b^4)*
sin(c)^4)*cos(d*x^2 + 2*c)^2 - 6*(b^5*cos(d*x^2 + 2*c)^3*sin(c) - 5*(2*a^3*
b^2 - a*b^4)*cos(c)^4 - 6*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^2 - (2*a^3*b^
2 - a*b^4)*sin(c)^4 - 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*cos(d*x^2 + 2*c)^
2 + (3*(4*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (4*a^2*b^3 - b^5)*sin(c)^3)*cos(
d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - 3*((16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^
4*sin(c) + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^2*sin(c)^3 + (16*a^4*b -
12*a^2*b^3 + b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + 3*(b^5*cos(d*x^2 + 2*c)^4*co
s(c) - 8*a*b^4*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + (16*a^4*b - 12*a^2*b^3 +
b^5)*cos(c)^5 + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^3*sin(c)^2 + (16*a^4
*b - 12*a^2*b^3 + b^5)*cos(c)*sin(c)^4 + 2*((4*a^2*b^3 - b^5)*cos(c)^3 + 3*
(4*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 16*((2*a^3*b^2 - a*
b^4)*cos(c)^3*sin(c) + (2*a^3*b^2 - a*b^4)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c
))*sin(d*x^2 + 2*c))*sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.31

$$\int \frac{x}{a+b\sin(c+dx^2)}\,dx=\frac{\pi\left\lfloor\frac{dx^2+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(\frac{1}{2}dx^2+\frac{1}{2}c)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}$$

```
[In] integrate(x/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] (pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^2 + 1/2*
c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.67

$$\int \frac{x}{a+b\sin(c+dx^2)}\,dx = -\frac{\ln\left(-x\,e^{d\,x^2\,1i}\,e^{c\,1i}\,2i-\frac{2\,x\,\left(b\,1i+a\,e^{d\,x^2\,1i}\,e^{c\,1i}\right)}{\sqrt{a+b}\,\sqrt{b-a}}\right)-\ln\left(-x\,e^{d\,x^2\,1i}\,e^{c\,1i}\,2i+\frac{2\,x\,\left(b\,1i+a\,e^{d\,x^2\,1i}\,e^{c\,1i}\right)}{\sqrt{a+b}\,\sqrt{b-a}}\right)}{2\,d\,\sqrt{a+b}\,\sqrt{b-a}}$$

```
[In] int(x/(a + b*sin(c + d*x^2)),x)
```

```
[Out] -(log(- x*exp(d*x^2*1i)*exp(c*1i)*2i - (2*x*(b*1i + a*exp(d*x^2*1i)*exp(c*1
i)))/((a + b)^(1/2)*(b - a)^(1/2))) - log((2*x*(b*1i + a*exp(d*x^2*1i)*exp(
c*1i)))/((a + b)^(1/2)*(b - a)^(1/2)) - x*exp(d*x^2*1i)*exp(c*1i)*2i))/(2*d
*(a + b)^(1/2)*(b - a)^(1/2))
```

3.38 $\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sin(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*Sin[c + d*x^2])),x]
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*x^2])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sin[c + d*x^2])),x]
[Out] Integrate[1/(x*(a + b*Sin[c + d*x^2])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sin\left(d\,x^2+c\right)\right)}dx$$

```
[In] int(1/x/(a+b*sin(d*x^2+c)),x)
[Out] int(1/x/(a+b*sin(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] integral(1/(b*x*sin(d*x^2 + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x**2+c)),x)
[Out] Integral(1/(x*(a + b*sin(c + d*x**2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 6.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)}\,dx$$

```
[In] int(1/(x*(a + b*sin(c + d*x^2))),x)
[Out] int(1/(x*(a + b*sin(c + d*x^2))), x)
```

3.39 $\int \frac{1}{x^3(a+b\sin(c+dx^2))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\text{Int}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/(a+b*sin(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

```
[In] Int[1/(x^3*(a + b*Sin[c + d*x^2])),x]
[Out] Defer[Int][1/(x^3*(a + b*Sin[c + d*x^2])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b \sin (c + dx^2))} \, dx = \int \frac{1}{x^3 (a + b \sin (c + dx^2))} \, dx$$

```
[In] Integrate[1/(x^3*(a + b*Sin[c + d*x^2])),x]
[Out] Integrate[1/(x^3*(a + b*Sin[c + d*x^2])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a + b \sin (d x^2 + c))} dx$$

```
[In] int(1/x^3/(a+b*sin(d*x^2+c)),x)
[Out] int(1/x^3/(a+b*sin(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^3 (a + b \sin (c + dx^2))} \, dx = \int \frac{1}{(b \sin (dx^2 + c) + a)x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] integral(1/(b*x^3*sin(d*x^2 + c) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 (a + b \sin (c + dx^2))} \, dx = \int \frac{1}{x^3 (a + b \sin (c + dx^2))} \, dx$$

```
[In] integrate(1/x**3/(a+b*sin(d*x**2+c)),x)
[Out] Integral(1/(x**3*(a + b*sin(c + d*x**2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\sin(c + dx^2))} dx = \int \frac{1}{(b\sin(dx^2 + c) + a)x^3} dx$$

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\sin(c + dx^2))} dx = \int \frac{1}{(b\sin(dx^2 + c) + a)x^3} dx$$

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 6.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\sin(c + dx^2))} dx = \int \frac{1}{x^3 \left(a + b \sin(d\,x^2 + c)\right)} dx$$

```
[In] int(1/(x^3*(a + b*sin(c + d*x^2))),x)
[Out] int(1/(x^3*(a + b*sin(c + d*x^2))), x)
```

3.40 $\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx = \text{Int}\left(\frac{x^2}{a+b\sin(c+dx^2)},x\right)$$

```
[Out] Unintegrable(x^2/(a+b*sin(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx = \int \frac{x^2}{a+b\sin(c+dx^2)}\,dx$$

```
[In] Int[x^2/(a + b*Sin[c + d*x^2]),x]
[Out] Defer[Int][x^2/(a + b*Sin[c + d*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{a+b\sin(c+dx^2)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx=\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx$$

```
[In] Integrate[x^2/(a + b*Sin[c + d*x^2]),x]
[Out] Integrate[x^2/(a + b*Sin[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\sin(d\,x^2+c)}dx$$

```
[In] int(x^2/(a+b*sin(d*x^2+c)),x)
[Out] int(x^2/(a+b*sin(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx=\int \frac{x^2}{b\sin(dx^2+c)+a}\,dx$$

```
[In] integrate(x^2/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] integral(x^2/(b*sin(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx=\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx$$

```
[In] integrate(x**2/(a+b*sin(d*x**2+c)),x)
[Out] Integral(x**2/(a + b*sin(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx=\int \frac{x^2}{b\sin(dx^2+c)+a}\,dx$$

```
[In] integrate(x^2/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate(x^2/(b*sin(d*x^2 + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx=\int \frac{x^2}{b\sin(dx^2+c)+a}\,dx$$

```
[In] integrate(x^2/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^2/(b*sin(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 5.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx=\int \frac{x^2}{a+b\,\sin\left(d\,x^2+c\right)}\,dx$$

```
[In] int(x^2/(a + b*sin(c + d*x^2)),x)
[Out] int(x^2/(a + b*sin(c + d*x^2)), x)
```

3.41 $\int \frac{1}{a+b\sin(c+dx^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx = \mathrm{Int}\left(\frac{1}{a+b\sin(c+dx^2)},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx = \int \frac{1}{a+b\sin(c+dx^2)}\,dx$$

```
[In] Int[(a + b*Sin[c + d*x^2])^(-1),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x^2])^(-1), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{a+b\sin(c+dx^2)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx = \int \frac{1}{a+b\sin(c+dx^2)}\,dx$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])^(-1),x]
[Out] Integrate[(a + b*Sin[c + d*x^2])^(-1), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin(d\,x^2+c)}dx$$

```
[In] int(1/(a+b*sin(d*x^2+c)),x)
[Out] int(1/(a+b*sin(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx = \int \frac{1}{b\sin(dx^2+c)+a}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] integral(1/(b*sin(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx = \int \frac{1}{a+b\sin(c+dx^2)}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x**2+c)),x)
[Out] Integral(1/(a + b*sin(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx=\int \frac{1}{b\sin(dx^2+c)+a}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate(1/(b*sin(d*x^2 + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx=\int \frac{1}{b\sin(dx^2+c)+a}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate(1/(b*sin(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 5.96 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx=\int \frac{1}{a+b\,\sin(d\,x^2+c)}\,dx$$

```
[In] int(1/(a + b*sin(c + d*x^2)),x)
[Out] int(1/(a + b*sin(c + d*x^2)), x)
```

3.42 $\int \frac{1}{x^2(a+b\sin(c+dx^2))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\text{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sin(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

```
[In] Int[1/(x^2*(a + b*Sin[c + d*x^2])),x]
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*x^2])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b \sin (c + dx^2))} \, dx = \int \frac{1}{x^2 (a + b \sin (c + dx^2))} \, dx$$

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*x^2])),x]
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*x^2])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a + b \sin (d\,x^2 + c))} dx$$

```
[In] int(1/x^2/(a+b*sin(d*x^2+c)),x)
[Out] int(1/x^2/(a+b*sin(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^2 (a + b \sin (c + dx^2))} \, dx = \int \frac{1}{(b \sin (dx^2 + c) + a)x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] integral(1/(b*x^2*sin(d*x^2 + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 (a + b \sin (c + dx^2))} \, dx = \int \frac{1}{x^2 (a + b \sin (c + dx^2))} \, dx$$

```
[In] integrate(1/x**2/(a+b*sin(d*x**2+c)),x)
[Out] Integral(1/(x**2*(a + b*sin(c + d*x**2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b\sin(c + dx^2))} \, dx = \int \frac{1}{(b\sin(dx^2 + c) + a)x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b\sin(c + dx^2))} \, dx = \int \frac{1}{(b\sin(dx^2 + c) + a)x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 6.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b\sin(c + dx^2))} \, dx = \int \frac{1}{x^2 \, (a + b \, \sin(d\, x^2 + c))} \, dx$$

```
[In] int(1/(x^2*(a + b*sin(c + d*x^2))),x)
[Out] int(1/(x^2*(a + b*sin(c + d*x^2))), x)
```

3.43 $\int \frac{x^5}{(a+b\sin(c+dx^2))^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 663

$$\begin{aligned}\int \frac{x^5}{(a+b\sin(c+dx^2))^2} dx = {} & \frac{ix^4}{2(a^2-b^2)d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} \\ & - \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} \\ & + \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} + \frac{i\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^3} \\ & - \frac{ax^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d^2} \\ & + \frac{i\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^3} + \frac{ax^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d^2} \\ & - \frac{ia\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d^3} + \frac{ia\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d^3} \\ & + \frac{bx^4\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))}\end{aligned}$$

```
[Out] 1/2*I*x^4/(a^2-b^2)/d-x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a
^2-b^2)/d^2-1/2*I*a*x^4*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2
-b^2)^(3/2)/d-x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/
```

```
d^2+1/2*I*a*x^4*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3
/2)/d+I*polylog(2,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d^3-a
*x^2*polylog(2,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^
2+I*polylog(2,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d^3+a*x^2
*polylog(2,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2-I*
a*polylog(3,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^3+I
*a*polylog(3,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^3+
1/2*b*x^4*cos(d*x^2+c)/(a^2-b^2)/d/(a+b*sin(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3460, 3405, 3404, 2296, 2221, 2611, 2320, 6724, 4615, 2317, 2438}

$$
\begin{aligned}
\int \frac{x^5}{(a+b\sin(c+dx^2))^2}\,dx = {} & \frac{i\operatorname{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a-\sqrt{a^2-b^2}}\right)}{d^3(a^2-b^2)}+\frac{i\operatorname{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a+\sqrt{a^2-b^2}}\right)}{d^3(a^2-b^2)} \\
& -\frac{ia\operatorname{PolyLog}\left(3,\frac{ibe^{i(dx^2+c)}}{a-\sqrt{a^2-b^2}}\right)}{d^3(a^2-b^2)^{3/2}}+\frac{ia\operatorname{PolyLog}\left(3,\frac{ibe^{i(dx^2+c)}}{a+\sqrt{a^2-b^2}}\right)}{d^3(a^2-b^2)^{3/2}} \\
& -\frac{ax^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a-\sqrt{a^2-b^2}}\right)}{d^2(a^2-b^2)^{3/2}}+\frac{ax^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a+\sqrt{a^2-b^2}}\right)}{d^2(a^2-b^2)^{3/2}} \\
& -\frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{d^2(a^2-b^2)}-\frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{d^2(a^2-b^2)} \\
& -\frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2d(a^2-b^2)^{3/2}}+\frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{2d(a^2-b^2)^{3/2}} \\
& +\frac{bx^4\cos(c+dx^2)}{2d(a^2-b^2)(a+b\sin(c+dx^2))}+\frac{ix^4}{2d(a^2-b^2)}
\end{aligned}
$$

```
[In] Int[x^5/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((I/2)*x^4)/((a^2 - b^2)*d) - (x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqr
t[a^2 - b^2])])/((a^2 - b^2)*d^2) - ((I/2)*a*x^4*Log[1 - (I*b*E^(I*(c + d*x
^2)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (x^2*Log[1 - (I*b*E^(
I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) + ((I/2)*a*x^4*Lo
g[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d)
 + (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^
2)*d^3) - (a*x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])
/((a^2 - b^2)^(3/2)*d^2) + (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[
```

```
a^2 - b^2])])/((a^2 - b^2)*d^3) + (a*x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))
/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - (I*a*PolyLog[3, (I*b*E^(
I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^3) + (I*a*Poly
Log[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d
^3) + (b*x^4*Cos[c + d*x^2])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x^2]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+b\sin(c+dx))^2}\,dx, x, x^2\right)$$

$$= \frac{bx^4\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} + \frac{a\text{Subst}\left(\int \frac{x^2}{a+b\sin(c+dx)}\,dx,x,x^2\right)}{2(a^2-b^2)} - \frac{b\text{Subst}\left(\int \frac{x\cos(c+dx)}{a+b\sin(c+dx)}\,dx,x,x^2\right)}{(a^2-b^2)d}$$

$$= \frac{ix^4}{2(a^2-b^2)d} + \frac{bx^4\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} + \frac{a\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx,x,x^2\right)}{a^2-b^2} - \frac{b\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{a-\sqrt{a^2-b^2}-ibe^{i(c+dx)}}\,dx,x,x^2\right)}{(a^2-b^2)d} - \frac{b\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{a+\sqrt{a^2-b^2}-ibe^{i(c+dx)}}\,dx,x,x^2\right)}{(a^2-b^2)d}$$

$$= \frac{ix^4}{2(a^2-b^2)d} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} + \frac{bx^4\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} - \frac{(iab)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx,x,x^2\right)}{(a^2-b^2)^{3/2}} + \frac{(iab)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx,x,x^2\right)}{(a^2-b^2)^{3/2}} + \frac{\text{Subst}\left(\int \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)\,dx,x,x^2\right)}{(a^2-b^2)d^2} + \frac{\text{Subst}\left(\int \log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)\,dx,x,x^2\right)}{(a^2-b^2)d^2}$$

$$= \frac{ix^4}{2(a^2-b^2)d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} - \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} + \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} + \frac{bx^4\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} - \frac{i\text{Subst}\left(\int\frac{\log\left(1-\frac{ibx}{a-\sqrt{a^2-b^2}}\right)}{x}\,dx,x,e^{i(c+dx^2)}\right)}{(a^2-b^2)d^3} - \frac{i\text{Subst}\left(\int\frac{\log\left(1-\frac{ibx}{a+\sqrt{a^2-b^2}}\right)}{x}\,dx,x,e^{i(c+dx^2)}\right)}{(a^2-b^2)d^3} + \frac{(ia)\text{Subst}\left(\int x\log\left(1-\frac{2ibe^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)\,dx,x,x^2\right)}{(a^2-b^2)^{3/2}d} - \frac{(ia)\text{Subst}\left(\int x\log\left(1-\frac{2ibe^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right)\,dx,x,x^2\right)}{(a^2-b^2)^{3/2}d}$$

$$= \frac{ix^4}{2(a^2-b^2)d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} - \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} + \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} + \frac{i\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^3} - \frac{ax^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d^2} + \frac{i\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^3} + \frac{ax^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d^2} + \frac{bx^4\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} + \frac{a\text{Subst}\left(\int\text{PolyLog}\left(2,\frac{2ibe^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)\,dx,x,x^2\right)}{(a^2-b^2)^{3/2}d^2} - \frac{a\text{Subst}\left(\int\text{PolyLog}\left(2,\frac{2ibe^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right)\,dx,x,x^2\right)}{(a^2-b^2)^{3/2}d^2}$$

$$= \frac{ix^4}{2(a^2-b^2)d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} - \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} + \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} + \frac{i\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^3} - \frac{ax^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d^2} + \frac{i\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^3} + \frac{ax^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d^2} + \frac{bx^4\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} - \frac{(ia)\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,\frac{ibx}{a-\sqrt{a^2-b^2}}\right)}{x}dx,x,e^{i(c+dx^2)}\right)}{(a^2-b^2)^{3/2}d^3} + \frac{(ia)\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,\frac{ibx}{a+\sqrt{a^2-b^2}}\right)}{x}dx,x,e^{i(c+dx^2)}\right)}{(a^2-b^2)^{3/2}d^3}$$

$$= \frac{ix^4}{2(a^2-b^2)d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} - \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} + \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} + \frac{i\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^3} - \frac{ax^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d^2} + \frac{i\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^3} + \frac{ax^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d^2} - \frac{ia\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d^3} + \frac{ia\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d^3} + \frac{bx^4\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))}$$

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{(a+b\sin(c+dx^2))^2}\,dx = \frac{id^2x^4 - 2dx^2\log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right) - \frac{iad^2x^4\log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 2dx^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right) + \frac{iad^2x^4\log\left(1-\frac{ibe^{i(c}}{a+\sqrt{}}\right.}{\sqrt{a^2-b^2}}}{}$$

```
[In] Integrate[x^5/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (I*d^2*x^4 - 2*d*x^2*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])
] - (I*a*d^2*x^4*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])])/S
qrt[a^2 - b^2] - 2*d*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^
2])] + (I*a*d^2*x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])
/Sqrt[a^2 - b^2] + (2*I - (2*a*d*x^2)/Sqrt[a^2 - b^2])*PolyLog[2, ((-I)*b*E
^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] + (2*I + (2*a*d*x^2)/Sqrt[a^2 - b
^2])*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])] - ((2*I)*a*P
olyLog[3, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2] +
 ((2*I)*a*PolyLog[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/Sqrt[a
^2 - b^2] + (b*d^2*x^4*Cos[c + d*x^2])/(a + b*Sin[c + d*x^2]))/(2*(a^2 - b^
2)*d^3)
```

Maple [F]

$$\int \frac{x^5}{(a+b\sin(d\,x^2+c))^2}dx$$

```
[In] int(x^5/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(x^5/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2469 vs. 2(565) = 1130.

Time = 0.53 (sec) , antiderivative size = 2469, normalized size of antiderivative = 3.72

$$\int \frac{x^5}{(a+b\sin(c+dx^2))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2*b - b^3)*d^2*x^4*cos(d*x^2 + c) + 2*(a*b^2*sin(d*x^2 + c) + a^2
*b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x^2 + c) + a*sin(d*x^2 +
c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2
*(a*b^2*sin(d*x^2 + c) + a^2*b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos
(d*x^2 + c) + a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sq
rt(-(a^2 - b^2)/b^2))/b) + 2*(a*b^2*sin(d*x^2 + c) + a^2*b)*sqrt(-(a^2 - b^
2)/b^2)*polylog(3, -(-I*a*cos(d*x^2 + c) + a*sin(d*x^2 + c) + (b*cos(d*x^2
+ c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*(a*b^2*sin(d*x^2
+ c) + a^2*b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x^2 + c) + a*s
in(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b
^2))/b) - 2*(-I*a^3 + I*a*b^2 + (-I*a^2*b + I*b^3)*sin(d*x^2 + c) + (-I*a*b
^2*d*x^2*sin(d*x^2 + c) - I*a^2*b*d*x^2)*sqrt(-(a^2 - b^2)/b^2))*dilog((I*a
*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*a^3 + I*a*b^2 + (-I*a^2*b + I*
b^3)*sin(d*x^2 + c) + (I*a*b^2*d*x^2*sin(d*x^2 + c) + I*a^2*b*d*x^2)*sqrt(-
(a^2 - b^2)/b^2))*dilog((I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x
^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(I*a^3
 - I*a*b^2 + (I*a^2*b - I*b^3)*sin(d*x^2 + c) + (I*a*b^2*d*x^2*sin(d*x^2 +
c) + I*a^2*b*d*x^2)*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(d*x^2 + c) - a*
sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/
b^2) - b)/b + 1) - 2*(I*a^3 - I*a*b^2 + (I*a^2*b - I*b^3)*sin(d*x^2 + c) +
(-I*a*b^2*d*x^2*sin(d*x^2 + c) - I*a^2*b*d*x^2)*sqrt(-(a^2 - b^2)/b^2))*dil
og((-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*
x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (2*(a^2*b - b^3)*c*sin(d*x^2
 + c) + 2*(a^3 - a*b^2)*c + (a*b^2*c^2*sin(d*x^2 + c) + a^2*b*c^2)*sqrt(-(a
^2 - b^2)/b^2))*log(2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(
a^2 - b^2)/b^2) + 2*I*a) + (2*(a^2*b - b^3)*c*sin(d*x^2 + c) + 2*(a^3 - a*b
^2)*c + (a*b^2*c^2*sin(d*x^2 + c) + a^2*b*c^2)*sqrt(-(a^2 - b^2)/b^2))*log(
2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*
I*a) + (2*(a^2*b - b^3)*c*sin(d*x^2 + c) + 2*(a^3 - a*b^2)*c - (a*b^2*c^2*s
in(d*x^2 + c) + a^2*b*c^2)*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x^2 + c)
+ 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (2*(a^2*b -
b^3)*c*sin(d*x^2 + c) + 2*(a^3 - a*b^2)*c - (a*b^2*c^2*sin(d*x^2 + c) + a^2
*b*c^2)*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (2*(a^3 - a*b^2)*d*x^2 + 2*(a^3
 - a*b^2)*c + 2*((a^2*b - b^3)*d*x^2 + (a^2*b - b^3)*c)*sin(d*x^2 + c) + (a
^2*b*d^2*x^4 - a^2*b*c^2 + (a*b^2*d^2*x^4 - a*b^2*c^2)*sin(d*x^2 + c))*sqrt
(-(a^2 - b^2)/b^2))*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*
x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (2*(a^3 - a
*b^2)*d*x^2 + 2*(a^3 - a*b^2)*c + 2*((a^2*b - b^3)*d*x^2 + (a^2*b - b^3)*c)
*sin(d*x^2 + c) - (a^2*b*d^2*x^4 - a^2*b*c^2 + (a*b^2*d^2*x^4 - a*b^2*c^2)*
sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))*log(-(I*a*cos(d*x^2 + c) - a*sin(d*
x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) -
 b)/b) - (2*(a^3 - a*b^2)*d*x^2 + 2*(a^3 - a*b^2)*c + 2*((a^2*b - b^3)*d*x^
2 + (a^2*b - b^3)*c)*sin(d*x^2 + c) + (a^2*b*d^2*x^4 - a^2*b*c^2 + (a*b^2*d
```

```
^2*x^4 - a*b^2*c^2)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))*log(-(-I*a*cos(
d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b) - (2*(a^3 - a*b^2)*d*x^2 + 2*(a^3 - a*b^2)*c +
2*((a^2*b - b^3)*d*x^2 + (a^2*b - b^3)*c)*sin(d*x^2 + c) - (a^2*b*d^2*x^4 -
 a^2*b*c^2 + (a*b^2*d^2*x^4 - a*b^2*c^2)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/
b^2))*log(-(-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*
b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/((a^4*b - 2*a^2*b^3 + b^5
)*d^3*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d^3)
```

Sympy [F]

$$\int \frac{x^5}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^5}{(a+b\sin(c+dx^2))^2}\,dx$$

```
[In] integrate(x**5/(a+b*sin(d*x**2+c))**2,x)
[Out] Integral(x**5/(a + b*sin(c + d*x**2))**2, x)
```

Maxima [F]

$$\int \frac{x^5}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^5}{(b\sin(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^5/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
[Out] (a*b*x^4*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + a*b*x^4*cos(d*x^2 + c) + ((a^2
*b^2 - b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 + c)^2 +
 4*(a^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 - b^4)*d*
sin(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3*b - a*
b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin(d*x^
2 + c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 + 2*c))*integrate(2*(2*a^2*d*x^5*co
s(d*x^2 + c)^2 + 2*a^2*d*x^5*sin(d*x^2 + c)^2 + a*b*d*x^5*sin(d*x^2 + c) -
2*a*b*x^3*cos(d*x^2 + c) - (a*b*d*x^5*sin(d*x^2 + c) + 2*a*b*x^3*cos(d*x^2
+ c))*cos(2*d*x^2 + 2*c) + (a*b*d*x^5*cos(d*x^2 + c) - 2*a*b*x^3*sin(d*x^2
+ c) - 2*b^2*x^3)*sin(2*d*x^2 + 2*c))/((a^2*b^2 - b^4)*d*cos(2*d*x^2 + 2*c)
^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*cos(d*x^2 +
 c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 - b^4)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4 -
a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2
 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d)*cos(
2*d*x^2 + 2*c)), x) + (a*b*x^4*sin(d*x^2 + c) + b^2*x^4)*sin(2*d*x^2 + 2*c)
)/((a^2*b^2 - b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 +
 c)^2 + 4*(a^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 -
b^4)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3
*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*s
in(d*x^2 + c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 + 2*c))
```

Giac [F]

$$\int \frac{x^5}{(a+b\sin(c+dx^2))^2} dx = \int \frac{x^5}{(b\sin(dx^2+c)+a)^2} dx$$

```
[In] integrate(x^5/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^5/(b*sin(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+b\sin(c+dx^2))^2} dx = \int \frac{x^5}{(a+b\sin(dx^2+c))^2} dx$$

```
[In] int(x^5/(a + b*sin(c + d*x^2))^2,x)
[Out] int(x^5/(a + b*sin(c + d*x^2))^2, x)
```

3.44 $\int \frac{x^3}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 324

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx = -\frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d}+\frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \frac{\log(a+b\sin(c+dx^2))}{2(a^2-b^2)d^2}-\frac{a\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d^2} + \frac{a\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d^2}+\frac{bx^2\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))}$$

```
[Out] -1/2*ln(a+b*sin(d*x^2+c))/(a^2-b^2)/d^2-1/2*I*a*x^2*ln(1-I*b*exp(I*(d*x^2+c
))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d+1/2*I*a*x^2*ln(1-I*b*exp(I*(d*x^2
+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d-1/2*a*polylog(2,I*b*exp(I*(d*x^
2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+1/2*a*polylog(2,I*b*exp(I*(d
*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+1/2*b*x^2*cos(d*x^2+c)/(a
^2-b^2)/d/(a+b*sin(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3460, 3405, 3404, 2296, 2221, 2317, 2438, 2747, 31}

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx = -\frac{a\,\text{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a-\sqrt{a^2-b^2}}\right)}{2d^2(a^2-b^2)^{3/2}}+\frac{a\,\text{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a+\sqrt{a^2-b^2}}\right)}{2d^2(a^2-b^2)^{3/2}} -\frac{\log(a+b\sin(c+dx^2))}{2d^2(a^2-b^2)}-\frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2d(a^2-b^2)^{3/2}} +\frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{2d(a^2-b^2)^{3/2}}+\frac{bx^2\cos(c+dx^2)}{2d(a^2-b^2)(a+b\sin(c+dx^2))}$$

```
[In] Int[x^3/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((-1/2*I)*a*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/((a
^2 - b^2)^(3/2)*d) + ((I/2)*a*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt
[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - Log[a + b*Sin[c + d*x^2]]/(2*(a^2 -
b^2)*d^2) - (a*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(
2*(a^2 - b^2)^(3/2)*d^2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[
a^2 - b^2])])/(2*(a^2 - b^2)^(3/2)*d^2) + (b*x^2*Cos[c + d*x^2])/(2*(a^2 -
b^2)*d*(a + b*Sin[c + d*x^2]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+b\sin(c+dx))^2}\,dx, x, x^2\right)$$

$$= \frac{bx^2\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} + \frac{a\text{Subst}\left(\int \frac{x}{a+b\sin(c+dx)}\,dx, x, x^2\right)}{2(a^2-b^2)} - \frac{b\text{Subst}\left(\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx, x, x^2\right)}{2(a^2-b^2)d}$$

$$= \frac{bx^2\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} + \frac{a\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^2\right)}{a^2-b^2} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx^2)\right)}{2(a^2-b^2)d^2}$$

$$= -\frac{\log(a+b\sin(c+dx^2))}{2(a^2-b^2)d^2} + \frac{bx^2\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} - \frac{(iab)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^2\right)}{(a^2-b^2)^{3/2}} + \frac{(iab)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^2\right)}{(a^2-b^2)^{3/2}}$$

$$= -\frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} + \frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \frac{\log(a+b\sin(c+dx^2))}{2(a^2-b^2)d^2} + \frac{bx^2\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} + \frac{(ia)\text{Subst}\left(\int \log\left(1-\frac{2ibe^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)dx, x, x^2\right)}{2(a^2-b^2)^{3/2}d} - \frac{(ia)\text{Subst}\left(\int \log\left(1-\frac{2ibe^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right)dx, x, x^2\right)}{2(a^2-b^2)^{3/2}d}$$

$$= -\frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} + \frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \frac{\log(a+b\sin(c+dx^2))}{2(a^2-b^2)d^2} + \frac{bx^2\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} + \frac{a\text{Subst}\left(\int \frac{\log\left(1-\frac{2ibx}{2a-2\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{2(a^2-b^2)^{3/2}d^2} - \frac{a\text{Subst}\left(\int \frac{\log\left(1-\frac{2ibx}{2a+2\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{2(a^2-b^2)^{3/2}d^2}$$

$$= -\frac{iax^2 \log\left(1 - \frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2} d} + \frac{iax^2 \log\left(1 - \frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2} d} - \frac{\log(a+b\sin(c+dx^2))}{2(a^2-b^2) d^2} - \frac{a \operatorname{PolyLog}\left(2, \frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2} d^2} + \frac{a \operatorname{PolyLog}\left(2, \frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2} d^2} + \frac{bx^2 \cos(c+dx^2)}{2(a^2-b^2) d (a+b\sin(c+dx^2))}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2} dx$$
$$= \frac{-\frac{iadx^2 \log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{iadx^2 \log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - \frac{\log(a+b\sin(c+dx^2))}{a^2-b^2} - \frac{a \operatorname{PolyLog}\left(2,-\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{a \operatorname{PolyLog}\left(2,\frac{ibe}{a+}\right.}{(a^2-b^2)}}{2d^2}$$

```
[In] Integrate[x^3/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (((-I)*a*d*x^2*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])])/(a^
2 - b^2)^(3/2) + (I*a*d*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 -
 b^2])])/(a^2 - b^2)^(3/2) - Log[a + b*Sin[c + d*x^2]]/(a^2 - b^2) - (a*Pol
yLog[2, ((-I)*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])])/(a^2 - b^2)^(3/
2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/(a^2 - b
^2)^(3/2) + (b*d*x^2*Cos[c + d*x^2])/((a^2 - b^2)*(a + b*Sin[c + d*x^2])))/
(2*d^2)
```

Maple [F]

$$\int \frac{x^3}{(a+b\sin(d\,x^2+c))^2} dx$$

```
[In] int(x^3/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(x^3/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1509 vs. $2(274) = 548$.

Time = 0.46 (sec) , antiderivative size = 1509, normalized size of antiderivative = 4.66

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2*b - b^3)*d*x^2*cos(d*x^2 + c) + (I*a*b^2*sin(d*x^2 + c) + I*a^2
*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (
b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
 (-I*a*b^2*sin(d*x^2 + c) - I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(
d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b + 1) + (-I*a*b^2*sin(d*x^2 + c) - I*a^2*b)*sqrt(
-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*
x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (I*a*b^
2*sin(d*x^2 + c) + I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^2 +
c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) - (a^2*b*d*x^2 + a^2*b*c + (a*b^2*d*x^2 + a*b^2*c)*
sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x
^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b) + (a^2*b*d*x^2 + a^2*b*c + (a*b^2*d*x^2 + a*b^2*c)*sin(d*x^2 + c))*sq
rt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d
*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (a^2*b*d*x
^2 + a^2*b*c + (a*b^2*d*x^2 + a*b^2*c)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^
2)*log(-(-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*s
in(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (a^2*b*d*x^2 + a^2*b*c + (a
*b^2*d*x^2 + a*b^2*c)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos
(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sq
rt(-(a^2 - b^2)/b^2) - b)/b) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^2 + c)
+ (a*b^2*c*sin(d*x^2 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*
x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (a^
3 - a*b^2 + (a^2*b - b^3)*sin(d*x^2 + c) + (a*b^2*c*sin(d*x^2 + c) + a^2*b*
c)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^
2 + c) - (a*b^2*c*sin(d*x^2 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(-2*
b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*
a) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^2 + c) - (a*b^2*c*sin(d*x^2 + c)
+ a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^
2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a))/((a^4*b - 2*a^2*b^3 + b^5)*d^
2*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)
```

Sympy [F]

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx$$

```
[In] integrate(x**3/(a+b*sin(d*x**2+c))**2,x)
[Out] Integral(x**3/(a + b*sin(c + d*x**2))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^3}{(b\sin(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^3/(b*sin(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^3}{(a+b\,\sin(d\,x^2+c))^2}\,dx$$

```
[In] int(x^3/(a + b*sin(c + d*x^2))^2,x)
[Out] int(x^3/(a + b*sin(c + d*x^2))^2, x)
```

3.45 $\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 91

$$\int \frac{x}{(a+b\sin(c+dx^2))^2}\,dx=\frac{a\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}+\frac{b\cos(c+dx^2)}{2(a^2-b^2)\,d\,(a+b\sin(c+dx^2))}$$

```
[Out] a*arctan((b+a*tan(1/2*d*x^2+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d+1/2*
b*cos(d*x^2+c)/(a^2-b^2)/d/(a+b*sin(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3460, 2743, 12, 2739, 632, 210}

$$\int \frac{x}{(a+b\sin(c+dx^2))^2}\,dx=\frac{a\arctan\left(\frac{a\tan\left(\frac{1}{2}(c+dx^2)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{3/2}}+\frac{b\cos(c+dx^2)}{2d\,(a^2-b^2)\,(a+b\sin(c+dx^2))}$$

```
[In] Int[x/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (a*ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)*d
) + (b*Cos[c + d*x^2])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x^2]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+b\sin(c+dx))^2}\,dx, x, x^2\right)$$

$$= \frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)} + \frac{\text{Subst}\left(\int \frac{a}{a+b\sin(c+dx)}\,dx, x, x^2\right)}{2\left(a^2-b^2\right)}$$

$$= \frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)} + \frac{a\text{Subst}\left(\int \frac{1}{a+b\sin(c+dx)}\,dx, x, x^2\right)}{2\left(a^2-b^2\right)}$$

$$= \frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)} + \frac{a\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}\left(c+dx^2\right)\right)\right)}{\left(a^2-b^2\right)d}$$

$$= \frac{b\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} - \frac{(2a)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{(a^2-b^2)d}$$

$$= \frac{a\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d} + \frac{b\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\sin(c+dx^2))^2}\,dx = \frac{\frac{2a\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{b\cos(c+dx^2)}{a+b\sin(c+dx^2)}}{2(a-b)(a+b)d}$$

```
[In] Integrate[x/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a*ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 (b*Cos[c + d*x^2])/(a + b*Sin[c + d*x^2]))/(2*(a - b)*(a + b)*d)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{\frac{\frac{2b^2\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{a(a^2-b^2)}+\frac{2b}{a^2-b^2}}{\left(\tan^2\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)a+2b\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}}{2d}$
default	$\frac{\frac{\frac{2b^2\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{a(a^2-b^2)}+\frac{2b}{a^2-b^2}}{\left(\tan^2\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)a+2b\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}}{2d}$
risch	$\frac{ib+a\,e^{i(dx^2+c)}}{(a^2-b^2)d\left(b\,e^{2i(dx^2+c)}-b+2ia\,e^{i(dx^2+c)}\right)}-\frac{a\ln\left(e^{i(dx^2+c)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{a\ln\left(e^{i(dx^2+c)}+\frac{ia\sqrt{-a^2-}}{b\sqrt{-}}\right.}{2\sqrt{-a^2+b^2}\,(a+b)(}$

```
[In] int(x/(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(2*(b^2/a/(a^2-b^2)*tan(1/2*d*x^2+1/2*c)+b/(a^2-b^2))/(tan(1/2*d*x^2+
1/2*c)^2*a+2*b*tan(1/2*d*x^2+1/2*c)+a)+2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*
tan(1/2*d*x^2+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.02

$$\int \frac{x}{(a+b\sin(c+dx^2))^2}\,dx = \left[\frac{(ab\sin(dx^2+c)+a^2)\sqrt{-a^2+b^2}\log\left(-\frac{(2a^2-b^2)\cos(dx^2+c)^2-2ab\sin(dx^2+c)-a^2-b^2-2(a\cos(dx^2+c)\sin(dx^2+c)+b\cos(dx}{b^2\cos(dx^2+c)^2-2ab\sin(dx^2+c)-a^2-b^2}\right.}{4((a^4b-2a^2b^3+b^5)d\sin(dx^2+c)+(a^5-2a^3b^2+ab^4)d)}\right.$$
$$\left.-\frac{(ab\sin(dx^2+c)+a^2)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx^2+c)+b}{\sqrt{a^2-b^2}\cos(dx^2+c)}\right)-(a^2b-b^3)\cos(dx^2+c)}{2((a^4b-2a^2b^3+b^5)d\sin(dx^2+c)+(a^5-2a^3b^2+ab^4)d)}\right]$$

```
[In] integrate(x/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*sin(d*x^2 + c) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d
*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2 - 2*(a*cos(d*x^2 + c)*sin(d*
x^2 + c) + b*cos(d*x^2 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x^2 + c)^2 - 2*a*
b*sin(d*x^2 + c) - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(d*x^2 + c))/((a^4*b -
2*a^2*b^3 + b^5)*d*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/2*((a*
b*sin(d*x^2 + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x^2 + c) + b)/(sqr
t(a^2 - b^2)*cos(d*x^2 + c))) - (a^2*b - b^3)*cos(d*x^2 + c))/((a^4*b - 2*a
^2*b^3 + b^5)*d*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. 2(71) = 142.

Time = 54.30 (sec) , antiderivative size = 2116, normalized size of antiderivative = 23.25

$$\int \frac{x}{(a+b\sin(c+dx^2))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Piecewise((zoo*x**2/sin(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((tan(c/2 +
 d*x**2/2)/(4*d) - 1/(4*d*tan(c/2 + d*x**2/2)))/b**2, Eq(a, 0)), (-3*tan(c/
2 + d*x**2/2)**2/(3*b**2*d*tan(c/2 + d*x**2/2)**3 - 9*b**2*d*tan(c/2 + d*x*
*2/2)**2 + 9*b**2*d*tan(c/2 + d*x**2/2) - 3*b**2*d) + 3*tan(c/2 + d*x**2/2)
/(3*b**2*d*tan(c/2 + d*x**2/2)**3 - 9*b**2*d*tan(c/2 + d*x**2/2)**2 + 9*b**
2*d*tan(c/2 + d*x**2/2) - 3*b**2*d) - 2/(3*b**2*d*tan(c/2 + d*x**2/2)**3 -
9*b**2*d*tan(c/2 + d*x**2/2)**2 + 9*b**2*d*tan(c/2 + d*x**2/2) - 3*b**2*d),
 Eq(a, -b)), (-3*tan(c/2 + d*x**2/2)**2/(3*b**2*d*tan(c/2 + d*x**2/2)**3 +
```

```
9*b**2*d*tan(c/2 + d*x**2/2)**2 + 9*b**2*d*tan(c/2 + d*x**2/2) + 3*b**2*d)
- 3*tan(c/2 + d*x**2/2)/(3*b**2*d*tan(c/2 + d*x**2/2)**3 + 9*b**2*d*tan(c/2
 + d*x**2/2)**2 + 9*b**2*d*tan(c/2 + d*x**2/2) + 3*b**2*d) - 2/(3*b**2*d*ta
n(c/2 + d*x**2/2)**3 + 9*b**2*d*tan(c/2 + d*x**2/2)**2 + 9*b**2*d*tan(c/2 +
 d*x**2/2) + 3*b**2*d), Eq(a, b)), (x**2/(2*(a + b*sin(c))**2), Eq(d, 0)),
(a**3*log(tan(c/2 + d*x**2/2) + b/a - sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x**
2/2)**2/(2*a**4*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)**2 + 2*a**4*d*sqrt
(-a**2 + b**2) + 4*a**3*b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2) - 2*a**2
*b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)**2 - 2*a**2*b**2*d*sqrt(-a**
2 + b**2) - 4*a*b**3*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)) + a**3*log(t
an(c/2 + d*x**2/2) + b/a - sqrt(-a**2 + b**2)/a)/(2*a**4*d*sqrt(-a**2 + b**
2)*tan(c/2 + d*x**2/2)**2 + 2*a**4*d*sqrt(-a**2 + b**2) + 4*a**3*b*d*sqrt(-
a**2 + b**2)*tan(c/2 + d*x**2/2) - 2*a**2*b**2*d*sqrt(-a**2 + b**2)*tan(c/2
 + d*x**2/2)**2 - 2*a**2*b**2*d*sqrt(-a**2 + b**2) - 4*a*b**3*d*sqrt(-a**2
+ b**2)*tan(c/2 + d*x**2/2)) - a**3*log(tan(c/2 + d*x**2/2) + b/a + sqrt(-a
**2 + b**2)/a)*tan(c/2 + d*x**2/2)**2/(2*a**4*d*sqrt(-a**2 + b**2)*tan(c/2
+ d*x**2/2)**2 + 2*a**4*d*sqrt(-a**2 + b**2) + 4*a**3*b*d*sqrt(-a**2 + b**2
)*tan(c/2 + d*x**2/2) - 2*a**2*b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2
)**2 - 2*a**2*b**2*d*sqrt(-a**2 + b**2) - 4*a*b**3*d*sqrt(-a**2 + b**2)*tan
(c/2 + d*x**2/2)) - a**3*log(tan(c/2 + d*x**2/2) + b/a + sqrt(-a**2 + b**2)
/a)/(2*a**4*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)**2 + 2*a**4*d*sqrt(-a*
*2 + b**2) + 4*a**3*b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2) - 2*a**2*b**
2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)**2 - 2*a**2*b**2*d*sqrt(-a**2 +
b**2) - 4*a*b**3*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)) + 2*a**2*b*log(t
an(c/2 + d*x**2/2) + b/a - sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x**2/2)/(2*a**
4*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)**2 + 2*a**4*d*sqrt(-a**2 + b**2)
 + 4*a**3*b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2) - 2*a**2*b**2*d*sqrt(-
a**2 + b**2)*tan(c/2 + d*x**2/2)**2 - 2*a**2*b**2*d*sqrt(-a**2 + b**2) - 4*
a*b**3*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)) - 2*a**2*b*log(tan(c/2 + d
*x**2/2) + b/a + sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x**2/2)/(2*a**4*d*sqrt(-
a**2 + b**2)*tan(c/2 + d*x**2/2)**2 + 2*a**4*d*sqrt(-a**2 + b**2) + 4*a**3*
b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2) - 2*a**2*b**2*d*sqrt(-a**2 + b**
2)*tan(c/2 + d*x**2/2)**2 - 2*a**2*b**2*d*sqrt(-a**2 + b**2) - 4*a*b**3*d*s
qrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)) + 2*a*b*sqrt(-a**2 + b**2)/(2*a**4*d
*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)**2 + 2*a**4*d*sqrt(-a**2 + b**2) +
4*a**3*b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2) - 2*a**2*b**2*d*sqrt(-a**
2 + b**2)*tan(c/2 + d*x**2/2)**2 - 2*a**2*b**2*d*sqrt(-a**2 + b**2) - 4*a*b
**3*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)) + 2*b**2*sqrt(-a**2 + b**2)*t
an(c/2 + d*x**2/2)/(2*a**4*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)**2 + 2*
a**4*d*sqrt(-a**2 + b**2) + 4*a**3*b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/
2) - 2*a**2*b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)**2 - 2*a**2*b**2*
d*sqrt(-a**2 + b**2) - 4*a*b**3*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)),
True))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.58

$$\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \frac{\left(\pi\left\lfloor\frac{dx^2+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{(a^2d-b^2d)\sqrt{a^2-b^2}} + \frac{b^2\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+ab}{(a^3d-ab^2d)\left(a\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+a\right)}$$

```
[In] integrate(x/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
[Out] (pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^2 + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a/((a^2*d - b^2*d)*sqrt(a^2 - b^2)) + (b^2*tan(1/
2*d*x^2 + 1/2*c) + a*b)/((a^3*d - a*b^2*d)*(a*tan(1/2*d*x^2 + 1/2*c)^2 + 2*
b*tan(1/2*d*x^2 + 1/2*c) + a))
```

Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.96

$$\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \frac{\frac{b}{a^2-b^2}+\frac{b^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{a\,(a^2-b^2)}}{d\,\left(a\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2+2\,b\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+a\right)} + \frac{a\,\mathrm{atan}\left(\frac{(a^2-b^2)\left(\frac{a^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{a\,\left(2\,a^2\,b-2\,b^3\right)}{2\,(a+b)^{3/2}\,\left(a^2-b^2\right)\,(a-b)^{3/2}}\right)}{a}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

```
[In] int(x/(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] (b/(a^2 - b^2) + (b^2*tan(c/2 + (d*x^2)/2))/(a*(a^2 - b^2)))/(d*(a + a*tan(
c/2 + (d*x^2)/2)^2 + 2*b*tan(c/2 + (d*x^2)/2))) + (a*atan(((a^2 - b^2)*((a^
2*tan(c/2 + (d*x^2)/2))/((a + b)^(3/2)*(a - b)^(3/2)) + (a*(2*a^2*b - 2*b^3
))/(2*(a + b)^(3/2)*(a^2 - b^2)*(a - b)^(3/2))))/a))/(d*(a + b)^(3/2)*(a -
b)^(3/2))
```

3.46 $\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sin(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x*(a + b*Sin[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sin[c + d*x^2])^2),x]
[Out] Integrate[1/(x*(a + b*Sin[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\sin\left(d\,x^2+c\right))^2}dx$$

```
[In] int(1/x/(a+b*sin(d*x^2+c))^2,x)
[Out] int(1/x/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*x*cos(d*x^2 + c)^2 - 2*a*b*x*sin(d*x^2 + c) - (a^2 + b^2)*
x), x)
```

Sympy [N/A]

Not integrable

Time = 26.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x**2+c))**2,x)
[Out] Integral(1/(x*(a + b*sin(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 3466, normalized size of antiderivative = 192.56

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
[Out] (a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*cos(2*c)*sin(2*d*x^2) - b^4*
cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) - 2*(a^3*b - a*
b^3)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c) - a*b^3*sin(2*d*x^2)*
sin(2*c) + a^3*b - a*b^3 + 2*(a^4 - a^2*b^2)*cos(c)*sin(d*x^2) + 2*(a^4 - a
^2*b^2)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + (a^4*b^2*d*x^2*cos(2*d*x^2 + 2*
c)^2 + a^4*b^2*d*x^2*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^
2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 -
2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*s
in(2*c)^2)*d*x^2*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(c
)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 +
 a^2*b^4)*sin(c)^2)*d*x^2*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^
2*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2 - 2*(2*((a^3*b^3 -
a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^2*cos(d*x^2
) - (a^2*b^4 - b^6)*d*x^2*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) +
 (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2
*b^4*d*x^2*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^2*sin(2*d*x^2)*sin(2*c) + 2*
(a^5*b - a^3*b^3)*d*x^2*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^2*cos(d
*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^2)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b
^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*cos(
d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*
sin(c))*d*x^2*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^2*sin(2*c))*sin(2*d*x^2) - 2
*(a^2*b^4*d*x^2*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^2*cos(2*d*x^2)*sin(2*c)
```

```
 - 2*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x^2*
sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*integrate(-2*(b^4*cos(2*c)*sin(2*d*x
^2) + b^4*cos(2*d*x^2)*sin(2*c) - 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) + 2*(
a^3*b - a*b^3)*sin(d*x^2)*sin(c) + (a^3*b*d*x^2*sin(d*x^2 + c) - a^3*b*cos(
d*x^2 + c))*cos(2*d*x^2 + 2*c) + (a^3*b - a*b^3 + (a*b^3*d*x^2*sin(2*c) + a
*b^3*cos(2*c))*cos(2*d*x^2) - 2*((a^4 - a^2*b^2)*d*x^2*cos(c) - (a^4 - a^2*
b^2)*sin(c))*cos(d*x^2) + (a*b^3*d*x^2*cos(2*c) - a*b^3*sin(2*c))*sin(2*d*x
^2) + 2*((a^4 - a^2*b^2)*d*x^2*sin(c) + (a^4 - a^2*b^2)*cos(c))*sin(d*x^2))
*cos(d*x^2 + c) - (a^3*b*d*x^2*cos(d*x^2 + c) + a^3*b*sin(d*x^2 + c) + a^2*
b^2)*sin(2*d*x^2 + 2*c) - ((a^3*b - a*b^3)*d*x^2 + (a*b^3*d*x^2*cos(2*c) -
a*b^3*sin(2*c))*cos(2*d*x^2) + 2*((a^4 - a^2*b^2)*d*x^2*sin(c) + (a^4 - a^2
*b^2)*cos(c))*cos(d*x^2) - (a*b^3*d*x^2*sin(2*c) + a*b^3*cos(2*c))*sin(2*d*
x^2) + 2*((a^4 - a^2*b^2)*d*x^2*cos(c) - (a^4 - a^2*b^2)*sin(c))*sin(d*x^2)
)*sin(d*x^2 + c))/(a^4*b^2*d*x^3*cos(2*d*x^2 + 2*c)^2 + a^4*b^2*d*x^3*sin(2
*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^3*cos(2*d*x^2)^2 +
4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c
)^2)*d*x^3*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^3*sin(2*d*x
^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^3*cos(c)*sin(d*x^2) + 4*((a^6 - 2
*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^3*
sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^3*cos(d*x^2)*sin(c) + (a^4
*b^2 - 2*a^2*b^4 + b^6)*d*x^3 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (
a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^3*cos(d*x^2) - (a^2*b^4 - b^6)*d*x^3*
cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c
)*sin(c))*d*x^3*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2*b^4*d*x^3*cos(2*d*x^2)*co
s(2*c) - a^2*b^4*d*x^3*sin(2*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^3*co
s(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^3*cos(d*x^2)*sin(c) + (a^4*b^2 -
a^2*b^4)*d*x^3)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c
) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^3*cos(d*x^2) + 2*((a^3*b^3 - a*b
^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^3*sin(d*x^2) +
 (a^2*b^4 - b^6)*d*x^3*sin(2*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x^3*cos(2*c)*s
in(2*d*x^2) + a^2*b^4*d*x^3*cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x
^3*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x^3*sin(d*x^2)*sin(c))*sin(2*d
*x^2 + 2*c)), x) + (a^3*b*sin(d*x^2 + c) + a^2*b^2)*sin(2*d*x^2 + 2*c) - (a
*b^3*cos(2*c)*sin(2*d*x^2) + a*b^3*cos(2*d*x^2)*sin(2*c) - 2*(a^4 - a^2*b^2
)*cos(d*x^2)*cos(c) + 2*(a^4 - a^2*b^2)*sin(d*x^2)*sin(c))*sin(d*x^2 + c))/
(a^4*b^2*d*x^2*cos(2*d*x^2 + 2*c)^2 + a^4*b^2*d*x^2*sin(2*d*x^2 + 2*c)^2 +
(b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^
2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*cos(d*x
^2)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^2*sin(2*d*x^2)^2 + 4*(a^5*b -
 2*a^3*b^3 + a*b^5)*d*x^2*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4
)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*sin(d*x^2)^2 + 4*(
a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 +
 b^6)*d*x^2 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*c
os(2*c)*sin(c))*d*x^2*cos(d*x^2) - (a^2*b^4 - b^6)*d*x^2*cos(2*c) - 2*((a^3
*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*si
```

```
n(d*x^2))*cos(2*d*x^2) - 2*(a^2*b^4*d*x^2*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d
*x^2*sin(2*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^2*cos(c)*sin(d*x^2) +
2*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^2)*co
s(2*d*x^2 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b
^5)*sin(2*c)*sin(c))*d*x^2*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c
) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^2*sin(d*x^2) + (a^2*b^4 - b^6)*d
*x^2*sin(2*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x^2*cos(2*c)*sin(2*d*x^2) + a^2*
b^4*d*x^2*cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2)*cos(
c) + 2*(a^5*b - a^3*b^3)*d*x^2*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 6.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

```
[In] int(1/(x*(a + b*sin(c + d*x^2))^2),x)
[Out] int(1/(x*(a + b*sin(c + d*x^2))^2), x)
```

3.47 $\int \frac{1}{x^3(a+b\sin(c+dx^2))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \operatorname{Int}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^3/(a+b*sin(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x^3*(a + b*Sin[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x^3*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \left(a+b\sin \left(c+dx^2\right)\right)^2} \, dx=\int \frac{1}{x^3 \left(a+b\sin \left(c+dx^2\right)\right)^2} \, dx$$

```
[In] Integrate[1/(x^3*(a + b*Sin[c + d*x^2])^2),x]
[Out] Integrate[1/(x^3*(a + b*Sin[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3}\left(a+b\sin \left(d\,x^{2}+c\right)\right)^{2}}d x$$

```
[In] int(1/x^3/(a+b*sin(d*x^2+c))^2,x)
[Out] int(1/x^3/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^3 \left(a+b\sin \left(c+dx^2\right)\right)^2} \, dx=\int \frac{1}{{\left(b \sin \left(dx^{2} + c\right) + a\right)}^{2} x^{3}} \, dx$$

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*x^3*cos(d*x^2 + c)^2 - 2*a*b*x^3*sin(d*x^2 + c) - (a^2 + b
^2)*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 39.90 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] integrate(1/x**3/(a+b*sin(d*x**2+c))**2,x)
[Out] Integral(1/(x**3*(a + b*sin(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 4.12 (sec) , antiderivative size = 3475, normalized size of antiderivative = 193.06

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
[Out] (a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*cos(2*c)*sin(2*d*x^2) - b^4*
cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) - 2*(a^3*b - a*
b^3)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c) - a*b^3*sin(2*d*x^2)*
sin(2*c) + a^3*b - a*b^3 + 2*(a^4 - a^2*b^2)*cos(c)*sin(d*x^2) + 2*(a^4 - a
^2*b^2)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + (a^4*b^2*d*x^4*cos(2*d*x^2 + 2*
c)^2 + a^4*b^2*d*x^4*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^
2)*d*x^4*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 -
2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^4*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*s
in(2*c)^2)*d*x^4*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^4*cos(c
)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 +
 a^2*b^4)*sin(c)^2)*d*x^4*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^
4*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^4 - 2*(2*((a^3*b^3 -
a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^4*cos(d*x^2
) - (a^2*b^4 - b^6)*d*x^4*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) +
 (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^4*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2
*b^4*d*x^4*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^4*sin(2*d*x^2)*sin(2*c) + 2*
(a^5*b - a^3*b^3)*d*x^4*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^4*cos(d
*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^4)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b
^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^4*cos(
d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*
sin(c))*d*x^4*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^4*sin(2*c))*sin(2*d*x^2) - 2
*(a^2*b^4*d*x^4*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^4*cos(2*d*x^2)*sin(2*c)
```

```
 - 2*(a^5*b - a^3*b^3)*d*x^4*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x^4*
sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*integrate(-2*(2*b^4*cos(2*c)*sin(2*d
*x^2) + 2*b^4*cos(2*d*x^2)*sin(2*c) - 4*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) +
 4*(a^3*b - a*b^3)*sin(d*x^2)*sin(c) + (a^3*b*d*x^2*sin(d*x^2 + c) - 2*a^3*
b*cos(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (2*a^3*b - 2*a*b^3 + (a*b^3*d*x^2*si
n(2*c) + 2*a*b^3*cos(2*c))*cos(2*d*x^2) - 2*((a^4 - a^2*b^2)*d*x^2*cos(c) -
 2*(a^4 - a^2*b^2)*sin(c))*cos(d*x^2) + (a*b^3*d*x^2*cos(2*c) - 2*a*b^3*sin
(2*c))*sin(2*d*x^2) + 2*((a^4 - a^2*b^2)*d*x^2*sin(c) + 2*(a^4 - a^2*b^2)*c
os(c))*sin(d*x^2))*cos(d*x^2 + c) - (a^3*b*d*x^2*cos(d*x^2 + c) + 2*a^3*b*s
in(d*x^2 + c) + 2*a^2*b^2)*sin(2*d*x^2 + 2*c) - ((a^3*b - a*b^3)*d*x^2 + (a
*b^3*d*x^2*cos(2*c) - 2*a*b^3*sin(2*c))*cos(2*d*x^2) + 2*((a^4 - a^2*b^2)*d
*x^2*sin(c) + 2*(a^4 - a^2*b^2)*cos(c))*cos(d*x^2) - (a*b^3*d*x^2*sin(2*c)
+ 2*a*b^3*cos(2*c))*sin(2*d*x^2) + 2*((a^4 - a^2*b^2)*d*x^2*cos(c) - 2*(a^4
 - a^2*b^2)*sin(c))*sin(d*x^2))*sin(d*x^2 + c))/(a^4*b^2*d*x^5*cos(2*d*x^2
+ 2*c)^2 + a^4*b^2*d*x^5*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2
*c)^2)*d*x^5*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^
6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^5*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b
^6*sin(2*c)^2)*d*x^5*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^5*c
os(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b
^2 + a^2*b^4)*sin(c)^2)*d*x^5*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*
d*x^5*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^5 - 2*(2*((a^3*b^
3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^5*cos(d
*x^2) - (a^2*b^4 - b^6)*d*x^5*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(
c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^5*sin(d*x^2))*cos(2*d*x^2) - 2*
(a^2*b^4*d*x^5*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^5*sin(2*d*x^2)*sin(2*c)
+ 2*(a^5*b - a^3*b^3)*d*x^5*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^5*c
os(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^5)*cos(2*d*x^2 + 2*c) - 2*(2*((a
^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^5*
cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2
*c)*sin(c))*d*x^5*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^5*sin(2*c))*sin(2*d*x^2)
 - 2*(a^2*b^4*d*x^5*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^5*cos(2*d*x^2)*sin(
2*c) - 2*(a^5*b - a^3*b^3)*d*x^5*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*
x^5*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c)), x) + (a^3*b*sin(d*x^2 + c) + a^
2*b^2)*sin(2*d*x^2 + 2*c) - (a*b^3*cos(2*c)*sin(2*d*x^2) + a*b^3*cos(2*d*x^
2)*sin(2*c) - 2*(a^4 - a^2*b^2)*cos(d*x^2)*cos(c) + 2*(a^4 - a^2*b^2)*sin(d
*x^2)*sin(c))*sin(d*x^2 + c))/(a^4*b^2*d*x^4*cos(2*d*x^2 + 2*c)^2 + a^4*b^2
*d*x^4*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^4*cos(2
*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^
2*b^4)*sin(c)^2)*d*x^4*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x
^4*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^4*cos(c)*sin(d*x^2) +
 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(
c)^2)*d*x^4*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^4*cos(d*x^2)*s
in(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^4 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*
sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^4*cos(d*x^2) - (a^2*b^4 -
 b^6)*d*x^4*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*
```

```
b^5)*sin(2*c)*sin(c))*d*x^4*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2*b^4*d*x^4*cos
(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^4*sin(2*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b
^3)*d*x^4*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^4*cos(d*x^2)*sin(c) +
 (a^4*b^2 - a^2*b^4)*d*x^4)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*co
s(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^4*cos(d*x^2) + 2*((a
^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^4*
sin(d*x^2) + (a^2*b^4 - b^6)*d*x^4*sin(2*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x^
4*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^4*cos(2*d*x^2)*sin(2*c) - 2*(a^5*b -
a^3*b^3)*d*x^4*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x^4*sin(d*x^2)*sin
(c))*sin(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a+b\sin(c+dx^2))^2}\,dx=\int \frac{1}{(b\sin(dx^2+c)+a)^2x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)^2*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 6.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a+b\sin(c+dx^2))^2}\,dx=\int \frac{1}{x^3\,(a+b\,\sin(d\,x^2+c))^2}\,dx$$

```
[In] int(1/(x^3*(a + b*sin(c + d*x^2))^2),x)
[Out] int(1/(x^3*(a + b*sin(c + d*x^2))^2), x)
```

3.48 $\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx = \text{Int}\left(\frac{x^2}{(a+b\sin(c+dx^2))^2},x\right)$$

```
[Out] Unintegrable(x^2/(a+b*sin(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx$$

```
[In] Int[x^2/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][x^2/(a + b*Sin[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx$$

```
[In] Integrate[x^2/(a + b*Sin[c + d*x^2])^2,x]
[Out] Integrate[x^2/(a + b*Sin[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+b\sin(d\,x^2+c))^2}dx$$

```
[In] int(x^2/(a+b*sin(d*x^2+c))^2,x)
[Out] int(x^2/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\sin(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(-x^2/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2), x)
```

Sympy [N/A]

Not integrable

Time = 40.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx$$

```
[In] integrate(x**2/(a+b*sin(d*x**2+c))**2,x)
[Out] Integral(x**2/(a + b*sin(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 922, normalized size of antiderivative = 51.22

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\sin(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] (a*b*x*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + a*b*x*cos(d*x^2 + c) + ((a^2*b^2
 - b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 + c)^2 + 4*(
a^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 - b^4)*d*sin(
2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)
*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin(d*x^2 +
c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 + 2*c))*integrate((4*a^2*d*x^2*cos(d*x^
2 + c)^2 + 4*a^2*d*x^2*sin(d*x^2 + c)^2 + 2*a*b*d*x^2*sin(d*x^2 + c) - a*b*
cos(d*x^2 + c) - (2*a*b*d*x^2*sin(d*x^2 + c) + a*b*cos(d*x^2 + c))*cos(2*d*
x^2 + 2*c) + (2*a*b*d*x^2*cos(d*x^2 + c) - a*b*sin(d*x^2 + c) - b^2)*sin(2*
d*x^2 + 2*c))/((a^2*b^2 - b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d
*cos(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) +
 (a^2*b^2 - b^4)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c
)^2 + 4*(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b
- a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 + 2*c)), x) + (a
*b*x*sin(d*x^2 + c) + b^2*x)*sin(2*d*x^2 + 2*c))/((a^2*b^2 - b^4)*d*cos(2*d
*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*
cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 - b^4)*d*sin(2*d*x^2 + 2*c)^2
+ 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*sin(d*x^2 + c)
 + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b
^4)*d)*cos(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\sin(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sin(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^2}{(a+b\,\sin(d\,x^2+c))^2}\,dx$$

```
[In] int(x^2/(a + b*sin(c + d*x^2))^2,x)
[Out] int(x^2/(a + b*sin(c + d*x^2))^2, x)
```

3.49 $\int \frac{1}{(a+b\sin(c+dx^2))^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{(a+b\sin(c+dx^2))^2} dx = \text{Int}\left(\frac{1}{(a+b\sin(c+dx^2))^2}, x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(a+b\sin(c+dx^2))^2} dx = \int \frac{1}{(a+b\sin(c+dx^2))^2} dx$$

```
[In] Int[(a + b*Sin[c + d*x^2])^(-2),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x^2])^(-2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(a+b\sin(c+dx^2))^2} dx$$

Mathematica [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{1}{(a+b\sin(c+dx^2))^2}\,dx$$

```
[In] Integrate[(a + b*Sin[c + d*x^2])^(-2),x]
[Out] Integrate[(a + b*Sin[c + d*x^2])^(-2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sin(d\,x^2+c))^2}dx$$

```
[In] int(1/(a+b*sin(d*x^2+c))^2,x)
[Out] int(1/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

$$\int \frac{1}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{1}{(b\sin(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2), x)
```

Sympy [N/A]

Not integrable

Time = 18.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{1}{(a+b\sin(c+dx^2))^2}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x**2+c))**2,x)
[Out] Integral((a + b*sin(c + d*x**2))**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 4.01 (sec) , antiderivative size = 3381, normalized size of antiderivative = 241.50

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] (a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*cos(2*c)*sin(2*d*x^2) - b^4*
cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) - 2*(a^3*b - a*
b^3)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c) - a*b^3*sin(2*d*x^2)*
sin(2*c) + a^3*b - a*b^3 + 2*(a^4 - a^2*b^2)*cos(c)*sin(d*x^2) + 2*(a^4 - a
^2*b^2)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + (a^4*b^2*d*x*cos(2*d*x^2 + 2*c)
^2 + a^4*b^2*d*x*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d
*x*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*
b^2 + a^2*b^4)*sin(c)^2)*d*x*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^
2)*d*x*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(c)*sin(d*x^2)
 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*si
n(c)^2)*d*x*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x^2)*sin
(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(
2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*cos(d*x^2) - (a^2*b^4 - b^6)*
d*x*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin
(2*c)*sin(c))*d*x*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2*b^4*d*x*cos(2*d*x^2)*co
s(2*c) - a^2*b^4*d*x*sin(2*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x*cos(c)
*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^
4)*d*x)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3
*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)
*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*sin(d*x^2) + (a^2*b^4 -
b^6)*d*x*sin(2*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x*cos(2*c)*sin(2*d*x^2) + a^
2*b^4*d*x*cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)*cos(c)
 + 2*(a^5*b - a^3*b^3)*d*x*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*integrate
(-(b^4*cos(2*c)*sin(2*d*x^2) + b^4*cos(2*d*x^2)*sin(2*c) - 2*(a^3*b - a*b^3
)*cos(d*x^2)*cos(c) + 2*(a^3*b - a*b^3)*sin(d*x^2)*sin(c) + (2*a^3*b*d*x^2*
sin(d*x^2 + c) - a^3*b*cos(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (a^3*b - a*b^3
+ (2*a*b^3*d*x^2*sin(2*c) + a*b^3*cos(2*c))*cos(2*d*x^2) - 2*(2*(a^4 - a^2*
b^2)*d*x^2*cos(c) - (a^4 - a^2*b^2)*sin(c))*cos(d*x^2) + (2*a*b^3*d*x^2*cos
(2*c) - a*b^3*sin(2*c))*sin(2*d*x^2) + 2*(2*(a^4 - a^2*b^2)*d*x^2*sin(c) +
(a^4 - a^2*b^2)*cos(c))*sin(d*x^2))*cos(d*x^2 + c) - (2*a^3*b*d*x^2*cos(d*x
^2 + c) + a^3*b*sin(d*x^2 + c) + a^2*b^2)*sin(2*d*x^2 + 2*c) - (2*(a^3*b -
a*b^3)*d*x^2 + (2*a*b^3*d*x^2*cos(2*c) - a*b^3*sin(2*c))*cos(2*d*x^2) + 2*(
2*(a^4 - a^2*b^2)*d*x^2*sin(c) + (a^4 - a^2*b^2)*cos(c))*cos(d*x^2) - (2*a*
b^3*d*x^2*sin(2*c) + a*b^3*cos(2*c))*sin(2*d*x^2) + 2*(2*(a^4 - a^2*b^2)*d*
x^2*cos(c) - (a^4 - a^2*b^2)*sin(c))*sin(d*x^2))*sin(d*x^2 + c))/(a^4*b^2*d
```

```
*x^2*cos(2*d*x^2 + 2*c)^2 + a^4*b^2*d*x^2*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2
*c)^2 + b^6*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^
4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*cos(d*x^2)^2 + (b
^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^2*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3
 + a*b^5)*d*x^2*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2
 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*sin(d*x^2)^2 + 4*(a^5*b - 2*
a^3*b^3 + a*b^5)*d*x^2*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^
2 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*si
n(c))*d*x^2*cos(d*x^2) - (a^2*b^4 - b^6)*d*x^2*cos(2*c) - 2*((a^3*b^3 - a*b
^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*sin(d*x^2))*
cos(2*d*x^2) - 2*(a^2*b^4*d*x^2*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^2*sin(2
*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^2*cos(c)*sin(d*x^2) + 2*(a^5*b -
 a^3*b^3)*d*x^2*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^2)*cos(2*d*x^2
+ 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*
c)*sin(c))*d*x^2*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b
^3 - a*b^5)*cos(2*c)*sin(c))*d*x^2*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^2*sin(2
*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x^2*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^2*
cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2)*cos(c) + 2*(a^
5*b - a^3*b^3)*d*x^2*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c)), x) + (a^3*b*si
n(d*x^2 + c) + a^2*b^2)*sin(2*d*x^2 + 2*c) - (a*b^3*cos(2*c)*sin(2*d*x^2) +
 a*b^3*cos(2*d*x^2)*sin(2*c) - 2*(a^4 - a^2*b^2)*cos(d*x^2)*cos(c) + 2*(a^4
 - a^2*b^2)*sin(d*x^2)*sin(c))*sin(d*x^2 + c))/(a^4*b^2*d*x*cos(2*d*x^2 + 2
*c)^2 + a^4*b^2*d*x*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2
)*d*x*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a
^4*b^2 + a^2*b^4)*sin(c)^2)*d*x*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*
c)^2)*d*x*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(c)*sin(d*x
^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)
*sin(c)^2)*d*x*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x^2)*
sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*s
in(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*cos(d*x^2) - (a^2*b^4 - b^
6)*d*x*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*
sin(2*c)*sin(c))*d*x*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2*b^4*d*x*cos(2*d*x^2)
*cos(2*c) - a^2*b^4*d*x*sin(2*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x*cos
(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2
*b^4)*d*x)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (
a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos
(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*sin(d*x^2) + (a^2*b^4
 - b^6)*d*x*sin(2*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x*cos(2*c)*sin(2*d*x^2) +
 a^2*b^4*d*x*cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)*cos
(c) + 2*(a^5*b - a^3*b^3)*d*x*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{1}{(b\sin(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x^2 + c) + a)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 5.98 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{1}{(a+b\,\sin(d\,x^2+c))^2}\,dx$$

```
[In] int(1/(a + b*sin(c + d*x^2))^2,x)
[Out] int(1/(a + b*sin(c + d*x^2))^2, x)
```

3.50 $\int \frac{1}{x^2(a+b\sin(c+dx^2))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sin(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x^2*(a + b*Sin[c + d*x^2])^2),x]
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d x^2\right)\right)^2} \, dx=\int \frac{1}{x^2 \left(a+b \sin \left(c+d x^2\right)\right)^2} \, dx$$

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*x^2])^2),x]
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \left(a +b \sin \left(d \,x^{2}+c \right)\right)^{2}}d x$$

```
[In] int(1/x^2/(a+b*sin(d*x^2+c))^2,x)
[Out] int(1/x^2/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d x^2\right)\right)^2} \, dx=\int { \frac {1}{{\left(b \sin \left(d x^{2} + c\right) + a\right)}^{2} x^{2}} \,d x }$$

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*x^2*cos(d*x^2 + c)^2 - 2*a*b*x^2*sin(d*x^2 + c) - (a^2 + b
^2)*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 44.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 (a+b\sin(c+dx^2))^2}\,dx=\int \frac{1}{x^2 (a+b\sin(c+dx^2))^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*sin(d*x**2+c))**2,x)
[Out] Integral(1/(x**2*(a + b*sin(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 4.14 (sec) , antiderivative size = 3486, normalized size of antiderivative = 193.67

$$\int \frac{1}{x^2 (a+b\sin(c+dx^2))^2}\,dx=\int \frac{1}{(b\sin(dx^2+c)+a)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
[Out] (a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*cos(2*c)*sin(2*d*x^2) - b^4*
cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) - 2*(a^3*b - a*
b^3)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c) - a*b^3*sin(2*d*x^2)*
sin(2*c) + a^3*b - a*b^3 + 2*(a^4 - a^2*b^2)*cos(c)*sin(d*x^2) + 2*(a^4 - a
^2*b^2)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + (a^4*b^2*d*x^3*cos(2*d*x^2 + 2*
c)^2 + a^4*b^2*d*x^3*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^
2)*d*x^3*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 -
2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^3*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*s
in(2*c)^2)*d*x^3*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^3*cos(c
)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 +
 a^2*b^4)*sin(c)^2)*d*x^3*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^
3*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^3 - 2*(2*((a^3*b^3 -
a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^3*cos(d*x^2
) - (a^2*b^4 - b^6)*d*x^3*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) +
 (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^3*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2
*b^4*d*x^3*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^3*sin(2*d*x^2)*sin(2*c) + 2*
(a^5*b - a^3*b^3)*d*x^3*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^3*cos(d
*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^3)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b
^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^3*cos(
d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*
sin(c))*d*x^3*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^3*sin(2*c))*sin(2*d*x^2) - 2
*(a^2*b^4*d*x^3*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^3*cos(2*d*x^2)*sin(2*c)
```

```
 - 2*(a^5*b - a^3*b^3)*d*x^3*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x^3*
sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*integrate(-(3*b^4*cos(2*c)*sin(2*d*x
^2) + 3*b^4*cos(2*d*x^2)*sin(2*c) - 6*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) + 6
*(a^3*b - a*b^3)*sin(d*x^2)*sin(c) + (2*a^3*b*d*x^2*sin(d*x^2 + c) - 3*a^3*
b*cos(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (3*a^3*b - 3*a*b^3 + (2*a*b^3*d*x^2*
sin(2*c) + 3*a*b^3*cos(2*c))*cos(2*d*x^2) - 2*(2*(a^4 - a^2*b^2)*d*x^2*cos(
c) - 3*(a^4 - a^2*b^2)*sin(c))*cos(d*x^2) + (2*a*b^3*d*x^2*cos(2*c) - 3*a*b
^3*sin(2*c))*sin(2*d*x^2) + 2*(2*(a^4 - a^2*b^2)*d*x^2*sin(c) + 3*(a^4 - a^
2*b^2)*cos(c))*sin(d*x^2))*cos(d*x^2 + c) - (2*a^3*b*d*x^2*cos(d*x^2 + c) +
 3*a^3*b*sin(d*x^2 + c) + 3*a^2*b^2)*sin(2*d*x^2 + 2*c) - (2*(a^3*b - a*b^3
)*d*x^2 + (2*a*b^3*d*x^2*cos(2*c) - 3*a*b^3*sin(2*c))*cos(2*d*x^2) + 2*(2*(
a^4 - a^2*b^2)*d*x^2*sin(c) + 3*(a^4 - a^2*b^2)*cos(c))*cos(d*x^2) - (2*a*b
^3*d*x^2*sin(2*c) + 3*a*b^3*cos(2*c))*sin(2*d*x^2) + 2*(2*(a^4 - a^2*b^2)*d
*x^2*cos(c) - 3*(a^4 - a^2*b^2)*sin(c))*sin(d*x^2))*sin(d*x^2 + c))/(a^4*b^
2*d*x^4*cos(2*d*x^2 + 2*c)^2 + a^4*b^2*d*x^4*sin(2*d*x^2 + 2*c)^2 + (b^6*co
s(2*c)^2 + b^6*sin(2*c)^2)*d*x^4*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2
*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^4*cos(d*x^2)^2 +
 (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^4*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*
b^3 + a*b^5)*d*x^4*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c
)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^4*sin(d*x^2)^2 + 4*(a^5*b -
 2*a^3*b^3 + a*b^5)*d*x^4*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d
*x^4 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)
*sin(c))*d*x^4*cos(d*x^2) - (a^2*b^4 - b^6)*d*x^4*cos(2*c) - 2*((a^3*b^3 -
a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^4*sin(d*x^2
))*cos(2*d*x^2) - 2*(a^2*b^4*d*x^4*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^4*si
n(2*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^4*cos(c)*sin(d*x^2) + 2*(a^5*
b - a^3*b^3)*d*x^4*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^4)*cos(2*d*x
^2 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin
(2*c)*sin(c))*d*x^4*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^
3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^4*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^4*si
n(2*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x^4*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x
^4*cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x^4*cos(d*x^2)*cos(c) + 2*
(a^5*b - a^3*b^3)*d*x^4*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c)), x) + (a^3*b
*sin(d*x^2 + c) + a^2*b^2)*sin(2*d*x^2 + 2*c) - (a*b^3*cos(2*c)*sin(2*d*x^2
) + a*b^3*cos(2*d*x^2)*sin(2*c) - 2*(a^4 - a^2*b^2)*cos(d*x^2)*cos(c) + 2*(
a^4 - a^2*b^2)*sin(d*x^2)*sin(c))*sin(d*x^2 + c))/(a^4*b^2*d*x^3*cos(2*d*x^
2 + 2*c)^2 + a^4*b^2*d*x^3*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin
(2*c)^2)*d*x^3*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (
a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^3*cos(d*x^2)^2 + (b^6*cos(2*c)^2 +
 b^6*sin(2*c)^2)*d*x^3*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^3
*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4
*b^2 + a^2*b^4)*sin(c)^2)*d*x^3*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5
)*d*x^3*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^3 - 2*(2*((a^3*
b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^3*cos
(d*x^2) - (a^2*b^4 - b^6)*d*x^3*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*co
```

```
s(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^3*sin(d*x^2))*cos(2*d*x^2) -
2*(a^2*b^4*d*x^3*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^3*sin(2*d*x^2)*sin(2*c
) + 2*(a^5*b - a^3*b^3)*d*x^3*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^3
*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^3)*cos(2*d*x^2 + 2*c) - 2*(2*(
(a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^
3*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos
(2*c)*sin(c))*d*x^3*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^3*sin(2*c))*sin(2*d*x^
2) - 2*(a^2*b^4*d*x^3*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^3*cos(2*d*x^2)*si
n(2*c) - 2*(a^5*b - a^3*b^3)*d*x^3*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*
d*x^3*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a+b\sin(c+dx^2))^2} \, dx = \int \frac{1}{(b\sin(dx^2+c)+a)^2 x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 6.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a+b\sin(c+dx^2))^2} \, dx = \int \frac{1}{x^2 (a+b\,\sin(d\,x^2+c))^2} \, dx$$

```
[In] int(1/(x^2*(a + b*sin(c + d*x^2))^2),x)
[Out] int(1/(x^2*(a + b*sin(c + d*x^2))^2), x)
```

3.51 $\int (ex)^m (a+b\sin(c+dx^2))^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx = \mathrm{Int}\left((ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p, x\right)$$

```
[Out] Unintegrable((e*x)^m*(a+b*sin(d*x^2+c))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx = \int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx$$

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^2])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Sin[c + d*x^2])^p, x]
```

Rubi steps

$$\text{integral} = \int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx=\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx$$

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m \left(a+b\sin\left(d\,x^2+c\right)\right)^p dx$$

```
[In] int((e*x)^m*(a+b*sin(d*x^2+c))^p,x)
[Out] int((e*x)^m*(a+b*sin(d*x^2+c))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx=\int (ex)^m \left(b\sin\left(dx^2+c\right)+a\right)^p\,dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*sin(d*x^2 + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 16.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx=\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx$$

```
[In] integrate((e*x)**m*(a+b*sin(d*x**2+c))**p,x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**2))**p, x)
```

Maxima [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \,dx=\int (ex)^m \left(b\sin\left(dx^2+c\right)+a\right)^p dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*sin(d*x^2 + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \,dx=\int (ex)^m \left(b\sin\left(dx^2+c\right)+a\right)^p dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*sin(d*x^2 + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 6.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \,dx=\int (e\,x)^m \left(a+b\,\sin\left(d\,x^2+c\right)\right)^p dx$$

```
[In] int((e*x)^m*(a + b*sin(c + d*x^2))^p,x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^2))^p, x)
```

3.52 $\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 444

$$\begin{aligned}
&\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3\,dx \\
&= \frac{a(2a^2+3b^2)\,(ex)^{1+m}}{2e(1+m)} + \frac{3ib(4a^2+b^2)\,e^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{16e} \\
&\quad - \frac{3ib(4a^2+b^2)\,e^{-ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},idx^2\right)}{16e} \\
&\quad + \frac{3\,2^{-\frac{7}{2}-\frac{m}{2}}ab^2e^{2ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-2idx^2\right)}{e} \\
&\quad + \frac{3\,2^{-\frac{7}{2}-\frac{m}{2}}ab^2e^{-2ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},2idx^2\right)}{e} \\
&\quad - \frac{i3^{-\frac{1}{2}-\frac{m}{2}}b^3e^{3ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-3idx^2\right)}{16e} \\
&\quad + \frac{i3^{-\frac{1}{2}-\frac{m}{2}}b^3e^{-3ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},3idx^2\right)}{16e}
\end{aligned}$$

```
[Out] 1/2*a*(2*a^2+3*b^2)*(e*x)^(1+m)/e/(1+m)+3/16*I*b*(4*a^2+b^2)*exp(I*c)*(e*x)
^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-I*d*x^2)/e-3/16*I*b*(4*a^2+
b^2)*(e*x)^(1+m)*(I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,I*d*x^2)/e/exp(I*c)
+3*2^(-7/2-1/2*m)*a*b^2*exp(2*I*c)*(e*x)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*GAMM
A(1/2+1/2*m,-2*I*d*x^2)/e+3*2^(-7/2-1/2*m)*a*b^2*(e*x)^(1+m)*(I*d*x^2)^(-1/
2-1/2*m)*GAMMA(1/2+1/2*m,2*I*d*x^2)/e/exp(2*I*c)-1/16*I*3^(-1/2-1/2*m)*b^3*
exp(3*I*c)*(e*x)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-3*I*d*x^2)/
e+1/16*I*3^(-1/2-1/2*m)*b^3*(e*x)^(1+m)*(I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/
2*m,3*I*d*x^2)/e/exp(3*I*c)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3484, 6, 3471, 2250, 3470}

$$\begin{aligned}\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 dx \\ = \frac{3ibe^{ic}(4a^2+b^2)\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-idx^2\right)}{16e} \\ -\frac{3ibe^{-ic}(4a^2+b^2)\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},idx^2\right)}{16e}+\frac{a(2a^2+3b^2)(ex)^{m+1}}{2e(m+1)} \\ +\frac{3ab^2e^{2ic}2^{-\frac{m}{2}-\frac{7}{2}}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-2idx^2\right)}{e} \\ +\frac{3ab^2e^{-2ic}2^{-\frac{m}{2}-\frac{7}{2}}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},2idx^2\right)}{e} \\ -\frac{ib^3e^{3ic}3^{-\frac{m}{2}-\frac{1}{2}}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-3idx^2\right)}{16e} \\ +\frac{ib^3e^{-3ic}3^{-\frac{m}{2}-\frac{1}{2}}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},3idx^2\right)}{16e}\end{aligned}$$

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^2])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + (((3*I)/16)*b*(4*a^2 + b^
2)*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*
x^2])/e - (((3*I)/16)*b*(4*a^2 + b^2)*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*
Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c)) + (3*2^(-7/2 - m/2)*a*b^2*E^((2*I)*c
)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-2*I)*d*x^2])/e
 + (3*2^(-7/2 - m/2)*a*b^2*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1 +
m)/2, (2*I)*d*x^2])/(e*E^((2*I)*c)) - ((I/16)*3^(-1/2 - m/2)*b^3*E^((3*I)*c
)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-3*I)*d*x^2])/e
 + ((I/16)*3^(-1/2 - m/2)*b^3*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1
 + m)/2, (3*I)*d*x^2])/(e*E^((3*I)*c))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
```

```
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3(ex)^m + \frac{3}{2}ab^2(ex)^m - \frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^2\right) + 3a^2b(ex)^m\sin\left(c+dx^2\right)\right.\\
&\qquad \left. + \frac{3}{4}b^3(ex)^m\sin\left(c+dx^2\right) - \frac{1}{4}b^3(ex)^m\sin\left(3c+3dx^2\right)\right)\,dx\\
&= \int \left(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m - \frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^2\right) + 3a^2b(ex)^m\sin\left(c+dx^2\right)\right.\\
&\qquad \left. + \frac{3}{4}b^3(ex)^m\sin\left(c+dx^2\right) - \frac{1}{4}b^3(ex)^m\sin\left(3c+3dx^2\right)\right)\,dx\\
&= \int \left(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m - \frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^2\right)\right.\\
&\qquad \left. + \left(3a^2b+\frac{3b^3}{4}\right)(ex)^m\sin\left(c+dx^2\right) - \frac{1}{4}b^3(ex)^m\sin\left(3c+3dx^2\right)\right)\,dx\\
&= \frac{a(2a^2+3b^2)(ex)^{1+m}}{2e(1+m)} - \frac{1}{2}\left(3ab^2\right)\int (ex)^m\cos\left(2c+2dx^2\right)\,dx\\
&\quad - \frac{1}{4}b^3\int (ex)^m\sin\left(3c+3dx^2\right)\,dx + \frac{1}{4}\left(3b\left(4a^2+b^2\right)\right)\int (ex)^m\sin\left(c+dx^2\right)\,dx
\end{aligned}$$

$$= \frac{a(2a^2+3b^2)(ex)^{1+m}}{2e(1+m)} - \frac{1}{4}(3ab^2)\int e^{-2ic-2idx^2}(ex)^m\,dx - \frac{1}{4}(3ab^2)\int e^{2ic+2idx^2}(ex)^m\,dx - \frac{1}{8}(ib^3)\int e^{-3ic-3idx^2}(ex)^m\,dx + \frac{1}{8}(ib^3)\int e^{3ic+3idx^2}(ex)^m\,dx + \frac{1}{8}(3ib(4a^2+b^2))\int e^{-ic-idx^2}(ex)^m\,dx - \frac{1}{8}(3ib(4a^2+b^2))\int e^{ic+idx^2}(ex)^m\,dx$$

$$= \frac{a(2a^2+3b^2)(ex)^{1+m}}{2e(1+m)} + \frac{3ib(4a^2+b^2)\,e^{ic}(ex)^{1+m}\,(-idx^2)^{\frac{1}{2}(-1-m)}\,\Gamma\left(\frac{1+m}{2},-idx^2\right)}{16e} - \frac{3ib(4a^2+b^2)\,e^{-ic}(ex)^{1+m}\,(idx^2)^{\frac{1}{2}(-1-m)}\,\Gamma\left(\frac{1+m}{2},idx^2\right)}{16e} + \frac{3\;2^{-\frac{7}{2}-\frac{m}{2}}ab^2e^{2ic}(ex)^{1+m}\,(-idx^2)^{\frac{1}{2}(-1-m)}\,\Gamma\left(\frac{1+m}{2},-2idx^2\right)}{e} + \frac{3\;2^{-\frac{7}{2}-\frac{m}{2}}ab^2e^{-2ic}(ex)^{1+m}\,(idx^2)^{\frac{1}{2}(-1-m)}\,\Gamma\left(\frac{1+m}{2},2idx^2\right)}{e} - \frac{i3^{-\frac{1}{2}-\frac{m}{2}}b^3e^{3ic}(ex)^{1+m}\,(-idx^2)^{\frac{1}{2}(-1-m)}\,\Gamma\left(\frac{1+m}{2},-3idx^2\right)}{16e} + \frac{i3^{-\frac{1}{2}-\frac{m}{2}}b^3e^{-3ic}(ex)^{1+m}\,(idx^2)^{\frac{1}{2}(-1-m)}\,\Gamma\left(\frac{1+m}{2},3idx^2\right)}{16e}$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.84

$$\int (ex)^m\left(a+b\sin\left(c+dx^2\right)\right)^3\,dx = \frac{1}{16}ix(ex)^m\left(-\frac{8ia(2a^2+3b^2)}{1+m} + 3b(4a^2+b^2)\,e^{ic}\left(-idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},-idx^2\right) - 3b(4a^2+b^2)\,e^{-ic}\left(idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},idx^2\right) - 3i2^{\frac{1}{2}-\frac{m}{2}}ab^2e^{2ic}\left(-idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},-2idx^2\right) - 3i2^{\frac{1}{2}-\frac{m}{2}}ab^2e^{-2ic}\left(idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},2idx^2\right) - 3^{-\frac{1}{2}-\frac{m}{2}}b^3e^{3ic}\left(-idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},-3idx^2\right) + 3^{-\frac{1}{2}-\frac{m}{2}}b^3e^{-3ic}\left(idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},3idx^2\right)\right)$$

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^3,x]
```

```
[Out] (I/16)*x*(e*x)^m*(((-8*I)*a*(2*a^2 + 3*b^2))/(1 + m) + 3*b*(4*a^2 + b^2)*E^
(I*c)*((-I)*d*x^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, (-I)*d*x^2] - (3*b*(4*a^2
+ b^2)*(I*d*x^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, I*d*x^2])/E^(I*c) - (3*I)*2^
(1/2 - m/2)*a*b^2*E^((2*I)*c)*((-I)*d*x^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, (-
2*I)*d*x^2] - ((3*I)*2^(1/2 - m/2)*a*b^2*(I*d*x^2)^(-1/2 - m/2)*Gamma[(1 +
m)/2, (2*I)*d*x^2])/E^((2*I)*c) - 3^(-1/2 - m/2)*b^3*E^((3*I)*c)*((-I)*d*x^
2)^(-1/2 - m/2)*Gamma[(1 + m)/2, (-3*I)*d*x^2] + (3^(-1/2 - m/2)*b^3*(I*d*x
^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, (3*I)*d*x^2])/E^((3*I)*c))
```

Maple [F]

$$\int \left(e x \right)^{m} \left(a +b \sin \left(d \,x^{2}+c \right)\right)^{3}d x$$

```
[In] int((e*x)^m*(a+b*sin(d*x^2+c))^3,x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^2+c))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.73

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3\,dx$$
$$= \frac{24\left(2\,a^3+3\,ab^2\right)(ex)^m\,dx+\left(b^3em+b^3e\right)e^{\left(-\frac{1}{2}\,(m-1)\log\left(\frac{3i\,d}{e^2}\right)-3i\,c\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},3i\,dx^2\right)-9\left(i\,ab^2em+i\,ab^2e\right)e}{}$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(24*(2*a^3 + 3*a*b^2)*(e*x)^m*d*x + (b^3*e*m + b^3*e)*e^(-1/2*(m - 1)*
log(3*I*d/e^2) - 3*I*c)*gamma(1/2*m + 1/2, 3*I*d*x^2) - 9*(I*a*b^2*e*m + I*
a*b^2*e)*e^(-1/2*(m - 1)*log(2*I*d/e^2) - 2*I*c)*gamma(1/2*m + 1/2, 2*I*d*x
^2) - 9*((4*a^2*b + b^3)*e*m + (4*a^2*b + b^3)*e)*e^(-1/2*(m - 1)*log(I*d/e
^2) - I*c)*gamma(1/2*m + 1/2, I*d*x^2) - 9*((4*a^2*b + b^3)*e*m + (4*a^2*b
+ b^3)*e)*e^(-1/2*(m - 1)*log(-I*d/e^2) + I*c)*gamma(1/2*m + 1/2, -I*d*x^2)
 - 9*(-I*a*b^2*e*m - I*a*b^2*e)*e^(-1/2*(m - 1)*log(-2*I*d/e^2) + 2*I*c)*ga
mma(1/2*m + 1/2, -2*I*d*x^2) + (b^3*e*m + b^3*e)*e^(-1/2*(m - 1)*log(-3*I*d
/e^2) + 3*I*c)*gamma(1/2*m + 1/2, -3*I*d*x^2))/(d*m + d)
```

Sympy [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 \, dx = \int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 \, dx$$

```
[In] integrate((e*x)**m*(a+b*sin(d*x**2+c))**3,x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**2))**3, x)
```

Maxima [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 \, dx = \int \left(b\sin\left(dx^2+c\right)+a\right)^3 (ex)^m \, dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^3,x, algorithm="maxima")
[Out] (e*x)^(m + 1)*a^3/(e*(m + 1)) + 1/8*(12*a*b^2*e^m*x*x^m - 12*(a*b^2*e^m*m +
 a*b^2*e^m)*integrate(x^m*cos(2*d*x^2 + 2*c), x) + 3*((4*a^2*b + b^3)*e^m*m
*sin(c) + (4*a^2*b + b^3)*e^m*sin(c))*integrate(x^m*cos(d*x^2), x) - 2*(b^3
*e^m*m + b^3*e^m)*integrate(x^m*sin(3*d*x^2 + 3*c), x) + 3*((4*a^2*b + b^3)
*e^m*m + (4*a^2*b + b^3)*e^m)*integrate(x^m*sin(d*x^2 + c), x) + 3*((4*a^2*
b + b^3)*e^m*m*cos(c) + (4*a^2*b + b^3)*e^m*cos(c))*integrate(x^m*sin(d*x^2
), x))/(m + 1)
```

Giac [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 \, dx = \int \left(b\sin\left(dx^2+c\right)+a\right)^3 (ex)^m \, dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^3,x, algorithm="giac")
[Out] integrate((b*sin(d*x^2 + c) + a)^3*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 \, dx = \int (e\,x)^m \left(a+b\,\sin\left(d\,x^2+c\right)\right)^3 dx$$

```
[In] int((e*x)^m*(a + b*sin(c + d*x^2))^3,x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^2))^3, x)
```

3.53 $\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 279

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{(2a^2+b^2)(ex)^{1+m}}{2e(1+m)}+\frac{iabe^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{2e}-\frac{iabe^{-ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},idx^2\right)}{2e}+\frac{2^{-\frac{7}{2}-\frac{m}{2}}b^2e^{2ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-2idx^2\right)}{e}+\frac{2^{-\frac{7}{2}-\frac{m}{2}}b^2e^{-2ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},2idx^2\right)}{e}$$

```
[Out] 1/2*(2*a^2+b^2)*(e*x)^(1+m)/e/(1+m)+1/2*I*a*b*exp(I*c)*(e*x)^(1+m)*(-I*d*x^
2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-I*d*x^2)/e-1/2*I*a*b*(e*x)^(1+m)*(I*d*x^2)
^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,I*d*x^2)/e/exp(I*c)+2^(-7/2-1/2*m)*b^2*exp(2*
I*c)*(e*x)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-2*I*d*x^2)/e+2^(-
7/2-1/2*m)*b^2*(e*x)^(1+m)*(I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,2*I*d*x^2
)/e/exp(2*I*c)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3484, 6, 3471, 2250, 3470}

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2 \, dx = \frac{(2a^2+b^2)\,(ex)^{m+1}}{2e(m+1)} + \frac{iabe^{ic}(-idx^2)^{\frac{1}{2}(-m-1)}\,(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-idx^2\right)}{2e} - \frac{iabe^{-ic}(idx^2)^{\frac{1}{2}(-m-1)}\,(ex)^{m+1}\Gamma\left(\frac{m+1}{2},idx^2\right)}{2e} + \frac{b^2e^{2ic}2^{-\frac{m}{2}-\frac{7}{2}}(-idx^2)^{\frac{1}{2}(-m-1)}\,(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-2idx^2\right)}{e} + \frac{b^2e^{-2ic}2^{-\frac{m}{2}-\frac{7}{2}}(idx^2)^{\frac{1}{2}(-m-1)}\,(ex)^{m+1}\Gamma\left(\frac{m+1}{2},2idx^2\right)}{e}$$

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a^2 + b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/2)*a*b*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2])/e - ((I/2)*a*b*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c)) + (2^(-7/2 - m/2)*b^2*E^((2*I)*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-2*I)*d*x^2])/e + (2^(-7/2 - m/2)*b^2*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (2*I)*d*x^2])/(e*E^((2*I)*c))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2, Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I + d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2(ex)^m + \frac{1}{2}b^2(ex)^m - \frac{1}{2}b^2(ex)^m \cos\left(2c+2dx^2\right) + 2ab(ex)^m \sin\left(c+dx^2\right)\right) dx \\
&= \int \left(\left(a^2+\frac{b^2}{2}\right)(ex)^m - \frac{1}{2}b^2(ex)^m \cos\left(2c+2dx^2\right) + 2ab(ex)^m \sin\left(c+dx^2\right)\right) dx \\
&= \frac{(2a^2+b^2)(ex)^{1+m}}{2e(1+m)} + (2ab)\int (ex)^m \sin\left(c+dx^2\right) dx - \frac{1}{2}b^2 \int (ex)^m \cos\left(2c+2dx^2\right) dx \\
&= \frac{(2a^2+b^2)(ex)^{1+m}}{2e(1+m)} + (iab)\int e^{-ic-idx^2}(ex)^m dx - (iab)\int e^{ic+idx^2}(ex)^m dx \\
&\quad - \frac{1}{4}b^2 \int e^{-2ic-2idx^2}(ex)^m dx - \frac{1}{4}b^2 \int e^{2ic+2idx^2}(ex)^m dx \\
&= \frac{(2a^2+b^2)(ex)^{1+m}}{2e(1+m)} + \frac{iabe^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{2e} \\
&\quad - \frac{iabe^{-ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},idx^2\right)}{2e} \\
&\quad + \frac{2^{-\frac{7}{2}-\frac{m}{2}}b^2e^{2ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-2idx^2\right)}{e} \\
&\quad + \frac{2^{-\frac{7}{2}-\frac{m}{2}}b^2e^{-2ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},2idx^2\right)}{e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.97

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2 dx$$
$$= \frac{2^{\frac{1}{2}(-7-m)}x(ex)^m\left(d^2x^4\right)^{\frac{1}{2}(-1-m)}\left(2^{\frac{7+m}{2}}a^2\left(d^2x^4\right)^{\frac{1+m}{2}} + 2^{\frac{5+m}{2}}b^2\left(d^2x^4\right)^{\frac{1+m}{2}} + b^2\left(idx^2\right)^{\frac{1+m}{2}}\cos(2c)\Gamma\left(\frac{1+m}{2},-2id\right.\right.}{}$$

[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^2,x]

[Out] (2^((-7 - m)/2)*x*(e*x)^m*(d^2*x^4)^((-1 - m)/2)*(2^((7 + m)/2)*a^2*(d^2*x^4)^((1 + m)/2) + 2^((5 + m)/2)*b^2*(d^2*x^4)^((1 + m)/2) + b^2*(I*d*x^2)^((1 + m)/2)*Cos[2*c]*Gamma[(1 + m)/2, (-2*I)*d*x^2] + b^2*m*(I*d*x^2)^((1 + m)/2)*Cos[2*c]*Gamma[(1 + m)/2, (-2*I)*d*x^2] + b^2*((-I)*d*x^2)^((1 + m)/2)*Cos[2*c]*Gamma[(1 + m)/2, (2*I)*d*x^2] + b^2*m*((-I)*d*x^2)^((1 + m)/2)*Cos[2*c]*Gamma[(1 + m)/2, (2*I)*d*x^2] - I*2^((5 + m)/2)*a*b*(1 + m)*((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, I*d*x^2]*(Cos[c] - I*Sin[c]) + I*2^((5 + m)/2)*a*b*(1 + m)*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2]*(Cos[c] + I*Sin[c]) + I*b^2*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-2*I)*d*x^2]*Sin[2*c] + I*b^2*m*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-2*I)*d*x^2]*Sin[2*c] - I*b^2*((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (2*I)*d*x^2]*Sin[2*c] - I*b^2*m*((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (2*I)*d*x^2]*Sin[2*c]))/(1 + m)

Maple [F]

$$\int (ex)^m \left(a+b\sin\left(d\,x^2+c\right)\right)^2 dx$$

[In] int((e*x)^m*(a+b*sin(d*x^2+c))^2,x)

[Out] int((e*x)^m*(a+b*sin(d*x^2+c))^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.71

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2 \, dx$$
$$= \frac{8\,(2\,a^2+b^2)(ex)^m\,dx + (-i\,b^2em - i\,b^2e)e^{\left(-\frac{1}{2}\,(m-1)\log\left(\frac{2i\,d}{e^2}\right)-2i\,c\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},2i\,dx^2\right) - 8\,(abem+abe)e^{\left(-\frac{1}{2}\,(m\right.}}{}$$

[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")

[Out] 1/16*(8*(2*a^2 + b^2)*(e*x)^m*d*x + (-I*b^2*e*m - I*b^2*e)*e^(-1/2*(m - 1)*log(2*I*d/e^2) - 2*I*c)*gamma(1/2*m + 1/2, 2*I*d*x^2) - 8*(a*b*e*m + a*b*e)*e^(-1/2*(m - 1)*log(I*d/e^2) - I*c)*gamma(1/2*m + 1/2, I*d*x^2) - 8*(a*b*e*m + a*b*e)*e^(-1/2*(m - 1)*log(-I*d/e^2) + I*c)*gamma(1/2*m + 1/2, -I*d*x^2) + (I*b^2*e*m + I*b^2*e)*e^(-1/2*(m - 1)*log(-2*I*d/e^2) + 2*I*c)*gamma(1/2*m + 1/2, -2*I*d*x^2))/(d*m + d)

Sympy [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate((e*x)**m*(a+b*sin(d*x**2+c))**2,x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**2))**2, x)
```

Maxima [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\int \left(b\sin\left(dx^2+c\right)+a\right)^2 (ex)^m\,dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
[Out] (e*x)^(m + 1)*a^2/(e*(m + 1)) + 1/2*(b^2*e^m*x*x^m - (b^2*e^m*m + b^2*e^m)*
integrate(x^m*cos(2*d*x^2 + 2*c), x) + 4*(a*b*e^m*m + a*b*e^m)*integrate(x^
m*sin(d*x^2 + c), x))/(m + 1)
```

Giac [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\int \left(b\sin\left(dx^2+c\right)+a\right)^2 (ex)^m\,dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x^2 + c) + a)^2*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\int (e\,x)^m\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2\,dx$$

```
[In] int((e*x)^m*(a + b*sin(c + d*x^2))^2,x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^2))^2, x)
```

3.54 $\int (ex)^m (a+b\sin(c+dx^2))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 134

$$\int (ex)^m (a+b\sin(c+dx^2))\,dx = \frac{a(ex)^{1+m}}{e(1+m)} + \frac{ibe^{ic}(ex)^{1+m}(-idx^2)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{4e} - \frac{ibe^{-ic}(ex)^{1+m}(idx^2)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},idx^2\right)}{4e}$$

```
[Out] a*(e*x)^(1+m)/e/(1+m)+1/4*I*b*exp(I*c)*(e*x)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*
GAMMA(1/2+1/2*m,-I*d*x^2)/e-1/4*I*b*(e*x)^(1+m)*(I*d*x^2)^(-1/2-1/2*m)*GAMM
A(1/2+1/2*m,I*d*x^2)/e/exp(I*c)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {14, 3470, 2250}

$$\int (ex)^m (a+b\sin(c+dx^2))\,dx = \frac{a(ex)^{m+1}}{e(m+1)} + \frac{ibe^{ic}(-idx^2)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-idx^2\right)}{4e} - \frac{ibe^{-ic}(idx^2)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},idx^2\right)}{4e}$$

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*(e*x)^(1 + m))/(e*(1 + m)) + ((I/4)*b*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^2)
^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2])/e - ((I/4)*b*(e*x)^(1 + m)*(I*d
*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a(ex)^m + b(ex)^m \sin\left(c+dx^2\right)\right)\,dx\\ &= \frac{a(ex)^{1+m}}{e(1+m)} + b\int (ex)^m \sin\left(c+dx^2\right)\,dx\\ &= \frac{a(ex)^{1+m}}{e(1+m)} + \frac{1}{2}(ib)\int e^{-ic-idx^2}(ex)^m\,dx - \frac{1}{2}(ib)\int e^{ic+idx^2}(ex)^m\,dx\\ &= \frac{a(ex)^{1+m}}{e(1+m)} + \frac{ibe^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{4e}\\ &\quad - \frac{ibe^{-ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},idx^2\right)}{4e}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.11

$$\int (ex)^m\left(a+b\sin\left(c+dx^2\right)\right)\,dx$$

$$= \frac{x(ex)^m\left(d^2x^4\right)^{\frac{1}{2}(-1-m)}\left(4a\left(d^2x^4\right)^{\frac{1+m}{2}} - ib(1+m)\left(-idx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},idx^2\right)(\cos(c)-i\sin(c)) + ib(1+m)}{4(1+m)}$$

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (x*(e*x)^m*(d^2*x^4)^((-1 - m)/2)*(4*a*(d^2*x^4)^((1 + m)/2) - I*b*(1 + m)*
((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, I*d*x^2]*(Cos[c] - I*Sin[c]) + I*
b*(1 + m)*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2]*(Cos[c] + I*Si
n[c])))/(4*(1 + m))
```

Maple [F]

$$\int (ex)^m \left(a + b\sin\left(d\,x^2 + c\right)\right) dx$$

```
[In] int((e*x)^m*(a+b*sin(d*x^2+c)),x)
[Out] int((e*x)^m*(a+b*sin(d*x^2+c)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int (ex)^m \left(a + b\sin\left(c + dx^2\right)\right)\,dx = \frac{4\,(ex)^m\,adx - (bem + be)e^{\left(-\frac{1}{2}\,(m-1)\log\left(\frac{i\,d}{e^2}\right) - i\,c\right)}\Gamma\left(\frac{1}{2}\,m + \frac{1}{2}, i\,dx^2\right) - (bem + be)e^{\left(-\frac{1}{2}\,(m-1)\log\left(-\frac{i\,d}{e^2}\right) + i\,c\right)}\Gamma\left(\frac{1}{2}\,m}{4\,(dm + d)}$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] 1/4*(4*(e*x)^m*a*d*x - (b*e*m + b*e)*e^(-1/2*(m - 1)*log(I*d/e^2) - I*c)*ga
mma(1/2*m + 1/2, I*d*x^2) - (b*e*m + b*e)*e^(-1/2*(m - 1)*log(-I*d/e^2) + I
*c)*gamma(1/2*m + 1/2, -I*d*x^2))/(d*m + d)
```

Sympy [F]

$$\int (ex)^m \left(a + b\sin\left(c + dx^2\right)\right)\,dx = \int (ex)^m \left(a + b\sin\left(c + dx^2\right)\right)\,dx$$

```
[In] integrate((e*x)**m*(a+b*sin(d*x**2+c)),x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**2)), x)
```

Maxima [F]

$$\int (ex)^m \left(a + b\sin\left(c + dx^2\right)\right)\,dx = \int \left(b\sin\left(dx^2 + c\right) + a\right)(ex)^m\,dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] b*e^m*integrate(x^m*sin(d*x^2 + c), x) + (e*x)^(m + 1)*a/(e*(m + 1))
```

Giac [F]

$$\int (ex)^m\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\int \left(b\sin\left(dx^2+c\right)+a\right)(ex)^m\,dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate((b*sin(d*x^2 + c) + a)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\int (e\,x)^m\left(a+b\,\sin\left(d\,x^2+c\right)\right)\,dx$$

```
[In] int((e*x)^m*(a + b*sin(c + d*x^2)),x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^2)), x)
```

3.55 $\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx = \text{Int}\left(\frac{(ex)^m}{a+b\sin(c+dx^2)},x\right)$$

```
[Out] Unintegrable((e*x)^m/(a+b*sin(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx = \int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx$$

```
[In] Int[(e*x)^m/(a + b*Sin[c + d*x^2]),x]
[Out] Defer[Int][(e*x)^m/(a + b*Sin[c + d*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx=\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx$$

```
[In] Integrate[(e*x)^m/(a + b*Sin[c + d*x^2]),x]
[Out] Integrate[(e*x)^m/(a + b*Sin[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^m}{a+b\sin(d\,x^2+c)}dx$$

```
[In] int((e*x)^m/(a+b*sin(d*x^2+c)),x)
[Out] int((e*x)^m/(a+b*sin(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx=\int \frac{(ex)^m}{b\sin(dx^2+c)+a}\,dx$$

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] integral((e*x)^m/(b*sin(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx=\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx$$

```
[In] integrate((e*x)**m/(a+b*sin(d*x**2+c)),x)
[Out] Integral((e*x)**m/(a + b*sin(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a + b\sin(c + dx^2)}\,dx = \int \frac{(ex)^m}{b\sin(dx^2 + c) + a}\,dx$$

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate((e*x)^m/(b*sin(d*x^2 + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a + b\sin(c + dx^2)}\,dx = \int \frac{(ex)^m}{b\sin(dx^2 + c) + a}\,dx$$

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate((e*x)^m/(b*sin(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 6.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a + b\sin(c + dx^2)}\,dx = \int \frac{(e\,x)^m}{a + b\,\sin(d\,x^2 + c)}\,dx$$

```
[In] int((e*x)^m/(a + b*sin(c + d*x^2)),x)
[Out] int((e*x)^m/(a + b*sin(c + d*x^2)), x)
```

3.56 $\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2} \, dx = \text{Int}\left(\frac{(ex)^m}{(a+b\sin(c+dx^2))^2}, x\right)$$

```
[Out] Unintegrable((e*x)^m/(a+b*sin(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2} \, dx = \int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2} \, dx$$

```
[In] Int[(e*x)^m/(a + b*Sin[c + d*x^2])^2,x]
[Out] Defer[Int][(e*x)^m/(a + b*Sin[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2} \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2}\,dx=\int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2}\,dx$$

```
[In] Integrate[(e*x)^m/(a + b*Sin[c + d*x^2])^2,x]
[Out] Integrate[(e*x)^m/(a + b*Sin[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^m}{(a+b\sin(d\,x^2+c))^2}dx$$

```
[In] int((e*x)^m/(a+b*sin(d*x^2+c))^2,x)
[Out] int((e*x)^m/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2}\,dx=\int \frac{(ex)^m}{(b\sin(dx^2+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(-(e*x)^m/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2)
, x)
```

Sympy [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] integrate((e*x)**m/(a+b*sin(d*x**2+c))**2,x)
[Out] Integral((e*x)**m/(a + b*sin(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 8.11 (sec) , antiderivative size = 3886, normalized size of antiderivative = 194.30

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{(ex)^m}{\left(b\sin\left(dx^2+c\right)+a\right)^2}\,dx$$

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
[Out] (a^3*b*e^m*x^m*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*e^m*x^m*cos(2*c)*sin
(2*d*x^2) - b^4*e^m*x^m*cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*e^m*x^m*c
os(d*x^2)*cos(c) - 2*(a^3*b - a*b^3)*e^m*x^m*sin(d*x^2)*sin(c) - (a*b^3*e^m
*x^m*cos(2*d*x^2)*cos(2*c) - a*b^3*e^m*x^m*sin(2*d*x^2)*sin(2*c) + 2*(a^4 -
 a^2*b^2)*e^m*x^m*cos(c)*sin(d*x^2) + 2*(a^4 - a^2*b^2)*e^m*x^m*cos(d*x^2)*
sin(c) + (a^3*b - a*b^3)*e^m*x^m)*cos(d*x^2 + c) - (a^4*b^2*d*x*cos(2*d*x^2
 + 2*c)^2 + a^4*b^2*d*x*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*
c)^2)*d*x*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 -
 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*si
n(2*c)^2)*d*x*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(c)*sin
(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*
b^4)*sin(c)^2)*d*x*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x
^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x - 2*(2*((a^3*b^3 - a*b^5)*cos(
c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*cos(d*x^2) - (a^2*b^4
- b^6)*d*x*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b
^5)*sin(2*c)*sin(c))*d*x*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2*b^4*d*x*cos(2*d*
x^2)*cos(2*c) - a^2*b^4*d*x*sin(2*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x
*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)*sin(c) + (a^4*b^2 -
 a^2*b^4)*d*x)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c)
 + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)
*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*sin(d*x^2) + (a^2
*b^4 - b^6)*d*x*sin(2*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x*cos(2*c)*sin(2*d*x^
```

```
2) + a^2*b^4*d*x*cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)
*cos(c) + 2*(a^5*b - a^3*b^3)*d*x*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*in
tegrate(-((b^4*e^m*m*sin(2*c) - b^4*e^m*sin(2*c))*x^m*cos(2*d*x^2) - 2*((a^
3*b - a*b^3)*e^m*m*cos(c) - (a^3*b - a*b^3)*e^m*cos(c))*x^m*cos(d*x^2) + (b
^4*e^m*m*cos(2*c) - b^4*e^m*cos(2*c))*x^m*sin(2*d*x^2) + 2*((a^3*b - a*b^3)
*e^m*m*sin(c) - (a^3*b - a*b^3)*e^m*sin(c))*x^m*sin(d*x^2) - (2*a^3*b*d*e^m
*x^2*x^m*sin(d*x^2 + c) + (a^3*b*e^m*m - a^3*b*e^m)*x^m*cos(d*x^2 + c))*cos
(2*d*x^2 + 2*c) - ((2*a*b^3*d*e^m*x^2*sin(2*c) - a*b^3*e^m*m*cos(2*c) + a*b
^3*e^m*cos(2*c))*x^m*cos(2*d*x^2) - 2*(2*(a^4 - a^2*b^2)*d*e^m*x^2*cos(c) +
 (a^4 - a^2*b^2)*e^m*m*sin(c) - (a^4 - a^2*b^2)*e^m*sin(c))*x^m*cos(d*x^2)
+ (2*a*b^3*d*e^m*x^2*cos(2*c) + a*b^3*e^m*m*sin(2*c) - a*b^3*e^m*sin(2*c))*
x^m*sin(2*d*x^2) + 2*(2*(a^4 - a^2*b^2)*d*e^m*x^2*sin(c) - (a^4 - a^2*b^2)*
e^m*m*cos(c) + (a^4 - a^2*b^2)*e^m*cos(c))*x^m*sin(d*x^2) - ((a^3*b - a*b^3
)*e^m*m - (a^3*b - a*b^3)*e^m)*x^m)*cos(d*x^2 + c) + (2*a^3*b*d*e^m*x^2*x^m
*cos(d*x^2 + c) - (a^3*b*e^m*m - a^3*b*e^m)*x^m*sin(d*x^2 + c) - (a^2*b^2*e
^m*m - a^2*b^2*e^m)*x^m)*sin(2*d*x^2 + 2*c) + (2*(a^3*b - a*b^3)*d*e^m*x^2*
x^m + (2*a*b^3*d*e^m*x^2*cos(2*c) + a*b^3*e^m*m*sin(2*c) - a*b^3*e^m*sin(2*
c))*x^m*cos(2*d*x^2) + 2*(2*(a^4 - a^2*b^2)*d*e^m*x^2*sin(c) - (a^4 - a^2*b
^2)*e^m*m*cos(c) + (a^4 - a^2*b^2)*e^m*cos(c))*x^m*cos(d*x^2) - (2*a*b^3*d*
e^m*x^2*sin(2*c) - a*b^3*e^m*m*cos(2*c) + a*b^3*e^m*cos(2*c))*x^m*sin(2*d*x
^2) + 2*(2*(a^4 - a^2*b^2)*d*e^m*x^2*cos(c) + (a^4 - a^2*b^2)*e^m*m*sin(c)
- (a^4 - a^2*b^2)*e^m*sin(c))*x^m*sin(d*x^2))*sin(d*x^2 + c))/(a^4*b^2*d*x^
2*cos(2*d*x^2 + 2*c)^2 + a^4*b^2*d*x^2*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)
^2 + b^6*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*
cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*cos(d*x^2)^2 + (b^6*
cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^2*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 +
a*b^5)*d*x^2*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 +
(a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3
*b^3 + a*b^5)*d*x^2*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2 -
 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c
))*d*x^2*cos(d*x^2) - (a^2*b^4 - b^6)*d*x^2*cos(2*c) - 2*((a^3*b^3 - a*b^5)
*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*sin(d*x^2))*cos
(2*d*x^2) - 2*(a^2*b^4*d*x^2*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^2*sin(2*d*
x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^2*cos(c)*sin(d*x^2) + 2*(a^5*b - a^
3*b^3)*d*x^2*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^2)*cos(2*d*x^2 + 2
*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*
sin(c))*d*x^2*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3
- a*b^5)*cos(2*c)*sin(c))*d*x^2*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^2*sin(2*c)
)*sin(2*d*x^2) - 2*(a^2*b^4*d*x^2*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^2*cos
(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2)*cos(c) + 2*(a^5*b
 - a^3*b^3)*d*x^2*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c)), x) + (a^3*b*e^m*x
^m*sin(d*x^2 + c) + a^2*b^2*e^m*x^m)*sin(2*d*x^2 + 2*c) - (a*b^3*e^m*x^m*co
s(2*c)*sin(2*d*x^2) + a*b^3*e^m*x^m*cos(2*d*x^2)*sin(2*c) - 2*(a^4 - a^2*b^
2)*e^m*x^m*cos(d*x^2)*cos(c) + 2*(a^4 - a^2*b^2)*e^m*x^m*sin(d*x^2)*sin(c))
*sin(d*x^2 + c))/(a^4*b^2*d*x*cos(2*d*x^2 + 2*c)^2 + a^4*b^2*d*x*sin(2*d*x^
```

```
2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x*cos(2*d*x^2)^2 + 4*((a^6
 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*
x*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x*sin(2*d*x^2)^2 + 4*(
a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^
2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x*sin(d*x^2)^2 +
4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4
+ b^6)*d*x - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*co
s(2*c)*sin(c))*d*x*cos(d*x^2) - (a^2*b^4 - b^6)*d*x*cos(2*c) - 2*((a^3*b^3
- a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x*sin(d*x^2
))*cos(2*d*x^2) - 2*(a^2*b^4*d*x*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x*sin(2*
d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x*cos(c)*sin(d*x^2) + 2*(a^5*b - a^
3*b^3)*d*x*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x)*cos(2*d*x^2 + 2*c)
- 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(
c))*d*x*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^
5)*cos(2*c)*sin(c))*d*x*sin(d*x^2) + (a^2*b^4 - b^6)*d*x*sin(2*c))*sin(2*d*
x^2) - 2*(a^2*b^4*d*x*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x*cos(2*d*x^2)*sin(
2*c) - 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x*
sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{(ex)^m}{(b\sin(dx^2+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate((e*x)^m/(b*sin(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{(e\,x)^m}{\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

```
[In] int((e*x)^m/(a + b*sin(c + d*x^2))^2,x)
[Out] int((e*x)^m/(a + b*sin(c + d*x^2))^2, x)
```

3.57 $\int x^5(a+b\sin(c+dx^3))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{ax^6}{6}-\frac{bx^3\cos\left(c+dx^3\right)}{3d}+\frac{b\sin\left(c+dx^3\right)}{3d^2}$$

```
[Out] 1/6*a*x^6-1/3*b*x^3*cos(d*x^3+c)/d+1/3*b*sin(d*x^3+c)/d^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3460, 3377, 2717}

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{ax^6}{6}+\frac{b\sin\left(c+dx^3\right)}{3d^2}-\frac{bx^3\cos\left(c+dx^3\right)}{3d}$$

```
[In] Int[x^5*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^6)/6 - (b*x^3*Cos[c + d*x^3])/(3*d) + (b*Sin[c + d*x^3])/(3*d^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax^5 + bx^5 \sin\left(c+dx^3\right)\right) \, dx \\
&= \frac{ax^6}{6} + b\int x^5 \sin\left(c+dx^3\right) \, dx \\
&= \frac{ax^6}{6} + \frac{1}{3} b\text{Subst}\left(\int x \sin(c+dx) \, dx, x, x^3\right) \\
&= \frac{ax^6}{6} - \frac{bx^3 \cos\left(c+dx^3\right)}{3d} + \frac{b\text{Subst}\left(\int \cos(c+dx) \, dx, x, x^3\right)}{3d} \\
&= \frac{ax^6}{6} - \frac{bx^3 \cos\left(c+dx^3\right)}{3d} + \frac{b\sin\left(c+dx^3\right)}{3d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$
\int x^5\left(a+b\sin\left(c+dx^3\right)\right) \, dx = \frac{ax^6}{6} - \frac{bx^3\cos\left(c+dx^3\right)}{3d} + \frac{b\sin\left(c+dx^3\right)}{3d^2}
$$

```
[In] Integrate[x^5*(a + b*Sin[c + d*x^3]),x]
[Out] (a*x^6)/6 - (b*x^3*Cos[c + d*x^3])/(3*d) + (b*Sin[c + d*x^3])/(3*d^2)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{a\,x^{6}}{6}-\frac{b\,x^{3}\cos\left(d\,x^{3}+c\right)}{3d}+\frac{b\sin\left(d\,x^{3}+c\right)}{3d^{2}}$	39
parallelrisch	$\frac{a\,x^{6}d^{2}-2x^{3}bd\cos\left(d\,x^{3}+c\right)+2b\sin\left(d\,x^{3}+c\right)}{6d^{2}}$	41
parts	$\frac{a\,x^{6}}{6}+\frac{\frac{2b\tan\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)}{3d^{2}}-\frac{b\,x^{3}}{3d}+\frac{b\,x^{3}\left(\tan^{2}\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)\right)}{3d}}{1+\tan^{2}\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)}$	75
norman	$\frac{\frac{a\,x^{6}}{6}+\frac{a\,x^{6}\left(\tan^{2}\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)\right)}{6}+\frac{2b\tan\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)}{3d^{2}}-\frac{b\,x^{3}}{3d}+\frac{b\,x^{3}\left(\tan^{2}\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)\right)}{3d}}{1+\tan^{2}\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)}$	93

```
[In] int(x^5*(a+b*sin(d*x^3+c)),x,method=_RETURNVERBOSE)
[Out] 1/6*a*x^6-1/3*b*x^3*cos(d*x^3+c)/d+1/3*b*sin(d*x^3+c)/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x^{5}\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx=\frac{ad^{2}x^{6}-2\,bdx^{3}\cos\left(dx^{3}+c\right)+2\,b\sin\left(dx^{3}+c\right)}{6\,d^{2}}$$

```
[In] integrate(x^5*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] 1/6*(a*d^2*x^6 - 2*b*d*x^3*cos(d*x^3 + c) + 2*b*sin(d*x^3 + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int x^{5}\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx=\begin{cases}\frac{ax^{6}}{6}-\frac{bx^{3}\cos\left(c+dx^{3}\right)}{3d}+\frac{b\sin\left(c+dx^{3}\right)}{3d^{2}} & \text{for}\ d\neq 0\\ \frac{x^{6}\left(a+b\sin\left(c\right)\right)}{6} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**5*(a+b*sin(d*x**3+c)),x)
[Out] Piecewise((a*x**6/6 - b*x**3*cos(c + d*x**3)/(3*d) + b*sin(c + d*x**3)/(3*d
**2), Ne(d, 0)), (x**6*(a + b*sin(c))/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int x^5 \left(a + b\sin\left(c + dx^3\right)\right) \, dx = \frac{1}{6}\,ax^6 - \frac{(dx^3\cos\left(dx^3 + c\right) - \sin\left(dx^3 + c\right))b}{3\,d^2}$$

```
[In] integrate(x^5*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] 1/6*a*x^6 - 1/3*(d*x^3*cos(d*x^3 + c) - sin(d*x^3 + c))*b/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int x^5 \left(a + b\sin\left(c + dx^3\right)\right) \, dx = \frac{\left(dx^3 + c\right)^2 a - 2\left(dx^3 + c\right)b\cos\left(dx^3 + c\right) + 2\,b\sin\left(dx^3 + c\right)}{6\,d^2} - \frac{(dx^3 + c)ac - bc\cos\left(dx^3 + c\right)}{3\,d^2}$$

```
[In] integrate(x^5*(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] 1/6*((d*x^3 + c)^2*a - 2*(d*x^3 + c)*b*cos(d*x^3 + c) + 2*b*sin(d*x^3 + c))
/d^2 - 1/3*((d*x^3 + c)*a*c - b*c*cos(d*x^3 + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int x^5 \left(a + b\sin\left(c + dx^3\right)\right) \, dx = \frac{a\,x^6}{6} + \frac{\frac{b\,\sin\left(d\,x^3+c\right)}{3} - \frac{b\,d\,x^3\,\cos\left(d\,x^3+c\right)}{3}}{d^2}$$

```
[In] int(x^5*(a + b*sin(c + d*x^3)),x)
[Out] (a*x^6)/6 + ((b*sin(c + d*x^3))/3 - (b*d*x^3*cos(c + d*x^3))/3)/d^2
```

3.58 $\int x^2(a+b\sin(c+dx^3))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{ax^3}{3}-\frac{b\cos\left(c+dx^3\right)}{3d}$$

```
[Out] 1/3*a*x^3-1/3*b*cos(d*x^3+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {14, 3460, 2718}

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{ax^3}{3}-\frac{b\cos\left(c+dx^3\right)}{3d}$$

```
[In] Int[x^2*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^3)/3 - (b*Cos[c + d*x^3])/(3*d)
```

<u>Rule 14</u>

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

<u>Rule 2718</u>

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^2 + bx^2 \sin\left(c + dx^3\right)\right) dx \\
&= \frac{ax^3}{3} + b \int x^2 \sin\left(c + dx^3\right) dx \\
&= \frac{ax^3}{3} + \frac{1}{3} b \text{Subst}\left(\int \sin(c + dx)\, dx, x, x^3\right) \\
&= \frac{ax^3}{3} - \frac{b \cos\left(c + dx^3\right)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int x^2 \left(a + b \sin\left(c + dx^3\right)\right) dx = \frac{ax^3}{3} - \frac{b \cos(c) \cos\left(dx^3\right)}{3d} + \frac{b \sin(c) \sin\left(dx^3\right)}{3d}$$

```
[In] Integrate[x^2*(a + b*Sin[c + d*x^3]),x]
[Out] (a*x^3)/3 - (b*Cos[c]*Cos[d*x^3])/(3*d) + (b*Sin[c]*Sin[d*x^3])/(3*d)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{a\,x^{3}}{3}-\frac{b\cos\left(d\,x^{3}+c\right)}{3d}$	22
parts	$\frac{a\,x^{3}}{3}-\frac{b\cos\left(d\,x^{3}+c\right)}{3d}$	22
derivativedivides	$\frac{\left(d\,x^{3}+c\right)a-b\cos\left(d\,x^{3}+c\right)}{3d}$	27
default	$\frac{\left(d\,x^{3}+c\right)a-b\cos\left(d\,x^{3}+c\right)}{3d}$	27
parallelrisch	$\frac{a\,x^{3}d-b\cos\left(d\,x^{3}+c\right)-b}{3d}$	27
norman	$\frac{\frac{a\,x^{3}}{3}-\frac{2b}{3d}+\frac{a\,x^{3}\left(\tan^{2}\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)\right)}{3}}{1+\tan^{2}\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)}$	51

```
[In] int(x^2*(a+b*sin(d*x^3+c)),x,method=_RETURNVERBOSE)
[Out] 1/3*a*x^3-1/3*b*cos(d*x^3+c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x^{2}\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx=\frac{adx^{3}-b\cos\left(dx^{3}+c\right)}{3\,d}$$

```
[In] integrate(x^2*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] 1/3*(a*d*x^3 - b*cos(d*x^3 + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int x^{2}\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx=\begin{cases}\frac{ax^{3}}{3}-\frac{b\cos\left(c+dx^{3}\right)}{3d} & \text{for } d\neq 0\\ \frac{x^{3}(a+b\sin\left(c\right))}{3} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(a+b*sin(d*x**3+c)),x)
[Out] Piecewise((a*x**3/3 - b*cos(c + d*x**3)/(3*d), Ne(d, 0)), (x**3*(a + b*sin(
c))/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x^2(a+b\sin(c+dx^3))\,dx = \frac{1}{3}\,ax^3 - \frac{b\cos(dx^3+c)}{3\,d}$$

```
[In] integrate(x^2*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] 1/3*a*x^3 - 1/3*b*cos(d*x^3 + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int x^2(a+b\sin(c+dx^3))\,dx = \frac{(dx^3+c)a - b\cos(dx^3+c)}{3\,d}$$

```
[In] integrate(x^2*(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] 1/3*((d*x^3 + c)*a - b*cos(d*x^3 + c))/d
```

Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x^2(a+b\sin(c+dx^3))\,dx = \frac{a\,x^3}{3} - \frac{b\,\cos(d\,x^3+c)}{3\,d}$$

```
[In] int(x^2*(a + b*sin(c + d*x^3)),x)
[Out] (a*x^3)/3 - (b*cos(c + d*x^3))/(3*d)
```

3.59 $\int \frac{a+b\sin(c+dx^3)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 31

$$\int \frac{a+b\sin(c+dx^3)}{x}\,dx = a\log(x) + \frac{1}{3}b\,\text{CosIntegral}\left(dx^3\right)\sin(c) + \frac{1}{3}b\cos(c)\text{Si}\left(dx^3\right)$$

```
[Out] a*ln(x)+1/3*b*cos(c)*Si(d*x^3)+1/3*b*Ci(d*x^3)*sin(c)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3458, 3457, 3456}

$$\int \frac{a+b\sin(c+dx^3)}{x}\,dx = a\log(x) + \frac{1}{3}b\sin(c)\,\text{CosIntegral}\left(dx^3\right) + \frac{1}{3}b\cos(c)\text{Si}\left(dx^3\right)$$

```
[In] Int[(a + b*Sin[c + d*x^3])/x,x]
```

```
[Out] a*Log[x] + (b*CosIntegral[d*x^3]*Sin[c])/3 + (b*Cos[c]*SinIntegral[d*x^3])/
3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{x}+\frac{b\sin(c+dx^3)}{x}\right)dx \\
&= a\log(x)+b\int \frac{\sin(c+dx^3)}{x}\,dx \\
&= a\log(x)+(b\cos(c))\int \frac{\sin(dx^3)}{x}\,dx+(b\sin(c))\int \frac{\cos(dx^3)}{x}\,dx \\
&= a\log(x)+\frac{1}{3}b\,\text{CosIntegral}\left(dx^3\right)\sin(c)+\frac{1}{3}b\cos(c)\text{Si}\left(dx^3\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{a+b\sin(c+dx^3)}{x}\,dx = a\log(x)+\frac{1}{3}b\left(\text{CosIntegral}\left(dx^3\right)\sin(c)+\cos(c)\text{Si}\left(dx^3\right)\right)$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x,x]
[Out] a*Log[x] + (b*(CosIntegral[d*x^3]*Sin[c] + Cos[c]*SinIntegral[d*x^3]))/3
```

Maple [F]

$$\int \frac{a+b\sin(d\,x^3+c)}{x}dx$$

```
[In] int((a+b*sin(d*x^3+c))/x,x)
[Out] int((a+b*sin(d*x^3+c))/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{a + b\sin(c + dx^3)}{x}\,dx = \frac{1}{3}\,b\,\mathrm{Ci}\left(dx^3\right)\sin(c) + \frac{1}{3}\,b\cos(c)\,\mathrm{Si}\left(dx^3\right) + a\log(x)$$

```
[In] integrate((a+b*sin(d*x^3+c))/x,x, algorithm="fricas")
[Out] 1/3*b*cos_integral(d*x^3)*sin(c) + 1/3*b*cos(c)*sin_integral(d*x^3) + a*log
(x)
```

Sympy [F]

$$\int \frac{a + b\sin(c + dx^3)}{x}\,dx = \int \frac{a + b\sin(c + dx^3)}{x}\,dx$$

```
[In] integrate((a+b*sin(d*x**3+c))/x,x)
[Out] Integral((a + b*sin(c + d*x**3))/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{a + b\sin(c + dx^3)}{x}\,dx$$
$$= -\frac{1}{6}\left(\left(i\,\mathrm{Ei}\left(i\,dx^3\right) - i\,\mathrm{Ei}\left(-i\,dx^3\right)\right)\cos(c) - \left(\mathrm{Ei}\left(i\,dx^3\right) + \mathrm{Ei}\left(-i\,dx^3\right)\right)\sin(c)\right)b + a\log(x)$$

```
[In] integrate((a+b*sin(d*x^3+c))/x,x, algorithm="maxima")
[Out] -1/6*((I*Ei(I*d*x^3) - I*Ei(-I*d*x^3))*cos(c) - (Ei(I*d*x^3) + Ei(-I*d*x^3)
)*sin(c))*b + a*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{a + b \sin(c + dx^3)}{x} dx = \frac{1}{3} b \operatorname{Ci}\left(dx^3\right) \sin(c) + \frac{1}{3} b \cos(c) \operatorname{Si}\left(dx^3\right) + \frac{1}{3} a \log\left(dx^3\right)$$

```
[In] integrate((a+b*sin(d*x^3+c))/x,x, algorithm="giac")
[Out] 1/3*b*cos_integral(d*x^3)*sin(c) + 1/3*b*cos(c)*sin_integral(d*x^3) + 1/3*a
*log(d*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sin(c + dx^3)}{x} dx = a \ln(x) + \frac{b \sin(c) \operatorname{cosint}(d x^3)}{3} + \frac{b \cos(c) \operatorname{sinint}(d x^3)}{3}$$

```
[In] int((a + b*sin(c + d*x^3))/x,x)
[Out] a*log(x) + (b*sin(c)*cosint(d*x^3))/3 + (b*cos(c)*sinint(d*x^3))/3
```

3.60 $\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 53

$$\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx = -\frac{a}{3x^3}+\frac{1}{3}bd\cos(c)\operatorname{CosIntegral}\left(dx^3\right) - \frac{b\sin(c+dx^3)}{3x^3}-\frac{1}{3}bd\sin(c)\operatorname{Si}\left(dx^3\right)$$

```
[Out] -1/3*a/x^3+1/3*b*d*Ci(d*x^3)*cos(c)-1/3*b*d*Si(d*x^3)*sin(c)-1/3*b*sin(d*x^
3+c)/x^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 3460, 3378, 3384, 3380, 3383}

$$\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx = -\frac{a}{3x^3}+\frac{1}{3}bd\cos(c)\operatorname{CosIntegral}\left(dx^3\right) -\frac{1}{3}bd\sin(c)\operatorname{Si}\left(dx^3\right)-\frac{b\sin(c+dx^3)}{3x^3}$$

```
[In] Int[(a + b*Sin[c + d*x^3])/x^4,x]
```

```
[Out] -1/3*a/x^3 + (b*d*Cos[c]*CosIntegral[d*x^3])/3 - (b*Sin[c + d*x^3])/(3*x^3)
 - (b*d*Sin[c]*SinIntegral[d*x^3])/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{x^4} + \frac{b \sin(c + dx^3)}{x^4}\right) dx \\
&= -\frac{a}{3x^3} + b \int \frac{\sin(c + dx^3)}{x^4} \, dx \\
&= -\frac{a}{3x^3} + \frac{1}{3} b \text{Subst}\left(\int \frac{\sin(c + dx)}{x^2} \, dx, x, x^3\right)
\end{aligned}$$

$$= -\frac{a}{3x^3} - \frac{b\sin(c+dx^3)}{3x^3} + \frac{1}{3}(bd)\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx, x, x^3\right)$$
$$= -\frac{a}{3x^3} - \frac{b\sin(c+dx^3)}{3x^3} + \frac{1}{3}(bd\cos(c))\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx, x, x^3\right) - \frac{1}{3}(bd\sin(c))\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx, x, x^3\right)$$
$$= -\frac{a}{3x^3} + \frac{1}{3}bd\cos(c)\,\text{CosIntegral}\left(dx^3\right) - \frac{b\sin(c+dx^3)}{3x^3} - \frac{1}{3}bd\sin(c)\text{Si}\left(dx^3\right)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx = -\frac{a-bdx^3\cos(c)\,\text{CosIntegral}(dx^3)+b\sin(c+dx^3)+bdx^3\sin(c)\text{Si}(dx^3)}{3x^3}$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^4,x]
[Out] -1/3*(a - b*d*x^3*Cos[c]*CosIntegral[d*x^3] + b*Sin[c + d*x^3] + b*d*x^3*Si
n[c]*SinIntegral[d*x^3])/x^3
```

Maple [F]

$$\int \frac{a+b\sin(d\,x^3+c)}{x^4}dx$$

```
[In] int((a+b*sin(d*x^3+c))/x^4,x)
[Out] int((a+b*sin(d*x^3+c))/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx = \frac{bdx^3\cos(c)\,\text{Ci}(dx^3) - bdx^3\sin(c)\,\text{Si}(dx^3) - b\sin(dx^3+c) - a}{3\,x^3}$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^4,x, algorithm="fricas")
[Out] 1/3*(b*d*x^3*cos(c)*cos_integral(d*x^3) - b*d*x^3*sin(c)*sin_integral(d*x^3
) - b*sin(d*x^3 + c) - a)/x^3
```

Sympy [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx = \int \frac{a+b\sin(c+dx^3)}{x^4}\,dx$$

```
[In] integrate((a+b*sin(d*x**3+c))/x**4,x)
[Out] Integral((a + b*sin(c + d*x**3))/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx = \frac{1}{6}\left(\left(\Gamma\left(-1, i\,dx^3\right)+\Gamma\left(-1,-i\,dx^3\right)\right)\cos(c)-\left(i\,\Gamma\left(-1,i\,dx^3\right)-i\,\Gamma\left(-1,-i\,dx^3\right)\right)\sin(c)\right)bd - \frac{a}{3\,x^3}$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^4,x, algorithm="maxima")
[Out] 1/6*((gamma(-1, I*d*x^3) + gamma(-1, -I*d*x^3))*cos(c) - (I*gamma(-1, I*d*x
^3) - I*gamma(-1, -I*d*x^3))*sin(c))*b*d - 1/3*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(45) = 90.

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.87

$$\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx = \frac{(dx^3+c)bd^2\cos(c)\,\mathrm{Ci}(dx^3) - bcd^2\cos(c)\,\mathrm{Ci}(dx^3) - (dx^3+c)bd^2\sin(c)\,\mathrm{Si}(dx^3) + bcd^2\sin(c)\,\mathrm{Si}(dx^3) -}{3\,d^2x^3}$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^4,x, algorithm="giac")
[Out] 1/3*((d*x^3 + c)*b*d^2*cos(c)*cos_integral(d*x^3) - b*c*d^2*cos(c)*cos_inte
gral(d*x^3) - (d*x^3 + c)*b*d^2*sin(c)*sin_integral(d*x^3) + b*c*d^2*sin(c)
*sin_integral(d*x^3) - b*d^2*sin(d*x^3 + c) - a*d^2)/(d^2*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx=\int \frac{a+b\,\sin\left(d\,x^3+c\right)}{x^4}\,dx$$

```
[In] int((a + b*sin(c + d*x^3))/x^4,x)
[Out] int((a + b*sin(c + d*x^3))/x^4, x)
```

3.61 $\int x^4(a+b\sin(c+dx^3))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int x^4(a+b\sin(c+dx^3))\,dx=\frac{ax^5}{5}-\frac{bx^2\cos(c+dx^3)}{3d}-\frac{be^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d(-idx^3)^{2/3}}-\frac{be^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d(idx^3)^{2/3}}$$

```
[Out] 1/5*a*x^5-1/3*b*x^2*cos(d*x^3+c)/d-1/9*b*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/d
/(-I*d*x^3)^(2/3)-1/9*b*x^2*GAMMA(2/3,I*d*x^3)/d/exp(I*c)/(I*d*x^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3466, 3471, 2250}

$$\int x^4(a+b\sin(c+dx^3))\,dx=\frac{ax^5}{5}-\frac{bx^2\cos(c+dx^3)}{3d}-\frac{be^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d(-idx^3)^{2/3}}-\frac{be^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d(idx^3)^{2/3}}$$

```
[In] Int[x^4*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^5)/5 - (b*x^2*Cos[c + d*x^3])/(3*d) - (b*E^(I*c)*x^2*Gamma[2/3, (-I)*d
*x^3])/(9*d*((-I)*d*x^3)^(2/3)) - (b*x^2*Gamma[2/3, I*d*x^3])/(9*d*E^(I*c)*
(I*d*x^3)^(2/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^4+bx^4\sin\left(c+dx^3\right)\right)\,dx\\
&= \frac{ax^5}{5}+b\int x^4\sin\left(c+dx^3\right)\,dx\\
&= \frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}+\frac{(2b)\int x\cos\left(c+dx^3\right)\,dx}{3d}\\
&= \frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}+\frac{b\int e^{-ic-idx^3}x\,dx}{3d}+\frac{b\int e^{ic+idx^3}x\,dx}{3d}\\
&= \frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}-\frac{be^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{be^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$
$$=\frac{dx^8\left(3(d^2x^6)^{2/3}\left(3adx^3-5b\cos\left(c+dx^3\right)\right)-5b(-idx^3)^{2/3}\Gamma\left(\frac{2}{3},idx^3\right)\left(\cos(c)-i\sin(c)\right)-5b(idx^3)^{2/3}\Gamma\left(\frac{2}{3}\right.\right.}{45\left(d^2x^6\right)^{5/3}}$$

```
[In] Integrate[x^4*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (d*x^8*(3*(d^2*x^6)^(2/3)*(3*a*d*x^3 - 5*b*Cos[c + d*x^3]) - 5*b*((-I)*d*x^
3)^(2/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) - 5*b*(I*d*x^3)^(2/3)*Gamm
a[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c])))/(45*(d^2*x^6)^(5/3))
```

Maple [F]

$$\int x^4\left(a+b\sin\left(d\,x^3+c\right)\right)dx$$

```
[In] int(x^4*(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(x^4*(a+b*sin(d*x^3+c)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$
$$=\frac{9\,ad^2x^5-15\,bdx^2\cos\left(dx^3+c\right)-5\left(-i\,b\cos\left(c\right)-b\sin\left(c\right)\right)\left(i\,d\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},i\,dx^3\right)-5\left(i\,b\cos\left(c\right)-b\sin\left(c\right)\right)\left(-i\right.}{45\,d^2}$$

```
[In] integrate(x^4*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/45*(9*a*d^2*x^5 - 15*b*d*x^2*cos(d*x^3 + c) - 5*(-I*b*cos(c) - b*sin(c))*
(I*d)^(1/3)*gamma(2/3, I*d*x^3) - 5*(I*b*cos(c) - b*sin(c))*(-I*d)^(1/3)*ga
mma(2/3, -I*d*x^3))/d^2
```

Sympy [F]

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$

```
[In] integrate(x**4*(a+b*sin(d*x**3+c)),x)
[Out] Integral(x**4*(a + b*sin(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{1}{5}\,ax^5 - \frac{\left(6\,dx^3\cos\left(dx^3+c\right)-\left(dx^3\right)^{\frac{1}{3}}\left(\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},i\,dx^3\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-i\,dx^3\right)\right)\cos\left(c\right)+\left(\left(\sqrt{3}+i\right)\Gamma\right.\right.\right.}{18\,d^2x}$$

```
[In] integrate(x^4*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] 1/5*a*x^5 - 1/18*(6*d*x^3*cos(d*x^3 + c) - (d*x^3)^(1/3)*(((I*sqrt(3) - 1)*
gamma(2/3, I*d*x^3) + (-I*sqrt(3) - 1)*gamma(2/3, -I*d*x^3))*cos(c) + ((sqr
t(3) + I)*gamma(2/3, I*d*x^3) + (sqrt(3) - I)*gamma(2/3, -I*d*x^3))*sin(c))
)*b/(d^2*x)
```

Giac [F]

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int\left(b\sin\left(dx^3+c\right)+a\right)x^4\,dx$$

```
[In] integrate(x^4*(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int x^4\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)\,dx$$

```
[In] int(x^4*(a + b*sin(c + d*x^3)),x)
[Out] int(x^4*(a + b*sin(c + d*x^3)), x)
```

3.62 $\int x(a+b\sin(c+dx^3))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{ax^2}{2}+\frac{ibe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{6\left(-idx^3\right)^{2/3}}-\frac{ibe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{6\left(idx^3\right)^{2/3}}$$

```
[Out] 1/2*a*x^2+1/6*I*b*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/(-I*d*x^3)^(2/3)-1/6*I*b
*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {14, 3470, 2250}

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{ax^2}{2}+\frac{ibe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{6\left(-idx^3\right)^{2/3}}-\frac{ibe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{6\left(idx^3\right)^{2/3}}$$

```
[In] Int[x*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^2)/2 + ((I/6)*b*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3)
 - ((I/6)*b*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax + bx\sin\left(c+dx^3\right)\right)\,dx \\
&= \frac{ax^2}{2} + b\int x\sin\left(c+dx^3\right)\,dx \\
&= \frac{ax^2}{2} + \frac{1}{2}(ib)\int e^{-ic-idx^3}x\,dx - \frac{1}{2}(ib)\int e^{ic+idx^3}x\,dx \\
&= \frac{ax^2}{2} + \frac{ibe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{6\left(-idx^3\right)^{2/3}} - \frac{ibe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{6\left(idx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx = \frac{x^2\left(3a(d^2x^6)^{2/3} + b(-idx^3)^{2/3}\Gamma\left(\frac{2}{3},idx^3\right)(-i\cos(c)-\sin(c)) + ib(idx^3)^{2/3}\Gamma\left(\frac{2}{3},-idx^3\right)(\cos(c)+i\sin(c))\right)}{6\left(d^2x^6\right)^{2/3}}$$

```
[In] Integrate[x*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (x^2*(3*a*(d^2*x^6)^(2/3) + b*((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3]*((-I)*
Cos[c] - Sin[c]) + I*b*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*S
in[c])))/(6*(d^2*x^6)^(2/3))
```

Maple [F]

$$\int x\left(a+b\sin\left(d\,x^{3}+c\right)\right)dx$$

```
[In] int(x*(a+b*sin(d*x^3+c)),x)
[Out] int(x*(a+b*sin(d*x^3+c)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int x\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx = \frac{3\,adx^{2}-(b\cos\left(c\right)-i\,b\sin\left(c\right))(i\,d)^{\frac{1}{3}}\,\Gamma\left(\frac{2}{3},i\,dx^{3}\right)-(b\cos\left(c\right)+i\,b\sin\left(c\right))(-i\,d)^{\frac{1}{3}}\,\Gamma\left(\frac{2}{3},-i\,dx^{3}\right)}{6\,d}$$

```
[In] integrate(x*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] 1/6*(3*a*d*x^2 - (b*cos(c) - I*b*sin(c))*(I*d)^(1/3)*gamma(2/3, I*d*x^3) -
(b*cos(c) + I*b*sin(c))*(-I*d)^(1/3)*gamma(2/3, -I*d*x^3))/d
```

Sympy [F]

$$\int x\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx=\int x\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx$$

```
[In] integrate(x*(a+b*sin(d*x**3+c)),x)
[Out] Integral(x*(a + b*sin(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02

$$\int x\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx=\frac{1}{2}\,ax^{2} - \frac{(dx^{3})^{\frac{1}{3}}\,\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},i\,dx^{3}\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-i\,dx^{3}\right)\right)\cos\left(c\right)-\left(\left(i\,\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},i\,dx^{3}\right)+\left(-i\,\sqrt{3}-1\right)\Gamma\right.\right.}{12\,dx}$$

```
[In] integrate(x*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] 1/2*a*x^2 - 1/12*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*d*x^3) + (sqrt(
3) - I)*gamma(2/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) - 1)*gamma(2/3, I*d*x^3)
 + (-I*sqrt(3) - 1)*gamma(2/3, -I*d*x^3))*sin(c))*b/(d*x)
```

Giac [F]

$$\int x(a+b\sin(c+dx^3))\,dx = \int (b\sin(dx^3+c)+a)x\,dx$$

```
[In] integrate(x*(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\sin(c+dx^3))\,dx = \int x\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)\,dx$$

```
[In] int(x*(a + b*sin(c + d*x^3)),x)
[Out] int(x*(a + b*sin(c + d*x^3)), x)
```

3.63 $\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx = -\frac{a}{x}-\frac{bde^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{2\left(-idx^3\right)^{2/3}}-\frac{bde^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{2\left(idx^3\right)^{2/3}}-\frac{b\sin(c+dx^3)}{x}$$

```
[Out] -a/x-1/2*b*d*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/(-I*d*x^3)^(2/3)-1/2*b*d*x^2*
GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(2/3)-b*sin(d*x^3+c)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3468, 3471, 2250}

$$\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx = -\frac{a}{x}-\frac{b\sin(c+dx^3)}{x}-\frac{be^{ic}dx^2\Gamma\left(\frac{2}{3},-idx^3\right)}{2\left(-idx^3\right)^{2/3}}-\frac{be^{-ic}dx^2\Gamma\left(\frac{2}{3},idx^3\right)}{2\left(idx^3\right)^{2/3}}$$

```
[In] Int[(a + b*Sin[c + d*x^3])/x^2,x]
```

```
[Out] -(a/x) - (b*d*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/(2*((-I)*d*x^3)^(2/3)) -
(b*d*x^2*Gamma[2/3, I*d*x^3])/(2*E^(I*c)*(I*d*x^3)^(2/3)) - (b*Sin[c + d*x^
3])/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{x^2}+\frac{b\sin\left(c+dx^3\right)}{x^2}\right)\,dx\\
&= -\frac{a}{x}+b\int \frac{\sin\left(c+dx^3\right)}{x^2}\,dx\\
&= -\frac{a}{x}-\frac{b\sin\left(c+dx^3\right)}{x}+(3bd)\int x\cos\left(c+dx^3\right)\,dx\\
&= -\frac{a}{x}-\frac{b\sin\left(c+dx^3\right)}{x}+\frac{1}{2}(3bd)\int e^{-ic-idx^3}x\,dx+\frac{1}{2}(3bd)\int e^{ic+idx^3}x\,dx\\
&= -\frac{a}{x}-\frac{bde^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{2\left(-idx^3\right)^{2/3}}-\frac{bde^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{2\left(idx^3\right)^{2/3}}-\frac{b\sin\left(c+dx^3\right)}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.19

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^2}\,dx$$
$$=\frac{-ib(-idx^3)^{5/3}\,\Gamma\left(\frac{2}{3},idx^3\right)(\cos(c)-i\sin(c))+ib(idx^3)^{5/3}\,\Gamma\left(\frac{2}{3},-idx^3\right)(\cos(c)+i\sin(c))-2(d^2x^6)^{2/3}\,(a+}{2x\left(d^2x^6\right)^{2/3}}$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^2,x]
```

```
[Out] ((-I)*b*((-I)*d*x^3)^(5/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*b*(I
*d*x^3)^(5/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 2*(d^2*x^6)^(2/3
)*(a + b*Sin[c + d*x^3]))/(2*x*(d^2*x^6)^(2/3))
```

Maple [F]

$$\int \frac{a+b\sin\left(d\,x^{3}+c\right)}{x^{2}}dx$$

```
[In] int((a+b*sin(d*x^3+c))/x^2,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{a+b\sin\left(c+dx^{3}\right)}{x^{2}}\,dx = \frac{(i\,bx\cos\left(c\right)+bx\sin\left(c\right))(i\,d)^{\frac{1}{3}}\,\Gamma\left(\frac{2}{3},i\,dx^{3}\right)+(-i\,bx\cos\left(c\right)+bx\sin\left(c\right))(-i\,d)^{\frac{1}{3}}\,\Gamma\left(\frac{2}{3},-i\,dx^{3}\right)-2\,b\sin\left(dx^{3}+\right.}{2\,x}$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*((I*b*x*cos(c) + b*x*sin(c))*(I*d)^(1/3)*gamma(2/3, I*d*x^3) + (-I*b*x*
cos(c) + b*x*sin(c))*(-I*d)^(1/3)*gamma(2/3, -I*d*x^3) - 2*b*sin(d*x^3 + c)
 - 2*a)/x
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+dx^{3}\right)}{x^{2}}\,dx = \int \frac{a+b\sin\left(c+dx^{3}\right)}{x^{2}}\,dx$$

```
[In] integrate((a+b*sin(d*x**3+c))/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \frac{a + b \sin\left(c + dx^3\right)}{x^2}\,dx =$$
$$-\frac{(dx^3)^{\frac{1}{3}}\left(\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3}, i\,dx^3\right) + \left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3}, -i\,dx^3\right)\right)\cos\left(c\right) + \left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{1}{3}, i\,dx^3\right) + \left(\sqrt{\right.\right.\right.}{12\,x}$$
$$-\frac{a}{x}$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^2,x, algorithm="maxima")
```

```
[Out] -1/12*(d*x^3)^(1/3)*(((I*sqrt(3) - 1)*gamma(-1/3, I*d*x^3) + (-I*sqrt(3) -
1)*gamma(-1/3, -I*d*x^3))*cos(c) + ((sqrt(3) + I)*gamma(-1/3, I*d*x^3) + (s
qrt(3) - I)*gamma(-1/3, -I*d*x^3))*sin(c))*b/x - a/x
```

Giac [F]

$$\int \frac{a + b \sin\left(c + dx^3\right)}{x^2}\,dx = \int \frac{b \sin\left(dx^3 + c\right) + a}{x^2}\,dx$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sin\left(c + dx^3\right)}{x^2}\,dx = \int \frac{a + b\,\sin\left(d\,x^3 + c\right)}{x^2}\,dx$$

```
[In] int((a + b*sin(c + d*x^3))/x^2,x)
```

```
[Out] int((a + b*sin(c + d*x^3))/x^2, x)
```

3.64 $\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 130

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx = -\frac{a}{4x^4}-\frac{3bd\cos(c+dx^3)}{4x}-\frac{3ibd^2e^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{8(-idx^3)^{2/3}}+\frac{3ibd^2e^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{8(idx^3)^{2/3}}-\frac{b\sin(c+dx^3)}{4x^4}$$

```
[Out] -1/4*a/x^4-3/4*b*d*cos(d*x^3+c)/x-3/8*I*b*d^2*exp(I*c)*x^2*GAMMA(2/3,-I*d*x
^3)/(-I*d*x^3)^(2/3)+3/8*I*b*d^2*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^
(2/3)-1/4*b*sin(d*x^3+c)/x^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {14, 3468, 3469, 3470, 2250}

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx = -\frac{a}{4x^4}-\frac{3ibe^{ic}d^2x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{8(-idx^3)^{2/3}}+\frac{3ibe^{-ic}d^2x^2\Gamma\left(\frac{2}{3},idx^3\right)}{8(idx^3)^{2/3}}-\frac{3bd\cos(c+dx^3)}{4x}-\frac{b\sin(c+dx^3)}{4x^4}$$

```
[In] Int[(a + b*Sin[c + d*x^3])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (3*b*d*Cos[c + d*x^3])/(4*x) - (((3*I)/8)*b*d^2*E^(I*c)*x^2*Ga
mma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3) + (((3*I)/8)*b*d^2*x^2*Gamma[2/3,
I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/3)) - (b*Sin[c + d*x^3])/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a}{x^5}+\frac{b\sin\left(c+dx^3\right)}{x^5}\right)\,dx\\
&= -\frac{a}{4x^4}+b\int \frac{\sin\left(c+dx^3\right)}{x^5}\,dx\\
&= -\frac{a}{4x^4}-\frac{b\sin\left(c+dx^3\right)}{4x^4}+\frac{1}{4}(3bd)\int \frac{\cos\left(c+dx^3\right)}{x^2}\,dx\\
&= -\frac{a}{4x^4}-\frac{3bd\cos\left(c+dx^3\right)}{4x}-\frac{b\sin\left(c+dx^3\right)}{4x^4}-\frac{1}{4}\left(9bd^2\right)\int x\sin\left(c+dx^3\right)\,dx\\
&= -\frac{a}{4x^4}-\frac{3bd\cos\left(c+dx^3\right)}{4x}-\frac{b\sin\left(c+dx^3\right)}{4x^4}\\
&\quad -\frac{1}{8}\left(9ibd^2\right)\int e^{-ic-idx^3}x\,dx+\frac{1}{8}\left(9ibd^2\right)\int e^{ic+idx^3}x\,dx
\end{aligned}
$$

$$= -\frac{a}{4x^4} - \frac{3bd\cos(c+dx^3)}{4x} - \frac{3ibd^2e^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{8(-idx^3)^{2/3}} + \frac{3ibd^2e^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{8(idx^3)^{2/3}} - \frac{b\sin(c+dx^3)}{4x^4}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx = \frac{3bd^2x^6(idx^3)^{2/3}\Gamma\left(\frac{2}{3},-idx^3\right)(-i\cos(c)+\sin(c)) + 3bd^2x^6(-idx^3)^{2/3}\Gamma\left(\frac{2}{3},idx^3\right)(i\cos(c)+\sin(c)) - 2(d^2x^6\cdots}{8x^4(d^2x^6)^{2/3}}$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^5,x]
```

```
[Out] (3*b*d^2*x^6*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*x^3]*((-I)*Cos[c] + Sin[c])
+ 3*b*d^2*x^6*((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3]*(I*Cos[c] + Sin[c]) -
2*(d^2*x^6)^(2/3)*(a + 3*b*d*x^3*Cos[c + d*x^3] + b*Sin[c + d*x^3]))/(8*x^4
*(d^2*x^6)^(2/3))
```

Maple [F]

$$\int \frac{a+b\sin(d\,x^3+c)}{x^5}dx$$

```
[In] int((a+b*sin(d*x^3+c))/x^5,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.78

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx = -\frac{6\,bdx^3\cos(dx^3+c) - 3\,(bdx^4\cos(c) - i\,bdx^4\sin(c))(i\,d)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},i\,dx^3\right) - 3\,(bdx^4\cos(c) + i\,bdx^4\sin(c))(-\cdots}{8\,x^4}$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(6*b*d*x^3*cos(d*x^3 + c) - 3*(b*d*x^4*cos(c) - I*b*d*x^4*sin(c))*(I*d
)^(1/3)*gamma(2/3, I*d*x^3) - 3*(b*d*x^4*cos(c) + I*b*d*x^4*sin(c))*(-I*d)^
(1/3)*gamma(2/3, -I*d*x^3) + 2*b*sin(d*x^3 + c) + 2*a)/x^4
```

Sympy [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx=\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx$$

```
[In] integrate((a+b*sin(d*x**3+c))/x**5,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.70

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx$$
$$=\frac{(dx^3)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{4}{3},i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{4}{3},-i\,dx^3\right)\right)\cos(c)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{4}{3},i\,dx^3\right)+\left(-i\sqrt{3}-\right.\right.}{12\,x}$$
$$-\frac{a}{4\,x^4}$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(-4/3, I*d*x^3) + (sqrt(3) - I)*gam
ma(-4/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) - 1)*gamma(-4/3, I*d*x^3) + (-I*sq
rt(3) - 1)*gamma(-4/3, -I*d*x^3))*sin(c))*b*d/x - 1/4*a/x^4
```

Giac [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx=\int \frac{b\sin(dx^3+c)+a}{x^5}\,dx$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx = \int \frac{a+b\,\sin(d\,x^3+c)}{x^5}\,dx$$

```
[In] int((a + b*sin(c + d*x^3))/x^5,x)
[Out] int((a + b*sin(c + d*x^3))/x^5, x)
```

3.65 $\int x^3(a+b\sin(c+dx^3))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 106

$$\int x^3(a+b\sin(c+dx^3))\,dx=\frac{ax^4}{4}-\frac{bx\cos(c+dx^3)}{3d}-\frac{be^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{18d\sqrt[3]{-idx^3}}-\frac{be^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{18d\sqrt[3]{idx^3}}$$

```
[Out] 1/4*a*x^4-1/3*b*x*cos(d*x^3+c)/d-1/18*b*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/d/(-
I*d*x^3)^(1/3)-1/18*b*x*GAMMA(1/3,I*d*x^3)/d/exp(I*c)/(I*d*x^3)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3466, 3437, 2239}

$$\int x^3(a+b\sin(c+dx^3))\,dx=\frac{ax^4}{4}-\frac{bx\cos(c+dx^3)}{3d}-\frac{be^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{18d\sqrt[3]{-idx^3}}-\frac{be^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{18d\sqrt[3]{idx^3}}$$

```
[In] Int[x^3*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^4)/4 - (b*x*Cos[c + d*x^3])/(3*d) - (b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3
])/(18*d*((-I)*d*x^3)^(1/3)) - (b*x*Gamma[1/3, I*d*x^3])/(18*d*E^(I*c)*(I*d
*x^3)^(1/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^3 + bx^3 \sin\left(c + dx^3\right)\right) \, dx \\
&= \frac{ax^4}{4} + b \int x^3 \sin\left(c + dx^3\right) \, dx \\
&= \frac{ax^4}{4} - \frac{bx \cos\left(c + dx^3\right)}{3d} + \frac{b \int \cos\left(c + dx^3\right) \, dx}{3d} \\
&= \frac{ax^4}{4} - \frac{bx \cos\left(c + dx^3\right)}{3d} + \frac{b \int e^{-ic-idx^3} \, dx}{6d} + \frac{b \int e^{ic+idx^3} \, dx}{6d} \\
&= \frac{ax^4}{4} - \frac{bx \cos\left(c + dx^3\right)}{3d} - \frac{be^{ic} x \Gamma\left(\frac{1}{3}, -idx^3\right)}{18d\sqrt[3]{-idx^3}} - \frac{be^{-ic} x \Gamma\left(\frac{1}{3}, idx^3\right)}{18d\sqrt[3]{idx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int x^3 \left(a + b \sin\left(c + dx^3\right)\right) \, dx$$
$$= \frac{dx^7 \left(3\sqrt[3]{d^2x^6}\left(3adx^3 - 4b\cos\left(c + dx^3\right)\right) - 2b\sqrt[3]{-idx^3}\Gamma\left(\frac{1}{3}, idx^3\right)\left(\cos(c) - i\sin(c)\right) - 2b\sqrt[3]{idx^3}\Gamma\left(\frac{1}{3}, -idx^3\right)\left(c\right.\right.}{36\left(d^2x^6\right)^{4/3}}$$

```
[In] Integrate[x^3*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (d*x^7*(3*(d^2*x^6)^(1/3)*(3*a*d*x^3 - 4*b*Cos[c + d*x^3]) - 2*b*((-I)*d*x^
3)^(1/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) - 2*b*(I*d*x^3)^(1/3)*Gamm
a[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c])))/(36*(d^2*x^6)^(4/3))
```

Maple [F]

$$\int x^{3}\left(a+b\sin\left(d\,x^{3}+c\right)\right)dx$$

[In] int(x^3*(a+b*sin(d*x^3+c)),x)

[Out] int(x^3*(a+b*sin(d*x^3+c)),x)

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int x^{3}\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx$$
$$=\frac{9\,ad^{2}x^{4}-12\,bdx\cos\left(dx^{3}+c\right)-2\left(-i\,b\cos\left(c\right)-b\sin\left(c\right)\right)(i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},i\,dx^{3}\right)-2\left(i\,b\cos\left(c\right)-b\sin\left(c\right)\right)(-i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},-i\,dx^{3}\right)}{36\,d^{2}}$$

[In] integrate(x^3*(a+b*sin(d*x^3+c)),x, algorithm="fricas")

[Out] 1/36*(9*a*d^2*x^4 - 12*b*d*x*cos(d*x^3 + c) - 2*(-I*b*cos(c) - b*sin(c))*(I*d)^(2/3)*gamma(1/3, I*d*x^3) - 2*(I*b*cos(c) - b*sin(c))*(-I*d)^(2/3)*gamma(1/3, -I*d*x^3))/d^2

Sympy [F]

$$\int x^{3}\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx=\int x^{3}\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx$$

[In] integrate(x**3*(a+b*sin(d*x**3+c)),x)

[Out] Integral(x**3*(a + b*sin(c + d*x**3)), x)

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04

$$\int x^{3}\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx=\frac{1}{4}\,ax^{4}$$
$$-\frac{\left(12\,(dx^{3})^{\frac{1}{3}}\,x\cos\left(dx^{3}+c\right)+\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},i\,dx^{3}\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-i\,dx^{3}\right)\right)\cos\left(c\right)+\left(\left(-i\,\sqrt{3}-1\right)\Gamma\left(\right.\right.\right.\right.}{36\,(dx^{3})^{\frac{1}{3}}\,d}$$

```
[In] integrate(x^3*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 - 1/36*(12*(d*x^3)^(1/3)*x*cos(d*x^3 + c) + (((sqrt(3) - I)*gamma
(1/3, I*d*x^3) + (sqrt(3) + I)*gamma(1/3, -I*d*x^3))*cos(c) + ((-I*sqrt(3)
- 1)*gamma(1/3, I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -I*d*x^3))*sin(c))*x)
*b/((d*x^3)^(1/3)*d)
```

Giac [F]

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int(b\sin\left(dx^3+c\right)+a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int x^3\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)\,dx$$

```
[In] int(x^3*(a + b*sin(c + d*x^3)),x)
```

```
[Out] int(x^3*(a + b*sin(c + d*x^3)), x)
```

3.66 $\int \left(a+b\sin\left(c+dx^3\right)\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 82

$$\int \left(a+b\sin\left(c+dx^3\right)\right)\,dx = ax+\frac{ibe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{6\sqrt[3]{-idx^3}}-\frac{ibe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{6\sqrt[3]{idx^3}}$$

```
[Out] a*x+1/6*I*b*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)-1/6*I*b*x*GAMMA
(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3436, 2239}

$$\int \left(a+b\sin\left(c+dx^3\right)\right)\,dx = ax+\frac{ibe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{6\sqrt[3]{-idx^3}}-\frac{ibe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{6\sqrt[3]{idx^3}}$$

```
[In] Int[a + b*Sin[c + d*x^3],x]
```

```
[Out] a*x + ((I/6)*b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) - ((I/6
)*b*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \sin\left(c+dx^3\right)\,dx\\
&= ax + \frac{1}{2}(ib)\int e^{-ic-idx^3}\,dx - \frac{1}{2}(ib)\int e^{ic+idx^3}\,dx\\
&= ax + \frac{ibe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{6\sqrt[3]{-idx^3}} - \frac{ibe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{6\sqrt[3]{idx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\int \left(a+b\sin\left(c+dx^3\right)\right)\,dx = ax - \frac{1}{2}ib\cos(c)\left(-\frac{x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}} + \frac{x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}\right) + \frac{1}{2}b\left(-\frac{x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}} - \frac{x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}\right)\sin(c)$$

```
[In] Integrate[a + b*Sin[c + d*x^3],x]
[Out] a*x - (I/2)*b*Cos[c]*(-1/3*(x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) +
(x*Gamma[1/3, I*d*x^3])/(3*(I*d*x^3)^(1/3))) + (b*(-1/3*(x*Gamma[1/3, (-I)*
d*x^3])/((-I)*d*x^3)^(1/3) - (x*Gamma[1/3, I*d*x^3])/(3*(I*d*x^3)^(1/3)))*S
in[c])/2
```

Maple [F]

$$\int \left(a+b\sin\left(d\,x^3+c\right)\right)dx$$

```
[In] int(a+b*sin(d*x^3+c),x)
[Out] int(a+b*sin(d*x^3+c),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

$$\int \left(a+b\sin \left(c+dx^{3}\right)\right)\,dx = \frac{6\,adx-(b\cos\left(c\right)-i\,b\sin\left(c\right))(i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},i\,dx^{3}\right)-(b\cos\left(c\right)+i\,b\sin\left(c\right))(-i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},-i\,dx^{3}\right)}{6\,d}$$

```
[In] integrate(a+b*sin(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*d*x - (b*cos(c) - I*b*sin(c))*(I*d)^(2/3)*gamma(1/3, I*d*x^3) - (b
*cos(c) + I*b*sin(c))*(-I*d)^(2/3)*gamma(1/3, -I*d*x^3))/d
```

Sympy [F]

$$\int \left(a+b\sin \left(c+dx^{3}\right)\right)\,dx = \int \left(a+b\sin \left(c+dx^{3}\right)\right)\,dx$$

```
[In] integrate(a+b*sin(d*x**3+c),x)
```

```
[Out] Integral(a + b*sin(c + d*x**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \left(a+b\sin \left(c+dx^{3}\right)\right)\,dx = \frac{\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},i\,dx^{3}\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-i\,dx^{3}\right)\right)\cos\left(c\right)-\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},i\,dx^{3}\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-i\,d\right.\right.\right.}{12\,\left(dx^{3}\right)^{\frac{1}{3}}} + ax$$

```
[In] integrate(a+b*sin(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/12*(((-I*sqrt(3) - 1)*gamma(1/3, I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -I
*d*x^3))*cos(c) - ((sqrt(3) - I)*gamma(1/3, I*d*x^3) + (sqrt(3) + I)*gamma(
1/3, -I*d*x^3))*sin(c))*b*x/(d*x^3)^(1/3) + a*x
```

Giac [F]

$$\int \left(a + b\sin\left(c + dx^3\right)\right) \, dx = \int b\sin\left(dx^3 + c\right) + a\,dx$$

```
[In] integrate(a+b*sin(d*x^3+c),x, algorithm="giac")
[Out] integrate(b*sin(d*x^3 + c) + a, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a + b\sin\left(c + dx^3\right)\right) \, dx = \int a + b\,\sin\left(d\,x^3 + c\right) \, dx$$

```
[In] int(a + b*sin(c + d*x^3),x)
[Out] int(a + b*sin(c + d*x^3), x)
```

3.67 $\int \frac{a+b\sin(c+dx^3)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{a+b\sin(c+dx^3)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{bde^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{4\sqrt[3]{-idx^3}}-\frac{bde^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{4\sqrt[3]{idx^3}}-\frac{b\sin(c+dx^3)}{2x^2}$$

```
[Out] -1/2*a/x^2-1/4*b*d*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)-1/4*b*d*
x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(1/3)-1/2*b*sin(d*x^3+c)/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3468, 3437, 2239}

$$\int \frac{a+b\sin(c+dx^3)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{be^{ic}dx\Gamma\left(\frac{1}{3},-idx^3\right)}{4\sqrt[3]{-idx^3}}-\frac{be^{-ic}dx\Gamma\left(\frac{1}{3},idx^3\right)}{4\sqrt[3]{idx^3}}-\frac{b\sin(c+dx^3)}{2x^2}$$

```
[In] Int[(a + b*Sin[c + d*x^3])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*d*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/(4*((-I)*d*x^3)^(1/3))
- (b*d*x*Gamma[1/3, I*d*x^3])/(4*E^(I*c)*(I*d*x^3)^(1/3)) - (b*Sin[c + d*x^
3])/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{x^3}+\frac{b\sin\left(c+dx^3\right)}{x^3}\right) dx\\
&= -\frac{a}{2x^2}+b\int \frac{\sin\left(c+dx^3\right)}{x^3}\,dx\\
&= -\frac{a}{2x^2}-\frac{b\sin\left(c+dx^3\right)}{2x^2}+\frac{1}{2}(3bd)\int \cos\left(c+dx^3\right)\,dx\\
&= -\frac{a}{2x^2}-\frac{b\sin\left(c+dx^3\right)}{2x^2}+\frac{1}{4}(3bd)\int e^{-ic-idx^3}\,dx+\frac{1}{4}(3bd)\int e^{ic+idx^3}\,dx\\
&= -\frac{a}{2x^2}-\frac{bde^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{4\sqrt[3]{-idx^3}}-\frac{bde^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{4\sqrt[3]{idx^3}}-\frac{b\sin\left(c+dx^3\right)}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.19

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx = \frac{-ib(-idx^3)^{4/3}\,\Gamma\left(\frac{1}{3},idx^3\right)(\cos(c)-i\sin(c))+ib(idx^3)^{4/3}\,\Gamma\left(\frac{1}{3},-idx^3\right)(\cos(c)+i\sin(c))-2\sqrt[3]{d^2x^6}(a+b\,\mathrm{si}}{4x^2\sqrt[3]{d^2x^6}}$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^3,x]
```

```
[Out] ((-I)*b*((-I)*d*x^3)^(4/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*b*(I
*d*x^3)^(4/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 2*(d^2*x^6)^(1/3
)*(a + b*Sin[c + d*x^3]))/(4*x^2*(d^2*x^6)^(1/3))
```

Maple [F]

$$\int \frac{a+b\sin\left(d\,x^{3}+c\right)}{x^{3}}dx$$

```
[In] int((a+b*sin(d*x^3+c))/x^3,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx = \frac{(i\,bx^2\cos\left(c\right)+bx^2\sin\left(c\right))(i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},i\,dx^3\right)+(-i\,bx^2\cos\left(c\right)+bx^2\sin\left(c\right))(-i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},-i\,dx^3\right)-2\,b\sin\left(d\right.}{4\,x^2}$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*((I*b*x^2*cos(c) + b*x^2*sin(c))*(I*d)^(2/3)*gamma(1/3, I*d*x^3) + (-I*
b*x^2*cos(c) + b*x^2*sin(c))*(-I*d)^(2/3)*gamma(1/3, -I*d*x^3) - 2*b*sin(d*
x^3 + c) - 2*a)/x^2
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx = \int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx$$

```
[In] integrate((a+b*sin(d*x**3+c))/x**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx = \frac{(dx^3)^{\frac{2}{3}}\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{2}{3},i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(-\frac{2}{3},-i\,dx^3\right)\right)\cos\left(c\right)-\left(\left(i\,\sqrt{3}+1\right)\Gamma\left(-\frac{2}{3},i\,dx^3\right)+\left(-i\,\sqrt{3}+\right.\right.\right.}{12\,x^2} - \frac{a}{2\,x^2}$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*(d*x^3)^(2/3)*(((sqrt(3) - I)*gamma(-2/3, I*d*x^3) + (sqrt(3) + I)*gam
ma(-2/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) + 1)*gamma(-2/3, I*d*x^3) + (-I*sq
rt(3) + 1)*gamma(-2/3, -I*d*x^3))*sin(c))*b/x^2 - 1/2*a/x^2
```

Giac [F]

$$\int \frac{a + b\sin(c + dx^3)}{x^3} \, dx = \int \frac{b\sin(dx^3 + c) + a}{x^3} \, dx$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sin(c + dx^3)}{x^3} \, dx = \int \frac{a + b\,\sin(d\,x^3 + c)}{x^3} \, dx$$

```
[In] int((a + b*sin(c + d*x^3))/x^3,x)
```

```
[Out] int((a + b*sin(c + d*x^3))/x^3, x)
```

3.68 $\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx=-\frac{a}{5x^5}-\frac{3bd\cos(c+dx^3)}{10x^2}-\frac{3ibd^2e^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{20\sqrt[3]{-idx^3}}+\frac{3ibd^2e^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{20\sqrt[3]{idx^3}}-\frac{b\sin(c+dx^3)}{5x^5}$$

```
[Out] -1/5*a/x^5-3/10*b*d*cos(d*x^3+c)/x^2-3/20*I*b*d^2*exp(I*c)*x*GAMMA(1/3,-I*d
*x^3)/(-I*d*x^3)^(1/3)+3/20*I*b*d^2*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)
^(1/3)-1/5*b*sin(d*x^3+c)/x^5
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {14, 3468, 3469, 3436, 2239}

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx=-\frac{a}{5x^5}-\frac{3ibe^{ic}d^2x\Gamma\left(\frac{1}{3},-idx^3\right)}{20\sqrt[3]{-idx^3}}+\frac{3ibe^{-ic}d^2x\Gamma\left(\frac{1}{3},idx^3\right)}{20\sqrt[3]{idx^3}}-\frac{b\sin(c+dx^3)}{5x^5}-\frac{3bd\cos(c+dx^3)}{10x^2}$$

```
[In] Int[(a + b*Sin[c + d*x^3])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (3*b*d*Cos[c + d*x^3])/(10*x^2) - (((3*I)/20)*b*d^2*E^(I*c)*x*
Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) + (((3*I)/20)*b*d^2*x*Gamma[1/3,
 I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3)) - (b*Sin[c + d*x^3])/(5*x^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a}{x^6}+\frac{b\sin\left(c+dx^3\right)}{x^6}\right)\,dx\\
&= -\frac{a}{5x^5}+b\int \frac{\sin\left(c+dx^3\right)}{x^6}\,dx\\
&= -\frac{a}{5x^5}-\frac{b\sin\left(c+dx^3\right)}{5x^5}+\frac{1}{5}(3bd)\int \frac{\cos\left(c+dx^3\right)}{x^3}\,dx\\
&= -\frac{a}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}-\frac{b\sin\left(c+dx^3\right)}{5x^5}-\frac{1}{10}\left(9bd^2\right)\int \sin\left(c+dx^3\right)\,dx\\
&= -\frac{a}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}-\frac{b\sin\left(c+dx^3\right)}{5x^5}\\
&\quad -\frac{1}{20}\left(9ibd^2\right)\int e^{-ic-idx^3}\,dx+\frac{1}{20}\left(9ibd^2\right)\int e^{ic+idx^3}\,dx
\end{aligned}
$$

$$= -\frac{a}{5x^5} - \frac{3bd\cos(c+dx^3)}{10x^2} - \frac{3ibd^2e^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{20\sqrt[3]{-idx^3}} + \frac{3ibd^2e^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{20\sqrt[3]{idx^3}} - \frac{b\sin(c+dx^3)}{5x^5}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx$$
$$= \frac{3bd^2x^6\sqrt[3]{idx^3}\Gamma\left(\frac{1}{3},-idx^3\right)(-i\cos(c)+\sin(c)) + 3bd^2x^6\sqrt[3]{-idx^3}\Gamma\left(\frac{1}{3},idx^3\right)(i\cos(c)+\sin(c)) - 2\sqrt[3]{d^2x^6}(2a}{20x^5\sqrt[3]{d^2x^6}}$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^6,x]
```

```
[Out] (3*b*d^2*x^6*(I*d*x^3)^(1/3)*Gamma[1/3, (-I)*d*x^3]*((-I)*Cos[c] + Sin[c])
+ 3*b*d^2*x^6*((-I)*d*x^3)^(1/3)*Gamma[1/3, I*d*x^3]*(I*Cos[c] + Sin[c]) -
2*(d^2*x^6)^(1/3)*(2*a + 3*b*d*x^3*Cos[c + d*x^3] + 2*b*Sin[c + d*x^3]))/(2
0*x^5*(d^2*x^6)^(1/3))
```

Maple [F]

$$\int \frac{a+b\sin(d\,x^3+c)}{x^6}dx$$

```
[In] int((a+b*sin(d*x^3+c))/x^6,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx =$$
$$-\frac{6\,bdx^3\cos(dx^3+c) - 3\,(bdx^5\cos(c) - i\,bdx^5\sin(c))(i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},i\,dx^3\right) - 3\,(bdx^5\cos(c) + i\,bdx^5\sin(c))(}{20\,x^5}$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(6*b*d*x^3*cos(d*x^3 + c) - 3*(b*d*x^5*cos(c) - I*b*d*x^5*sin(c))*(I*
d)^(2/3)*gamma(1/3, I*d*x^3) - 3*(b*d*x^5*cos(c) + I*b*d*x^5*sin(c))*(-I*d)
^(2/3)*gamma(1/3, -I*d*x^3) + 4*b*sin(d*x^3 + c) + 4*a)/x^5
```

Sympy [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx = \int \frac{a+b\sin(c+dx^3)}{x^6}\,dx$$

```
[In] integrate((a+b*sin(d*x**3+c))/x**6,x)
[Out] Integral((a + b*sin(c + d*x**3))/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx = -\frac{(dx^3)^{\frac{2}{3}}\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{5}{3},i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{5}{3},-i\,dx^3\right)\right)\cos(c)-\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{5}{3},i\,dx^3\right)+\left(\sqrt{3}\right.\right.\right.}{12\,x^2} - \frac{a}{5\,x^5}$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^6,x, algorithm="maxima")
[Out] -1/12*(d*x^3)^(2/3)*(((-I*sqrt(3) - 1)*gamma(-5/3, I*d*x^3) + (I*sqrt(3) -
1)*gamma(-5/3, -I*d*x^3))*cos(c) - ((sqrt(3) - I)*gamma(-5/3, I*d*x^3) + (s
qrt(3) + I)*gamma(-5/3, -I*d*x^3))*sin(c))*b*d/x^2 - 1/5*a/x^5
```

Giac [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx = \int \frac{b\sin(dx^3+c)+a}{x^6}\,dx$$

```
[In] integrate((a+b*sin(d*x^3+c))/x^6,x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx = \int \frac{a+b\,\sin(d\,x^3+c)}{x^6}\,dx$$

```
[In] int((a + b*sin(c + d*x^3))/x^6,x)
[Out] int((a + b*sin(c + d*x^3))/x^6, x)
```

3.69 $\int x^5(a+b\sin(c+dx^3))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{2abx^3\cos\left(c+dx^3\right)}{3d}+\frac{2ab\sin\left(c+dx^3\right)}{3d^2}-\frac{b^2x^3\cos\left(c+dx^3\right)\sin\left(c+dx^3\right)}{6d}+\frac{b^2\sin^2\left(c+dx^3\right)}{12d^2}$$

```
[Out] 1/6*a^2*x^6+1/12*b^2*x^6-2/3*a*b*x^3*cos(d*x^3+c)/d+2/3*a*b*sin(d*x^3+c)/d^
2-1/6*b^2*x^3*cos(d*x^3+c)*sin(d*x^3+c)/d+1/12*b^2*sin(d*x^3+c)^2/d^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3460, 3398, 3377, 2717, 3391, 30}

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{a^2x^6}{6}+\frac{2ab\sin\left(c+dx^3\right)}{3d^2}-\frac{2abx^3\cos\left(c+dx^3\right)}{3d}+\frac{b^2\sin^2\left(c+dx^3\right)}{12d^2}-\frac{b^2x^3\sin\left(c+dx^3\right)\cos\left(c+dx^3\right)}{6d}+\frac{b^2x^6}{12}$$

```
[In] Int[x^5*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (a^2*x^6)/6 + (b^2*x^6)/12 - (2*a*b*x^3*Cos[c + d*x^3])/(3*d) + (2*a*b*Sin[
c + d*x^3])/(3*d^2) - (b^2*x^3*Cos[c + d*x^3]*Sin[c + d*x^3])/(6*d) + (b^2*
Sin[c + d*x^3]^2)/(12*d^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x(a+b\sin(c+dx))^2\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(a^2x+2abx\sin(c+dx)+b^2x\sin^2(c+dx)\right)\,dx,x,x^3\right)\\
&= \frac{a^2x^6}{6}+\frac{1}{3}(2ab)\text{Subst}\left(\int x\sin(c+dx)\,dx,x,x^3\right)+\frac{1}{3}b^2\text{Subst}\left(\int x\sin^2(c+dx)\,dx,x,x^3\right)\\
&= \frac{a^2x^6}{6}-\frac{2abx^3\cos\left(c+dx^3\right)}{3d}-\frac{b^2x^3\cos\left(c+dx^3\right)\sin\left(c+dx^3\right)}{6d}+\frac{b^2\sin^2\left(c+dx^3\right)}{12d^2}\\
&\quad+\frac{1}{6}b^2\text{Subst}\left(\int x\,dx,x,x^3\right)+\frac{(2ab)\text{Subst}\left(\int\cos(c+dx)\,dx,x,x^3\right)}{3d}
\end{aligned}$$

$$= \frac{a^2x^6}{6} + \frac{b^2x^6}{12} - \frac{2abx^3\cos(c+dx^3)}{3d} + \frac{2ab\sin(c+dx^3)}{3d^2} - \frac{b^2x^3\cos(c+dx^3)\sin(c+dx^3)}{6d} + \frac{b^2\sin^2(c+dx^3)}{12d^2}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx = \frac{4a^2d^2x^6+2b^2d^2x^6-16abdx^3\cos(c+dx^3)-b^2\cos(2(c+dx^3))+16ab\sin(c+dx^3)-2b^2dx^3\sin(2(c+dx^3))}{24d^2}$$

```
[In] Integrate[x^5*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (4*a^2*d^2*x^6 + 2*b^2*d^2*x^6 - 16*a*b*d*x^3*Cos[c + d*x^3] - b^2*Cos[2*(c
 + d*x^3)] + 16*a*b*Sin[c + d*x^3] - 2*b^2*d*x^3*Sin[2*(c + d*x^3)])/(24*d^
2)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

method	result
risch	$\frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{2ab\,x^3\cos(d\,x^3+c)}{3d}+\frac{2ab\sin(d\,x^3+c)}{3d^2}-\frac{b^2\cos(2d\,x^3+2c)}{24d^2}-\frac{b^2x^3\sin(2d\,x^3+2c)}{12d}$
parallelrisch	$\frac{4a^2d^2x^6+2b^2d^2x^6-16ab\,x^3\cos(d\,x^3+c)d-2b^2x^3\sin(2d\,x^3+2c)d+16\sin(d\,x^3+c)ab-b^2\cos(2d\,x^3+2c)+b^2}{24d^2}$
parts	$\frac{a^2x^6}{6}+\frac{\frac{b^2x^6}{12}+\frac{x^6b^2\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{6}+\frac{x^6b^2\left(\tan^4\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{12}+\frac{b^2\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{3d^2}-\frac{b^2x^3\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3d}+\frac{b^2x^3\left(\tan^3\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)^2}$
default	$\frac{a^2x^6}{6}+\frac{b^2x^6}{6}+\frac{-\frac{b^2x^6}{6}-\frac{x^6b^2\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{3}-\frac{x^6b^2\left(\tan^4\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{6}+\frac{2b^2\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{3d^2}-\frac{2b^2x^3\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3d}+\frac{2b^2x^3}{}}{2\left(1+\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)^2}$
norman	$\frac{\left(\frac{a^2}{6}+\frac{b^2}{12}\right)x^6+\left(\frac{a^2}{3}+\frac{b^2}{6}\right)x^6\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{6}+\frac{b^2}{12}\right)x^6\left(\tan^4\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)+\frac{b^2\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{3d^2}-\frac{b^2x^3\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3d}+\frac{b^2x^3}{}}{\left(1+\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)^2}$

```
[In] int(x^5*(a+b*sin(d*x^3+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2*x^6+1/12*b^2*x^6-2/3*a*b*x^3*cos(d*x^3+c)/d+2/3*a*b*sin(d*x^3+c)/d^
2-1/24*b^2/d^2*cos(2*d*x^3+2*c)-1/12*b^2*x^3/d*sin(2*d*x^3+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx = \frac{(2\,a^2+b^2)d^2x^6-8\,abdx^3\cos\left(dx^3+c\right)-b^2\cos\left(dx^3+c\right)^2-2\left(b^2dx^3\cos\left(dx^3+c\right)-4\,ab\right)\sin\left(dx^3+c\right)}{12\,d^2}$$

```
[In] integrate(x^5*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
[Out] 1/12*((2*a^2 + b^2)*d^2*x^6 - 8*a*b*d*x^3*cos(d*x^3 + c) - b^2*cos(d*x^3 +
c)^2 - 2*(b^2*d*x^3*cos(d*x^3 + c) - 4*a*b)*sin(d*x^3 + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.34

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx = \begin{cases} \frac{a^2x^6}{6} - \frac{2abx^3\cos\left(c+dx^3\right)}{3d} + \frac{2ab\sin\left(c+dx^3\right)}{3d^2} + \frac{b^2x^6\sin^2\left(c+dx^3\right)}{12} + \frac{b^2x^6\cos^2\left(c+dx^3\right)}{12} - \frac{b^2x^3\sin\left(c+dx^3\right)\cos\left(c+dx^3\right)}{6d} - \frac{b^2\cos^2\left(\right.}{12} \\ \frac{x^6(a+b\sin\left(c\right))^2}{6} \end{cases}$$

```
[In] integrate(x**5*(a+b*sin(d*x**3+c))**2,x)
[Out] Piecewise((a**2*x**6/6 - 2*a*b*x**3*cos(c + d*x**3)/(3*d) + 2*a*b*sin(c + d
*x**3)/(3*d**2) + b**2*x**6*sin(c + d*x**3)**2/12 + b**2*x**6*cos(c + d*x**
3)**2/12 - b**2*x**3*sin(c + d*x**3)*cos(c + d*x**3)/(6*d) - b**2*cos(c + d
*x**3)**2/(12*d**2), Ne(d, 0)), (x**6*(a + b*sin(c))**2/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx = \frac{1}{6}\,a^2x^6 - \frac{2\left(dx^3\cos\left(dx^3+c\right)-\sin\left(dx^3+c\right)\right)ab}{3\,d^2} + \frac{\left(2\,d^2x^6-2\,dx^3\sin\left(2\,dx^3+2\,c\right)-\cos\left(2\,dx^3+2\,c\right)\right)b^2}{24\,d^2}$$

```
[In] integrate(x^5*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
[Out] 1/6*a^2*x^6 - 2/3*(d*x^3*cos(d*x^3 + c) - sin(d*x^3 + c))*a*b/d^2 + 1/24*(2
*d^2*x^6 - 2*d*x^3*sin(2*d*x^3 + 2*c) - cos(2*d*x^3 + 2*c))*b^2/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.54

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$=\frac{4\,(dx^3+c)^2a^2+2\,(dx^3+c)^2b^2-16\,(dx^3+c)ab\cos\left(dx^3+c\right)-2\,(dx^3+c)b^2\sin\left(2\,dx^3+2\,c\right)-b^2\cos\left(2\,dx\right.}{24\,d^2}$$
$$-\frac{4\,(dx^3+c)a^2c+\left(2\,dx^3+2\,c-\sin\left(2\,dx^3+2\,c\right)\right)b^2c-8\,abc\cos\left(dx^3+c\right)}{12\,d^2}$$

```
[In] integrate(x^5*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(4*(d*x^3 + c)^2*a^2 + 2*(d*x^3 + c)^2*b^2 - 16*(d*x^3 + c)*a*b*cos(d*
x^3 + c) - 2*(d*x^3 + c)*b^2*sin(2*d*x^3 + 2*c) - b^2*cos(2*d*x^3 + 2*c) +
16*a*b*sin(d*x^3 + c))/d^2 - 1/12*(4*(d*x^3 + c)*a^2*c + (2*d*x^3 + 2*c - s
in(2*d*x^3 + 2*c))*b^2*c - 8*a*b*c*cos(d*x^3 + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=$$
$$-\frac{b^2\cos\left(d\,x^3+c\right)^2-2\,a^2\,d^2\,x^6-b^2\,d^2\,x^6-8\,a\,b\,\sin\left(d\,x^3+c\right)+8\,a\,b\,d\,x^3\,\cos\left(d\,x^3+c\right)+2\,b^2\,d\,x^3\,\cos\left(d\,x\right.}{12\,d^2}$$

```
[In] int(x^5*(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] -(b^2*cos(c + d*x^3)^2 - 2*a^2*d^2*x^6 - b^2*d^2*x^6 - 8*a*b*sin(c + d*x^3)
 + 8*a*b*d*x^3*cos(c + d*x^3) + 2*b^2*d*x^3*cos(c + d*x^3)*sin(c + d*x^3))/
(12*d^2)
```

3.70 $\int x^2(a+b\sin(c+dx^3))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{2ab\cos\left(c+dx^3\right)}{3d}-\frac{b^2\cos\left(c+dx^3\right)\sin\left(c+dx^3\right)}{6d}$$

```
[Out] 1/6*(2*a^2+b^2)*x^3-2/3*a*b*cos(d*x^3+c)/d-1/6*b^2*cos(d*x^3+c)*sin(d*x^3+c
)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3460, 2723}

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{1}{6}x^3\left(2a^2+b^2\right)-\frac{2ab\cos\left(c+dx^3\right)}{3d}-\frac{b^2\sin\left(c+dx^3\right)\cos\left(c+dx^3\right)}{6d}$$

```
[In] Int[x^2*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^3)/6 - (2*a*b*Cos[c + d*x^3])/(3*d) - (b^2*Cos[c + d*x^3]*
Sin[c + d*x^3])/(6*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int (a+b\sin(c+dx))^2\,dx, x, x^3\right)\\ &= \frac{1}{6}\left(2a^2+b^2\right)x^3 - \frac{2ab\cos\left(c+dx^3\right)}{3d} - \frac{b^2\cos\left(c+dx^3\right)\sin\left(c+dx^3\right)}{6d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\begin{aligned}&\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx\\ &= -\frac{-2(2a^2+b^2)\left(c+dx^3\right)+8ab\cos\left(c+dx^3\right)+b^2\sin\left(2(c+dx^3)\right)}{12d}\end{aligned}$$

```
[In] Integrate[x^2*(a + b*Sin[c + d*x^3])^2,x]
[Out] -1/12*(-2*(2*a^2 + b^2)*(c + d*x^3) + 8*a*b*Cos[c + d*x^3] + b^2*Sin[2*(c +
 d*x^3)])/d
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result
risch	$\frac{x^{3}a^{2}}{3}+\frac{x^{3}b^{2}}{6}-\frac{2ab\cos\left(d\,x^{3}+c\right)}{3d}-\frac{b^{2}\sin\left(2d\,x^{3}+2c\right)}{12d}$
parallelrisch	$\frac{4a^{2}d\,x^{3}+2b^{2}d\,x^{3}-8ab\cos\left(d\,x^{3}+c\right)-\sin\left(2d\,x^{3}+2c\right)b^{2}-8ab}{12d}$
parts	$\frac{x^{3}a^{2}}{3}+\frac{b^{2}\left(-\frac{\cos\left(d\,x^{3}+c\right)\sin\left(d\,x^{3}+c\right)}{2}+\frac{d\,x^{3}}{2}+\frac{c}{2}\right)}{3d}-\frac{2ab\cos\left(d\,x^{3}+c\right)}{3d}$
derivativedivides	$\frac{b^{2}\left(-\frac{\cos\left(d\,x^{3}+c\right)\sin\left(d\,x^{3}+c\right)}{2}+\frac{d\,x^{3}}{2}+\frac{c}{2}\right)-2ab\cos\left(d\,x^{3}+c\right)+a^{2}\left(d\,x^{3}+c\right)}{3d}$
default	$\frac{b^{2}\left(-\frac{\cos\left(d\,x^{3}+c\right)\sin\left(d\,x^{3}+c\right)}{2}+\frac{d\,x^{3}}{2}+\frac{c}{2}\right)-2ab\cos\left(d\,x^{3}+c\right)+a^{2}\left(d\,x^{3}+c\right)}{3d}$
norman	$\frac{\left(\frac{a^{2}}{3}+\frac{b^{2}}{6}\right)x^{3}+\left(\frac{a^{2}}{3}+\frac{b^{2}}{6}\right)x^{3}\left(\tan^{4}\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)\right)+\left(\frac{2a^{2}}{3}+\frac{b^{2}}{3}\right)x^{3}\left(\tan^{2}\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)\right)-\frac{4ab}{3d}-\frac{b^{2}\tan\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)}{3d}+\frac{b^{2}\left(\tan^{3}\left(\frac{d\,x^{3}}{2}\cdots\right.\right.}{3d}\cdots}{\left(1+\tan^{2}\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)\right)^{2}}$

[In] int(x^2*(a+b*sin(d*x^3+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/3*x^3*a^2+1/6*x^3*b^2-2/3*a*b*cos(d*x^3+c)/d-1/12*b^2/d*sin(2*d*x^3+2*c)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int x^{2}\left(a+b\sin\left(c+dx^{3}\right)\right)^{2}\,dx$$
$$=\frac{(2\,a^{2}+b^{2})dx^{3}-b^{2}\cos\left(dx^{3}+c\right)\sin\left(dx^{3}+c\right)-4\,ab\cos\left(dx^{3}+c\right)}{6\,d}$$

[In] integrate(x^2*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")

[Out] 1/6*((2*a^2 + b^2)*d*x^3 - b^2*cos(d*x^3 + c)*sin(d*x^3 + c) - 4*a*b*cos(d*
x^3 + c))/d

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx = \begin{cases} \frac{a^2x^3}{3}-\frac{2ab\cos\left(c+dx^3\right)}{3d}+\frac{b^2x^3\sin^2\left(c+dx^3\right)}{6}+\frac{b^2x^3\cos^2\left(c+dx^3\right)}{6}-\frac{b^2\sin\left(c+dx^3\right)\cos\left(c+dx^3\right)}{6d} & \text{for } d\neq 0 \\ \frac{x^3(a+b\sin(c))^2}{3} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Piecewise((a**2*x**3/3 - 2*a*b*cos(c + d*x**3)/(3*d) + b**2*x**3*sin(c + d*
x**3)**2/6 + b**2*x**3*cos(c + d*x**3)**2/6 - b**2*sin(c + d*x**3)*cos(c +
d*x**3)/(6*d), Ne(d, 0)), (x**3*(a + b*sin(c))**2/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx = \frac{1}{3}\,a^2x^3+\frac{\left(2\,dx^3-\sin\left(2\,dx^3+2\,c\right)\right)b^2}{12\,d}-\frac{2\,ab\cos\left(dx^3+c\right)}{3\,d}$$

```
[In] integrate(x^2*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/12*(2*d*x^3 - sin(2*d*x^3 + 2*c))*b^2/d - 2/3*a*b*cos(d*x^3
 + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx = \frac{4\left(dx^3+c\right)a^2+\left(2\,dx^3+2\,c-\sin\left(2\,dx^3+2\,c\right)\right)b^2-8\,ab\cos\left(dx^3+c\right)}{12\,d}$$

```
[In] integrate(x^2*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(4*(d*x^3 + c)*a^2 + (2*d*x^3 + 2*c - sin(2*d*x^3 + 2*c))*b^2 - 8*a*b*
cos(d*x^3 + c))/d
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{a^2\,x^3}{3}+\frac{b^2\,x^3}{6}-\frac{b^2\,\sin\left(2\,d\,x^3+2\,c\right)}{12\,d}-\frac{2\,a\,b\,\cos\left(d\,x^3+c\right)}{3\,d}$$

```
[In] int(x^2*(a + b*sin(c + d*x^3))^2,x)
[Out] (a^2*x^3)/3 + (b^2*x^3)/6 - (b^2*sin(2*c + 2*d*x^3))/(12*d) - (2*a*b*cos(c
+ d*x^3))/(3*d)
```

3.71 $\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx = -\frac{1}{6}b^2\cos(2c)\,\text{CosIntegral}\left(2dx^3\right) + \frac{1}{2}\left(2a^2+b^2\right)\log(x) + \frac{2}{3}ab\,\text{CosIntegral}\left(dx^3\right)\sin(c) + \frac{2}{3}ab\cos(c)\text{Si}\left(dx^3\right) + \frac{1}{6}b^2\sin(2c)\text{Si}\left(2dx^3\right)$$

```
[Out] -1/6*b^2*Ci(2*d*x^3)*cos(2*c)+1/2*(2*a^2+b^2)*ln(x)+2/3*a*b*cos(c)*Si(d*x^3
)+2/3*a*b*Ci(d*x^3)*sin(c)+1/6*b^2*Si(2*d*x^3)*sin(2*c)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3484, 6, 3459, 3457, 3456, 3458}

$$\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx = \frac{1}{2}\left(2a^2+b^2\right)\log(x) + \frac{2}{3}ab\sin(c)\,\text{CosIntegral}\left(dx^3\right) + \frac{2}{3}ab\cos(c)\text{Si}\left(dx^3\right) - \frac{1}{6}b^2\cos(2c)\,\text{CosIntegral}\left(2dx^3\right) + \frac{1}{6}b^2\sin(2c)\text{Si}\left(2dx^3\right)$$

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x,x]
```

```
[Out] -1/6*(b^2*Cos[2*c]*CosIntegral[2*d*x^3]) + ((2*a^2 + b^2)*Log[x])/2 + (2*a*
b*CosIntegral[d*x^3]*Sin[c])/3 + (2*a*b*Cos[c]*SinIntegral[d*x^3])/3 + (b^2
*Sin[2*c]*SinIntegral[2*d*x^3])/6
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{x}+\frac{b^2}{2x}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x}+\frac{2ab\sin\left(c+dx^3\right)}{x}\right)dx\\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x}+\frac{2ab\sin\left(c+dx^3\right)}{x}\right)dx\\
&= \frac{1}{2}\left(2a^2+b^2\right)\log(x)+(2ab)\int \frac{\sin\left(c+dx^3\right)}{x}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^3\right)}{x}\,dx\\
&= \frac{1}{2}\left(2a^2+b^2\right)\log(x)+(2ab\cos(c))\int \frac{\sin\left(dx^3\right)}{x}\,dx-\frac{1}{2}\left(b^2\cos(2c)\right)\int \frac{\cos\left(2dx^3\right)}{x}\,dx\\
&\quad+(2ab\sin(c))\int \frac{\cos\left(dx^3\right)}{x}\,dx+\frac{1}{2}\left(b^2\sin(2c)\right)\int \frac{\sin\left(2dx^3\right)}{x}\,dx
\end{aligned}
$$

$$= -\frac{1}{6}b^2\cos(2c)\,\text{CosIntegral}\left(2dx^3\right) + \frac{1}{2}\left(2a^2+b^2\right)\log(x) + \frac{2}{3}ab\,\text{CosIntegral}\left(dx^3\right)\sin(c) + \frac{2}{3}ab\cos(c)\text{Si}\left(dx^3\right) + \frac{1}{6}b^2\sin(2c)\text{Si}\left(2dx^3\right)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx = \frac{1}{2}\left(2a^2+b^2\right)\log(x) - \frac{1}{6}b\left(b\cos(2c)\,\text{CosIntegral}\left(2dx^3\right) - 4a\,\text{CosIntegral}\left(dx^3\right)\sin(c) - 4a\cos(c)\text{Si}\left(dx^3\right) - b\sin(2c)\text{Si}\left(2dx^3\right)\right)$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x,x]
[Out] ((2*a^2 + b^2)*Log[x])/2 - (b*(b*Cos[2*c]*CosIntegral[2*d*x^3] - 4*a*CosInt
egral[d*x^3]*Sin[c] - 4*a*Cos[c]*SinIntegral[d*x^3] - b*Sin[2*c]*SinIntegra
l[2*d*x^3]))/6
```

Maple [F]

$$\int \frac{(a+b\sin(d\,x^3+c))^2}{x}dx$$

```
[In] int((a+b*sin(d*x^3+c))^2/x,x)
[Out] int((a+b*sin(d*x^3+c))^2/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx = -\frac{1}{6}\,b^2\cos(2\,c)\,\text{Ci}\left(2\,dx^3\right) + \frac{2}{3}\,ab\,\text{Ci}\left(dx^3\right)\sin(c) + \frac{1}{6}\,b^2\sin(2\,c)\,\text{Si}\left(2\,dx^3\right) + \frac{2}{3}\,ab\cos(c)\,\text{Si}\left(dx^3\right) + \frac{1}{2}\left(2\,a^2+b^2\right)\log(x)$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x,x, algorithm="fricas")
[Out] -1/6*b^2*cos(2*c)*cos_integral(2*d*x^3) + 2/3*a*b*cos_integral(d*x^3)*sin(c
) + 1/6*b^2*sin(2*c)*sin_integral(2*d*x^3) + 2/3*a*b*cos(c)*sin_integral(d*
x^3) + 1/2*(2*a^2 + b^2)*log(x)
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx = \int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx$$

```
[In] integrate((a+b*sin(d*x**3+c))**2/x,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\begin{aligned}
&\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx \\
&= -\frac{1}{3}\left(\left(i\,\mathrm{Ei}\left(i\,dx^3\right)-i\,\mathrm{Ei}\left(-i\,dx^3\right)\right)\cos(c)-\left(\mathrm{Ei}\left(i\,dx^3\right)+\mathrm{Ei}\left(-i\,dx^3\right)\right)\sin(c)\right)ab \\
&\quad -\frac{1}{12}\left(\left(\mathrm{Ei}\left(2i\,dx^3\right)+\mathrm{Ei}\left(-2i\,dx^3\right)\right)\cos(2\,c)-\left(-i\,\mathrm{Ei}\left(2i\,dx^3\right)+i\,\mathrm{Ei}\left(-2i\,dx^3\right)\right)\sin(2\,c)-6\,\log(x)\right)b^2 \\
&\quad + a^2\log(x)
\end{aligned}$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x,x, algorithm="maxima")
```

```
[Out] -1/3*((I*Ei(I*d*x^3) - I*Ei(-I*d*x^3))*cos(c) - (Ei(I*d*x^3) + Ei(-I*d*x^3)
)*sin(c))*a*b - 1/12*((Ei(2*I*d*x^3) + Ei(-2*I*d*x^3))*cos(2*c) - (-I*Ei(2*
I*d*x^3) + I*Ei(-2*I*d*x^3))*sin(2*c) - 6*log(x))*b^2 + a^2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\begin{aligned}
\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx &= -\frac{1}{6}\,b^2\cos(2\,c)\,\mathrm{Ci}\left(2\,dx^3\right)+\frac{2}{3}\,ab\,\mathrm{Ci}\left(dx^3\right)\sin(c) \\
&\quad +\frac{2}{3}\,ab\cos(c)\,\mathrm{Si}\left(dx^3\right)-\frac{1}{6}\,b^2\sin(2\,c)\,\mathrm{Si}\left(-2\,dx^3\right) \\
&\quad +\frac{1}{3}\,a^2\log\left(dx^3\right)+\frac{1}{6}\,b^2\log\left(dx^3\right)
\end{aligned}$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x,x, algorithm="giac")
```

```
[Out] -1/6*b^2*cos(2*c)*cos_integral(2*d*x^3) + 2/3*a*b*cos_integral(d*x^3)*sin(c
) + 2/3*a*b*cos(c)*sin_integral(d*x^3) - 1/6*b^2*sin(2*c)*sin_integral(-2*d
*x^3) + 1/3*a^2*log(d*x^3) + 1/6*b^2*log(d*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx = \int \frac{(a+b\,\sin(d\,x^3+c))^2}{x}\,dx$$

```
[In] int((a + b*sin(c + d*x^3))^2/x,x)
[Out] int((a + b*sin(c + d*x^3))^2/x, x)
```

3.72 $\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 122

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx = -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos(2(c+dx^3))}{6x^3} + \frac{2}{3}abd\cos(c)\operatorname{CosIntegral}(dx^3) + \frac{1}{3}b^2d\operatorname{CosIntegral}(2dx^3)\sin(2c)-\frac{2ab\sin(c+dx^3)}{3x^3} - \frac{2}{3}abd\sin(c)\operatorname{Si}(dx^3)+\frac{1}{3}b^2d\cos(2c)\operatorname{Si}(2dx^3)$$

```
[Out] 1/6*(-2*a^2-b^2)/x^3+2/3*a*b*d*Ci(d*x^3)*cos(c)+1/6*b^2*cos(2*d*x^3+2*c)/x^
3+1/3*b^2*d*cos(2*c)*Si(2*d*x^3)-2/3*a*b*d*Si(d*x^3)*sin(c)+1/3*b^2*d*Ci(2*
d*x^3)*sin(2*c)-2/3*a*b*sin(d*x^3+c)/x^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3484, 6, 3461, 3378, 3384, 3380, 3383, 3460}

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx = -\frac{2a^2+b^2}{6x^3}+\frac{2}{3}abd\cos(c)\operatorname{CosIntegral}(dx^3) - \frac{2}{3}abd\sin(c)\operatorname{Si}(dx^3)-\frac{2ab\sin(c+dx^3)}{3x^3} + \frac{1}{3}b^2d\sin(2c)\operatorname{CosIntegral}(2dx^3) + \frac{1}{3}b^2d\cos(2c)\operatorname{Si}(2dx^3)+\frac{b^2\cos(2(c+dx^3))}{6x^3}$$

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^4,x]
```

```
[Out] -1/6*(2*a^2 + b^2)/x^3 + (b^2*Cos[2*(c + d*x^3)])/(6*x^3) + (2*a*b*d*Cos[c]
*CosIntegral[d*x^3])/3 + (b^2*d*CosIntegral[2*d*x^3]*Sin[2*c])/3 - (2*a*b*S
in[c + d*x^3])/(3*x^3) - (2*a*b*d*Sin[c]*SinIntegral[d*x^3])/3 + (b^2*d*Cos
[2*c]*SinIntegral[2*d*x^3])/3
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{x^4}+\frac{b^2}{2x^4}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^4}+\frac{2ab\sin\left(c+dx^3\right)}{x^4}\right)\,dx\\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^4}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^4}+\frac{2ab\sin\left(c+dx^3\right)}{x^4}\right)\,dx\\
&= -\frac{2a^2+b^2}{6x^3}+(2ab)\int \frac{\sin\left(c+dx^3\right)}{x^4}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^3\right)}{x^4}\,dx\\
&= -\frac{2a^2+b^2}{6x^3}+\frac{1}{3}(2ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,x^3\right)-\frac{1}{6}b^2\text{Subst}\left(\int \frac{\cos(2c+2dx)}{x^2}\,dx,x,x^3\right)\\
&= -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos\left(2\left(c+dx^3\right)\right)}{6x^3}-\frac{2ab\sin\left(c+dx^3\right)}{3x^3}\\
&\quad+\frac{1}{3}(2abd)\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx,x,x^3\right)\\
&\quad+\frac{1}{3}\left(b^2d\right)\text{Subst}\left(\int \frac{\sin(2c+2dx)}{x}\,dx,x,x^3\right)\\
&= -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos\left(2\left(c+dx^3\right)\right)}{6x^3}-\frac{2ab\sin\left(c+dx^3\right)}{3x^3}\\
&\quad+\frac{1}{3}(2abd\cos(c))\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx,x,x^3\right)\\
&\quad+\frac{1}{3}\left(b^2d\cos(2c)\right)\text{Subst}\left(\int \frac{\sin(2dx)}{x}\,dx,x,x^3\right)\\
&\quad-\frac{1}{3}(2abd\sin(c))\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx,x,x^3\right)\\
&\quad+\frac{1}{3}\left(b^2d\sin(2c)\right)\text{Subst}\left(\int \frac{\cos(2dx)}{x}\,dx,x,x^3\right)\\
&= -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos\left(2\left(c+dx^3\right)\right)}{6x^3}+\frac{2}{3}abd\cos(c)\,\text{CosIntegral}\left(dx^3\right)\\
&\quad+\frac{1}{3}b^2d\,\text{CosIntegral}\left(2dx^3\right)\sin(2c)-\frac{2ab\sin\left(c+dx^3\right)}{3x^3}\\
&\quad-\frac{2}{3}abd\sin(c)\text{Si}\left(dx^3\right)+\frac{1}{3}b^2d\cos(2c)\text{Si}\left(2dx^3\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx$$
$$= \frac{-2a^2-b^2+b^2\cos(2(c+dx^3))+4abdx^3\cos(c)\,\text{CosIntegral}(dx^3)+2b^2dx^3\,\text{CosIntegral}(2dx^3)\sin(2c)-4a}{6x^3}$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^4,x]
```

```
[Out] (-2*a^2 - b^2 + b^2*Cos[2*(c + d*x^3)] + 4*a*b*d*x^3*Cos[c]*CosIntegral[d*x
^3] + 2*b^2*d*x^3*CosIntegral[2*d*x^3]*Sin[2*c] - 4*a*b*Sin[c + d*x^3] - 4*
a*b*d*x^3*Sin[c]*SinIntegral[d*x^3] + 2*b^2*d*x^3*Cos[2*c]*SinIntegral[2*d*
x^3])/(6*x^3)
```

Maple [F]

$$\int \frac{(a+b\sin(d\,x^3+c))^2}{x^4}dx$$

```
[In] int((a+b*sin(d*x^3+c))^2/x^4,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx$$
$$= \frac{2\,abdx^3\cos(c)\,\text{Ci}(dx^3)+b^2dx^3\,\text{Ci}(2\,dx^3)\sin(2\,c)+b^2dx^3\cos(2\,c)\,\text{Si}(2\,dx^3)-2\,abdx^3\sin(c)\,\text{Si}(dx^3)+b^2}{3\,x^3}$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*b*d*x^3*cos(c)*cos_integral(d*x^3) + b^2*d*x^3*cos_integral(2*d*x^
3)*sin(2*c) + b^2*d*x^3*cos(2*c)*sin_integral(2*d*x^3) - 2*a*b*d*x^3*sin(c)
*sin_integral(d*x^3) + b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 -
b^2)/x^3
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx = \int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx$$

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**4,x)
[Out] Integral((a + b*sin(c + d*x**3))**2/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx$$
$$= \frac{1}{3}\left(\left(\Gamma\left(-1, i\,dx^3\right)+\Gamma\left(-1, -i\,dx^3\right)\right)\cos(c) - \left(i\,\Gamma\left(-1, i\,dx^3\right) - i\,\Gamma\left(-1, -i\,dx^3\right)\right)\sin(c)\right)abd$$
$$+ \frac{\left(\left((i\,\Gamma(-1, 2i\,dx^3) - i\,\Gamma(-1, -2i\,dx^3))\cos(2c) + (\Gamma(-1, 2i\,dx^3) + \Gamma(-1, -2i\,dx^3))\sin(2c)\right)dx^3 - 1\right)b^2}{6x^3}$$
$$- \frac{a^2}{3x^3}$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^4,x, algorithm="maxima")
[Out] 1/3*((gamma(-1, I*d*x^3) + gamma(-1, -I*d*x^3))*cos(c) - (I*gamma(-1, I*d*x
^3) - I*gamma(-1, -I*d*x^3))*sin(c))*a*b*d + 1/6*(((I*gamma(-1, 2*I*d*x^3)
- I*gamma(-1, -2*I*d*x^3))*cos(2*c) + (gamma(-1, 2*I*d*x^3) + gamma(-1, -2*
I*d*x^3))*sin(2*c))*d*x^3 - 1)*b^2/x^3 - 1/3*a^2/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(109) = 218.

Time = 0.31 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.85

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx$$
$$= \frac{4(dx^3+c)abd^2\cos(c)\,\mathrm{Ci}(dx^3) - 4abcd^2\cos(c)\,\mathrm{Ci}(dx^3) + 2(dx^3+c)b^2d^2\,\mathrm{Ci}(2dx^3)\sin(2c) - 2b^2cd^2\,\mathrm{Ci}}{}$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^4,x, algorithm="giac")
[Out] 1/6*(4*(d*x^3 + c)*a*b*d^2*cos(c)*cos_integral(d*x^3) - 4*a*b*c*d^2*cos(c)*
cos_integral(d*x^3) + 2*(d*x^3 + c)*b^2*d^2*cos_integral(2*d*x^3)*sin(2*c)
```

```
- 2*b^2*c*d^2*cos_integral(2*d*x^3)*sin(2*c) - 4*(d*x^3 + c)*a*b*d^2*sin(c)
*sin_integral(d*x^3) + 4*a*b*c*d^2*sin(c)*sin_integral(d*x^3) - 2*(d*x^3 +
c)*b^2*d^2*cos(2*c)*sin_integral(-2*d*x^3) + 2*b^2*c*d^2*cos(2*c)*sin_integ
ral(-2*d*x^3) + b^2*d^2*cos(2*d*x^3 + 2*c) - 4*a*b*d^2*sin(d*x^3 + c) - 2*a
^2*d^2 - b^2*d^2)/(d^2*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx = \int \frac{(a+b\,\sin(d\,x^3+c))^2}{x^4}\,dx$$

```
[In] int((a + b*sin(c + d*x^3))^2/x^4,x)
[Out] int((a + b*sin(c + d*x^3))^2/x^4, x)
```

3.73 $\int x^4(a+b\sin(c+dx^3))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 249

$$\int x^4(a+b\sin(c+dx^3))^2\,dx = \frac{1}{10}(2a^2+b^2)x^5 - \frac{2abx^2\cos(c+dx^3)}{3d} - \frac{2abe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d(-idx^3)^{2/3}} - \frac{2abe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d(idx^3)^{2/3}} + \frac{ib^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{36\ 2^{2/3}d(-idx^3)^{2/3}} - \frac{ib^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{36\ 2^{2/3}d(idx^3)^{2/3}} - \frac{b^2x^2\sin(2c+2dx^3)}{12d}$$

```
[Out] 1/10*(2*a^2+b^2)*x^5-2/3*a*b*x^2*cos(d*x^3+c)/d-2/9*a*b*exp(I*c)*x^2*GAMMA(
2/3,-I*d*x^3)/d/(-I*d*x^3)^(2/3)-2/9*a*b*x^2*GAMMA(2/3,I*d*x^3)/d/exp(I*c)/
(I*d*x^3)^(2/3)+1/72*I*b^2*exp(2*I*c)*x^2*GAMMA(2/3,-2*I*d*x^3)*2^(1/3)/d/(
-I*d*x^3)^(2/3)-1/72*I*b^2*x^2*GAMMA(2/3,2*I*d*x^3)*2^(1/3)/d/exp(2*I*c)/(I
*d*x^3)^(2/3)-1/12*b^2*x^2*sin(2*d*x^3+2*c)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3484, 6, 3467, 3470, 2250, 3466, 3471}

$$\int x^4(a+b\sin(c+dx^3))^2\,dx = \frac{1}{10}x^5(2a^2+b^2) - \frac{2abx^2\cos(c+dx^3)}{3d} - \frac{2abe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d(-idx^3)^{2/3}} - \frac{2abe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d(idx^3)^{2/3}} - \frac{b^2x^2\sin(2c+2dx^3)}{12d} + \frac{ib^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{36\ 2^{2/3}d(-idx^3)^{2/3}} - \frac{ib^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{36\ 2^{2/3}d(idx^3)^{2/3}}$$

```
[In] Int[x^4*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^5)/10 - (2*a*b*x^2*Cos[c + d*x^3])/(3*d) - (2*a*b*E^(I*c)*
x^2*Gamma[2/3, (-I)*d*x^3])/(9*d*((-I)*d*x^3)^(2/3)) - (2*a*b*x^2*Gamma[2/3
, I*d*x^3])/(9*d*E^(I*c)*(I*d*x^3)^(2/3)) + ((I/36)*b^2*E^((2*I)*c)*x^2*Gam
ma[2/3, (-2*I)*d*x^3])/(2^(2/3)*d*((-I)*d*x^3)^(2/3)) - ((I/36)*b^2*x^2*Gam
ma[2/3, (2*I)*d*x^3])/(2^(2/3)*d*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (b^2*x^2*Si
n[2*c + 2*d*x^3])/(12*d)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2x^4+\frac{b^2x^4}{2}-\frac{1}{2}b^2x^4\cos\left(2c+2dx^3\right)+2abx^4\sin\left(c+dx^3\right)\right)\,dx\\
&= \int \left(\left(a^2+\frac{b^2}{2}\right)x^4-\frac{1}{2}b^2x^4\cos\left(2c+2dx^3\right)+2abx^4\sin\left(c+dx^3\right)\right)\,dx\\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5+(2ab)\int x^4\sin\left(c+dx^3\right)\,dx-\frac{1}{2}b^2\int x^4\cos\left(2c+2dx^3\right)\,dx\\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}-\frac{b^2x^2\sin\left(2c+2dx^3\right)}{12d}\\
&\quad+\frac{(4ab)\int x\cos\left(c+dx^3\right)\,dx}{3d}+\frac{b^2\int x\sin\left(2c+2dx^3\right)\,dx}{6d}\\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}-\frac{b^2x^2\sin\left(2c+2dx^3\right)}{12d}+\frac{(2ab)\int e^{-ic-idx^3}x\,dx}{3d}\\
&\quad+\frac{(2ab)\int e^{ic+idx^3}x\,dx}{3d}+\frac{(ib^2)\int e^{-2ic-2idx^3}x\,dx}{12d}-\frac{(ib^2)\int e^{2ic+2idx^3}x\,dx}{12d}\\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}-\frac{2abe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{2abe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}\\
&\quad+\frac{ib^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{36\ 2^{2/3}d\left(-idx^3\right)^{2/3}}-\frac{ib^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{36\ 2^{2/3}d\left(idx^3\right)^{2/3}}-\frac{b^2x^2\sin\left(2c+2dx^3\right)}{12d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.36

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$=\frac{dx^8\left(72a^2dx^3(d^2x^6)^{2/3}+36b^2dx^3(d^2x^6)^{2/3}-240ab(d^2x^6)^{2/3}\cos\left(c+dx^3\right)+5i\sqrt[3]{2}b^2(idx^3)^{2/3}\cos(2c)\Gamma\left(\frac{2}{3},-\right.\right.}{}$$

```
[In] Integrate[x^4*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (d*x^8*(72*a^2*d*x^3*(d^2*x^6)^(2/3) + 36*b^2*d*x^3*(d^2*x^6)^(2/3) - 240*a
*b*(d^2*x^6)^(2/3)*Cos[c + d*x^3] + (5*I)*2^(1/3)*b^2*(I*d*x^3)^(2/3)*Cos[2
*c]*Gamma[2/3, (-2*I)*d*x^3] - (5*I)*2^(1/3)*b^2*((-I)*d*x^3)^(2/3)*Cos[2*c
```

```
]*Gamma[2/3, (2*I)*d*x^3] - 80*a*b*((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3]*(
Cos[c] - I*Sin[c]) - 80*a*b*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c]
+ I*Sin[c]) - 5*2^(1/3)*b^2*(I*d*x^3)^(2/3)*Gamma[2/3, (-2*I)*d*x^3]*Sin[2*
c] - 5*2^(1/3)*b^2*((-I)*d*x^3)^(2/3)*Gamma[2/3, (2*I)*d*x^3]*Sin[2*c] - 30
*b^2*(d^2*x^6)^(2/3)*Sin[2*(c + d*x^3)]))/(360*(d^2*x^6)^(5/3))
```

Maple [F]

$$\int x^{4}\left(a+b\sin\left(d\,x^{3}+c\right)\right)^{2}dx$$

```
[In] int(x^4*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x^4*(a+b*sin(d*x^3+c))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.75

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$=\frac{36\left(2\,a^2+b^2\right)d^2x^5-60\,b^2dx^2\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)-240\,abdx^2\cos\left(dx^3+c\right)-5\left(b^2\cos\left(2\,c\right)-i\,b^2\sin\right.}{}$$

```
[In] integrate(x^4*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/360*(36*(2*a^2 + b^2)*d^2*x^5 - 60*b^2*d*x^2*cos(d*x^3 + c)*sin(d*x^3 + c
) - 240*a*b*d*x^2*cos(d*x^3 + c) - 5*(b^2*cos(2*c) - I*b^2*sin(2*c))*(2*I*d
)^(1/3)*gamma(2/3, 2*I*d*x^3) - 80*(-I*a*b*cos(c) - a*b*sin(c))*(I*d)^(1/3)
*gamma(2/3, I*d*x^3) - 80*(I*a*b*cos(c) - a*b*sin(c))*(-I*d)^(1/3)*gamma(2/
3, -I*d*x^3) - 5*(b^2*cos(2*c) + I*b^2*sin(2*c))*(-2*I*d)^(1/3)*gamma(2/3,
-2*I*d*x^3))/d^2
```

Sympy [F]

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\int x^4\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

```
[In] integrate(x**4*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(x**4*(a + b*sin(c + d*x**3))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.94

$$\int x^4 (a + b\sin(c + dx^3))^2 \, dx = \frac{1}{5} a^2 x^5 - \frac{\left(6\,dx^3 \cos(dx^3 + c) - (dx^3)^{\frac{1}{3}} \left(\left((i\sqrt{3} - 1)\Gamma\left(\frac{2}{3}, i\,dx^3\right) + (-i\sqrt{3} - 1)\Gamma\left(\frac{2}{3}, -i\,dx^3\right)\right)\cos(c) + \left((\sqrt{3} + i)\right.\right.\right.}{9\,d^2 x} + \frac{\left(72\,d^2 x^6 - 60\,dx^3 \sin(2\,dx^3 + 2\,c) - 5 \cdot 2^{\frac{1}{3}} (dx^3)^{\frac{1}{3}} \left(\left((\sqrt{3} + i)\Gamma\left(\frac{2}{3}, 2i\,dx^3\right) + (\sqrt{3} - i)\Gamma\left(\frac{2}{3}, -2i\,dx^3\right)\right)\cos\right.\right.}{720\,d^2 x}$$

```
[In] integrate(x^4*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 - 1/9*(6*d*x^3*cos(d*x^3 + c) - (d*x^3)^(1/3)*(((I*sqrt(3) - 1)
*gamma(2/3, I*d*x^3) + (-I*sqrt(3) - 1)*gamma(2/3, -I*d*x^3))*cos(c) + ((sq
rt(3) + I)*gamma(2/3, I*d*x^3) + (sqrt(3) - I)*gamma(2/3, -I*d*x^3))*sin(c)
))*a*b/(d^2*x) + 1/720*(72*d^2*x^6 - 60*d*x^3*sin(2*d*x^3 + 2*c) - 5*2^(1/3
)*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(2/3, 2*I*d*x^3) + (sqrt(3) - I)*gamma
(2/3, -2*I*d*x^3))*cos(2*c) + ((-I*sqrt(3) + 1)*gamma(2/3, 2*I*d*x^3) + (I*
sqrt(3) + 1)*gamma(2/3, -2*I*d*x^3))*sin(2*c)))*b^2/(d^2*x)
```

Giac [F]

$$\int x^4 (a + b\sin(c + dx^3))^2 \, dx = \int (b\sin(dx^3 + c) + a)^2 x^4 \, dx$$

```
[In] integrate(x^4*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a + b\sin(c + dx^3))^2 \, dx = \int x^4 \left(a + b\,\sin\left(d\,x^3 + c\right)\right)^2 dx$$

```
[In] int(x^4*(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] int(x^4*(a + b*sin(c + d*x^3))^2, x)
```

3.74 $\int x(a+b\sin(c+dx^3))^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 193

$$\int x(a+b\sin(c+dx^3))^2\,dx = \frac{1}{4}(2a^2+b^2)\,x^2 + \frac{iabe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{3(-idx^3)^{2/3}} - \frac{iabe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{3(idx^3)^{2/3}} + \frac{b^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{12\ 2^{2/3}(-idx^3)^{2/3}} + \frac{b^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{12\ 2^{2/3}(idx^3)^{2/3}}$$

```
[Out] 1/4*(2*a^2+b^2)*x^2+1/3*I*a*b*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/(-I*d*x^3)^(
2/3)-1/3*I*a*b*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(2/3)+1/24*b^2*exp
(2*I*c)*x^2*GAMMA(2/3,-2*I*d*x^3)*2^(1/3)/(-I*d*x^3)^(2/3)+1/24*b^2*x^2*GAM
MA(2/3,2*I*d*x^3)*2^(1/3)/exp(2*I*c)/(I*d*x^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3484, 6, 3471, 2250, 3470}

$$\int x(a+b\sin(c+dx^3))^2\,dx = \frac{1}{4}x^2(2a^2+b^2) + \frac{iabe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{3(-idx^3)^{2/3}} - \frac{iabe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{3(idx^3)^{2/3}} + \frac{b^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{12\ 2^{2/3}(-idx^3)^{2/3}} + \frac{b^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{12\ 2^{2/3}(idx^3)^{2/3}}$$

```
[In] Int[x*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^2)/4 + ((I/3)*a*b*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((-I
)*d*x^3)^(2/3) - ((I/3)*a*b*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/
3)) + (b^2*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I)*d*x^3])/(12*2^(2/3)*((-I)*d*x^
```

```
3)^(2/3)) + (b^2*x^2*Gamma[2/3, (2*I)*d*x^3])/(12*2^(2/3)*E^((2*I)*c)*(I*d*
x^3)^(2/3))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2x+\frac{b^2x}{2}-\frac{1}{2}b^2x\cos\left(2c+2dx^3\right)+2abx\sin\left(c+dx^3\right)\right)\,dx\\
&= \int \left(\left(a^2+\frac{b^2}{2}\right)x-\frac{1}{2}b^2x\cos\left(2c+2dx^3\right)+2abx\sin\left(c+dx^3\right)\right)\,dx\\
&= \frac{1}{4}\left(2a^2+b^2\right)x^2+(2ab)\int x\sin\left(c+dx^3\right)\,dx-\frac{1}{2}b^2\int x\cos\left(2c+2dx^3\right)\,dx\\
&= \frac{1}{4}\left(2a^2+b^2\right)x^2+(iab)\int e^{-ic-idx^3}x\,dx-(iab)\int e^{ic+idx^3}x\,dx\\
&\quad -\frac{1}{4}b^2\int e^{-2ic-2idx^3}x\,dx-\frac{1}{4}b^2\int e^{2ic+2idx^3}x\,dx
\end{aligned}
$$

$$= \frac{1}{4}(2a^2+b^2)\,x^2 + \frac{iabe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{3\left(-idx^3\right)^{2/3}} - \frac{iabe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{3\left(idx^3\right)^{2/3}} + \frac{b^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{12\ 2^{2/3}\left(-idx^3\right)^{2/3}} + \frac{b^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{12\ 2^{2/3}\left(idx^3\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.30

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$= \frac{1}{24}\left(6(2a^2+b^2)\,x^2 - \frac{8ab\cos(c)\left(\sqrt[3]{-idx^3}\Gamma\left(\frac{2}{3},-idx^3\right)+\sqrt[3]{idx^3}\Gamma\left(\frac{2}{3},idx^3\right)\right)}{dx} - \frac{8abx^2\left(\left(idx^3\right)^{2/3}\Gamma\left(\frac{2}{3},-idx^3\right)+\left(-idx^3\right)^{2/3}\Gamma\left(\frac{2}{3},idx^3\right)\right)\sin(c)}{\left(d^2x^6\right)^{2/3}} + \frac{\sqrt[3]{2}b^2x^2\Gamma\left(\frac{2}{3},2idx^3\right)(\cos(2c)-i\sin(2c))}{\left(idx^3\right)^{2/3}} + \frac{\sqrt[3]{2}b^2x^2\Gamma\left(\frac{2}{3},-2idx^3\right)(\cos(2c)+i\sin(2c))}{\left(-idx^3\right)^{2/3}}\right)$$

```
[In] Integrate[x*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (6*(2*a^2 + b^2)*x^2 - (8*a*b*Cos[c]*(((-I)*d*x^3)^(1/3)*Gamma[2/3, (-I)*d*
x^3] + (I*d*x^3)^(1/3)*Gamma[2/3, I*d*x^3]))/(d*x) - (8*a*b*x^2*((I*d*x^3)^
(2/3)*Gamma[2/3, (-I)*d*x^3] + ((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3])*Sin[
c])/(d^2*x^6)^(2/3) + (2^(1/3)*b^2*x^2*Gamma[2/3, (2*I)*d*x^3]*(Cos[2*c] -
I*Sin[2*c]))/(I*d*x^3)^(2/3) + (2^(1/3)*b^2*x^2*Gamma[2/3, (-2*I)*d*x^3]*(C
os[2*c] + I*Sin[2*c]))/((-I)*d*x^3)^(2/3))/24
```

Maple [F]

$$\int x\left(a+b\sin\left(d\,x^3+c\right)\right)^2dx$$

```
[In] int(x*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x*(a+b*sin(d*x^3+c))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.73

$$\int x(a+b\sin(c+dx^3))^2 dx = \frac{6(2a^2+b^2)dx^2+(-ib^2\cos(2c)-b^2\sin(2c))(2id)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},2idx^3\right)-8(ab\cos(c)-iab\sin(c))(id)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},\right.}{2}$$

```
[In] integrate(x*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
[Out] 1/24*(6*(2*a^2 + b^2)*d*x^2 + (-I*b^2*cos(2*c) - b^2*sin(2*c))*(2*I*d)^(1/3
)*gamma(2/3, 2*I*d*x^3) - 8*(a*b*cos(c) - I*a*b*sin(c))*(I*d)^(1/3)*gamma(2
/3, I*d*x^3) - 8*(a*b*cos(c) + I*a*b*sin(c))*(-I*d)^(1/3)*gamma(2/3, -I*d*x
^3) + (I*b^2*cos(2*c) - b^2*sin(2*c))*(-2*I*d)^(1/3)*gamma(2/3, -2*I*d*x^3)
)/d
```

Sympy [F]

$$\int x(a+b\sin(c+dx^3))^2 dx = \int x(a+b\sin(c+dx^3))^2 dx$$

```
[In] integrate(x*(a+b*sin(d*x**3+c))**2,x)
[Out] Integral(x*(a + b*sin(c + d*x**3))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03

$$\int x(a+b\sin(c+dx^3))^2 dx = \frac{1}{2}a^2x^2 - \frac{(dx^3)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},idx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-idx^3\right)\right)\cos(c)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},idx^3\right)+\left(-i\sqrt{3}-1\right)\right.}{6dx} + \frac{\left(12dx^3-2^{\frac{1}{3}}(dx^3)^{\frac{1}{3}}\left(\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},2idx^3\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-2idx^3\right)\right)\cos(2c)+\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},2\right.\right.\right.}{48dx}$$

```
[In] integrate(x*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
[Out] 1/2*a^2*x^2 - 1/6*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*d*x^3) + (sqrt
(3) - I)*gamma(2/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) - 1)*gamma(2/3, I*d*x^3
```

```
) + (-I*sqrt(3) - 1)*gamma(2/3, -I*d*x^3))*sin(c))*a*b/(d*x) + 1/48*(12*d*x
^3 - 2^(1/3)*(d*x^3)^(1/3)*(((I*sqrt(3) - 1)*gamma(2/3, 2*I*d*x^3) + (-I*sq
rt(3) - 1)*gamma(2/3, -2*I*d*x^3))*cos(2*c) + ((sqrt(3) + I)*gamma(2/3, 2*I
*d*x^3) + (sqrt(3) - I)*gamma(2/3, -2*I*d*x^3))*sin(2*c)))*b^2/(d*x)
```

Giac [F]

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\int\left(b\sin\left(dx^3+c\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\int x\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2\,dx$$

```
[In] int(x*(a + b*sin(c + d*x^3))^2,x)
[Out] int(x*(a + b*sin(c + d*x^3))^2, x)
```

3.75 $\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 231

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx = \frac{-2a^2-b^2}{2x}+\frac{b^2\cos(2c+2dx^3)}{2x}-\frac{abde^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{(-idx^3)^{2/3}} -\frac{abde^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{(idx^3)^{2/3}}+\frac{ib^2de^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{2\ 2^{2/3}(-idx^3)^{2/3}} -\frac{ib^2de^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{2\ 2^{2/3}(idx^3)^{2/3}}-\frac{2ab\sin(c+dx^3)}{x}$$

```
[Out] 1/2*(-2*a^2-b^2)/x+1/2*b^2*cos(2*d*x^3+2*c)/x-a*b*d*exp(I*c)*x^2*GAMMA(2/3,
-I*d*x^3)/(-I*d*x^3)^(2/3)-a*b*d*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^
(2/3)+1/4*I*b^2*d*exp(2*I*c)*x^2*GAMMA(2/3,-2*I*d*x^3)*2^(1/3)/(-I*d*x^3)^(
2/3)-1/4*I*b^2*d*x^2*GAMMA(2/3,2*I*d*x^3)*2^(1/3)/exp(2*I*c)/(I*d*x^3)^(2/3
)-2*a*b*sin(d*x^3+c)/x
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3484, 6, 3469, 3470, 2250, 3468, 3471}

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx = -\frac{2a^2+b^2}{2x}-\frac{2ab\sin(c+dx^3)}{x}-\frac{abe^{ic}dx^2\Gamma\left(\frac{2}{3},-idx^3\right)}{(-idx^3)^{2/3}} -\frac{abe^{-ic}dx^2\Gamma\left(\frac{2}{3},idx^3\right)}{(idx^3)^{2/3}}+\frac{b^2\cos(2c+2dx^3)}{2x} +\frac{ib^2e^{2ic}dx^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{2\ 2^{2/3}(-idx^3)^{2/3}}-\frac{ib^2e^{-2ic}dx^2\Gamma\left(\frac{2}{3},2idx^3\right)}{2\ 2^{2/3}(idx^3)^{2/3}}$$

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^2,x]
```

```
[Out] -1/2*(2*a^2 + b^2)/x + (b^2*Cos[2*c + 2*d*x^3])/(2*x) - (a*b*d*E^(I*c)*x^2*
Gamma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3) - (a*b*d*x^2*Gamma[2/3, I*d*x^3]
)/(E^(I*c)*(I*d*x^3)^(2/3)) + ((I/2)*b^2*d*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I
)*d*x^3])/(2^(2/3)*((-I)*d*x^3)^(2/3)) - ((I/2)*b^2*d*x^2*Gamma[2/3, (2*I)*
d*x^3])/(2^(2/3)*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (2*a*b*Sin[c + d*x^3])/x
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2}{x^2}+\frac{b^2}{2x^2}-\frac{b^2\cos(2c+2dx^3)}{2x^2}+\frac{2ab\sin(c+dx^3)}{x^2}\right)dx$$

$$= \int \left(\frac{a^2+\frac{b^2}{2}}{x^2}-\frac{b^2\cos(2c+2dx^3)}{2x^2}+\frac{2ab\sin(c+dx^3)}{x^2}\right)dx$$

$$= -\frac{2a^2+b^2}{2x}+(2ab)\int\frac{\sin(c+dx^3)}{x^2}\,dx-\frac{1}{2}b^2\int\frac{\cos(2c+2dx^3)}{x^2}\,dx$$

$$= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos(2c+2dx^3)}{2x}-\frac{2ab\sin(c+dx^3)}{x} + (6abd)\int x\cos\left(c+dx^3\right)\,dx+\left(3b^2d\right)\int x\sin\left(2c+2dx^3\right)\,dx$$

$$= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos(2c+2dx^3)}{2x}-\frac{2ab\sin(c+dx^3)}{x}+(3abd)\int e^{-ic-idx^3}x\,dx + (3abd)\int e^{ic+idx^3}x\,dx+\frac{1}{2}\left(3ib^2d\right)\int e^{-2ic-2idx^3}x\,dx-\frac{1}{2}\left(3ib^2d\right)\int e^{2ic+2idx^3}x\,dx$$

$$= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos(2c+2dx^3)}{2x}-\frac{abde^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{\left(-idx^3\right)^{2/3}}-\frac{abde^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{\left(idx^3\right)^{2/3}} + \frac{ib^2de^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{2\,2^{2/3}\left(-idx^3\right)^{2/3}}-\frac{ib^2de^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{2\,2^{2/3}\left(idx^3\right)^{2/3}}-\frac{2ab\sin(c+dx^3)}{x}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.44

$$\int\frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx = \frac{-4a^2(d^2x^6)^{2/3}-2b^2(d^2x^6)^{2/3}+2b^2(d^2x^6)^{2/3}\cos(2(c+dx^3))+\sqrt[3]{2}b^2(idx^3)^{5/3}\cos(2c)\Gamma\left(\frac{2}{3},-2idx^3\right)+\sqrt[3]{2}\cdots}{}$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^2,x]
```

```
[Out] (-4*a^2*(d^2*x^6)^(2/3) - 2*b^2*(d^2*x^6)^(2/3) + 2*b^2*(d^2*x^6)^(2/3)*Cos
[2*(c + d*x^3)] + 2^(1/3)*b^2*(I*d*x^3)^(5/3)*Cos[2*c]*Gamma[2/3, (-2*I)*d*
x^3] + 2^(1/3)*b^2*((-I)*d*x^3)^(5/3)*Cos[2*c]*Gamma[2/3, (2*I)*d*x^3] - (4
*I)*a*b*((-I)*d*x^3)^(5/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (4*I)*
```

```
a*b*(I*d*x^3)^(5/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) + I*2^(1/3)*
b^2*(I*d*x^3)^(5/3)*Gamma[2/3, (-2*I)*d*x^3]*Sin[2*c] - I*2^(1/3)*b^2*((-I)
*d*x^3)^(5/3)*Gamma[2/3, (2*I)*d*x^3]*Sin[2*c] - 8*a*b*(d^2*x^6)^(2/3)*Sin[
c + d*x^3])/(4*x*(d^2*x^6)^(2/3))
```

Maple [F]

$$\int \frac{(a + b\sin(d\,x^3 + c))^2}{x^2}dx$$

```
[In] int((a+b*sin(d*x^3+c))^2/x^2,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.75

$$\int \frac{(a + b\sin(c + dx^3))^2}{x^2}\,dx$$
$$= \frac{4\,b^2\cos(dx^3 + c)^2 - 8\,ab\sin(dx^3 + c) - (b^2x\cos(2\,c) - i\,b^2x\sin(2\,c))(2i\,d)^{\frac{1}{3}}\,\Gamma\left(\frac{2}{3}, 2i\,dx^3\right) - 4\,(-i\,abx\cos(c$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*cos(d*x^3 + c)^2 - 8*a*b*sin(d*x^3 + c) - (b^2*x*cos(2*c) - I*b^
2*x*sin(2*c))*(2*I*d)^(1/3)*gamma(2/3, 2*I*d*x^3) - 4*(-I*a*b*x*cos(c) - a*
b*x*sin(c))*(I*d)^(1/3)*gamma(2/3, I*d*x^3) - 4*(I*a*b*x*cos(c) - a*b*x*sin
(c))*(-I*d)^(1/3)*gamma(2/3, -I*d*x^3) - (b^2*x*cos(2*c) + I*b^2*x*sin(2*c)
)*(-2*I*d)^(1/3)*gamma(2/3, -2*I*d*x^3) - 4*a^2 - 4*b^2)/x
```

Sympy [F]

$$\int \frac{(a + b\sin(c + dx^3))^2}{x^2}\,dx = \int \frac{(a + b\sin(c + dx^3))^2}{x^2}\,dx$$

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.81

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx = -\frac{(dx^3)^{\frac{1}{3}}\left(\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3},i\,dx^3\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3},-i\,dx^3\right)\right)\cos(c)+\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{1}{3},i\,dx^3\right)+(\sqrt{}\right.\right.}{6x} + \frac{\left(2^{\frac{1}{3}}(dx^3)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{1}{3},2i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{1}{3},-2i\,dx^3\right)\right)\cos(2c)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3},2i\,dx^3\right)\right.\right.\right.}{24x} - \frac{a^2}{x}$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*(d*x^3)^(1/3)*(((I*sqrt(3) - 1)*gamma(-1/3, I*d*x^3) + (-I*sqrt(3) - 1
)*gamma(-1/3, -I*d*x^3))*cos(c) + ((sqrt(3) + I)*gamma(-1/3, I*d*x^3) + (sq
rt(3) - I)*gamma(-1/3, -I*d*x^3))*sin(c))*a*b/x + 1/24*(2^(1/3)*(d*x^3)^(1/
3)*(((sqrt(3) + I)*gamma(-1/3, 2*I*d*x^3) + (sqrt(3) - I)*gamma(-1/3, -2*I*
d*x^3))*cos(2*c) - ((I*sqrt(3) - 1)*gamma(-1/3, 2*I*d*x^3) + (-I*sqrt(3) -
1)*gamma(-1/3, -2*I*d*x^3))*sin(2*c)) - 12)*b^2/x - a^2/x
```

Giac [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx = \int \frac{(b\sin(dx^3+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx = \int \frac{(a+b\,\sin(d\,x^3+c))^2}{x^2}\,dx$$

```
[In] int((a + b*sin(c + d*x^3))^2/x^2,x)
```

```
[Out] int((a + b*sin(c + d*x^3))^2/x^2, x)
```

3.76 $\int \frac{(a+b\sin(c+dx^3))^2}{x^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 285

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^3))^2}{x^5} dx = & \frac{-2a^2-b^2}{8x^4} - \frac{3abd\cos(c+dx^3)}{2x} + \frac{b^2\cos(2c+2dx^3)}{8x^4} \\ & - \frac{3iabd^2e^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{4(-idx^3)^{2/3}} + \frac{3iabd^2e^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{4(idx^3)^{2/3}} \\ & - \frac{3b^2d^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{4\ 2^{2/3}(-idx^3)^{2/3}} - \frac{3b^2d^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{4\ 2^{2/3}(idx^3)^{2/3}} \\ & - \frac{ab\sin(c+dx^3)}{2x^4} - \frac{3b^2d\sin(2c+2dx^3)}{4x}\end{aligned}$$

```
[Out] 1/8*(-2*a^2-b^2)/x^4-3/2*a*b*d*cos(d*x^3+c)/x+1/8*b^2*cos(2*d*x^3+2*c)/x^4-
3/4*I*a*b*d^2*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/(-I*d*x^3)^(2/3)+3/4*I*a*b*d
^2*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(2/3)-3/8*b^2*d^2*exp(2*I*c)*x
^2*GAMMA(2/3,-2*I*d*x^3)*2^(1/3)/(-I*d*x^3)^(2/3)-3/8*b^2*d^2*x^2*GAMMA(2/3
,2*I*d*x^3)*2^(1/3)/exp(2*I*c)/(I*d*x^3)^(2/3)-1/2*a*b*sin(d*x^3+c)/x^4-3/4
*b^2*d*sin(2*d*x^3+2*c)/x
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used

$= \{3484, 6, 3469, 3468, 3471, 2250, 3470\}$

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx = -\frac{2a^2+b^2}{8x^4} - \frac{3iabe^{ic}d^2x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{4\left(-idx^3\right)^{2/3}} + \frac{3iabe^{-ic}d^2x^2\Gamma\left(\frac{2}{3},idx^3\right)}{4\left(idx^3\right)^{2/3}} - \frac{3abd\cos(c+dx^3)}{2x} - \frac{ab\sin(c+dx^3)}{2x^4} - \frac{3b^2e^{2ic}d^2x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{4\,2^{2/3}\left(-idx^3\right)^{2/3}} - \frac{3b^2e^{-2ic}d^2x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{4\,2^{2/3}\left(idx^3\right)^{2/3}} - \frac{3b^2d\sin(2c+2dx^3)}{4x} + \frac{b^2\cos(2c+2dx^3)}{8x^4}$$

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^5,x]
```

```
[Out] -1/8*(2*a^2 + b^2)/x^4 - (3*a*b*d*Cos[c + d*x^3])/(2*x) + (b^2*Cos[2*c + 2*
d*x^3])/(8*x^4) - (((3*I)/4)*a*b*d^2*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((
-I)*d*x^3)^(2/3) + (((3*I)/4)*a*b*d^2*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*
d*x^3)^(2/3)) - (3*b^2*d^2*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I)*d*x^3])/(4*2^(
2/3)*((-I)*d*x^3)^(2/3)) - (3*b^2*d^2*x^2*Gamma[2/3, (2*I)*d*x^3])/(4*2^(2/
3)*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (a*b*Sin[c + d*x^3])/(2*x^4) - (3*b^2*d*S
in[2*c + 2*d*x^3])/(4*x)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{x^5}+\frac{b^2}{2x^5}-\frac{b^2\cos(2c+2dx^3)}{2x^5}+\frac{2ab\sin(c+dx^3)}{x^5}\right)dx \\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^5}-\frac{b^2\cos(2c+2dx^3)}{2x^5}+\frac{2ab\sin(c+dx^3)}{x^5}\right)dx \\
&= -\frac{2a^2+b^2}{8x^4}+(2ab)\int \frac{\sin(c+dx^3)}{x^5}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^3)}{x^5}\,dx \\
&= -\frac{2a^2+b^2}{8x^4}+\frac{b^2\cos(2c+2dx^3)}{8x^4}-\frac{ab\sin(c+dx^3)}{2x^4} \\
&\quad +\frac{1}{2}(3abd)\int \frac{\cos(c+dx^3)}{x^2}\,dx+\frac{1}{4}(3b^2d)\int \frac{\sin(2c+2dx^3)}{x^2}\,dx \\
&= -\frac{2a^2+b^2}{8x^4}-\frac{3abd\cos(c+dx^3)}{2x}+\frac{b^2\cos(2c+2dx^3)}{8x^4}-\frac{ab\sin(c+dx^3)}{2x^4} \\
&\quad -\frac{3b^2d\sin(2c+2dx^3)}{4x}-\frac{1}{2}(9abd^2)\int x\sin(c+dx^3)\,dx+\frac{1}{2}(9b^2d^2)\int x\cos(2c \\
&\quad +2dx^3)\,dx \\
&= -\frac{2a^2+b^2}{8x^4}-\frac{3abd\cos(c+dx^3)}{2x}+\frac{b^2\cos(2c+2dx^3)}{8x^4}-\frac{ab\sin(c+dx^3)}{2x^4} \\
&\quad -\frac{3b^2d\sin(2c+2dx^3)}{4x}-\frac{1}{4}(9iabd^2)\int e^{-ic-idx^3}x\,dx+\frac{1}{4}(9iabd^2)\int e^{ic+idx^3}x\,dx \\
&\quad +\frac{1}{4}(9b^2d^2)\int e^{-2ic-2idx^3}x\,dx+\frac{1}{4}(9b^2d^2)\int e^{2ic+2idx^3}x\,dx
\end{aligned}$$

$$= -\frac{2a^2+b^2}{8x^4} - \frac{3abd\cos(c+dx^3)}{2x} + \frac{b^2\cos(2c+2dx^3)}{8x^4} - \frac{3iabd^2e^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{4(-idx^3)^{2/3}} + \frac{3iabd^2e^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{4(idx^3)^{2/3}} - \frac{3b^2d^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{4\,2^{2/3}(-idx^3)^{2/3}} - \frac{3b^2d^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{4\,2^{2/3}(idx^3)^{2/3}} - \frac{ab\sin(c+dx^3)}{2x^4} - \frac{3b^2d\sin(2c+2dx^3)}{4x}$$

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx =$$
$$-\frac{2a^2+b^2+12abdx^3\cos(c+dx^3)-b^2\cos(2(c+dx^3))-3\sqrt[3]{2}b^2(idx^3)^{4/3}\cos(2c)\Gamma\left(\frac{2}{3},2idx^3\right)+6iab(idx}{}$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^5,x]
```

```
[Out] -1/8*(2*a^2 + b^2 + 12*a*b*d*x^3*Cos[c + d*x^3] - b^2*Cos[2*(c + d*x^3)] -
3*2^(1/3)*b^2*(I*d*x^3)^(4/3)*Cos[2*c]*Gamma[2/3, (2*I)*d*x^3] + (6*I)*a*b*
(I*d*x^3)^(4/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (6*I)*a*b*(I*d*x^
3)^(2/3)*(d^2*x^6)^(1/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 3*2^(
1/3)*b^2*((-I)*d*x^3)^(4/3)*Gamma[2/3, (-2*I)*d*x^3]*(Cos[2*c] + I*Sin[2*c]
) + (3*I)*2^(1/3)*b^2*(I*d*x^3)^(4/3)*Gamma[2/3, (2*I)*d*x^3]*Sin[2*c] + 4*
a*b*Sin[c + d*x^3] + 6*b^2*d*x^3*Sin[2*(c + d*x^3)])/x^4
```

Maple [F]

$$\int \frac{(a+b\sin(d\,x^3+c))^2}{x^5}dx$$

```
[In] int((a+b*sin(d*x^3+c))^2/x^5,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.81

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx =$$
$$-\frac{12\,abdx^3\cos(dx^3+c)-2\,b^2\cos(dx^3+c)^2+3\,(-i\,b^2dx^4\cos(2\,c)-b^2dx^4\sin(2\,c))(2i\,d)^{\frac{1}{3}}\,\Gamma\left(\frac{2}{3},2i\,dx^3\right)}{}$$

[In] integrate((a+b*sin(d*x^3+c))^2/x^5,x, algorithm="fricas")

[Out] -1/8*(12*a*b*d*x^3*cos(d*x^3 + c) - 2*b^2*cos(d*x^3 + c)^2 + 3*(-I*b^2*d*x^4*cos(2*c) - b^2*d*x^4*sin(2*c))*(2*I*d)^(1/3)*gamma(2/3, 2*I*d*x^3) - 6*(a*b*d*x^4*cos(c) - I*a*b*d*x^4*sin(c))*(I*d)^(1/3)*gamma(2/3, I*d*x^3) - 6*(a*b*d*x^4*cos(c) + I*a*b*d*x^4*sin(c))*(-I*d)^(1/3)*gamma(2/3, -I*d*x^3) + 3*(I*b^2*d*x^4*cos(2*c) - b^2*d*x^4*sin(2*c))*(-2*I*d)^(1/3)*gamma(2/3, -2*I*d*x^3) + 2*a^2 + 2*b^2 + 4*(3*b^2*d*x^3*cos(d*x^3 + c) + a*b)*sin(d*x^3 + c))/x^4

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx = \int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx$$

[In] integrate((a+b*sin(d*x**3+c))**2/x**5,x)

[Out] Integral((a + b*sin(c + d*x**3))**2/x**5, x)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx = \frac{(dx^3)^{\frac{1}{3}}\left(\left((\sqrt{3}+i)\Gamma\left(-\frac{4}{3},i\,dx^3\right)+(\sqrt{3}-i)\Gamma\left(-\frac{4}{3},-i\,dx^3\right)\right)\cos(c)-\left((i\sqrt{3}-1)\Gamma\left(-\frac{4}{3},i\,dx^3\right)+(-i\sqrt{3}-\right.\right.}{6\,x} - \frac{\left(2\cdot 2^{\frac{1}{3}}(dx^3)^{\frac{1}{3}}\left(\left((-i\sqrt{3}+1)\Gamma\left(-\frac{4}{3},2i\,dx^3\right)+(i\sqrt{3}+1)\Gamma\left(-\frac{4}{3},-2i\,dx^3\right)\right)\cos(2c)-\left((\sqrt{3}+i)\Gamma\left(-\frac{4}{3},2i\right.\right.\right.}{24\,x^4} - \frac{a^2}{4\,x^4}$$

[In] integrate((a+b*sin(d*x^3+c))^2/x^5,x, algorithm="maxima")

[Out] 1/6*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(-4/3, I*d*x^3) + (sqrt(3) - I)*gamma(-4/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) - 1)*gamma(-4/3, I*d*x^3) + (-I*sqrt(3) - 1)*gamma(-4/3, -I*d*x^3))*sin(c))*a*b*d/x - 1/24*(2*2^(1/3)*(d*x^3)^(1/3)*(((-I*sqrt(3) + 1)*gamma(-4/3, 2*I*d*x^3) + (I*sqrt(3) + 1)*gamma(-4/3, -2*I*d*x^3))*cos(2*c) - ((sqrt(3) + I)*gamma(-4/3, 2*I*d*x^3) + (sqrt(3) - I)*gamma(-4/3, -2*I*d*x^3))*sin(2*c))*d*x^3 + 3)*b^2/x^4 - 1/4*a^2/x^4

Giac [F]

$$\int \frac{(a + b\sin(c + dx^3))^2}{x^5} \, dx = \int \frac{(b\sin(dx^3 + c) + a)^2}{x^5} \, dx$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^5,x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\sin(c + dx^3))^2}{x^5} \, dx = \int \frac{(a + b\,\sin(d\,x^3 + c))^2}{x^5} \, dx$$

```
[In] int((a + b*sin(c + d*x^3))^2/x^5,x)
[Out] int((a + b*sin(c + d*x^3))^2/x^5, x)
```

3.77 $\int x^3(a+b\sin(c+dx^3))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 237

$$\int x^3(a+b\sin(c+dx^3))^2\,dx=\frac{1}{8}(2a^2+b^2)\,x^4-\frac{2abx\cos(c+dx^3)}{3d}-\frac{abe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{9d\sqrt[3]{-idx^3}}-\frac{abe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{9d\sqrt[3]{idx^3}}+\frac{ib^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{idx^3}}-\frac{b^2x\sin(2c+2dx^3)}{12d}$$

```
[Out] 1/8*(2*a^2+b^2)*x^4-2/3*a*b*x*cos(d*x^3+c)/d-1/9*a*b*exp(I*c)*x*GAMMA(1/3,-
I*d*x^3)/d/(-I*d*x^3)^(1/3)-1/9*a*b*x*GAMMA(1/3,I*d*x^3)/d/exp(I*c)/(I*d*x^
3)^(1/3)+1/144*I*b^2*exp(2*I*c)*x*GAMMA(1/3,-2*I*d*x^3)*2^(2/3)/d/(-I*d*x^3
)^(1/3)-1/144*I*b^2*x*GAMMA(1/3,2*I*d*x^3)*2^(2/3)/d/exp(2*I*c)/(I*d*x^3)^(
1/3)-1/12*b^2*x*sin(2*d*x^3+2*c)/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3484, 6, 3467, 3436, 2239, 3466, 3437}

$$\int x^3(a+b\sin(c+dx^3))^2\,dx=\frac{1}{8}x^4(2a^2+b^2)-\frac{2abx\cos(c+dx^3)}{3d}-\frac{abe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{9d\sqrt[3]{-idx^3}}-\frac{abe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{9d\sqrt[3]{idx^3}}-\frac{b^2x\sin(2c+2dx^3)}{12d}+\frac{ib^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{idx^3}}$$

```
[In] Int[x^3*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^4)/8 - (2*a*b*x*Cos[c + d*x^3])/(3*d) - (a*b*E^(I*c)*x*Gam
ma[1/3, (-I)*d*x^3])/(9*d*((-I)*d*x^3)^(1/3)) - (a*b*x*Gamma[1/3, I*d*x^3])
/(9*d*E^(I*c)*(I*d*x^3)^(1/3)) + ((I/72)*b^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I
)*d*x^3])/(2^(1/3)*d*((-I)*d*x^3)^(1/3)) - ((I/72)*b^2*x*Gamma[1/3, (2*I)*d
*x^3])/(2^(1/3)*d*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (b^2*x*Sin[2*c + 2*d*x^3])
/(12*d)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2x^3+\frac{b^2x^3}{2}-\frac{1}{2}b^2x^3\cos\left(2c+2dx^3\right)+2abx^3\sin\left(c+dx^3\right)\right)dx\\
&= \int \left(\left(a^2+\frac{b^2}{2}\right)x^3-\frac{1}{2}b^2x^3\cos\left(2c+2dx^3\right)+2abx^3\sin\left(c+dx^3\right)\right)dx\\
&= \frac{1}{8}\left(2a^2+b^2\right)x^4+(2ab)\int x^3\sin\left(c+dx^3\right)dx-\frac{1}{2}b^2\int x^3\cos\left(2c+2dx^3\right)dx\\
&= \frac{1}{8}\left(2a^2+b^2\right)x^4-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{b^2x\sin\left(2c+2dx^3\right)}{12d}\\
&\quad+\frac{(2ab)\int\cos\left(c+dx^3\right)dx}{3d}+\frac{b^2\int\sin\left(2c+2dx^3\right)dx}{12d}\\
&= \frac{1}{8}\left(2a^2+b^2\right)x^4-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{b^2x\sin\left(2c+2dx^3\right)}{12d}+\frac{(ab)\int e^{-ic-idx^3}dx}{3d}\\
&\quad+\frac{(ab)\int e^{ic+idx^3}dx}{3d}+\frac{(ib^2)\int e^{-2ic-2idx^3}dx}{24d}-\frac{(ib^2)\int e^{2ic+2idx^3}dx}{24d}\\
&= \frac{1}{8}\left(2a^2+b^2\right)x^4-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{abe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{9d\sqrt[3]{-idx^3}}-\frac{abe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{9d\sqrt[3]{idx^3}}\\
&\quad+\frac{ib^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{idx^3}}-\frac{b^2x\sin\left(2c+2dx^3\right)}{12d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.43

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)^2dx$$
$$=\frac{dx^7\left(36a^2dx^3\sqrt[3]{d^2x^6}+18b^2dx^3\sqrt[3]{d^2x^6}-96ab\sqrt[3]{d^2x^6}\cos\left(c+dx^3\right)+i2^{2/3}b^2\sqrt[3]{idx^3}\cos(2c)\Gamma\left(\frac{1}{3},-2idx^3\right)-i2^{2}\right.}{}$$

```
[In] Integrate[x^3*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (d*x^7*(36*a^2*d*x^3*(d^2*x^6)^(1/3) + 18*b^2*d*x^3*(d^2*x^6)^(1/3) - 96*a*
b*(d^2*x^6)^(1/3)*Cos[c + d*x^3] + I*2^(2/3)*b^2*(I*d*x^3)^(1/3)*Cos[2*c]*G
amma[1/3, (-2*I)*d*x^3] - I*2^(2/3)*b^2*((-I)*d*x^3)^(1/3)*Cos[2*c]*Gamma[1
/3, (2*I)*d*x^3] - 16*a*b*((-I)*d*x^3)^(1/3)*Gamma[1/3, I*d*x^3]*(Cos[c] -
```

```
I*Sin[c]) - 16*a*b*(I*d*x^3)^(1/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c
]) - 2^(2/3)*b^2*(I*d*x^3)^(1/3)*Gamma[1/3, (-2*I)*d*x^3]*Sin[2*c] - 2^(2/3
)*b^2*((-I)*d*x^3)^(1/3)*Gamma[1/3, (2*I)*d*x^3]*Sin[2*c] - 12*b^2*(d^2*x^6
)^(1/3)*Sin[2*(c + d*x^3)]))/(144*(d^2*x^6)^(4/3))
```

Maple [F]

$$\int x^{3}\left(a+b\sin\left(d\,x^{3}+c\right)\right)^{2}dx$$

```
[In] int(x^3*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x^3*(a+b*sin(d*x^3+c))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.77

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$=\frac{18\,(2\,a^2+b^2)d^2x^4-24\,b^2dx\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)-96\,abdx\cos\left(dx^3+c\right)-(b^2\cos\left(2\,c\right)-i\,b^2\sin\left(2\,c\right.}{}$$

```
[In] integrate(x^3*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/144*(18*(2*a^2 + b^2)*d^2*x^4 - 24*b^2*d*x*cos(d*x^3 + c)*sin(d*x^3 + c)
- 96*a*b*d*x*cos(d*x^3 + c) - (b^2*cos(2*c) - I*b^2*sin(2*c))*(2*I*d)^(2/3)
*gamma(1/3, 2*I*d*x^3) - 16*(-I*a*b*cos(c) - a*b*sin(c))*(I*d)^(2/3)*gamma(
1/3, I*d*x^3) - 16*(I*a*b*cos(c) - a*b*sin(c))*(-I*d)^(2/3)*gamma(1/3, -I*d
*x^3) - (b^2*cos(2*c) + I*b^2*sin(2*c))*(-2*I*d)^(2/3)*gamma(1/3, -2*I*d*x^
3))/d^2
```

Sympy [F]

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\int x^3\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

```
[In] integrate(x**3*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(x**3*(a + b*sin(c + d*x**3))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.01

$$\int x^3(a+b\sin(c+dx^3))^2\,dx = \frac{1}{4}a^2x^4 - \frac{2^{\frac{2}{3}}\left(\left(\left((i\sqrt{3}+1)\Gamma\left(\frac{1}{3},2i\,dx^3\right)+(-i\sqrt{3}+1)\Gamma\left(\frac{1}{3},-2i\,dx^3\right)\right)\cos(2c)+\left((\sqrt{3}-i)\Gamma\left(\frac{1}{3},2i\,dx^3\right)+(\sqrt{3}+i\right.\right.\right.}{288\,(dx^3)^{\frac{1}{3}}d} - \frac{\left(12\,(dx^3)^{\frac{1}{3}}x\cos(dx^3+c)+\left(\left((\sqrt{3}-i)\Gamma\left(\frac{1}{3},i\,dx^3\right)+(\sqrt{3}+i)\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\cos(c)+\left((-i\sqrt{3}-1)\Gamma\left(\frac{1}{3}\right.\right.\right.\right.}{18\,(dx^3)^{\frac{1}{3}}d}$$

```
[In] integrate(x^3*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 - 1/288*2^(2/3)*((((I*sqrt(3) + 1)*gamma(1/3, 2*I*d*x^3) + (-I*
sqrt(3) + 1)*gamma(1/3, -2*I*d*x^3))*cos(2*c) + ((sqrt(3) - I)*gamma(1/3, 2
*I*d*x^3) + (sqrt(3) + I)*gamma(1/3, -2*I*d*x^3))*sin(2*c))*x - 6*2^(1/3)*(
3*d*x^4 - 2*x*sin(2*d*x^3 + 2*c))*(d*x^3)^(1/3))*b^2/((d*x^3)^(1/3)*d) - 1/
18*(12*(d*x^3)^(1/3)*x*cos(d*x^3 + c) + (((sqrt(3) - I)*gamma(1/3, I*d*x^3)
 + (sqrt(3) + I)*gamma(1/3, -I*d*x^3))*cos(c) + ((-I*sqrt(3) - 1)*gamma(1/3
, I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -I*d*x^3))*sin(c))*x)*a*b/((d*x^3)^
(1/3)*d)
```

Giac [F]

$$\int x^3(a+b\sin(c+dx^3))^2\,dx = \int (b\sin(dx^3+c)+a)^2x^3\,dx$$

```
[In] integrate(x^3*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\sin(c+dx^3))^2\,dx = \int x^3\,(a+b\,\sin(d\,x^3+c))^2\,dx$$

```
[In] int(x^3*(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] int(x^3*(a + b*sin(c + d*x^3))^2, x)
```

3.78 $\int (a+b\sin(c+dx^3))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 183

$$\int (a+b\sin(c+dx^3))^2\,dx=\frac{1}{2}(2a^2+b^2)\,x+\frac{iabe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{iabe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}+\frac{b^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{-idx^3}}+\frac{b^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{idx^3}}$$

```
[Out] 1/2*(2*a^2+b^2)*x+1/3*I*a*b*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)
-1/3*I*a*b*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(1/3)+1/24*b^2*exp(2*I*c
)*x*GAMMA(1/3,-2*I*d*x^3)*2^(2/3)/(-I*d*x^3)^(1/3)+1/24*b^2*x*GAMMA(1/3,2*I
*d*x^3)*2^(2/3)/exp(2*I*c)/(I*d*x^3)^(1/3)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3438, 3437, 2239, 3436}

$$\int (a+b\sin(c+dx^3))^2\,dx=\frac{1}{2}x(2a^2+b^2)+\frac{iabe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{iabe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}+\frac{b^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{-idx^3}}+\frac{b^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{idx^3}}$$

```
[In] Int[(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 + ((I/3)*a*b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*
x^3)^(1/3) - ((I/3)*a*b*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3)) +
(b^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I)*d*x^3])/(12*2^(1/3)*((-I)*d*x^3)^(1/3)
```

```
) + (b^2*x*Gamma[1/3, (2*I)*d*x^3])/(12*2^(1/3)*E^((2*I)*c)*(I*d*x^3)^(1/3)
)
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2+\frac{b^2}{2}-\frac{1}{2}b^2\cos\left(2c+2dx^3\right)+2ab\sin\left(c+dx^3\right)\right)\,dx \\
&= \frac{1}{2}\left(2a^2+b^2\right)x+(2ab)\int \sin\left(c+dx^3\right)\,dx-\frac{1}{2}b^2\int \cos\left(2c+2dx^3\right)\,dx \\
&= \frac{1}{2}\left(2a^2+b^2\right)x+(iab)\int e^{-ic-idx^3}\,dx-(iab)\int e^{ic+idx^3}\,dx \\
&\quad -\frac{1}{4}b^2\int e^{-2ic-2idx^3}\,dx-\frac{1}{4}b^2\int e^{2ic+2idx^3}\,dx \\
&= \frac{1}{2}\left(2a^2+b^2\right)x+\frac{iabe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{iabe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}} \\
&\quad +\frac{b^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{-idx^3}}+\frac{b^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{idx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.25

$$\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{1}{24}x\left(12\left(2a^2+b^2\right)-8iab\cos(c)\left(-\frac{\Gamma\left(\frac{1}{3},-idx^3\right)}{\sqrt[3]{-idx^3}}+\frac{\Gamma\left(\frac{1}{3},idx^3\right)}{\sqrt[3]{idx^3}}\right)+8ab\left(-\frac{\Gamma\left(\frac{1}{3},-idx^3\right)}{\sqrt[3]{-idx^3}}-\frac{\Gamma\left(\frac{1}{3},idx^3\right)}{\sqrt[3]{idx^3}}\right)\sin(c)+\frac{2^{2/3}b^2\Gamma\left(\frac{1}{3},2idx^3\right)\left(\cos(2c)-i\sin(2c)\right)}{\sqrt[3]{idx^3}}+\frac{2^{2/3}b^2\Gamma\left(\frac{1}{3},-2idx^3\right)\left(\cos(2c)+i\sin(2c)\right)}{\sqrt[3]{-idx^3}}\right)$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (x*(12*(2*a^2 + b^2) - (8*I)*a*b*Cos[c]*(-(Gamma[1/3, (-I)*d*x^3]/((-I)*d*x
^3)^(1/3)) + Gamma[1/3, I*d*x^3]/(I*d*x^3)^(1/3)) + 8*a*b*(-(Gamma[1/3, (-I
)*d*x^3]/((-I)*d*x^3)^(1/3)) - Gamma[1/3, I*d*x^3]/(I*d*x^3)^(1/3))*Sin[c]
+ (2^(2/3)*b^2*Gamma[1/3, (2*I)*d*x^3]*(Cos[2*c] - I*Sin[2*c]))/(I*d*x^3)^(
1/3) + (2^(2/3)*b^2*Gamma[1/3, (-2*I)*d*x^3]*(Cos[2*c] + I*Sin[2*c]))/((-I)
*d*x^3)^(1/3)))/24
```

Maple [F]

$$\int \left(a+b\sin\left(d\,x^3+c\right)\right)^2dx$$

```
[In] int((a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

$$\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$=\frac{12\left(2\,a^2+b^2\right)dx+\left(-i\,b^2\cos\left(2\,c\right)-b^2\sin\left(2\,c\right)\right)(2i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},2i\,dx^3\right)-8\left(ab\cos\left(c\right)-i\,ab\sin\left(c\right)\right)(i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},\right.}{2}$$

```
[In] integrate((a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*(2*a^2 + b^2)*d*x + (-I*b^2*cos(2*c) - b^2*sin(2*c))*(2*I*d)^(2/3)
*gamma(1/3, 2*I*d*x^3) - 8*(a*b*cos(c) - I*a*b*sin(c))*(I*d)^(2/3)*gamma(1/
3, I*d*x^3) - 8*(a*b*cos(c) + I*a*b*sin(c))*(-I*d)^(2/3)*gamma(1/3, -I*d*x^
3) + (I*b^2*cos(2*c) - b^2*sin(2*c))*(-2*I*d)^(2/3)*gamma(1/3, -2*I*d*x^3))
/d
```

Sympy [F]

$$\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

```
[In] integrate((a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05

$$\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$=\frac{\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\cos\left(c\right)-\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-i\,dx\right.\right.\right.}{6\,\left(dx^3\right)^{\frac{1}{3}}}$$
$$+\frac{2^{\frac{2}{3}}\left(\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},2i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-2i\,dx^3\right)\right)\cos\left(2\,c\right)+\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},2i\,dx^3\right)+\left(i\sqrt{3}-1\right.\right.\right.\right.}{48\,\left(dx^3\right)^{\frac{1}{3}}}$$
$$+a^2x$$

```
[In] integrate((a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(((-I*sqrt(3) - 1)*gamma(1/3, I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -I*
d*x^3))*cos(c) - ((sqrt(3) - I)*gamma(1/3, I*d*x^3) + (sqrt(3) + I)*gamma(1
/3, -I*d*x^3))*sin(c))*a*b*x/(d*x^3)^(1/3) + 1/48*2^(2/3)*((((sqrt(3) - I)*
gamma(1/3, 2*I*d*x^3) + (sqrt(3) + I)*gamma(1/3, -2*I*d*x^3))*cos(2*c) + ((
-I*sqrt(3) - 1)*gamma(1/3, 2*I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -2*I*d*x
^3))*sin(2*c))*x + 12*2^(1/3)*(d*x^3)^(1/3)*x)*b^2/(d*x^3)^(1/3) + a^2*x
```

Giac [F]

$$\int \left(a+b\sin \left(c+dx^3\right)\right)^2\,dx=\int \left(b\sin \left(dx^3+c\right)+a\right)^2\,dx$$

```
[In] integrate((a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin \left(c+dx^3\right)\right)^2\,dx=\int \left(a+b\,\sin \left(d\,x^3+c\right)\right)^2\,dx$$

```
[In] int((a + b*sin(c + d*x^3))^2,x)
[Out] int((a + b*sin(c + d*x^3))^2, x)
```

3.79 $\int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 227

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx = \frac{-2a^2-b^2}{4x^2}+\frac{b^2\cos(2c+2dx^3)}{4x^2}-\frac{abde^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{2\sqrt[3]{-idx^3}}-\frac{abde^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{2\sqrt[3]{idx^3}}+\frac{ib^2de^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{ib^2de^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{idx^3}}-\frac{ab\sin(c+dx^3)}{x^2}$$

```
[Out] 1/4*(-2*a^2-b^2)/x^2+1/4*b^2*cos(2*d*x^3+2*c)/x^2-1/2*a*b*d*exp(I*c)*x*GAMM
A(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)-1/2*a*b*d*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I
*d*x^3)^(1/3)+1/8*I*b^2*d*exp(2*I*c)*x*GAMMA(1/3,-2*I*d*x^3)*2^(2/3)/(-I*d*
x^3)^(1/3)-1/8*I*b^2*d*x*GAMMA(1/3,2*I*d*x^3)*2^(2/3)/exp(2*I*c)/(I*d*x^3)^
(1/3)-a*b*sin(d*x^3+c)/x^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3484, 6, 3469, 3436, 2239, 3468, 3437}

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx = -\frac{2a^2+b^2}{4x^2}-\frac{abe^{ic}dx\Gamma\left(\frac{1}{3},-idx^3\right)}{2\sqrt[3]{-idx^3}}-\frac{abe^{-ic}dx\Gamma\left(\frac{1}{3},idx^3\right)}{2\sqrt[3]{idx^3}}-\frac{ab\sin(c+dx^3)}{x^2}+\frac{ib^2e^{2ic}dx\Gamma\left(\frac{1}{3},-2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}dx\Gamma\left(\frac{1}{3},2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{idx^3}}+\frac{b^2\cos(2c+2dx^3)}{4x^2}$$

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^3,x]
```

```
[Out] -1/4*(2*a^2 + b^2)/x^2 + (b^2*Cos[2*c + 2*d*x^3])/(4*x^2) - (a*b*d*E^(I*c)*
x*Gamma[1/3, (-I)*d*x^3])/(2*((-I)*d*x^3)^(1/3)) - (a*b*d*x*Gamma[1/3, I*d*
x^3])/(2*E^(I*c)*(I*d*x^3)^(1/3)) + ((I/4)*b^2*d*E^((2*I)*c)*x*Gamma[1/3, (
-2*I)*d*x^3])/(2^(1/3)*((-I)*d*x^3)^(1/3)) - ((I/4)*b^2*d*x*Gamma[1/3, (2*I
)*d*x^3])/(2^(1/3)*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (a*b*Sin[c + d*x^3])/x^2
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{x^3}+\frac{b^2}{2x^3}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^3}+\frac{2ab\sin\left(c+dx^3\right)}{x^3}\right)dx\\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^3}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^3}+\frac{2ab\sin\left(c+dx^3\right)}{x^3}\right)dx\\
&= -\frac{2a^2+b^2}{4x^2}+(2ab)\int \frac{\sin\left(c+dx^3\right)}{x^3}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^3\right)}{x^3}\,dx\\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{4x^2}-\frac{ab\sin\left(c+dx^3\right)}{x^2}\\
&\quad+(3abd)\int \cos\left(c+dx^3\right)\,dx+\frac{1}{2}\left(3b^2d\right)\int \sin\left(2c+2dx^3\right)\,dx\\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{4x^2}-\frac{ab\sin\left(c+dx^3\right)}{x^2}+\frac{1}{2}(3abd)\int e^{-ic-idx^3}\,dx\\
&\quad+\frac{1}{2}(3abd)\int e^{ic+idx^3}\,dx+\frac{1}{4}\left(3ib^2d\right)\int e^{-2ic-2idx^3}\,dx-\frac{1}{4}\left(3ib^2d\right)\int e^{2ic+2idx^3}\,dx\\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{4x^2}-\frac{abde^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{2\sqrt[3]{-idx^3}}-\frac{abde^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{2\sqrt[3]{idx^3}}\\
&\quad+\frac{ib^2de^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{ib^2de^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{idx^3}}-\frac{ab\sin\left(c+dx^3\right)}{x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.47

$$\begin{aligned}
\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^3}\,dx &= \frac{-2a^2-b^2}{4x^2}+\frac{b^2\cos(2c)\cos\left(2dx^3\right)}{4x^2}\\
&\quad+\frac{3}{2}abd\cos(c)\left(-\frac{x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}\right)\\
&\quad-\frac{ab\cos\left(dx^3\right)\sin(c)}{x^2}\\
&\quad+\frac{3}{2}iabd\left(-\frac{x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}+\frac{x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}\right)\sin(c)\\
&\quad-\frac{b^2\left(idx^3\right)^{2/3}\Gamma\left(\frac{1}{3},2idx^3\right)\left(\cos(2c)-i\sin(2c)\right)}{4\sqrt[3]{2}x^2}\\
&\quad+\frac{ib^2dx\Gamma\left(\frac{1}{3},-2idx^3\right)\left(\cos(2c)+i\sin(2c)\right)}{4\sqrt[3]{2}\sqrt[3]{-idx^3}}\\
&\quad-\frac{ab\cos(c)\sin\left(dx^3\right)}{x^2}-\frac{b^2\sin(2c)\sin\left(2dx^3\right)}{4x^2}
\end{aligned}$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^3,x]
```

```
[Out] (-2*a^2 - b^2)/(4*x^2) + (b^2*Cos[2*c]*Cos[2*d*x^3])/(4*x^2) + (3*a*b*d*Cos
[c]*(-1/3*(x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) - (x*Gamma[1/3, I*d
*x^3])/(3*(I*d*x^3)^(1/3))))/2 - (a*b*Cos[d*x^3]*Sin[c])/x^2 + ((3*I)/2)*a*
b*d*(-1/3*(x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) + (x*Gamma[1/3, I*d
*x^3])/(3*(I*d*x^3)^(1/3)))*Sin[c] - (b^2*(I*d*x^3)^(2/3)*Gamma[1/3, (2*I)*
d*x^3]*(Cos[2*c] - I*Sin[2*c]))/(4*2^(1/3)*x^2) + ((I/4)*b^2*d*x*Gamma[1/3,
 (-2*I)*d*x^3]*(Cos[2*c] + I*Sin[2*c]))/(2^(1/3)*((-I)*d*x^3)^(1/3)) - (a*b
*Cos[c]*Sin[d*x^3])/x^2 - (b^2*Sin[2*c]*Sin[2*d*x^3])/(4*x^2)
```

Maple [F]

$$\int \frac{(a+b\sin(d\,x^3+c))^2}{x^3}dx$$

```
[In] int((a+b*sin(d*x^3+c))^2/x^3,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx = \frac{4\,b^2\cos(dx^3+c)^2 - 8\,ab\sin(dx^3+c) - (b^2x^2\cos(2\,c) - i\,b^2x^2\sin(2\,c))(2i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3}, 2i\,dx^3\right) - 4\,(-i\,abx^2\,c$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^2*cos(d*x^3 + c)^2 - 8*a*b*sin(d*x^3 + c) - (b^2*x^2*cos(2*c) - I*
b^2*x^2*sin(2*c))*(2*I*d)^(2/3)*gamma(1/3, 2*I*d*x^3) - 4*(-I*a*b*x^2*cos(c
) - a*b*x^2*sin(c))*(I*d)^(2/3)*gamma(1/3, I*d*x^3) - 4*(I*a*b*x^2*cos(c) -
 a*b*x^2*sin(c))*(-I*d)^(2/3)*gamma(1/3, -I*d*x^3) - (b^2*x^2*cos(2*c) + I*
b^2*x^2*sin(2*c))*(-2*I*d)^(2/3)*gamma(1/3, -2*I*d*x^3) - 4*a^2 - 4*b^2)/x^
2
```

Sympy [F]

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^3}\,dx$$

[In] integrate((a+b*sin(d*x**3+c))**2/x**3,x)

[Out] Integral((a + b*sin(c + d*x**3))**2/x**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^3}\,dx$$
$$=\frac{\left(dx^3\right)^{\frac{2}{3}}\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{2}{3},i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(-\frac{2}{3},-i\,dx^3\right)\right)\cos\left(c\right)-\left(\left(i\sqrt{3}+1\right)\Gamma\left(-\frac{2}{3},i\,dx^3\right)+\left(-i\sqrt{3}+\right.\right.\right.}{6\,x^2}$$
$$-\frac{\left(2^{\frac{2}{3}}\left(dx^3\right)^{\frac{2}{3}}\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{2}{3},2i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{2}{3},-2i\,dx^3\right)\right)\cos\left(2\,c\right)-\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{2}{3},2i\,dx^3\right.\right.\right.\right.}{24\,x^2}$$
$$-\frac{a^2}{2\,x^2}$$

[In] integrate((a+b*sin(d*x^3+c))^2/x^3,x, algorithm="maxima")

[Out] 1/6*(d*x^3)^(2/3)*(((sqrt(3) - I)*gamma(-2/3, I*d*x^3) + (sqrt(3) + I)*gamm
a(-2/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) + 1)*gamma(-2/3, I*d*x^3) + (-I*sqr
t(3) + 1)*gamma(-2/3, -I*d*x^3))*sin(c))*a*b/x^2 - 1/24*(2^(2/3)*(d*x^3)^(2
/3)*(((-I*sqrt(3) - 1)*gamma(-2/3, 2*I*d*x^3) + (I*sqrt(3) - 1)*gamma(-2/3,
 -2*I*d*x^3))*cos(2*c) - ((sqrt(3) - I)*gamma(-2/3, 2*I*d*x^3) + (sqrt(3) +
 I)*gamma(-2/3, -2*I*d*x^3))*sin(2*c)) + 6)*b^2/x^2 - 1/2*a^2/x^2

Giac [F]

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\sin\left(dx^3+c\right)+a\right)^2}{x^3}\,dx$$

[In] integrate((a+b*sin(d*x^3+c))^2/x^3,x, algorithm="giac")

[Out] integrate((b*sin(d*x^3 + c) + a)^2/x^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx = \int \frac{\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}{x^3}\,dx$$

```
[In] int((a + b*sin(c + d*x^3))^2/x^3,x)
[Out] int((a + b*sin(c + d*x^3))^2/x^3, x)
```

3.80 $\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 277

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx = \frac{-2a^2-b^2}{10x^5} - \frac{3abd\cos(c+dx^3)}{5x^2} + \frac{b^2\cos(2c+2dx^3)}{10x^5} - \frac{3iabd^2e^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{10\sqrt[3]{-idx^3}} + \frac{3iabd^2e^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{10\sqrt[3]{idx^3}} - \frac{3b^2d^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{-idx^3}} - \frac{3b^2d^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{idx^3}} - \frac{2ab\sin(c+dx^3)}{5x^5} - \frac{3b^2d\sin(2c+2dx^3)}{10x^2}$$

```
[Out] 1/10*(-2*a^2-b^2)/x^5-3/5*a*b*d*cos(d*x^3+c)/x^2+1/10*b^2*cos(2*d*x^3+2*c)/
x^5-3/10*I*a*b*d^2*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)+3/10*I*a
*b*d^2*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(1/3)-3/20*b^2*d^2*exp(2*I*c
)*x*GAMMA(1/3,-2*I*d*x^3)*2^(2/3)/(-I*d*x^3)^(1/3)-3/20*b^2*d^2*x*GAMMA(1/3
,2*I*d*x^3)*2^(2/3)/exp(2*I*c)/(I*d*x^3)^(1/3)-2/5*a*b*sin(d*x^3+c)/x^5-3/1
0*b^2*d*sin(2*d*x^3+2*c)/x^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used

= {3484, 6, 3469, 3468, 3437, 2239, 3436}

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^6}\,dx = -\frac{2a^2+b^2}{10x^5} - \frac{3iabe^{ic}d^2x\Gamma\left(\frac{1}{3},-idx^3\right)}{10\sqrt[3]{-idx^3}} + \frac{3iabe^{-ic}d^2x\Gamma\left(\frac{1}{3},idx^3\right)}{10\sqrt[3]{idx^3}} - \frac{2ab\sin\left(c+dx^3\right)}{5x^5} - \frac{3abd\cos\left(c+dx^3\right)}{5x^2} - \frac{3b^2e^{2ic}d^2x\Gamma\left(\frac{1}{3},-2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{-idx^3}} - \frac{3b^2e^{-2ic}d^2x\Gamma\left(\frac{1}{3},2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{idx^3}} + \frac{b^2\cos\left(2c+2dx^3\right)}{10x^5} - \frac{3b^2d\sin\left(2c+2dx^3\right)}{10x^2}$$

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^6,x]
```

```
[Out] -1/10*(2*a^2 + b^2)/x^5 - (3*a*b*d*Cos[c + d*x^3])/(5*x^2) + (b^2*Cos[2*c +
 2*d*x^3])/(10*x^5) - (((3*I)/10)*a*b*d^2*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])
/((-I)*d*x^3)^(1/3) + (((3*I)/10)*a*b*d^2*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(
I*d*x^3)^(1/3)) - (3*b^2*d^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I)*d*x^3])/(10*2^
(1/3)*((-I)*d*x^3)^(1/3)) - (3*b^2*d^2*x*Gamma[1/3, (2*I)*d*x^3])/(10*2^(1/
3)*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (2*a*b*Sin[c + d*x^3])/(5*x^5) - (3*b^2*d
*Sin[2*c + 2*d*x^3])/(10*x^2)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{x^6}+\frac{b^2}{2x^6}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^6}+\frac{2ab\sin\left(c+dx^3\right)}{x^6}\right)\,dx\\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^6}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^6}+\frac{2ab\sin\left(c+dx^3\right)}{x^6}\right)\,dx\\
&= -\frac{2a^2+b^2}{10x^5}+(2ab)\int \frac{\sin\left(c+dx^3\right)}{x^6}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^3\right)}{x^6}\,dx\\
&= -\frac{2a^2+b^2}{10x^5}+\frac{b^2\cos\left(2c+2dx^3\right)}{10x^5}-\frac{2ab\sin\left(c+dx^3\right)}{5x^5}\\
&\quad+\frac{1}{5}(6abd)\int \frac{\cos\left(c+dx^3\right)}{x^3}\,dx+\frac{1}{5}\left(3b^2d\right)\int \frac{\sin\left(2c+2dx^3\right)}{x^3}\,dx\\
&= -\frac{2a^2+b^2}{10x^5}-\frac{3abd\cos\left(c+dx^3\right)}{5x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{10x^5}-\frac{2ab\sin\left(c+dx^3\right)}{5x^5}\\
&\quad-\frac{3b^2d\sin\left(2c+2dx^3\right)}{10x^2}-\frac{1}{5}\left(9abd^2\right)\int \sin\left(c+dx^3\right)\,dx+\frac{1}{5}\left(9b^2d^2\right)\int \cos\left(2c+2dx^3\right)\,dx\\
&= -\frac{2a^2+b^2}{10x^5}-\frac{3abd\cos\left(c+dx^3\right)}{5x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{10x^5}-\frac{2ab\sin\left(c+dx^3\right)}{5x^5}\\
&\quad-\frac{3b^2d\sin\left(2c+2dx^3\right)}{10x^2}-\frac{1}{10}\left(9iabd^2\right)\int e^{-ic-idx^3}\,dx+\frac{1}{10}\left(9iabd^2\right)\int e^{ic+idx^3}\,dx\\
&\quad+\frac{1}{10}\left(9b^2d^2\right)\int e^{-2ic-2idx^3}\,dx+\frac{1}{10}\left(9b^2d^2\right)\int e^{2ic+2idx^3}\,dx
\end{aligned}$$

$$= -\frac{2a^2+b^2}{10x^5} - \frac{3abd\cos(c+dx^3)}{5x^2} + \frac{b^2\cos(2c+2dx^3)}{10x^5} - \frac{3iabd^2e^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{10\sqrt[3]{-idx^3}} + \frac{3iabd^2e^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{10\sqrt[3]{idx^3}} - \frac{3b^2d^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{-idx^3}} - \frac{3b^2d^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{idx^3}} - \frac{2ab\sin(c+dx^3)}{5x^5} - \frac{3b^2d\sin(2c+2dx^3)}{10x^2}$$

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx = -\frac{4a^2+2b^2+12abdx^3\cos(c+dx^3)-2b^2\cos(2(c+dx^3))-3\,2^{2/3}b^2(idx^3)^{5/3}\cos(2c)\Gamma\left(\frac{1}{3},2idx^3\right)+6iab(}{}$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^6,x]
```

```
[Out] -1/20*(4*a^2 + 2*b^2 + 12*a*b*d*x^3*Cos[c + d*x^3] - 2*b^2*Cos[2*(c + d*x^3
)] - 3*2^(2/3)*b^2*(I*d*x^3)^(5/3)*Cos[2*c]*Gamma[1/3, (2*I)*d*x^3] + (6*I)
*a*b*(I*d*x^3)^(5/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (6*I)*a*b*(I
*d*x^3)^(1/3)*(d^2*x^6)^(2/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) -
3*2^(2/3)*b^2*((-I)*d*x^3)^(5/3)*Gamma[1/3, (-2*I)*d*x^3]*(Cos[2*c] + I*Sin
[2*c]) + (3*I)*2^(2/3)*b^2*(I*d*x^3)^(5/3)*Gamma[1/3, (2*I)*d*x^3]*Sin[2*c]
 + 8*a*b*Sin[c + d*x^3] + 6*b^2*d*x^3*Sin[2*(c + d*x^3)])/x^5
```

Maple [F]

$$\int \frac{(a+b\sin(d\,x^3+c))^2}{x^6}dx$$

```
[In] int((a+b*sin(d*x^3+c))^2/x^6,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx = -\frac{12\,abdx^3\cos(dx^3+c)-4\,b^2\cos(dx^3+c)^2+3\,(-i\,b^2dx^5\cos(2\,c)-b^2dx^5\sin(2\,c))(2i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},2i\,dx^3\right)}{}$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(12*a*b*d*x^3*cos(d*x^3 + c) - 4*b^2*cos(d*x^3 + c)^2 + 3*(-I*b^2*d*x
^5*cos(2*c) - b^2*d*x^5*sin(2*c))*(2*I*d)^(2/3)*gamma(1/3, 2*I*d*x^3) - 6*(
a*b*d*x^5*cos(c) - I*a*b*d*x^5*sin(c))*(I*d)^(2/3)*gamma(1/3, I*d*x^3) - 6*
(a*b*d*x^5*cos(c) + I*a*b*d*x^5*sin(c))*(-I*d)^(2/3)*gamma(1/3, -I*d*x^3) +
 3*(I*b^2*d*x^5*cos(2*c) - b^2*d*x^5*sin(2*c))*(-2*I*d)^(2/3)*gamma(1/3, -2
*I*d*x^3) + 4*a^2 + 4*b^2 + 4*(3*b^2*d*x^3*cos(d*x^3 + c) + 2*a*b)*sin(d*x^
3 + c))/x^5
```

Sympy [F]

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^6}\,dx=\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^6}\,dx$$

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**6,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.70

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^6}\,dx=$$
$$-\frac{\left(dx^3\right)^{\frac{2}{3}}\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{5}{3},i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{5}{3},-i\,dx^3\right)\right)\cos\left(c\right)-\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{5}{3},i\,dx^3\right)+\left(\sqrt{3}\right.\right.\right.}{6\,x^2}$$
$$-\frac{\left(5\cdot 2^{\frac{2}{3}}\left(dx^3\right)^{\frac{2}{3}}\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{5}{3},2i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(-\frac{5}{3},-2i\,dx^3\right)\right)\cos\left(2\,c\right)+\left(\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{5}{3},2i\,d\right.\right.\right.\right.}{60\,x^5}$$
$$-\frac{a^2}{5\,x^5}$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/6*(d*x^3)^(2/3)*(((-I*sqrt(3) - 1)*gamma(-5/3, I*d*x^3) + (I*sqrt(3) - 1
)*gamma(-5/3, -I*d*x^3))*cos(c) - ((sqrt(3) - I)*gamma(-5/3, I*d*x^3) + (sq
rt(3) + I)*gamma(-5/3, -I*d*x^3))*sin(c))*a*b*d/x^2 - 1/60*(5*2^(2/3)*(d*x^
3)^(2/3)*(((sqrt(3) - I)*gamma(-5/3, 2*I*d*x^3) + (sqrt(3) + I)*gamma(-5/3,
 -2*I*d*x^3))*cos(2*c) + ((-I*sqrt(3) - 1)*gamma(-5/3, 2*I*d*x^3) + (I*sqrt
(3) - 1)*gamma(-5/3, -2*I*d*x^3))*sin(2*c))*d*x^3 + 6)*b^2/x^5 - 1/5*a^2/x^
5
```

Giac [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx = \int \frac{(b\sin(dx^3+c)+a)^2}{x^6}\,dx$$

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^6,x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx = \int \frac{(a+b\,\sin(d\,x^3+c))^2}{x^6}\,dx$$

```
[In] int((a + b*sin(c + d*x^3))^2/x^6,x)
[Out] int((a + b*sin(c + d*x^3))^2/x^6, x)
```

3.81 $\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 245

$$\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx = -\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}+\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d} - \frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d^2}+\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d^2}$$

```
[Out] -1/3*I*x^3*ln(1-I*b*exp(I*(d*x^3+c))/(a-(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)
+1/3*I*x^3*ln(1-I*b*exp(I*(d*x^3+c))/(a+(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)
-1/3*polylog(2,I*b*exp(I*(d*x^3+c))/(a-(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2
)+1/3*polylog(2,I*b*exp(I*(d*x^3+c))/(a+(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3460, 3404, 2296, 2221, 2317, 2438}

$$\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx = -\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i(dx^3+c)}}{a-\sqrt{a^2-b^2}}\right)}{3d^2\sqrt{a^2-b^2}}+\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i(dx^3+c)}}{a+\sqrt{a^2-b^2}}\right)}{3d^2\sqrt{a^2-b^2}} - \frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}+\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{\sqrt{a^2-b^2}+a}\right)}{3d\sqrt{a^2-b^2}}$$

```
[In] Int[x^5/(a + b*Sin[c + d*x^3]),x]
```

```
[Out] ((-1/3*I)*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/(Sqrt
[a^2 - b^2]*d) + ((I/3)*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 -
 b^2])])/(Sqrt[a^2 - b^2]*d) - PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a - Sqrt
[a^2 - b^2])]/(3*Sqrt[a^2 - b^2]*d^2) + PolyLog[2, (I*b*E^(I*(c + d*x^3)))/
(a + Sqrt[a^2 - b^2])]/(3*Sqrt[a^2 - b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{a+b\sin(c+dx)}\,dx, x, x^3\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^3\right)\\
&= -\frac{(2ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^3\right)}{3\sqrt{a^2-b^2}}+\frac{(2ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^3\right)}{3\sqrt{a^2-b^2}}\\
&= -\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}+\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}\\
&\quad+\frac{i\text{Subst}\left(\int \log\left(1-\frac{2ibe^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)\,dx, x, x^3\right)}{3\sqrt{a^2-b^2}d}\\
&\quad-\frac{i\text{Subst}\left(\int \log\left(1-\frac{2ibe^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right)\,dx, x, x^3\right)}{3\sqrt{a^2-b^2}d}\\
&= -\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}+\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}\\
&\quad+\frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{2ibx}{2a-2\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+dx^3)}\right)}{3\sqrt{a^2-b^2}d^2}\\
&\quad-\frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{2ibx}{2a+2\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+dx^3)}\right)}{3\sqrt{a^2-b^2}d^2}\\
&= -\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}+\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}\\
&\quad-\frac{\text{PolyLog}\left(2, \frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d^2}+\frac{\text{PolyLog}\left(2, \frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx = \frac{-idx^3\left(\log\left(1+\frac{ibe^{i(c+dx^3)}}{-a+\sqrt{a^2-b^2}}\right)-\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)\right)-\operatorname{PolyLog}\left(2,-\frac{ibe^{i(c+dx^3)}}{-a+\sqrt{a^2-b^2}}\right)+\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d^2}$$

```
[In] Integrate[x^5/(a + b*Sin[c + d*x^3]),x]
```

```
[Out] ((-I)*d*x^3*(Log[1 + (I*b*E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])] - Log[
1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 - b^2])]) - PolyLog[2, ((-I)*b*E^
(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])] + PolyLog[2, (I*b*E^(I*(c + d*x^3)
))/(a + Sqrt[a^2 - b^2])])/(3*Sqrt[a^2 - b^2]*d^2)
```

Maple [F]

$$\int \frac{x^5}{a+b\sin(d\,x^3+c)}dx$$

```
[In] int(x^5/(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(x^5/(a+b*sin(d*x^3+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(199) = 398.

Time = 0.42 (sec) , antiderivative size = 1041, normalized size of antiderivative = 4.25

$$\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(b*c*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^3 + c) + 2*I*b*sin(d*x^3 +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + b*c*sqrt(-(a^2 - b^2)/b^2)*log(
2*b*cos(d*x^3 + c) - 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*
I*a) - b*c*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x^3 + c) + 2*I*b*sin(d*x^3
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - b*c*sqrt(-(a^2 - b^2)/b^2)*lo
g(-2*b*cos(d*x^3 + c) - 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a) - I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^3 + c) - a*sin(d*x^
3 + c) + (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b
```

```
)/b + 1) + I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^3 + c) - a*sin(d*x
^3 + c) - (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) + I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^3 + c) - a*sin(d
*x^3 + c) + (b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) - I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^3 + c) - a*sin
(d*x^3 + c) - (b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b + 1) + (b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^3
+ c) - a*sin(d*x^3 + c) + (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^
2 - b^2)/b^2) - b)/b) - (b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*co
s(d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(
-(-I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) + (b*cos(d*x^3 + c) - I*b*sin(d*x^
3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/
b^2)*log(-(-I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) - I*b
*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/((a^2 - b^2)*d^2)
```

Sympy [F]

$$\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx = \int \frac{x^5}{a+b\sin(c+dx^3)}\,dx$$

[In] integrate(x**5/(a+b*sin(d*x**3+c)),x)

[Out] Integral(x**5/(a + b*sin(c + d*x**3)), x)

Maxima [F]

$$\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx = \int \frac{x^5}{b\sin(dx^3+c)+a}\,dx$$

[In] integrate(x^5/(a+b*sin(d*x^3+c)),x, algorithm="maxima")

[Out] integrate(x^5/(b*sin(d*x^3 + c) + a), x)

Giac [F]

$$\int \frac{x^5}{a + b\sin(c + dx^3)}\,dx = \int \frac{x^5}{b\sin(dx^3 + c) + a}\,dx$$

```
[In] integrate(x^5/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(x^5/(b*sin(d*x^3 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{a + b\sin(c + dx^3)}\,dx = \int \frac{x^5}{a + b\,\sin(d\,x^3 + c)}\,dx$$

```
[In] int(x^5/(a + b*sin(c + d*x^3)),x)
[Out] int(x^5/(a + b*sin(c + d*x^3)), x)
```

3.82 $\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx = \frac{2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^3)\right)}{\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}$$

```
[Out] 2/3*arctan((b+a*tan(1/2*d*x^3+1/2*c))/(a^2-b^2)^(1/2))/d/(a^2-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3460, 2739, 632, 210}

$$\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx = \frac{2\arctan\left(\frac{a\tan\left(\frac{1}{2}(c+dx^3)\right)+b}{\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}$$

```
[In] Int[x^2/(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/(3*Sqrt[a^2 - b^2]*d
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{a+b\sin(c+dx)}\,dx, x, x^3\right) \\
&= \frac{2\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx^3)\right)\right)}{3d} \\
&= -\frac{4\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+dx^3)\right)\right)}{3d} \\
&= \frac{2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^3)\right)}{\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx = \frac{2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^3)\right)}{\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}$$

```
[In] Integrate[x^2/(a + b*Sin[c + d*x^3]),x]
[Out] (2*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/(3*Sqrt[a^2 - b^2]*d
)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{2a\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}$	49
default	$\frac{2\arctan\left(\frac{2a\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}$	49
risch	$-\frac{\ln\left(\mathrm{e}^{i\left(d\,x^3+c\right)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{3\sqrt{-a^2+b^2}\,d}+\frac{\ln\left(\mathrm{e}^{i\left(d\,x^3+c\right)}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{b\sqrt{-a^2+b^2}}\right)}{3\sqrt{-a^2+b^2}\,d}$	138

```
[In] int(x^2/(a+b*sin(d*x^3+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x^3+1/2*c)+2*b)/(a^2-b^2)^(
1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 208, normalized size of antiderivative = 4.08

$$\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(dx^3+c)^2-2ab\sin(dx^3+c)-a^2-b^2+2\left(a\cos(dx^3+c)\sin(dx^3+c)+b\cos(dx^3+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx^3+c)^2-2ab\sin(dx^3+c)-a^2-b^2}\right)}{6(a^2-b^2)d}, -\frac{\arctan\left(-\frac{a\sin(dx^3+c)+b}{\sqrt{a^2-b^2}\cos(dx^3+c)}\right)}{3\sqrt{a^2-b^2}d}\right]$$

```
[In] integrate(x^2/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^
3 + c) - a^2 - b^2 + 2*(a*cos(d*x^3 + c)*sin(d*x^3 + c) + b*cos(d*x^3 + c))
*sqrt(-a^2 + b^2))/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2
))/((a^2 - b^2)*d), -1/3*arctan(-(a*sin(d*x^3 + c) + b)/(sqrt(a^2 - b^2)*co
s(d*x^3 + c)))/(sqrt(a^2 - b^2)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(41) = 82$.

Time = 6.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.37

$$\int \frac{x^2}{a + b\sin(c + dx^3)}\,dx = \begin{cases} \frac{\tilde{\infty}x^3}{\sin(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx^3}{2}\right)\right)}{3bd} & \text{for } a = 0 \\ \frac{x^3}{3(a + b\sin(c))} & \text{for } d = 0 \\ \frac{2}{3bd\tan\left(\frac{c}{2} + \frac{dx^3}{2}\right) - 3bd} & \text{for } a = -b \\ -\frac{2}{3bd\tan\left(\frac{c}{2} + \frac{dx^3}{2}\right) + 3bd} & \text{for } a = b \\ \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx^3}{2}\right) + \frac{b}{a} - \frac{\sqrt{-a^2 + b^2}}{a}\right)}{3d\sqrt{-a^2 + b^2}} - \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx^3}{2}\right) + \frac{b}{a} + \frac{\sqrt{-a^2 + b^2}}{a}\right)}{3d\sqrt{-a^2 + b^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2/(a+b*sin(d*x**3+c)),x)
```

```
[Out] Piecewise((zoo*x**3/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c/2 +
 d*x**3/2))/(3*b*d), Eq(a, 0)), (x**3/(3*(a + b*sin(c))), Eq(d, 0)), (2/(3*
b*d*tan(c/2 + d*x**3/2) - 3*b*d), Eq(a, -b)), (-2/(3*b*d*tan(c/2 + d*x**3/2
) + 3*b*d), Eq(a, b)), (log(tan(c/2 + d*x**3/2) + b/a - sqrt(-a**2 + b**2)/
a)/(3*d*sqrt(-a**2 + b**2)) - log(tan(c/2 + d*x**3/2) + b/a + sqrt(-a**2 +
b**2)/a)/(3*d*sqrt(-a**2 + b**2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8078 vs. $2(44) = 88$.

Time = 23.03 (sec) , antiderivative size = 8078, normalized size of antiderivative = 158.39

$$\int \frac{x^2}{a + b\sin(c + dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/3*arctan2(-2*(4*(a^2*b^4 - b^6)*cos(d*x^3 + 2*c)^4*cos(c)*sin(c) - 4*(a^2
*b^4 - b^6)*cos(c)*sin(d*x^3 + 2*c)^4*sin(c) - 4*((a^3*b^3 - a*b^5)*cos(c)^
3 + 3*(a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^3 - 4*(3*(a^3*b^3
 - a*b^5)*cos(c)^2*sin(c) + (a^3*b^3 - a*b^5)*sin(c)^3 + ((a^2*b^4 - b^6)*c
os(c)^2 - (a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^3 +
4*((4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 +
```

b^6)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c)^2 - 4*((4*a^4*b^2 - 5*a^2*b^4 + b^6
)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)*sin(c)^3 + 3*((a^3
*b^3 - a*b^5)*cos(c)^3 - (a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c
))*sin(d*x^3 + 2*c)^2 - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^5 + 2*(2*a^
5*b - 3*a^3*b^3 + a*b^5)*cos(c)^3*sin(c)^2 + (2*a^5*b - 3*a^3*b^3 + a*b^5)*
cos(c)*sin(c)^4)*cos(d*x^3 + 2*c) - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)
^4*sin(c) + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^2*sin(c)^3 + (2*a^5*b -
3*a^3*b^3 + a*b^5)*sin(c)^5 + ((a^2*b^4 - b^6)*cos(c)^2 - (a^2*b^4 - b^6)*s
in(c)^2)*cos(d*x^3 + 2*c)^3 - 3*((a^3*b^3 - a*b^5)*cos(c)^2*sin(c) - (a^3*b
^3 - a*b^5)*sin(c)^3)*cos(d*x^3 + 2*c)^2 + ((4*a^4*b^2 - 5*a^2*b^4 + b^6)*c
os(c)^4 - (4*a^4*b^2 - 5*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^3 + 2*c))*sin(d*x
^3 + 2*c) + (b^5*cos(d*x^3 + 2*c)^5*cos(c) - 4*a*b^4*cos(d*x^3 + 2*c)^4*cos
(c)*sin(c) + b^5*sin(d*x^3 + 2*c)^5*sin(c) + (b^5*cos(d*x^3 + 2*c)*cos(c) +
4*a*b^4*cos(c)*sin(c))*sin(d*x^3 + 2*c)^4 + 2*((2*a^2*b^3 - b^5)*cos(c)^3
+ 3*(2*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^3 + 2*(b^5*cos(d*x^
3 + 2*c)^2*sin(c) + 3*(2*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (2*a^2*b^3 - b^5)
sin(c)^3 + 2(a*b^4*cos(c)^2 - a*b^4*sin(c)^2)*cos(d*x^3 + 2*c))*sin(d*x^3
+ 2*c)^3 - 4*((4*a^3*b^2 - 3*a*b^4)*cos(c)^3*sin(c) + (4*a^3*b^2 - 3*a*b^4
)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c)^2 + 2*(b^5*cos(d*x^3 + 2*c)^3*cos(c) +
2*(4*a^3*b^2 - 3*a*b^4)*cos(c)^3*sin(c) + 2*(4*a^3*b^2 - 3*a*b^4)*cos(c)*si
n(c)^3 + 3*((2*a^2*b^3 - b^5)*cos(c)^3 - (2*a^2*b^3 - b^5)*cos(c)*sin(c)^2)
*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2 + ((8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)
^5 + 2*(8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)^3*sin(c)^2 + (8*a^4*b - 8*a^2*b^3
+ b^5)*cos(c)*sin(c)^4)*cos(d*x^3 + 2*c) + (b^5*cos(d*x^3 + 2*c)^4*sin(c)
+ (8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)^4*sin(c) + 2*(8*a^4*b - 8*a^2*b^3 + b^
5)*cos(c)^2*sin(c)^3 + (8*a^4*b - 8*a^2*b^3 + b^5)*sin(c)^5 + 4*(a*b^4*cos(
c)^2 - a*b^4*sin(c)^2)*cos(d*x^3 + 2*c)^3 - 6*((2*a^2*b^3 - b^5)*cos(c)^2*s
in(c) - (2*a^2*b^3 - b^5)*sin(c)^3)*cos(d*x^3 + 2*c)^2 + 4*((4*a^3*b^2 - 3*
a*b^4)*cos(c)^4 - (4*a^3*b^2 - 3*a*b^4)*sin(c)^4)*cos(d*x^3 + 2*c))*sin(d*x
^3 + 2*c))*sqrt(a^2 - b^2))/(b^6*cos(d*x^3 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*
x^3 + 2*c)^5 + b^6*sin(d*x^3 + 2*c)^6 - 6*a*b^5*cos(d*x^3 + 2*c)^5*sin(c) +
(32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^6 + 3*(32*a^6 - 48*a^4*b^2
+ 18*a^2*b^4 - b^6)*cos(c)^4*sin(c)^2 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^
4 - b^6)*cos(c)^2*sin(c)^4 + (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*sin(c
)^6 + 3*((2*a^2*b^4 - b^6)*cos(c)^2 + 5*(2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x
^3 + 2*c)^4 + 3*(b^6*cos(d*x^3 + 2*c)^2 - 2*a*b^5*cos(d*x^3 + 2*c)*sin(c) +
5*(2*a^2*b^4 - b^6)*cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^3 + 2*c
)^4 - 4*(3*(4*a^3*b^3 - 3*a*b^5)*cos(c)^2*sin(c) + 5*(4*a^3*b^3 - 3*a*b^5)*
sin(c)^3)*cos(d*x^3 + 2*c)^3 + 4*(3*a*b^5*cos(d*x^3 + 2*c)^2*cos(c) + 5*(4*
a^3*b^3 - 3*a*b^5)*cos(c)^3 - 6*(2*a^2*b^4 - b^6)*cos(d*x^3 + 2*c)*cos(c)*s
in(c) + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*sin(d*x^3 + 2*c)^3 + 3*((8
*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(
c)^2*sin(c)^2 + 5*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^3 + 2*c)^
2 + 3*(b^6*cos(d*x^3 + 2*c)^4 - 4*a*b^5*cos(d*x^3 + 2*c)^3*sin(c) + 5*(8*a^
4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^

```
2*sin(c)^2 + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4 + 6*((2*a^2*b^4 - b^6)*
cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 4*(3*(4*a^3*b^3
 - 3*a*b^5)*cos(c)^2*sin(c) + (4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^3 + 2
*c))*sin(d*x^3 + 2*c)^2 - 6*((16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^4*sin
(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^2*sin(c)^3 + (16*a^5*b - 2
0*a^3*b^3 + 5*a*b^5)*sin(c)^5)*cos(d*x^3 + 2*c) + 6*(a*b^5*cos(d*x^3 + 2*c)
^4*cos(c) + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^5 - 4*(2*a^2*b^4 - b^6
)*cos(d*x^3 + 2*c)^3*cos(c)*sin(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*co
s(c)^3*sin(c)^2 + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)*sin(c)^4 + 2*((4
*a^3*b^3 - 3*a*b^5)*cos(c)^3 + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*cos
(d*x^3 + 2*c)^2 - 4*((8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (8*a^4
*b^2 - 8*a^2*b^4 + b^6)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)
 - 2*(3*b^5*cos(c)*sin(d*x^3 + 2*c)^5 - 3*b^5*cos(d*x^3 + 2*c)^5*sin(c) + (
16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^6 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)
*cos(c)^4*sin(c)^2 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^2*sin(c)^4 +
(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*sin(c)^6 + 3*(a*b^4*cos(c)^2 + 5*a*b^4*sin(
c)^2)*cos(d*x^3 + 2*c)^4 + 3*(5*a*b^4*cos(c)^2 - b^5*cos(d*x^3 + 2*c)*sin(c
) + a*b^4*sin(c)^2)*sin(d*x^3 + 2*c)^4 - 2*(3*(4*a^2*b^3 - b^5)*cos(c)^2*si
n(c) + 5*(4*a^2*b^3 - b^5)*sin(c)^3)*cos(d*x^3 + 2*c)^3 + 2*(3*b^5*cos(d*x^
3 + 2*c)^2*cos(c) - 12*a*b^4*cos(d*x^3 + 2*c)*cos(c)*sin(c) + 5*(4*a^2*b^3
- b^5)*cos(c)^3 + 3*(4*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*sin(d*x^3 + 2*c)^3 +
 6*((2*a^3*b^2 - a*b^4)*cos(c)^4 + 6*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^2
+ 5*(2*a^3*b^2 - a*b^4)*sin(c)^4)*cos(d*x^3 + 2*c)^2 - 6*(b^5*cos(d*x^3 + 2
*c)^3*sin(c) - 5*(2*a^3*b^2 - a*b^4)*cos(c)^4 - 6*(2*a^3*b^2 - a*b^4)*cos(c
)^2*sin(c)^2 - (2*a^3*b^2 - a*b^4)*sin(c)^4 - 3*(a*b^4*cos(c)^2 + a*b^4*sin
(c)^2)*cos(d*x^3 + 2*c)^2 + (3*(4*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (4*a^2*b
^3 - b^5)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2 - 3*((16*a^4*b - 1
2*a^2*b^3 + b^5)*cos(c)^4*sin(c) + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^2
*sin(c)^3 + (16*a^4*b - 12*a^2*b^3 + b^5)*sin(c)^5)*cos(d*x^3 + 2*c) + 3*(b
^5*cos(d*x^3 + 2*c)^4*cos(c) - 8*a*b^4*cos(d*x^3 + 2*c)^3*cos(c)*sin(c) + (
16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^5 + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos
(c)^3*sin(c)^2 + (16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)*sin(c)^4 + 2*((4*a^2*
b^3 - b^5)*cos(c)^3 + 3*(4*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)
^2 - 16*((2*a^3*b^2 - a*b^4)*cos(c)^3*sin(c) + (2*a^3*b^2 - a*b^4)*cos(c)*s
in(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c))*sqrt(a^2 - b^2)), (b^6*cos(d*x
^3 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*x^3 + 2*c)^5 + b^6*sin(d*x^3 + 2*c)^6 -
6*a*b^5*cos(d*x^3 + 2*c)^5*sin(c) + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^6
+ 3*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4*sin(c)^2 + 3*(8*a^4*b^2 - 8*a^2*
b^4 + b^6)*cos(c)^2*sin(c)^4 + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^6 + ((4
*a^2*b^4 - b^6)*cos(c)^2 + 5*(4*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c)^4
 + (3*b^6*cos(d*x^3 + 2*c)^2 - 6*a*b^5*cos(d*x^3 + 2*c)*sin(c) + 5*(4*a^2*b
^4 - b^6)*cos(c)^2 + (4*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^3 + 2*c)^4 - 4*(3*
(2*a^3*b^3 - a*b^5)*cos(c)^2*sin(c) + 5*(2*a^3*b^3 - a*b^5)*sin(c)^3)*cos(d
*x^3 + 2*c)^3 + 4*(3*a*b^5*cos(d*x^3 + 2*c)^2*cos(c) + 5*(2*a^3*b^3 - a*b^5
)*cos(c)^3 - 2*(4*a^2*b^4 - b^6)*cos(d*x^3 + 2*c)*cos(c)*sin(c) + 3*(2*a^3*
```

b^3 - a*b^5)*cos(c)*sin(c)^2)*sin(d*x^3 + 2*c)^3 + ((8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^2*sin(c)^2 + 5*(8*a^4*b^2 - 4*a^2*b^4 - b^6)*sin(c)^4)*cos(d*x^3 + 2*c)^2 + (3*b^6*cos(d*x^3 + 2*c)^4 - 12*a*b^5*cos(d*x^3 + 2*c)^3*sin(c) + 5*(8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^2*sin(c)^2 + (8*a^4*b^2 - 4*a^2*b^4 - b^6)*sin(c)^4 + 6*((4*a^2*b^4 - b^6)*cos(c)^2 + (4*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 12*(3*(2*a^3*b^3 - a*b^5)*cos(c)^2*sin(c) + (2*a^3*b^3 - a*b^5)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2 - 2*((8*a^5*b - 5*a*b^5)*cos(c)^4*sin(c) + 2*(8*a^5*b - 5*a*b^5)*cos(c)^2*sin(c)^3 + (8*a^5*b - 5*a*b^5)*sin(c)^5)*cos(d*x^3 + 2*c) + 2*(3*a*b^5*cos(d*x^3 + 2*c)^4*cos(c) + (8*a^5*b - 5*a*b^5)*cos(c)^5 - 4*(4*a^2*b^4 - b^6)*cos(d*x^3 + 2*c)^3*cos(c)*sin(c) + 2*(8*a^5*b - 5*a*b^5)*cos(c)^3*sin(c)^2 + (8*a^5*b - 5*a*b^5)*cos(c)*sin(c)^4 + 6*((2*a^3*b^3 - a*b^5)*cos(c)^3 + 3*(2*a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 4*((8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^3*sin(c) + (8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c) - 4*(b^5*cos(c)*sin(d*x^3 + 2*c)^5 - b^5*cos(d*x^3 + 2*c)^5*sin(c) + (2*a^3*b^2 - a*b^4)*cos(c)^6 + 3*(2*a^3*b^2 - a*b^4)*cos(c)^4*sin(c)^2 + 3*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^4 + (2*a^3*b^2 - a*b^4)*sin(c)^6 + (a*b^4*cos(c)^2 + 5*a*b^4*sin(c)^2)*cos(d*x^3 + 2*c)^4 + (5*a*b^4*cos(c)^2 - b^5*cos(d*x^3 + 2*c)*sin(c) + a*b^4*sin(c)^2)*sin(d*x^3 + 2*c)^4 - 2*(3*a^2*b^3*cos(c)^2*sin(c) + 5*a^2*b^3*sin(c)^3)*cos(d*x^3 + 2*c)^3 + 2*(b^5*cos(d*x^3 + 2*c)^2*cos(c) + 5*a^2*b^3*cos(c)^3 - 4*a*b^4*cos(d*x^3 + 2*c)*cos(c)*sin(c) + 3*a^2*b^3*cos(c)*sin(c)^2)*sin(d*x^3 + 2*c)^3 + 2*(a^3*b^2*cos(c)^4 + 6*a^3*b^2*cos(c)^2*sin(c)^2 + 5*a^3*b^2*sin(c)^4)*cos(d*x^3 + 2*c)^2 + 2*(5*a^3*b^2*cos(c)^4 - b^5*cos(d*x^3 + 2*c)^3*sin(c) + 6*a^3*b^2*cos(c)^2*sin(c)^2 + a^3*b^2*sin(c)^4 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 3*(3*a^2*b^3*cos(c)^2*sin(c) + a^2*b^3*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2 - ((4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)^4*sin(c) + 2*(4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)^2*sin(c)^3 + (4*a^4*b + 2*a^2*b^3 - b^5)*sin(c)^5)*cos(d*x^3 + 2*c) + (b^5*cos(d*x^3 + 2*c)^4*cos(c) - 8*a*b^4*cos(d*x^3 + 2*c)^3*cos(c)*sin(c) + (4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)^5 + 2*(4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)^3*sin(c)^2 + (4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)*sin(c)^4 + 6*(a^2*b^3*cos(c)^3 + 3*a^2*b^3*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 16*(a^3*b^2*cos(c)^3*sin(c) + a^3*b^2*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c))*sqrt(a^2 - b^2))/(b^6*cos(d*x^3 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*x^3 + 2*c)^5 + b^6*sin(d*x^3 + 2*c)^6 - 6*a*b^5*cos(d*x^3 + 2*c)^5*sin(c) + (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^6 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^4*sin(c)^2 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^2*sin(c)^4 + (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*sin(c)^6 + 3*((2*a^2*b^4 - b^6)*cos(c)^2 + 5*(2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c)^4 + 3*(b^6*cos(d*x^3 + 2*c)^2 - 2*a*b^5*cos(d*x^3 + 2*c)*sin(c) + 5*(2*a^2*b^4 - b^6)*cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^3 + 2*c)^4 - 4*(3*(4*a^3*b^3 - 3*a*b^5)*cos(c)^2*sin(c) + 5*(4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^3 + 2*c)^3 + 4*(3*a*b^5*cos(d*x^3 + 2*c)^2*cos(c) + 5*(4*a^3*b^3 - 3*a*b^5)*cos(c

```
)^3 - 6*(2*a^2*b^4 - b^6)*cos(d*x^3 + 2*c)*cos(c)*sin(c) + 3*(4*a^3*b^3 - 3
*a*b^5)*cos(c)*sin(c)^2)*sin(d*x^3 + 2*c)^3 + 3*((8*a^4*b^2 - 8*a^2*b^4 + b
^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^2*sin(c)^2 + 5*(8*a^4
*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^3 + 2*c)^2 + 3*(b^6*cos(d*x^3 + 2
*c)^4 - 4*a*b^5*cos(d*x^3 + 2*c)^3*sin(c) + 5*(8*a^4*b^2 - 8*a^2*b^4 + b^6)
*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^2*sin(c)^2 + (8*a^4*b^2
- 8*a^2*b^4 + b^6)*sin(c)^4 + 6*((2*a^2*b^4 - b^6)*cos(c)^2 + (2*a^2*b^4 -
b^6)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 4*(3*(4*a^3*b^3 - 3*a*b^5)*cos(c)^2*sin
(c) + (4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2
- 6*((16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^4*sin(c) + 2*(16*a^5*b - 20*a
^3*b^3 + 5*a*b^5)*cos(c)^2*sin(c)^3 + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*sin
(c)^5)*cos(d*x^3 + 2*c) + 6*(a*b^5*cos(d*x^3 + 2*c)^4*cos(c) + (16*a^5*b -
20*a^3*b^3 + 5*a*b^5)*cos(c)^5 - 4*(2*a^2*b^4 - b^6)*cos(d*x^3 + 2*c)^3*cos
(c)*sin(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^3*sin(c)^2 + (16*a^
5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)*sin(c)^4 + 2*((4*a^3*b^3 - 3*a*b^5)*cos(
c)^3 + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 4*((8*
a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*
cos(c)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c) - 2*(3*b^5*cos(c)*sin(d
*x^3 + 2*c)^5 - 3*b^5*cos(d*x^3 + 2*c)^5*sin(c) + (16*a^5 - 16*a^3*b^2 + 3*
a*b^4)*cos(c)^6 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^4*sin(c)^2 + 3*(
16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^2*sin(c)^4 + (16*a^5 - 16*a^3*b^2 + 3
*a*b^4)*sin(c)^6 + 3*(a*b^4*cos(c)^2 + 5*a*b^4*sin(c)^2)*cos(d*x^3 + 2*c)^4
 + 3*(5*a*b^4*cos(c)^2 - b^5*cos(d*x^3 + 2*c)*sin(c) + a*b^4*sin(c)^2)*sin(
d*x^3 + 2*c)^4 - 2*(3*(4*a^2*b^3 - b^5)*cos(c)^2*sin(c) + 5*(4*a^2*b^3 - b^
5)*sin(c)^3)*cos(d*x^3 + 2*c)^3 + 2*(3*b^5*cos(d*x^3 + 2*c)^2*cos(c) - 12*a
*b^4*cos(d*x^3 + 2*c)*cos(c)*sin(c) + 5*(4*a^2*b^3 - b^5)*cos(c)^3 + 3*(4*a
^2*b^3 - b^5)*cos(c)*sin(c)^2)*sin(d*x^3 + 2*c)^3 + 6*((2*a^3*b^2 - a*b^4)*
cos(c)^4 + 6*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^2 + 5*(2*a^3*b^2 - a*b^4)*
sin(c)^4)*cos(d*x^3 + 2*c)^2 - 6*(b^5*cos(d*x^3 + 2*c)^3*sin(c) - 5*(2*a^3*
b^2 - a*b^4)*cos(c)^4 - 6*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^2 - (2*a^3*b^
2 - a*b^4)*sin(c)^4 - 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*cos(d*x^3 + 2*c)^
2 + (3*(4*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (4*a^2*b^3 - b^5)*sin(c)^3)*cos(
d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2 - 3*((16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^
4*sin(c) + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^2*sin(c)^3 + (16*a^4*b -
12*a^2*b^3 + b^5)*sin(c)^5)*cos(d*x^3 + 2*c) + 3*(b^5*cos(d*x^3 + 2*c)^4*co
s(c) - 8*a*b^4*cos(d*x^3 + 2*c)^3*cos(c)*sin(c) + (16*a^4*b - 12*a^2*b^3 +
b^5)*cos(c)^5 + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^3*sin(c)^2 + (16*a^4
*b - 12*a^2*b^3 + b^5)*cos(c)*sin(c)^4 + 2*((4*a^2*b^3 - b^5)*cos(c)^3 + 3*
(4*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 16*((2*a^3*b^2 - a*
b^4)*cos(c)^3*sin(c) + (2*a^3*b^2 - a*b^4)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c
))*sin(d*x^3 + 2*c))*sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx=\frac{2\left(\pi\left\lfloor\frac{dx^3+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx^3+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{3\sqrt{a^2-b^2}d}$$

```
[In] integrate(x^2/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] 2/3*(pi*floor(1/2*(d*x^3 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^3 +
1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.67

$$\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx=-\frac{\ln\left(-x^2\,\mathrm{e}^{d\,x^3\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}-\frac{2\,x^2\,\left(b\,1\mathrm{i}+a\,\mathrm{e}^{d\,x^3\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{b-a}}\right)-\ln\left(-x^2\,\mathrm{e}^{d\,x^3\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}+\frac{2\,x^2\,\left(b\,1\mathrm{i}+a\,\mathrm{e}^{d\,x^3\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{b-a}}\right)}{3\,d\,\sqrt{a+b}\,\sqrt{b-a}}$$

```
[In] int(x^2/(a + b*sin(c + d*x^3)),x)
```

```
[Out] -(log(- x^2*exp(d*x^3*1i)*exp(c*1i)*2i - (2*x^2*(b*1i + a*exp(d*x^3*1i)*exp
(c*1i)))/((a + b)^(1/2)*(b - a)^(1/2))) - log((2*x^2*(b*1i + a*exp(d*x^3*1i
)*exp(c*1i)))/((a + b)^(1/2)*(b - a)^(1/2)) - x^2*exp(d*x^3*1i)*exp(c*1i)*2
i))/(3*d*(a + b)^(1/2)*(b - a)^(1/2))
```

3.83 $\int \frac{1}{x(a+b\sin(c+dx^3))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx = \operatorname{Int}\left(\frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sin(d*x^3+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx = \int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*Sin[c + d*x^3])),x]
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*x^3])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sin[c + d*x^3])),x]
[Out] Integrate[1/(x*(a + b*Sin[c + d*x^3])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sin\left(d\,x^{3}+c\right)\right)}dx$$

```
[In] int(1/x/(a+b*sin(d*x^3+c)),x)
[Out] int(1/x/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral(1/(b*x*sin(d*x^3 + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x**3+c)),x)
[Out] Integral(1/(x*(a + b*sin(c + d*x**3))), x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^3+c\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^3+c\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 6.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)}\,dx$$

```
[In] int(1/(x*(a + b*sin(c + d*x^3))),x)
[Out] int(1/(x*(a + b*sin(c + d*x^3))), x)
```

3.84 $\int \frac{1}{x^4(a+b\sin(c+dx^3))} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^4 (a + b\sin(c + dx^3))} dx = \text{Int}\left(\frac{1}{x^4 (a + b\sin(c + dx^3))}, x\right)$$

```
[Out] Unintegrable(1/x^4/(a+b*sin(d*x^3+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^4 (a + b\sin(c + dx^3))} dx = \int \frac{1}{x^4 (a + b\sin(c + dx^3))} dx$$

```
[In] Int[1/(x^4*(a + b*Sin[c + d*x^3])),x]
[Out] Defer[Int][1/(x^4*(a + b*Sin[c + d*x^3])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^4 (a + b\sin(c + dx^3))} dx$$

Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

```
[In] Integrate[1/(x^4*(a + b*Sin[c + d*x^3])),x]
[Out] Integrate[1/(x^4*(a + b*Sin[c + d*x^3])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a+b\sin\left(d\,x^3+c\right)\right)}dx$$

```
[In] int(1/x^4/(a+b*sin(d*x^3+c)),x)
[Out] int(1/x^4/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^3+c\right)+a)x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral(1/(b*x^4*sin(d*x^3 + c) + a*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

```
[In] integrate(1/x**4/(a+b*sin(d*x**3+c)),x)
[Out] Integral(1/(x**4*(a + b*sin(c + d*x**3))), x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^3+c\right)+a)x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^3+c\right)+a)x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 6.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x^4\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)}\,dx$$

```
[In] int(1/(x^4*(a + b*sin(c + d*x^3))),x)
[Out] int(1/(x^4*(a + b*sin(c + d*x^3))), x)
```

3.85 $\int \frac{x}{a+b\sin(c+dx^3)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx = \text{Int}\left(\frac{x}{a+b\sin(c+dx^3)},x\right)$$

```
[Out] Unintegrable(x/(a+b*sin(d*x^3+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx = \int \frac{x}{a+b\sin(c+dx^3)}\,dx$$

```
[In] Int[x/(a + b*Sin[c + d*x^3]),x]
[Out] Defer[Int][x/(a + b*Sin[c + d*x^3]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{a+b\sin(c+dx^3)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx=\int \frac{x}{a+b\sin(c+dx^3)}\,dx$$

```
[In] Integrate[x/(a + b*Sin[c + d*x^3]),x]
[Out] Integrate[x/(a + b*Sin[c + d*x^3]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\sin(d\,x^3+c)}dx$$

```
[In] int(x/(a+b*sin(d*x^3+c)),x)
[Out] int(x/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx=\int \frac{x}{b\sin(dx^3+c)+a}\,dx$$

```
[In] integrate(x/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral(x/(b*sin(d*x^3 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx=\int \frac{x}{a+b\sin(c+dx^3)}\,dx$$

```
[In] integrate(x/(a+b*sin(d*x**3+c)),x)
[Out] Integral(x/(a + b*sin(c + d*x**3)), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx=\int \frac{x}{b\sin\left(dx^3+c\right)+a}\,dx$$

```
[In] integrate(x/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate(x/(b*sin(d*x^3 + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx=\int \frac{x}{b\sin\left(dx^3+c\right)+a}\,dx$$

```
[In] integrate(x/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(x/(b*sin(d*x^3 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 5.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx=\int \frac{x}{a+b\,\sin\left(d\,x^3+c\right)}\,dx$$

```
[In] int(x/(a + b*sin(c + d*x^3)),x)
[Out] int(x/(a + b*sin(c + d*x^3)), x)
```

3.86 $\int \frac{1}{x^2(a+b\sin(c+dx^3))} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2(a+b\sin(c+dx^3))} dx = \operatorname{Int}\left(\frac{1}{x^2(a+b\sin(c+dx^3))},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sin(d*x^3+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2(a+b\sin(c+dx^3))} dx = \int \frac{1}{x^2(a+b\sin(c+dx^3))} dx$$

```
[In] Int[1/(x^2*(a + b*Sin[c + d*x^3])),x]
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*x^3])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2(a+b\sin(c+dx^3))} dx$$

Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\,(a+b\sin\,(c+dx^3))}\,dx = \int \frac{1}{x^2\,(a+b\sin\,(c+dx^3))}\,dx$$

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*x^3])),x]
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*x^3])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\,(a+b\sin\,(d\,x^3+c))}dx$$

```
[In] int(1/x^2/(a+b*sin(d*x^3+c)),x)
[Out] int(1/x^2/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^2\,(a+b\sin\,(c+dx^3))}\,dx = \int \frac{1}{(b\sin\,(dx^3+c)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral(1/(b*x^2*sin(d*x^3 + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\,(a+b\sin\,(c+dx^3))}\,dx = \int \frac{1}{x^2\,(a+b\sin\,(c+dx^3))}\,dx$$

```
[In] integrate(1/x**2/(a+b*sin(d*x**3+c)),x)
[Out] Integral(1/(x**2*(a + b*sin(c + d*x**3))), x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b \sin(c + dx^3))} dx = \int \frac{1}{(b \sin(dx^3 + c) + a)x^2} dx$$

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b \sin(c + dx^3))} dx = \int \frac{1}{(b \sin(dx^3 + c) + a)x^2} dx$$

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 6.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b \sin(c + dx^3))} dx = \int \frac{1}{x^2 \ (a + b \ \sin(d\, x^3 + c))} dx$$

```
[In] int(1/(x^2*(a + b*sin(c + d*x^3))),x)
[Out] int(1/(x^2*(a + b*sin(c + d*x^3))), x)
```

3.87 $\int \frac{1}{a+b\sin(c+dx^3)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx = \text{Int}\left(\frac{1}{a+b\sin(c+dx^3)},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(d*x^3+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx = \int \frac{1}{a+b\sin(c+dx^3)}\,dx$$

```
[In] Int[(a + b*Sin[c + d*x^3])^(-1),x]
[Out] Defer[Int][(a + b*Sin[c + d*x^3])^(-1), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{a+b\sin(c+dx^3)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx=\int \frac{1}{a+b\sin(c+dx^3)}\,dx$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])^(-1),x]
[Out] Integrate[(a + b*Sin[c + d*x^3])^(-1), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin(d\,x^3+c)}dx$$

```
[In] int(1/(a+b*sin(d*x^3+c)),x)
[Out] int(1/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx=\int \frac{1}{b\sin(dx^3+c)+a}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral(1/(b*sin(d*x^3 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx=\int \frac{1}{a+b\sin(c+dx^3)}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x**3+c)),x)
[Out] Integral(1/(a + b*sin(c + d*x**3)), x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx = \int \frac{1}{b\sin(dx^3+c)+a}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate(1/(b*sin(d*x^3 + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx = \int \frac{1}{b\sin(dx^3+c)+a}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(1/(b*sin(d*x^3 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 5.90 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx = \int \frac{1}{a+b\,\sin(d\,x^3+c)}\,dx$$

```
[In] int(1/(a + b*sin(c + d*x^3)),x)
[Out] int(1/(a + b*sin(c + d*x^3)), x)
```

3.88 $\int \frac{1}{x^3(a+b\sin(c+dx^3))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 (a+b\sin(c+dx^3))}\,dx = \text{Int}\left(\frac{1}{x^3 (a+b\sin(c+dx^3))},x\right)$$

```
[Out] Unintegrable(1/x^3/(a+b*sin(d*x^3+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3 (a+b\sin(c+dx^3))}\,dx = \int \frac{1}{x^3 (a+b\sin(c+dx^3))}\,dx$$

```
[In] Int[1/(x^3*(a + b*Sin[c + d*x^3])),x]
```

```
[Out] Defer[Int][1/(x^3*(a + b*Sin[c + d*x^3])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^3 (a+b\sin(c+dx^3))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b \sin (c + dx^3))} \, dx = \int \frac{1}{x^3 (a + b \sin (c + dx^3))} \, dx$$

```
[In] Integrate[1/(x^3*(a + b*Sin[c + d*x^3])),x]
[Out] Integrate[1/(x^3*(a + b*Sin[c + d*x^3])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a + b \sin (d x^3 + c))} dx$$

```
[In] int(1/x^3/(a+b*sin(d*x^3+c)),x)
[Out] int(1/x^3/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^3 (a + b \sin (c + dx^3))} \, dx = \int \frac{1}{(b \sin (dx^3 + c) + a)x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral(1/(b*x^3*sin(d*x^3 + c) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 3.99 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 (a + b \sin (c + dx^3))} \, dx = \int \frac{1}{x^3 (a + b \sin (c + dx^3))} \, dx$$

```
[In] integrate(1/x**3/(a+b*sin(d*x**3+c)),x)
[Out] Integral(1/(x**3*(a + b*sin(c + d*x**3))), x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\sin(c + dx^3))}\,dx = \int \frac{1}{(b\sin(dx^3 + c) + a)x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\sin(c + dx^3))}\,dx = \int \frac{1}{(b\sin(dx^3 + c) + a)x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 6.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\sin(c + dx^3))}\,dx = \int \frac{1}{x^3\,(a + b\,\sin(d\,x^3 + c))}\,dx$$

```
[In] int(1/(x^3*(a + b*sin(c + d*x^3))),x)
[Out] int(1/(x^3*(a + b*sin(c + d*x^3))), x)
```

3.89 $\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 324

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx = -\frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d}+\frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d} - \frac{\log(a+b\sin(c+dx^3))}{3(a^2-b^2)d^2}-\frac{a\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d^2} + \frac{a\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d^2}+\frac{bx^3\cos(c+dx^3)}{3(a^2-b^2)d(a+b\sin(c+dx^3))}$$

```
[Out] -1/3*ln(a+b*sin(d*x^3+c))/(a^2-b^2)/d^2-1/3*I*a*x^3*ln(1-I*b*exp(I*(d*x^3+c
))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d+1/3*I*a*x^3*ln(1-I*b*exp(I*(d*x^3
+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d-1/3*a*polylog(2,I*b*exp(I*(d*x^
3+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+1/3*a*polylog(2,I*b*exp(I*(d
*x^3+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+1/3*b*x^3*cos(d*x^3+c)/(a
^2-b^2)/d/(a+b*sin(d*x^3+c))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3460, 3405, 3404, 2296, 2221, 2317, 2438, 2747, 31}

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx = -\frac{a\,\text{PolyLog}\left(2,\frac{ibe^{i(dx^3+c)}}{a-\sqrt{a^2-b^2}}\right)}{3d^2(a^2-b^2)^{3/2}} + \frac{a\,\text{PolyLog}\left(2,\frac{ibe^{i(dx^3+c)}}{a+\sqrt{a^2-b^2}}\right)}{3d^2(a^2-b^2)^{3/2}} - \frac{\log(a+b\sin(c+dx^3))}{3d^2(a^2-b^2)} - \frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3d(a^2-b^2)^{3/2}} + \frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{\sqrt{a^2-b^2}+a}\right)}{3d(a^2-b^2)^{3/2}} + \frac{bx^3\cos(c+dx^3)}{3d(a^2-b^2)(a+b\sin(c+dx^3))}$$

```
[In] Int[x^5/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((-1/3*I)*a*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/((a
^2 - b^2)^(3/2)*d) + ((I/3)*a*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt
[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - Log[a + b*Sin[c + d*x^3]]/(3*(a^2 -
b^2)*d^2) - (a*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/(
3*(a^2 - b^2)^(3/2)*d^2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[
a^2 - b^2])])/(3*(a^2 - b^2)^(3/2)*d^2) + (b*x^3*Cos[c + d*x^3])/(3*(a^2 -
b^2)*d*(a + b*Sin[c + d*x^3]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+b\sin(c+dx))^2}\,dx, x, x^3\right)$$

$$= \frac{bx^3\cos(c+dx^3)}{3(a^2-b^2)d(a+b\sin(c+dx^3))} + \frac{a\text{Subst}\left(\int \frac{x}{a+b\sin(c+dx)}\,dx, x, x^3\right)}{3(a^2-b^2)} - \frac{b\text{Subst}\left(\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx, x, x^3\right)}{3(a^2-b^2)d}$$

$$= \frac{bx^3\cos(c+dx^3)}{3(a^2-b^2)d(a+b\sin(c+dx^3))} + \frac{(2a)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^3\right)}{3(a^2-b^2)} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx^3)\right)}{3(a^2-b^2)d^2}$$

$$= -\frac{\log(a+b\sin(c+dx^3))}{3(a^2-b^2)d^2} + \frac{bx^3\cos(c+dx^3)}{3(a^2-b^2)d(a+b\sin(c+dx^3))} - \frac{(2iab)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^3\right)}{3(a^2-b^2)^{3/2}} + \frac{(2iab)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^3\right)}{3(a^2-b^2)^{3/2}}$$

$$= -\frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d} + \frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d} - \frac{\log(a+b\sin(c+dx^3))}{3(a^2-b^2)d^2} + \frac{bx^3\cos(c+dx^3)}{3(a^2-b^2)d(a+b\sin(c+dx^3))} + \frac{(ia)\text{Subst}\left(\int \log\left(1-\frac{2ibe^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)dx, x, x^3\right)}{3(a^2-b^2)^{3/2}d} - \frac{(ia)\text{Subst}\left(\int \log\left(1-\frac{2ibe^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right)dx, x, x^3\right)}{3(a^2-b^2)^{3/2}d}$$

$$= -\frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d} + \frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d} - \frac{\log(a+b\sin(c+dx^3))}{3(a^2-b^2)d^2} + \frac{bx^3\cos(c+dx^3)}{3(a^2-b^2)d(a+b\sin(c+dx^3))} + \frac{a\text{Subst}\left(\int \frac{\log\left(1-\frac{2ibx}{2a-2\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+dx^3)}\right)}{3(a^2-b^2)^{3/2}d^2} - \frac{a\text{Subst}\left(\int \frac{\log\left(1-\frac{2ibx}{2a+2\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+dx^3)}\right)}{3(a^2-b^2)^{3/2}d^2}$$

$$= -\frac{iax^3 \log\left(1 - \frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2} d} + \frac{iax^3 \log\left(1 - \frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2} d} - \frac{\log(a+b\sin(c+dx^3))}{3(a^2-b^2) d^2} - \frac{a \operatorname{PolyLog}\left(2, \frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2} d^2} + \frac{a \operatorname{PolyLog}\left(2, \frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2} d^2} + \frac{bx^3 \cos(c+dx^3)}{3(a^2-b^2) d (a+b\sin(c+dx^3))}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\, dx$$

$$= \frac{-\frac{iadx^3 \log\left(1+\frac{ibe^{i(c+dx^3)}}{-a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{iadx^3 \log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - \frac{\log(a+b\sin(c+dx^3))}{a^2-b^2} - \frac{a \operatorname{PolyLog}\left(2,-\frac{ibe^{i(c+dx^3)}}{-a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{a \operatorname{PolyLog}\left(2, \cdots\right)}{(a^2-b^2\cdots}}{3d^2}$$

```
[In] Integrate[x^5/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (((-I)*a*d*x^3*Log[1 + (I*b*E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])])/(a^
2 - b^2)^(3/2) + (I*a*d*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 -
 b^2])])/(a^2 - b^2)^(3/2) - Log[a + b*Sin[c + d*x^3]]/(a^2 - b^2) - (a*Pol
yLog[2, ((-I)*b*E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])])/(a^2 - b^2)^(3/
2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 - b^2])])/(a^2 - b
^2)^(3/2) + (b*d*x^3*Cos[c + d*x^3])/((a^2 - b^2)*(a + b*Sin[c + d*x^3])))/
(3*d^2)
```

Maple [F]

$$\int \frac{x^5}{(a+b\sin(d\,x^3+c))^2} dx$$

```
[In] int(x^5/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x^5/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1509 vs. $2(274) = 548$.

Time = 0.47 (sec) , antiderivative size = 1509, normalized size of antiderivative = 4.66

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2} \, dx = \text{Too large to display}$$

```
[In] integrate(x^5/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(a^2*b - b^3)*d*x^3*cos(d*x^3 + c) + (I*a*b^2*sin(d*x^3 + c) + I*a^2
*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) + (
b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
 (-I*a*b^2*sin(d*x^3 + c) - I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(
d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b + 1) + (-I*a*b^2*sin(d*x^3 + c) - I*a^2*b)*sqrt(
-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) + (b*cos(d*
x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (I*a*b^
2*sin(d*x^3 + c) + I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^3 +
c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) - (a^2*b*d*x^3 + a^2*b*c + (a*b^2*d*x^3 + a*b^2*c)*
sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^3 + c) - a*sin(d*x
^3 + c) + (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b) + (a^2*b*d*x^3 + a^2*b*c + (a*b^2*d*x^3 + a*b^2*c)*sin(d*x^3 + c))*sq
rt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d
*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (a^2*b*d*x
^3 + a^2*b*c + (a*b^2*d*x^3 + a*b^2*c)*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^
2)*log(-(-I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) + (b*cos(d*x^3 + c) - I*b*s
in(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (a^2*b*d*x^3 + a^2*b*c + (a
*b^2*d*x^3 + a*b^2*c)*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos
(d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sq
rt(-(a^2 - b^2)/b^2) - b)/b) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^3 + c)
+ (a*b^2*c*sin(d*x^3 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*
x^3 + c) + 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (a^
3 - a*b^2 + (a^2*b - b^3)*sin(d*x^3 + c) + (a*b^2*c*sin(d*x^3 + c) + a^2*b*
c)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x^3 + c) - 2*I*b*sin(d*x^3 + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^
3 + c) - (a*b^2*c*sin(d*x^3 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(-2*
b*cos(d*x^3 + c) + 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*
a) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^3 + c) - (a*b^2*c*sin(d*x^3 + c)
+ a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x^3 + c) - 2*I*b*sin(d*x^
3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a))/((a^4*b - 2*a^2*b^3 + b^5)*d^
2*sin(d*x^3 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)
```

Sympy [F]

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx$$

```
[In] integrate(x**5/(a+b*sin(d*x**3+c))**2,x)
[Out] Integral(x**5/(a + b*sin(c + d*x**3))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x^5}{(b\sin(dx^3+c)+a)^2}\,dx$$

```
[In] integrate(x^5/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate(x^5/(b*sin(d*x^3 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x^5}{(a+b\,\sin(d\,x^3+c))^2}\,dx$$

```
[In] int(x^5/(a + b*sin(c + d*x^3))^2,x)
[Out] int(x^5/(a + b*sin(c + d*x^3))^2, x)
```

3.90 $\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx=\frac{2a\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^3)\right)}{\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d}+\frac{b\cos(c+dx^3)}{3(a^2-b^2)d(a+b\sin(c+dx^3))}$$

```
[Out] 2/3*a*arctan((b+a*tan(1/2*d*x^3+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d+
1/3*b*cos(d*x^3+c)/(a^2-b^2)/d/(a+b*sin(d*x^3+c))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3460, 2743, 12, 2739, 632, 210}

$$\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx=\frac{2a\arctan\left(\frac{a\tan\left(\frac{1}{2}(c+dx^3)\right)+b}{\sqrt{a^2-b^2}}\right)}{3d(a^2-b^2)^{3/2}}+\frac{b\cos(c+dx^3)}{3d(a^2-b^2)(a+b\sin(c+dx^3))}$$

```
[In] Int[x^2/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (2*a*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/(3*(a^2 - b^2)^(3/
2)*d) + (b*Cos[c + d*x^3])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x^3]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+b\sin(c+dx))^2}\,dx, x, x^3\right) \\
&= \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)} + \frac{\text{Subst}\left(\int \frac{a}{a+b\sin(c+dx)}\,dx, x, x^3\right)}{3\left(a^2-b^2\right)} \\
&= \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)} + \frac{a\text{Subst}\left(\int \frac{1}{a+b\sin(c+dx)}\,dx, x, x^3\right)}{3\left(a^2-b^2\right)} \\
&= \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)} + \frac{(2a)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}\left(c+dx^3\right)\right)\right)}{3\left(a^2-b^2\right)d}
\end{aligned}$$

$$= \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)} - \frac{(4a)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)\right)}{3\left(a^2-b^2\right)d}$$

$$= \frac{2a\arctan\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)}{\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d} + \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \frac{\frac{2a\arctan\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{b\cos(c+dx^3)}{a+b\sin(c+dx^3)}}{3(a-b)(a+b)d}$$

```
[In] Integrate[x^2/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 (b*Cos[c + d*x^3])/(a + b*Sin[c + d*x^3]))/(3*(a - b)*(a + b)*d)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{\frac{\frac{2b^2\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)}{a\left(a^2-b^2\right)}+\frac{2b}{a^2-b^2}}{\left(\tan^2\left(\frac{dx^3}{2}+\frac{c}{2}\right)\right)a+2b\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}}{3d}$
default	$\frac{\frac{\frac{2b^2\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)}{a\left(a^2-b^2\right)}+\frac{2b}{a^2-b^2}}{\left(\tan^2\left(\frac{dx^3}{2}+\frac{c}{2}\right)\right)a+2b\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}}{3d}$
risch	$\frac{\frac{2ib}{3}+\frac{2a\,e^{i\left(dx^3+c\right)}}{3}}{(a^2-b^2)d\left(b\,e^{2i\left(dx^3+c\right)}-b+2ia\,e^{i\left(dx^3+c\right)}\right)}-\frac{a\ln\left(e^{i\left(dx^3+c\right)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{3\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{a\ln\left(e^{i\left(dx^3+c\right)}+\frac{ia\sqrt{-a^2+b^{}}}{b\sqrt{-a^{}}}\right.}{3\sqrt{-a^2+b^2}\,(a+b)(a-}$

```
[In] int(x^2/(a+b*sin(d*x^3+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(2*(b^2/a/(a^2-b^2)*tan(1/2*d*x^3+1/2*c)+b/(a^2-b^2))/(tan(1/2*d*x^3+
1/2*c)^2*a+2*b*tan(1/2*d*x^3+1/2*c)+a)+2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*
tan(1/2*d*x^3+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.89

$$\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx = \left[\frac{(ab\sin(dx^3+c)+a^2)\sqrt{-a^2+b^2}\log\left(-\frac{(2a^2-b^2)\cos(dx^3+c)^2-2ab\sin(dx^3+c)-a^2-b^2-2(a\cos(dx^3+c)\sin(dx^3+c)+b\cos(\ldots}{b^2\cos(dx^3+c)^2-2ab\sin(dx^3+c)-a^2-b^2}\right.}{6((a^4b-2a^2b^3+b^5)d\sin(dx^3+c)+(a^5-2a^3b^2+ab^4)d)}\right.$$
$$\left.-\frac{(ab\sin(dx^3+c)+a^2)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx^3+c)+b}{\sqrt{a^2-b^2}\cos(dx^3+c)}\right)-(a^2b-b^3)\cos(dx^3+c)}{3((a^4b-2a^2b^3+b^5)d\sin(dx^3+c)+(a^5-2a^3b^2+ab^4)d)}\right]$$

```
[In] integrate(x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] [1/6*((a*b*sin(d*x^3 + c) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d
*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2 - 2*(a*cos(d*x^3 + c)*sin(d*
x^3 + c) + b*cos(d*x^3 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x^3 + c)^2 - 2*a*
b*sin(d*x^3 + c) - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(d*x^3 + c))/((a^4*b -
2*a^2*b^3 + b^5)*d*sin(d*x^3 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/3*((a*
b*sin(d*x^3 + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x^3 + c) + b)/(sqr
t(a^2 - b^2)*cos(d*x^3 + c))) - (a^2*b - b^3)*cos(d*x^3 + c))/((a^4*b - 2*a
^2*b^3 + b^5)*d*sin(d*x^3 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. 2(75) = 150.

Time = 73.19 (sec) , antiderivative size = 2116, normalized size of antiderivative = 22.51

$$\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x**2/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Piecewise((zoo*x**3/sin(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((tan(c/2 +
 d*x**3/2)/(6*d) - 1/(6*d*tan(c/2 + d*x**3/2)))/b**2, Eq(a, 0)), (-6*tan(c/
2 + d*x**3/2)**2/(9*b**2*d*tan(c/2 + d*x**3/2)**3 - 27*b**2*d*tan(c/2 + d*x
**3/2)**2 + 27*b**2*d*tan(c/2 + d*x**3/2) - 9*b**2*d) + 6*tan(c/2 + d*x**3/
2)/(9*b**2*d*tan(c/2 + d*x**3/2)**3 - 27*b**2*d*tan(c/2 + d*x**3/2)**2 + 27
*b**2*d*tan(c/2 + d*x**3/2) - 9*b**2*d) - 4/(9*b**2*d*tan(c/2 + d*x**3/2)**
3 - 27*b**2*d*tan(c/2 + d*x**3/2)**2 + 27*b**2*d*tan(c/2 + d*x**3/2) - 9*b*
*2*d), Eq(a, -b)), (-6*tan(c/2 + d*x**3/2)**2/(9*b**2*d*tan(c/2 + d*x**3/2)
```

```
**3 + 27*b**2*d*tan(c/2 + d*x**3/2)**2 + 27*b**2*d*tan(c/2 + d*x**3/2) + 9*
b**2*d) - 6*tan(c/2 + d*x**3/2)/(9*b**2*d*tan(c/2 + d*x**3/2)**3 + 27*b**2*
d*tan(c/2 + d*x**3/2)**2 + 27*b**2*d*tan(c/2 + d*x**3/2) + 9*b**2*d) - 4/(9
*b**2*d*tan(c/2 + d*x**3/2)**3 + 27*b**2*d*tan(c/2 + d*x**3/2)**2 + 27*b**2
*d*tan(c/2 + d*x**3/2) + 9*b**2*d), Eq(a, b)), (x**3/(3*(a + b*sin(c))**2),
 Eq(d, 0)), (a**3*log(tan(c/2 + d*x**3/2) + b/a - sqrt(-a**2 + b**2)/a)*tan
(c/2 + d*x**3/2)**2/(3*a**4*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 + 3
*a**4*d*sqrt(-a**2 + b**2) + 6*a**3*b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3
/2) - 3*a**2*b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 - 3*a**2*b**2
*d*sqrt(-a**2 + b**2) - 6*a*b**3*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2))
+ a**3*log(tan(c/2 + d*x**3/2) + b/a - sqrt(-a**2 + b**2)/a)/(3*a**4*d*sqrt
(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 + 3*a**4*d*sqrt(-a**2 + b**2) + 6*a**
3*b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2) - 3*a**2*b**2*d*sqrt(-a**2 + b
**2)*tan(c/2 + d*x**3/2)**2 - 3*a**2*b**2*d*sqrt(-a**2 + b**2) - 6*a*b**3*d
*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)) - a**3*log(tan(c/2 + d*x**3/2) + b
/a + sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x**3/2)**2/(3*a**4*d*sqrt(-a**2 + b*
*2)*tan(c/2 + d*x**3/2)**2 + 3*a**4*d*sqrt(-a**2 + b**2) + 6*a**3*b*d*sqrt(
-a**2 + b**2)*tan(c/2 + d*x**3/2) - 3*a**2*b**2*d*sqrt(-a**2 + b**2)*tan(c/
2 + d*x**3/2)**2 - 3*a**2*b**2*d*sqrt(-a**2 + b**2) - 6*a*b**3*d*sqrt(-a**2
 + b**2)*tan(c/2 + d*x**3/2)) - a**3*log(tan(c/2 + d*x**3/2) + b/a + sqrt(-
a**2 + b**2)/a)/(3*a**4*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 + 3*a**
4*d*sqrt(-a**2 + b**2) + 6*a**3*b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)
- 3*a**2*b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 - 3*a**2*b**2*d*s
qrt(-a**2 + b**2) - 6*a*b**3*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)) + 2*
a**2*b*log(tan(c/2 + d*x**3/2) + b/a - sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x*
*3/2)/(3*a**4*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 + 3*a**4*d*sqrt(-
a**2 + b**2) + 6*a**3*b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2) - 3*a**2*b
**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 - 3*a**2*b**2*d*sqrt(-a**2
+ b**2) - 6*a*b**3*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)) - 2*a**2*b*log
(tan(c/2 + d*x**3/2) + b/a + sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x**3/2)/(3*a
**4*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 + 3*a**4*d*sqrt(-a**2 + b**
2) + 6*a**3*b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2) - 3*a**2*b**2*d*sqrt
(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 - 3*a**2*b**2*d*sqrt(-a**2 + b**2) -
6*a*b**3*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)) + 2*a*b*sqrt(-a**2 + b**
2)/(3*a**4*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 + 3*a**4*d*sqrt(-a**
2 + b**2) + 6*a**3*b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2) - 3*a**2*b**2
*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 - 3*a**2*b**2*d*sqrt(-a**2 + b
**2) - 6*a*b**3*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)) + 2*b**2*sqrt(-a*
*2 + b**2)*tan(c/2 + d*x**3/2)/(3*a**4*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**
3/2)**2 + 3*a**4*d*sqrt(-a**2 + b**2) + 6*a**3*b*d*sqrt(-a**2 + b**2)*tan(c
/2 + d*x**3/2) - 3*a**2*b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 -
3*a**2*b**2*d*sqrt(-a**2 + b**2) - 6*a*b**3*d*sqrt(-a**2 + b**2)*tan(c/2 +
d*x**3/2)), True))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.55

$$\begin{aligned}\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx &= \frac{2\left(\pi\left\lfloor\frac{dx^3+c}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx^3+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{3\,(a^2d-b^2d)\sqrt{a^2-b^2}} \\ &\quad + \frac{2\left(b^2\tan\left(\frac{1}{2}dx^3+\frac{1}{2}c\right)+ab\right)}{3\,(a^3d-ab^2d)\left(a\tan\left(\frac{1}{2}dx^3+\frac{1}{2}c\right)^2+2\,b\tan\left(\frac{1}{2}dx^3+\frac{1}{2}c\right)+a\right)}\end{aligned}$$

```
[In] integrate(x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] 2/3*(pi*floor(1/2*(d*x^3 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^3 +
1/2*c) + b)/sqrt(a^2 - b^2)))*a/((a^2*d - b^2*d)*sqrt(a^2 - b^2)) + 2/3*(b^
2*tan(1/2*d*x^3 + 1/2*c) + a*b)/((a^3*d - a*b^2*d)*(a*tan(1/2*d*x^3 + 1/2*c
)^2 + 2*b*tan(1/2*d*x^3 + 1/2*c) + a))
```

Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.98

$$\begin{aligned}\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx &= \frac{\frac{2\,b}{a^2-b^2}+\frac{2\,b^2\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{a\,(a^2-b^2)}}{d\left(3\,a\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2+6\,b\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)+3\,a\right)} \\ &\quad + \frac{2\,a\,\mathrm{atan}\left(\frac{3\,(a^2-b^2)\left(\frac{2\,a^2\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3\,(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{2\,a\,\left(3\,a^2\,b-3\,b^3\right)}{9\,(a+b)^{3/2}\,\left(a^2-b^2\right)\,(a-b)^{3/2}}\right)}{2\,a}\right)}{3\,d\,(a+b)^{3/2}\,(a-b)^{3/2}}\end{aligned}$$

```
[In] int(x^2/(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] ((2*b)/(a^2 - b^2) + (2*b^2*tan(c/2 + (d*x^3)/2))/(a*(a^2 - b^2)))/(d*(3*a
+ 3*a*tan(c/2 + (d*x^3)/2)^2 + 6*b*tan(c/2 + (d*x^3)/2))) + (2*a*atan((3*(a
^2 - b^2)*((2*a^2*tan(c/2 + (d*x^3)/2))/(3*(a + b)^(3/2)*(a - b)^(3/2)) + (
2*a*(3*a^2*b - 3*b^3))/(9*(a + b)^(3/2)*(a^2 - b^2)*(a - b)^(3/2))))/(2*a))
)/(3*d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.91 $\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

```
[In] Int[1/(x*(a + b*Sin[c + d*x^3])^2),x]
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sin[c + d*x^3])^2),x]
[Out] Integrate[1/(x*(a + b*Sin[c + d*x^3])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x{\left(a+b\sin\left(d\,x^{3}+c\right)\right)}^{2}}dx$$

```
[In] int(1/x/(a+b*sin(d*x^3+c))^2,x)
[Out] int(1/x/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2 x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*x*cos(d*x^3 + c)^2 - 2*a*b*x*sin(d*x^3 + c) - (a^2 + b^2)*
x), x)
```

Sympy [N/A]

Not integrable

Time = 25.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(1/(x*(a + b*sin(c + d*x**3))**2), x)
```

Maxima [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 2696, normalized size of antiderivative = 149.78

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2 x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*
x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) - 2*
a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin(c)
 - a*b)*cos(d*x^3 + c) + 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*s
in(2*c)^2)*d*x^3*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*
b^2)*sin(c)^2)*d*x^3*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2
- b^4)*sin(2*c)^2)*d*x^3*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(c)*si
n(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^3*si
n(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*
x^3 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(
c))*d*x^3*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^3*cos(2*c) - 2*((a^3*b - a*b^3)*
cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^3*sin(d*x^3))*cos(2*
d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*s
in(c))*d*x^3*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b
^3)*cos(2*c)*sin(c))*d*x^3*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^3*sin(2*c))*sin
(2*d*x^3))*integrate(-2*(b^4*cos(2*c)*sin(2*d*x^3) + b^4*cos(2*d*x^3)*sin(2
*c) - 2*(a^3*b - a*b^3)*cos(d*x^3)*cos(c) + 2*(a^3*b - a*b^3)*sin(d*x^3)*si
n(c) + (a^3*b*d*x^3*sin(d*x^3 + c) - a^3*b*cos(d*x^3 + c))*cos(2*d*x^3 + 2*
c) + (a^3*b - a*b^3 + (a*b^3*d*x^3*sin(2*c) + a*b^3*cos(2*c))*cos(2*d*x^3)
- 2*((a^4 - a^2*b^2)*d*x^3*cos(c) - (a^4 - a^2*b^2)*sin(c))*cos(d*x^3) + (a
*b^3*d*x^3*cos(2*c) - a*b^3*sin(2*c))*sin(2*d*x^3) + 2*((a^4 - a^2*b^2)*d*x
^3*sin(c) + (a^4 - a^2*b^2)*cos(c))*sin(d*x^3))*cos(d*x^3 + c) - (a^3*b*d*x
```

```
^3*cos(d*x^3 + c) + a^3*b*sin(d*x^3 + c) + a^2*b^2)*sin(2*d*x^3 + 2*c) - ((
a^3*b - a*b^3)*d*x^3 + (a*b^3*d*x^3*cos(2*c) - a*b^3*sin(2*c))*cos(2*d*x^3)
 + 2*((a^4 - a^2*b^2)*d*x^3*sin(c) + (a^4 - a^2*b^2)*cos(c))*cos(d*x^3) - (
a*b^3*d*x^3*sin(2*c) + a*b^3*cos(2*c))*sin(2*d*x^3) + 2*((a^4 - a^2*b^2)*d*
x^3*cos(c) - (a^4 - a^2*b^2)*sin(c))*sin(d*x^3))*sin(d*x^3 + c))/(a^4*b^2*d
*x^4*cos(2*d*x^3 + 2*c)^2 + a^4*b^2*d*x^4*sin(2*d*x^3 + 2*c)^2 + (b^6*cos(2
*c)^2 + b^6*sin(2*c)^2)*d*x^4*cos(2*d*x^3)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^
4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^4*cos(d*x^3)^2 + (b
^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^4*sin(2*d*x^3)^2 + 4*(a^5*b - 2*a^3*b^3
 + a*b^5)*d*x^4*cos(c)*sin(d*x^3) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2
 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^4*sin(d*x^3)^2 + 4*(a^5*b - 2*
a^3*b^3 + a*b^5)*d*x^4*cos(d*x^3)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^
4 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*si
n(c))*d*x^4*cos(d*x^3) - (a^2*b^4 - b^6)*d*x^4*cos(2*c) - 2*((a^3*b^3 - a*b
^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^4*sin(d*x^3))*
cos(2*d*x^3) - 2*(a^2*b^4*d*x^4*cos(2*d*x^3)*cos(2*c) - a^2*b^4*d*x^4*sin(2
*d*x^3)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^4*cos(c)*sin(d*x^3) + 2*(a^5*b -
 a^3*b^3)*d*x^4*cos(d*x^3)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^4)*cos(2*d*x^3
+ 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*
c)*sin(c))*d*x^4*cos(d*x^3) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b
^3 - a*b^5)*cos(2*c)*sin(c))*d*x^4*sin(d*x^3) + (a^2*b^4 - b^6)*d*x^4*sin(2
*c))*sin(2*d*x^3) - 2*(a^2*b^4*d*x^4*cos(2*c)*sin(2*d*x^3) + a^2*b^4*d*x^4*
cos(2*d*x^3)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x^4*cos(d*x^3)*cos(c) + 2*(a^
5*b - a^3*b^3)*d*x^4*sin(d*x^3)*sin(c))*sin(2*d*x^3 + 2*c)), x) + 2*(2*a^2*
cos(d*x^3)*cos(c) + a*b*cos(2*c)*sin(2*d*x^3) + a*b*cos(2*d*x^3)*sin(2*c) -
 2*a^2*sin(d*x^3)*sin(c))*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2 + (a
^2*b^2 - b^4)*sin(2*c)^2)*d*x^3*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^
2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^3*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c
)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^3*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*
d*x^3*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin
(c)^2)*d*x^3*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(d*x^3)*sin(c) + (a^
2*b^2 - b^4)*d*x^3 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3
)*cos(2*c)*sin(c))*d*x^3*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^3*cos(2*c) - 2*((
a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^3*sin
(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*
b^3)*sin(2*c)*sin(c))*d*x^3*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c)
 - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^3*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^
3*sin(2*c))*sin(2*d*x^3))
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a+b\sin(c+dx^3))^2}\,dx = \int \frac{1}{(b\sin(dx^3+c)+a)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 6.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a+b\sin(c+dx^3))^2}\,dx = \int \frac{1}{x\,(a+b\,\sin(d\,x^3+c))^2}\,dx$$

```
[In] int(1/(x*(a + b*sin(c + d*x^3))^2),x)
[Out] int(1/(x*(a + b*sin(c + d*x^3))^2), x)
```

3.92 $\int \frac{1}{x^4(a+b\sin(c+dx^3))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^4/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

```
[In] Int[1/(x^4*(a + b*Sin[c + d*x^3])^2),x]
[Out] Defer[Int][1/(x^4*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 (a + b\sin(c + dx^3))^2} dx = \int \frac{1}{x^4 (a + b\sin(c + dx^3))^2} dx$$

```
[In] Integrate[1/(x^4*(a + b*Sin[c + d*x^3])^2),x]
[Out] Integrate[1/(x^4*(a + b*Sin[c + d*x^3])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (a + b\sin(d\,x^3 + c))^2} dx$$

```
[In] int(1/x^4/(a+b*sin(d*x^3+c))^2,x)
[Out] int(1/x^4/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^4 (a + b\sin(c + dx^3))^2} dx = \int \frac{1}{(b\sin(dx^3 + c) + a)^2 x^4} dx$$

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*x^4*cos(d*x^3 + c)^2 - 2*a*b*x^4*sin(d*x^3 + c) - (a^2 + b
^2)*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 50.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4 (a + b\sin(c + dx^3))^2} dx = \int \frac{1}{x^4 (a + b\sin(c + dx^3))^2} dx$$

```
[In] integrate(1/x**4/(a+b*sin(d*x**3+c))**2,x)
[Out] Integral(1/(x**4*(a + b*sin(c + d*x**3))**2), x)
```

Maxima [N/A]

Not integrable

Time = 4.24 (sec) , antiderivative size = 2705, normalized size of antiderivative = 150.28

$$\int \frac{1}{x^4 (a + b\sin(c + dx^3))^2} dx = \int \frac{1}{(b\sin(dx^3 + c) + a)^2 x^4} dx$$

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*
x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) - 2*
a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin(c)
 - a*b)*cos(d*x^3 + c) + 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*s
in(2*c)^2)*d*x^6*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*
b^2)*sin(c)^2)*d*x^6*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2
- b^4)*sin(2*c)^2)*d*x^6*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(c)*si
n(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^6*si
n(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*
x^6 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(
c))*d*x^6*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^6*cos(2*c) - 2*((a^3*b - a*b^3)*
cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^6*sin(d*x^3))*cos(2*
d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*s
in(c))*d*x^6*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b
^3)*cos(2*c)*sin(c))*d*x^6*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^6*sin(2*c))*sin
(2*d*x^3))*integrate(-2*(2*b^4*cos(2*c)*sin(2*d*x^3) + 2*b^4*cos(2*d*x^3)*s
in(2*c) - 4*(a^3*b - a*b^3)*cos(d*x^3)*cos(c) + 4*(a^3*b - a*b^3)*sin(d*x^3
)*sin(c) + (a^3*b*d*x^3*sin(d*x^3 + c) - 2*a^3*b*cos(d*x^3 + c))*cos(2*d*x^
3 + 2*c) + (2*a^3*b - 2*a*b^3 + (a*b^3*d*x^3*sin(2*c) + 2*a*b^3*cos(2*c))*c
os(2*d*x^3) - 2*((a^4 - a^2*b^2)*d*x^3*cos(c) - 2*(a^4 - a^2*b^2)*sin(c))*c
os(d*x^3) + (a*b^3*d*x^3*cos(2*c) - 2*a*b^3*sin(2*c))*sin(2*d*x^3) + 2*((a^
4 - a^2*b^2)*d*x^3*sin(c) + 2*(a^4 - a^2*b^2)*cos(c))*sin(d*x^3))*cos(d*x^3
```

```
 + c) - (a^3*b*d*x^3*cos(d*x^3 + c) + 2*a^3*b*sin(d*x^3 + c) + 2*a^2*b^2)*s
in(2*d*x^3 + 2*c) - ((a^3*b - a*b^3)*d*x^3 + (a*b^3*d*x^3*cos(2*c) - 2*a*b^
3*sin(2*c))*cos(2*d*x^3) + 2*((a^4 - a^2*b^2)*d*x^3*sin(c) + 2*(a^4 - a^2*b
^2)*cos(c))*cos(d*x^3) - (a*b^3*d*x^3*sin(2*c) + 2*a*b^3*cos(2*c))*sin(2*d*
x^3) + 2*((a^4 - a^2*b^2)*d*x^3*cos(c) - 2*(a^4 - a^2*b^2)*sin(c))*sin(d*x^
3))*sin(d*x^3 + c))/(a^4*b^2*d*x^7*cos(2*d*x^3 + 2*c)^2 + a^4*b^2*d*x^7*sin
(2*d*x^3 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^7*cos(2*d*x^3)^2
+ 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin
(c)^2)*d*x^7*cos(d*x^3)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^7*sin(2*d
*x^3)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^7*cos(c)*sin(d*x^3) + 4*((a^6 -
 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^
7*sin(d*x^3)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^7*cos(d*x^3)*sin(c) + (a
^4*b^2 - 2*a^2*b^4 + b^6)*d*x^7 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) -
 (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^7*cos(d*x^3) - (a^2*b^4 - b^6)*d*x^
7*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2
*c)*sin(c))*d*x^7*sin(d*x^3))*cos(2*d*x^3) - 2*(a^2*b^4*d*x^7*cos(2*d*x^3)*
cos(2*c) - a^2*b^4*d*x^7*sin(2*d*x^3)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^7*
cos(c)*sin(d*x^3) + 2*(a^5*b - a^3*b^3)*d*x^7*cos(d*x^3)*sin(c) + (a^4*b^2
- a^2*b^4)*d*x^7)*cos(2*d*x^3 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos
(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^7*cos(d*x^3) + 2*((a^3*b^3 - a
*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^7*sin(d*x^3)
 + (a^2*b^4 - b^6)*d*x^7*sin(2*c))*sin(2*d*x^3) - 2*(a^2*b^4*d*x^7*cos(2*c)
*sin(2*d*x^3) + a^2*b^4*d*x^7*cos(2*d*x^3)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d
*x^7*cos(d*x^3)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x^7*sin(d*x^3)*sin(c))*sin(2
*d*x^3 + 2*c)), x) + 2*(2*a^2*cos(d*x^3)*cos(c) + a*b*cos(2*c)*sin(2*d*x^3)
 + a*b*cos(2*d*x^3)*sin(2*c) - 2*a^2*sin(d*x^3)*sin(c))*sin(d*x^3 + c))/(((
a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^6*cos(2*d*x^3)^
2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^6*cos(d*x^3
)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^6*sin(2
*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*
cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^6*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)
*d*x^6*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^6 + 2*(2*((a^3*b - a*b^3)*co
s(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^6*cos(d*x^3) - (a^2*b^
2 - b^4)*d*x^6*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b
^3)*sin(2*c)*sin(c))*d*x^6*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)
*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^6*cos(d*x^3) + 2*((
a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^6*sin
(d*x^3) + (a^2*b^2 - b^4)*d*x^6*sin(2*c))*sin(2*d*x^3))
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 (a + b\sin(c + dx^3))^2} \, dx = \int \frac{1}{(b\sin(dx^3 + c) + a)^2 x^4} \, dx$$

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)^2*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 6.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 (a + b\sin(c + dx^3))^2} \, dx = \int \frac{1}{x^4 (a + b\,\sin(d\,x^3 + c))^2} \, dx$$

```
[In] int(1/(x^4*(a + b*sin(c + d*x^3))^2),x)
[Out] int(1/(x^4*(a + b*sin(c + d*x^3))^2), x)
```

3.93 $\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \operatorname{Int}\left(\frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable(x/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

```
[In] Int[x/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] Defer[Int][x/(a + b*Sin[c + d*x^3])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x}{(a+b\sin(c+dx^3))^2}\,dx$$

```
[In] Integrate[x/(a + b*Sin[c + d*x^3])^2,x]
[Out] Integrate[x/(a + b*Sin[c + d*x^3])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\sin(d\,x^3+c))^2}dx$$

```
[In] int(x/(a+b*sin(d*x^3+c))^2,x)
[Out] int(x/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int \frac{x}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x}{(b\sin(dx^3+c)+a)^2}\,dx$$

```
[In] integrate(x/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
[Out] integral(-x/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2), x)
```

Sympy [N/A]

Not integrable

Time = 24.65 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x}{(a+b\sin(c+dx^3))^2}\,dx$$

```
[In] integrate(x/(a+b*sin(d*x**3+c))**2,x)
[Out] Integral(x/(a + b*sin(c + d*x**3))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\sin(c+dx^3))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x}{(b\sin(dx^3+c)+a)^2}\,dx$$

```
[In] integrate(x/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x/(b*sin(d*x^3 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x}{(a+b\,\sin(d\,x^3+c))^2}\,dx$$

```
[In] int(x/(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] int(x/(a + b*sin(c + d*x^3))^2, x)
```

3.94 $\int \frac{1}{x^2(a+b\sin(c+dx^3))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

```
[In] Int[1/(x^2*(a + b*Sin[c + d*x^3])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b\sin(c + dx^3))^2} \, dx = \int \frac{1}{x^2 (a + b\sin(c + dx^3))^2} \, dx$$

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*x^3])^2),x]
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*x^3])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a + b\sin(d\,x^3 + c))^2} dx$$

```
[In] int(1/x^2/(a+b*sin(d*x^3+c))^2,x)
[Out] int(1/x^2/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^2 (a + b\sin(c + dx^3))^2} \, dx = \int \frac{1}{(b\sin(dx^3 + c) + a)^2 x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*x^2*cos(d*x^3 + c)^2 - 2*a*b*x^2*sin(d*x^3 + c) - (a^2 + b
^2)*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 35.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*sin(d*x**3+c))**2,x)
[Out] Integral(1/(x**2*(a + b*sin(c + d*x**3))**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 6.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

```
[In] int(1/(x^2*(a + b*sin(c + d*x^3))^2),x)
[Out] int(1/(x^2*(a + b*sin(c + d*x^3))^2), x)
```

3.95 $\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2} dx = \text{Int}\left(\frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}, x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2} dx = \int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2} dx$$

```
[In] Int[(a + b*Sin[c + d*x^3])^(-2),x]
[Out] Defer[Int][(a + b*Sin[c + d*x^3])^(-2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 5.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx$$

```
[In] Integrate[(a + b*Sin[c + d*x^3])^(-2),x]
[Out] Integrate[(a + b*Sin[c + d*x^3])^(-2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sin(d\,x^3+c))^2}dx$$

```
[In] int(1/(a+b*sin(d*x^3+c))^2,x)
[Out] int(1/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

$$\int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{1}{(b\sin(dx^3+c)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2), x)
```

Sympy [N/A]

Not integrable

Time = 16.64 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x**3+c))**2,x)
[Out] Integral((a + b*sin(c + d*x**3))**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 2171, normalized size of antiderivative = 155.07

$$\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*
x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) - 2*
a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin(c)
 - a*b)*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*s
in(2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*
b^2)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2
 - b^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(c)*si
n(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^2*si
n(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*
x^2 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(
c))*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^2*cos(2*c) - 2*((a^3*b - a*b^3)*
cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^2*sin(d*x^3))*cos(2*
d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*s
in(c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b
^3)*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^2*sin(2*c))*sin
(2*d*x^3))*integrate(-2/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d
*x^3) + 2*b^2*cos(2*d*x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) - (2*a*b - (3
*a*b*d*x^3*sin(2*c) + 2*a*b*cos(2*c))*cos(2*d*x^3) - 2*(3*a^2*d*x^3*cos(c)
 - 2*a^2*sin(c))*cos(d*x^3) - (3*a*b*d*x^3*cos(2*c) - 2*a*b*sin(2*c))*sin(2*
d*x^3) + 2*(3*a^2*d*x^3*sin(c) + 2*a^2*cos(c))*sin(d*x^3))*cos(d*x^3 + c) +
 (3*a*b*d*x^3 - (3*a*b*d*x^3*cos(2*c) - 2*a*b*sin(2*c))*cos(2*d*x^3) + 2*(3
*a^2*d*x^3*sin(c) + 2*a^2*cos(c))*cos(d*x^3) + (3*a*b*d*x^3*sin(2*c) + 2*a*
b*cos(2*c))*sin(2*d*x^3) + 2*(3*a^2*d*x^3*cos(c) - 2*a^2*sin(c))*sin(d*x^3)
)*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2
)*d*x^3*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(
c)^2)*d*x^3*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*si
n(2*c)^2)*d*x^3*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(c)*sin(d*x^3)
 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^3*sin(d*x^3)^
2 + 4*(a^3*b - a*b^3)*d*x^3*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^3 + 2*(
2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^3
*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^3*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*
cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^3*sin(d*x^3))*cos(2*d*x^3) +
2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*
x^3*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2
*c)*sin(c))*d*x^3*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^3*sin(2*c))*sin(2*d*x^3)
```

```
), x) + 2*(2*a^2*cos(d*x^3)*cos(c) + a*b*cos(2*c)*sin(2*d*x^3) + a*b*cos(2*
d*x^3)*sin(2*c) - 2*a^2*sin(d*x^3)*sin(c))*sin(d*x^3 + c))/(((a^2*b^2 - b^4
)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 -
 a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b
^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4
*(a^3*b - a*b^3)*d*x^2*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a
^4 - a^2*b^2)*sin(c)^2)*d*x^2*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*
x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^2 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c)
 - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^
2*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*
sin(c))*d*x^2*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos
(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3
)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^
2*b^2 - b^4)*d*x^2*sin(2*c))*sin(2*d*x^3))
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx=\int \frac{1}{(b\sin(dx^3+c)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 5.96 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx=\int \frac{1}{(a+b\,\sin(d\,x^3+c))^2}\,dx$$

```
[In] int(1/(a + b*sin(c + d*x^3))^2,x)
[Out] int(1/(a + b*sin(c + d*x^3))^2, x)
```

3.96 $\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^3/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

```
[In] Int[1/(x^3*(a + b*Sin[c + d*x^3])^2),x]
```

```
[Out] Defer[Int][1/(x^3*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b \sin (c + dx^3))^2} \, dx = \int \frac{1}{x^3 (a + b \sin (c + dx^3))^2} \, dx$$

```
[In] Integrate[1/(x^3*(a + b*Sin[c + d*x^3])^2),x]
[Out] Integrate[1/(x^3*(a + b*Sin[c + d*x^3])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a + b \sin (d\,x^3 + c))^2} dx$$

```
[In] int(1/x^3/(a+b*sin(d*x^3+c))^2,x)
[Out] int(1/x^3/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^3 (a + b \sin (c + dx^3))^2} \, dx = \int \frac{1}{(b \sin (dx^3 + c) + a)^2 x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*x^3*cos(d*x^3 + c)^2 - 2*a*b*x^3*sin(d*x^3 + c) - (a^2 + b
^2)*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 49.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

```
[In] integrate(1/x**3/(a+b*sin(d*x**3+c))**2,x)
[Out] Integral(1/(x**3*(a + b*sin(c + d*x**3))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 2171, normalized size of antiderivative = 120.61

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*
x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) - 2*
a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin(c)
 - a*b)*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*s
in(2*c)^2)*d*x^5*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*
b^2)*sin(c)^2)*d*x^5*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2
- b^4)*sin(2*c)^2)*d*x^5*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(c)*si
n(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^5*si
n(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*
x^5 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(
c))*d*x^5*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^5*cos(2*c) - 2*((a^3*b - a*b^3)*
cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^5*sin(d*x^3))*cos(2*
d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*s
in(c))*d*x^5*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b
^3)*cos(2*c)*sin(c))*d*x^5*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^5*sin(2*c))*sin
(2*d*x^3))*integrate(-2/3*(10*a*b*cos(d*x^3)*cos(c) + 5*b^2*cos(2*c)*sin(2*
d*x^3) + 5*b^2*cos(2*d*x^3)*sin(2*c) - 10*a*b*sin(d*x^3)*sin(c) - (5*a*b -
(3*a*b*d*x^3*sin(2*c) + 5*a*b*cos(2*c))*cos(2*d*x^3) - 2*(3*a^2*d*x^3*cos(c
) - 5*a^2*sin(c))*cos(d*x^3) - (3*a*b*d*x^3*cos(2*c) - 5*a*b*sin(2*c))*sin(
2*d*x^3) + 2*(3*a^2*d*x^3*sin(c) + 5*a^2*cos(c))*sin(d*x^3))*cos(d*x^3 + c)
 + (3*a*b*d*x^3 - (3*a*b*d*x^3*cos(2*c) - 5*a*b*sin(2*c))*cos(2*d*x^3) + 2*
(3*a^2*d*x^3*sin(c) + 5*a^2*cos(c))*cos(d*x^3) + (3*a*b*d*x^3*sin(2*c) + 5*
```

```
a*b*cos(2*c))*sin(2*d*x^3) + 2*(3*a^2*d*x^3*cos(c) - 5*a^2*sin(c))*sin(d*x^
3))*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)
^2)*d*x^6*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*si
n(c)^2)*d*x^6*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*
sin(2*c)^2)*d*x^6*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(c)*sin(d*x^3
) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^6*sin(d*x^3
)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^6 + 2
*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x
^6*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^6*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c
)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^6*sin(d*x^3))*cos(2*d*x^3)
+ 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*
d*x^6*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos
(2*c)*sin(c))*d*x^6*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^6*sin(2*c))*sin(2*d*x^
3)), x) + 2*(2*a^2*cos(d*x^3)*cos(c) + a*b*cos(2*c)*sin(2*d*x^3) + a*b*cos(
2*d*x^3)*sin(2*c) - 2*a^2*sin(d*x^3)*sin(c))*sin(d*x^3 + c))/(((a^2*b^2 - b
^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^5*cos(2*d*x^3)^2 + 4*((a^4
 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^5*cos(d*x^3)^2 + ((a^2
*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^5*sin(2*d*x^3)^2 +
 4*(a^3*b - a*b^3)*d*x^5*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 +
(a^4 - a^2*b^2)*sin(c)^2)*d*x^5*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(
d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^5 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*
c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^5*cos(d*x^3) - (a^2*b^2 - b^4)*d*
x^5*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c
)*sin(c))*d*x^5*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*c
os(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^5*cos(d*x^3) + 2*((a^3*b - a*b
^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^5*sin(d*x^3) + (
a^2*b^2 - b^4)*d*x^5*sin(2*c))*sin(2*d*x^3))
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)^2*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 6.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a+b\sin(c+dx^3))^2} \, dx = \int \frac{1}{x^3 \left(a+b\,\sin\left(d\,x^3+c\right)\right)^2} \,dx$$

```
[In] int(1/(x^3*(a + b*sin(c + d*x^3))^2),x)
[Out] int(1/(x^3*(a + b*sin(c + d*x^3))^2), x)
```

3.97 $\int (ex)^m (a+b\sin(c+dx^3))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p \, dx = \text{Int}\left((ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p, x\right)$$

```
[Out] Unintegrable((e*x)^m*(a+b*sin(d*x^3+c))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p \, dx = \int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p \, dx$$

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^3])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Sin[c + d*x^3])^p, x]
```

Rubi steps

$$\text{integral} = \int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^p \, dx=\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^p \, dx$$

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(e x \right)^{m} \left(a +b \sin \left(d \,x^{3}+c \right)\right)^{p}d x$$

```
[In] int((e*x)^m*(a+b*sin(d*x^3+c))^p,x)
[Out] int((e*x)^m*(a+b*sin(d*x^3+c))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^p \, dx=\int { \left(e x\right)^{m} {\left(b \sin \left(d x^{3} + c\right) + a\right)}^{p} \,d x }$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*sin(d*x^3 + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 17.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^p \, dx=\int \left(e x\right)^{m} \left(a + b \sin{\left(c + d x^{3} \right)}\right)^{p}\, dx$$

```
[In] integrate((e*x)**m*(a+b*sin(d*x**3+c))**p,x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**3))**p, x)
```

Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p \, dx = \int (ex)^m \left(b\sin\left(dx^3+c\right)+a\right)^p dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*sin(d*x^3 + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p \, dx = \int (ex)^m \left(b\sin\left(dx^3+c\right)+a\right)^p dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*sin(d*x^3 + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 6.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p \, dx = \int (e\,x)^m \left(a+b\,\sin\left(d\,x^3+c\right)\right)^p dx$$

```
[In] int((e*x)^m*(a + b*sin(c + d*x^3))^p,x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^3))^p, x)
```

3.98 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 442

$$\begin{aligned}
&\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3 \, dx \\
&= \frac{a(2a^2+3b^2)\,(ex)^{1+m}}{2e(1+m)} + \frac{ib(4a^2+b^2)\,e^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{8e} \\
&\quad - \frac{ib(4a^2+b^2)\,e^{-ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},idx^3\right)}{8e} \\
&\quad + \frac{2^{-\frac{7}{3}-\frac{m}{3}}ab^2e^{2ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-2idx^3\right)}{e} \\
&\quad + \frac{2^{-\frac{7}{3}-\frac{m}{3}}ab^2e^{-2ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},2idx^3\right)}{e} \\
&\quad - \frac{i3^{-\frac{4}{3}-\frac{m}{3}}b^3e^{3ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-3idx^3\right)}{8e} \\
&\quad + \frac{i3^{-\frac{4}{3}-\frac{m}{3}}b^3e^{-3ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},3idx^3\right)}{8e}
\end{aligned}$$

```
[Out] 1/2*a*(2*a^2+3*b^2)*(e*x)^(1+m)/e/(1+m)+1/8*I*b*(4*a^2+b^2)*exp(I*c)*(e*x)^
(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-I*d*x^3)/e-1/8*I*b*(4*a^2+b^
2)*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,I*d*x^3)/e/exp(I*c)+2
^(-7/3-1/3*m)*a*b^2*exp(2*I*c)*(e*x)^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/
3+1/3*m,-2*I*d*x^3)/e+2^(-7/3-1/3*m)*a*b^2*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*
m)*GAMMA(1/3+1/3*m,2*I*d*x^3)/e/exp(2*I*c)-1/8*I*3^(-4/3-1/3*m)*b^3*exp(3*I
*c)*(e*x)^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-3*I*d*x^3)/e+1/8*I
*3^(-4/3-1/3*m)*b^3*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,3*I*
d*x^3)/e/exp(3*I*c)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3484, 6, 3471, 2250, 3470}

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3 dx = \frac{ibe^{ic}(4a^2+b^2)\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},-idx^3\right)}{8e} - \frac{ibe^{-ic}(4a^2+b^2)\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},idx^3\right)}{8e} + \frac{a(2a^2+3b^2)(ex)^{m+1}}{2e(m+1)} + \frac{ab^2e^{2ic}2^{-\frac{m}{3}-\frac{7}{3}}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},-2idx^3\right)}{e} + \frac{ab^2e^{-2ic}2^{-\frac{m}{3}-\frac{7}{3}}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},2idx^3\right)}{e} - \frac{ib^3e^{3ic}3^{-\frac{m}{3}-\frac{4}{3}}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},-3idx^3\right)}{8e} + \frac{ib^3e^{-3ic}3^{-\frac{m}{3}-\frac{4}{3}}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},3idx^3\right)}{8e}$$

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^3])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/8)*b*(4*a^2 + b^2)*E^
(I*c)*(e*x)^(1 + m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])
/e - ((I/8)*b*(4*a^2 + b^2)*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 +
 m)/3, I*d*x^3])/(e*E^(I*c)) + (2^(-7/3 - m/3)*a*b^2*E^((2*I)*c)*(e*x)^(1 +
 m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-2*I)*d*x^3])/e + (2^(-7/3
- m/3)*a*b^2*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (2*I)*d*
x^3])/(e*E^((2*I)*c)) - ((I/8)*3^(-4/3 - m/3)*b^3*E^((3*I)*c)*(e*x)^(1 + m)
*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-3*I)*d*x^3])/e + ((I/8)*3^(-4
/3 - m/3)*b^3*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (3*I)*d
*x^3])/(e*E^((3*I)*c))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
```

```
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3(ex)^m + \frac{3}{2}ab^2(ex)^m - \frac{3}{2}ab^2(ex)^m \cos\left(2c+2dx^3\right) + 3a^2b(ex)^m \sin\left(c+dx^3\right)\right. \\
&\qquad \left. + \frac{3}{4}b^3(ex)^m \sin\left(c+dx^3\right) - \frac{1}{4}b^3(ex)^m \sin\left(3c+3dx^3\right)\right) dx \\
&= \int \left(\left(a^3 + \frac{3ab^2}{2}\right)(ex)^m - \frac{3}{2}ab^2(ex)^m \cos\left(2c+2dx^3\right) + 3a^2b(ex)^m \sin\left(c+dx^3\right)\right. \\
&\qquad \left. + \frac{3}{4}b^3(ex)^m \sin\left(c+dx^3\right) - \frac{1}{4}b^3(ex)^m \sin\left(3c+3dx^3\right)\right) dx \\
&= \int \left(\left(a^3 + \frac{3ab^2}{2}\right)(ex)^m - \frac{3}{2}ab^2(ex)^m \cos\left(2c+2dx^3\right)\right. \\
&\qquad \left. + \left(3a^2b + \frac{3b^3}{4}\right)(ex)^m \sin\left(c+dx^3\right) - \frac{1}{4}b^3(ex)^m \sin\left(3c+3dx^3\right)\right) dx \\
&= \frac{a(2a^2+3b^2)(ex)^{1+m}}{2e(1+m)} - \frac{1}{2}(3ab^2)\int (ex)^m \cos\left(2c+2dx^3\right) dx \\
&\quad - \frac{1}{4}b^3 \int (ex)^m \sin\left(3c+3dx^3\right) dx + \frac{1}{4}\left(3b\left(4a^2+b^2\right)\right)\int (ex)^m \sin\left(c+dx^3\right) dx
\end{aligned}$$

$$= \frac{a(2a^2+3b^2)(ex)^{1+m}}{2e(1+m)} - \frac{1}{4}(3ab^2)\int e^{-2ic-2idx^3}(ex)^m\,dx - \frac{1}{4}(3ab^2)\int e^{2ic+2idx^3}(ex)^m\,dx - \frac{1}{8}(ib^3)\int e^{-3ic-3idx^3}(ex)^m\,dx + \frac{1}{8}(ib^3)\int e^{3ic+3idx^3}(ex)^m\,dx + \frac{1}{8}(3ib(4a^2+b^2))\int e^{-ic-idx^3}(ex)^m\,dx - \frac{1}{8}(3ib(4a^2+b^2))\int e^{ic+idx^3}(ex)^m\,dx$$

$$= \frac{a(2a^2+3b^2)(ex)^{1+m}}{2e(1+m)} + \frac{ib(4a^2+b^2)e^{ic}(ex)^{1+m}(-idx^3)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{8e} - \frac{ib(4a^2+b^2)e^{-ic}(ex)^{1+m}(idx^3)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},idx^3\right)}{8e} + \frac{2^{-\frac{7}{3}-\frac{m}{3}}ab^2e^{2ic}(ex)^{1+m}(-idx^3)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-2idx^3\right)}{e} + \frac{2^{-\frac{7}{3}-\frac{m}{3}}ab^2e^{-2ic}(ex)^{1+m}(idx^3)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},2idx^3\right)}{e} - \frac{i3^{-\frac{4}{3}-\frac{m}{3}}b^3e^{3ic}(ex)^{1+m}(-idx^3)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-3idx^3\right)}{8e} + \frac{i3^{-\frac{4}{3}-\frac{m}{3}}b^3e^{-3ic}(ex)^{1+m}(idx^3)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},3idx^3\right)}{8e}$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.84

$$\int (ex)^m\left(a+b\sin\left(c+dx^3\right)\right)^3\,dx = \frac{1}{24}ix(ex)^m\left(-\frac{12ia(2a^2+3b^2)}{1+m} + 3b(4a^2+b^2)e^{ic}(-idx^3)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},-idx^3\right) - 3b(4a^2+b^2)e^{-ic}(idx^3)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},idx^3\right) - 3i2^{\frac{2}{3}-\frac{m}{3}}ab^2e^{2ic}(-idx^3)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},-2idx^3\right) - 3i2^{\frac{2}{3}-\frac{m}{3}}ab^2e^{-2ic}(idx^3)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},2idx^3\right) - 3^{-\frac{1}{3}-\frac{m}{3}}b^3e^{3ic}(-idx^3)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},-3idx^3\right) + 3^{-\frac{1}{3}-\frac{m}{3}}b^3e^{-3ic}(idx^3)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},3idx^3\right)\right)$$

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^3,x]
```

```
[Out] (I/24)*x*(e*x)^m*(((-12*I)*a*(2*a^2 + 3*b^2))/(1 + m) + 3*b*(4*a^2 + b^2)*E
^(I*c)*((-I)*d*x^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, (-I)*d*x^3] - (3*b*(4*a^2
 + b^2)*(I*d*x^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, I*d*x^3])/E^(I*c) - (3*I)*2
^(2/3 - m/3)*a*b^2*E^((2*I)*c)*((-I)*d*x^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, (
-2*I)*d*x^3] - ((3*I)*2^(2/3 - m/3)*a*b^2*(I*d*x^3)^(-1/3 - m/3)*Gamma[(1 +
 m)/3, (2*I)*d*x^3])/E^((2*I)*c) - 3^(-1/3 - m/3)*b^3*E^((3*I)*c)*((-I)*d*x
^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, (-3*I)*d*x^3] + (3^(-1/3 - m/3)*b^3*(I*d*
x^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, (3*I)*d*x^3])/E^((3*I)*c))
```

Maple [F]

$$\int (ex)^m \left(a + b\sin\left(d\,x^3 + c\right)\right)^3 dx$$

```
[In] int((e*x)^m*(a+b*sin(d*x^3+c))^3,x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^3+c))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.14 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.79

$$\int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)^3 \, dx$$
$$= \frac{36\,(2\,a^3 + 3\,ab^2)(ex)^m\,dx + (b^3e^2m + b^3e^2)e^{\left(-\frac{1}{3}\,(m-2)\log\left(\frac{3i\,d}{e^3}\right) - 3i\,c\right)}\Gamma\left(\frac{1}{3}\,m + \frac{1}{3}, 3i\,dx^3\right) - 9\,(i\,ab^2e^2m + i\,ab^2}{}$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^3,x, algorithm="fricas")
```

```
[Out] 1/72*(36*(2*a^3 + 3*a*b^2)*(e*x)^m*d*x + (b^3*e^2*m + b^3*e^2)*e^(-1/3*(m -
 2)*log(3*I*d/e^3) - 3*I*c)*gamma(1/3*m + 1/3, 3*I*d*x^3) - 9*(I*a*b^2*e^2*
m + I*a*b^2*e^2)*e^(-1/3*(m - 2)*log(2*I*d/e^3) - 2*I*c)*gamma(1/3*m + 1/3,
 2*I*d*x^3) - 9*((4*a^2*b + b^3)*e^2*m + (4*a^2*b + b^3)*e^2)*e^(-1/3*(m -
2)*log(I*d/e^3) - I*c)*gamma(1/3*m + 1/3, I*d*x^3) - 9*((4*a^2*b + b^3)*e^2
*m + (4*a^2*b + b^3)*e^2)*e^(-1/3*(m - 2)*log(-I*d/e^3) + I*c)*gamma(1/3*m
+ 1/3, -I*d*x^3) - 9*(-I*a*b^2*e^2*m - I*a*b^2*e^2)*e^(-1/3*(m - 2)*log(-2*
I*d/e^3) + 2*I*c)*gamma(1/3*m + 1/3, -2*I*d*x^3) + (b^3*e^2*m + b^3*e^2)*e^
(-1/3*(m - 2)*log(-3*I*d/e^3) + 3*I*c)*gamma(1/3*m + 1/3, -3*I*d*x^3))/(d*m
 + d)
```

Sympy [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3\,dx=\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3\,dx$$

```
[In] integrate((e*x)**m*(a+b*sin(d*x**3+c))**3,x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**3))**3, x)
```

Maxima [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3\,dx=\int \left(b\sin\left(dx^3+c\right)+a\right)^3(ex)^m\,dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^3,x, algorithm="maxima")
[Out] (e*x)^(m + 1)*a^3/(e*(m + 1)) + 1/8*(12*a*b^2*e^m*x*x^m - 12*(a*b^2*e^m*m +
 a*b^2*e^m)*integrate(x^m*cos(2*d*x^3 + 2*c), x) + 3*((4*a^2*b + b^3)*e^m*m
*sin(c) + (4*a^2*b + b^3)*e^m*sin(c))*integrate(x^m*cos(d*x^3), x) - 2*(b^3
*e^m*m + b^3*e^m)*integrate(x^m*sin(3*d*x^3 + 3*c), x) + 3*((4*a^2*b + b^3)
*e^m*m + (4*a^2*b + b^3)*e^m)*integrate(x^m*sin(d*x^3 + c), x) + 3*((4*a^2*
b + b^3)*e^m*m*cos(c) + (4*a^2*b + b^3)*e^m*cos(c))*integrate(x^m*sin(d*x^3
), x))/(m + 1)
```

Giac [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3\,dx=\int \left(b\sin\left(dx^3+c\right)+a\right)^3(ex)^m\,dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^3,x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)^3*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3\,dx=\int (e\,x)^m\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)^3\,dx$$

```
[In] int((e*x)^m*(a + b*sin(c + d*x^3))^3,x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^3))^3, x)
```

3.99 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 285

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 \, dx = \frac{(2a^2+b^2)(ex)^{1+m}}{2e(1+m)} + \frac{iabe^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{3e} - \frac{iabe^{-ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},idx^3\right)}{3e} + \frac{2^{-\frac{7}{3}-\frac{m}{3}}b^2e^{2ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-2idx^3\right)}{3e} + \frac{2^{-\frac{7}{3}-\frac{m}{3}}b^2e^{-2ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},2idx^3\right)}{3e}$$

```
[Out] 1/2*(2*a^2+b^2)*(e*x)^(1+m)/e/(1+m)+1/3*I*a*b*exp(I*c)*(e*x)^(1+m)*(-I*d*x^
3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-I*d*x^3)/e-1/3*I*a*b*(e*x)^(1+m)*(I*d*x^3)
^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,I*d*x^3)/e/exp(I*c)+1/3*2^(-7/3-1/3*m)*b^2*ex
p(2*I*c)*(e*x)^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-2*I*d*x^3)/e+
1/3*2^(-7/3-1/3*m)*b^2*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,2
*I*d*x^3)/e/exp(2*I*c)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3484, 6, 3471, 2250, 3470}

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 dx = \frac{(2a^2+b^2)(ex)^{m+1}}{2e(m+1)} + \frac{iabe^{ic}(-idx^3)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},-idx^3\right)}{3e} - \frac{iabe^{-ic}(idx^3)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},idx^3\right)}{3e} + \frac{b^2e^{2ic}2^{-\frac{m}{3}-\frac{7}{3}}(-idx^3)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},-2idx^3\right)}{3e} + \frac{b^2e^{-2ic}2^{-\frac{m}{3}-\frac{7}{3}}(idx^3)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},2idx^3\right)}{3e}$$

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/3)*a*b*E^(I*c)*(e*x)^(1 +
 m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])/e - ((I/3)*a*b*
(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, I*d*x^3])/(e*E^(I*c))
 + (2^(-7/3 - m/3)*b^2*E^((2*I)*c)*(e*x)^(1 + m)*((-I)*d*x^3)^((-1 - m)/3)*
Gamma[(1 + m)/3, (-2*I)*d*x^3])/(3*e) + (2^(-7/3 - m/3)*b^2*(e*x)^(1 + m)*(
I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (2*I)*d*x^3])/(3*e*E^((2*I)*c))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2(ex)^m + \frac{1}{2}b^2(ex)^m - \frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^3\right) + 2ab(ex)^m\sin\left(c+dx^3\right)\right)\,dx\\
&= \int \left(\left(a^2+\frac{b^2}{2}\right)(ex)^m - \frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^3\right) + 2ab(ex)^m\sin\left(c+dx^3\right)\right)\,dx\\
&= \frac{(2a^2+b^2)\,(ex)^{1+m}}{2e(1+m)} + (2ab)\int (ex)^m\sin\left(c+dx^3\right)\,dx - \frac{1}{2}b^2\int (ex)^m\cos\left(2c+2dx^3\right)\,dx\\
&= \frac{(2a^2+b^2)\,(ex)^{1+m}}{2e(1+m)} + (iab)\int e^{-ic-idx^3}(ex)^m\,dx - (iab)\int e^{ic+idx^3}(ex)^m\,dx\\
&\quad - \frac{1}{4}b^2\int e^{-2ic-2idx^3}(ex)^m\,dx - \frac{1}{4}b^2\int e^{2ic+2idx^3}(ex)^m\,dx\\
&= \frac{(2a^2+b^2)\,(ex)^{1+m}}{2e(1+m)} + \frac{iabe^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{3e}\\
&\quad - \frac{iabe^{-ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},idx^3\right)}{3e}\\
&\quad + \frac{2^{-\frac{7}{3}-\frac{m}{3}}b^2e^{2ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-2idx^3\right)}{3e}\\
&\quad + \frac{2^{-\frac{7}{3}-\frac{m}{3}}b^2e^{-2ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},2idx^3\right)}{3e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.95

$$\int (ex)^m\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

$$= \frac{2^{\frac{1}{3}(-7-m)}x(ex)^m\left(d^2x^6\right)^{\frac{1}{3}(-1-m)}\left(3\,2^{\frac{7+m}{3}}a^2\left(d^2x^6\right)^{\frac{1+m}{3}} + 3\,2^{\frac{4+m}{3}}b^2\left(d^2x^6\right)^{\frac{1+m}{3}} + b^2\left(idx^3\right)^{\frac{1+m}{3}}\cos(2c)\Gamma\left(\frac{1+m}{3},-\right.\right.}{}$$

[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^2,x]

[Out] (2^((-7 - m)/3)*x*(e*x)^m*(d^2*x^6)^((-1 - m)/3)*(3*2^((7 + m)/3)*a^2*(d^2*x^6)^((1 + m)/3) + 3*2^((4 + m)/3)*b^2*(d^2*x^6)^((1 + m)/3) + b^2*(I*d*x^3)^((1 + m)/3)*Cos[2*c]*Gamma[(1 + m)/3, (-2*I)*d*x^3] + b^2*m*(I*d*x^3)^((1 + m)/3)*Cos[2*c]*Gamma[(1 + m)/3, (-2*I)*d*x^3] + b^2*((-I)*d*x^3)^((1 + m)/3)*Cos[2*c]*Gamma[(1 + m)/3, (2*I)*d*x^3] + b^2*m*((-I)*d*x^3)^((1 + m)/3)*Cos[2*c]*Gamma[(1 + m)/3, (2*I)*d*x^3] - I*2^((7 + m)/3)*a*b*(1 + m)*((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*2^((7 + m)/3)*a*b*(1 + m)*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) + I*b^2*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-2*I)*d*x^3]*Sin[2*c] + I*b^2*m*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-2*I)*d*x^3]*Sin[2*c] - I*b^2*((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (2*I)*d*x^3]*Sin[2*c] - I*b^2*m*((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (2*I)*d*x^3]*Sin[2*c]))/(3*(1 + m))

Maple [F]

$$\int (ex)^m \left(a + b\sin\left(d\,x^3 + c\right)\right)^2 dx$$

[In] int((e*x)^m*(a+b*sin(d*x^3+c))^2,x)

[Out] int((e*x)^m*(a+b*sin(d*x^3+c))^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.75

$$\int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)^2 \, dx$$
$$= \frac{12\,(2\,a^2 + b^2)(ex)^m\,dx + (-i\,b^2e^2m - i\,b^2e^2)e^{\left(-\frac{1}{3}\,(m-2)\log\left(\frac{2i\,d}{e^3}\right)-2i\,c\right)}\Gamma\left(\frac{1}{3}\,m + \frac{1}{3}, 2i\,dx^3\right) - 8\,(abe^2m + abe^2)e^{\cdots}}{\cdots}$$

[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")

[Out] 1/24*(12*(2*a^2 + b^2)*(e*x)^m*d*x + (-I*b^2*e^2*m - I*b^2*e^2)*e^(-1/3*(m - 2)*log(2*I*d/e^3) - 2*I*c)*gamma(1/3*m + 1/3, 2*I*d*x^3) - 8*(a*b*e^2*m + a*b*e^2)*e^(-1/3*(m - 2)*log(I*d/e^3) - I*c)*gamma(1/3*m + 1/3, I*d*x^3) - 8*(a*b*e^2*m + a*b*e^2)*e^(-1/3*(m - 2)*log(-I*d/e^3) + I*c)*gamma(1/3*m + 1/3, -I*d*x^3) + (I*b^2*e^2*m + I*b^2*e^2)*e^(-1/3*(m - 2)*log(-2*I*d/e^3) + 2*I*c)*gamma(1/3*m + 1/3, -2*I*d*x^3))/(d*m + d)

Sympy [F]

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^2 \, dx = \int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^2 \, dx$$

```
[In] integrate((e*x)**m*(a+b*sin(d*x**3+c))**2,x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**3))**2, x)
```

Maxima [F]

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^2 \, dx = \int \left(b\sin \left(dx^3+c\right)+a\right)^2 (ex)^m \, dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
[Out] (e*x)^(m + 1)*a^2/(e*(m + 1)) + 1/2*(b^2*e^m*x*x^m - (b^2*e^m*m + b^2*e^m)*
integrate(x^m*cos(2*d*x^3 + 2*c), x) + 4*(a*b*e^m*m + a*b*e^m)*integrate(x^
m*sin(d*x^3 + c), x))/(m + 1)
```

Giac [F]

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^2 \, dx = \int \left(b\sin \left(dx^3+c\right)+a\right)^2 (ex)^m \, dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)^2*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^2 \, dx = \int {\left(e\,x\right)}^m\,{\left(a+b\,\sin \left(d\,x^3+c\right)\right)}^2 \,dx$$

```
[In] int((e*x)^m*(a + b*sin(c + d*x^3))^2,x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^3))^2, x)
```

3.100 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 134

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right) dx = \frac{a(ex)^{1+m}}{e(1+m)}+\frac{ibe^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{6e} - \frac{ibe^{-ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},idx^3\right)}{6e}$$

```
[Out] a*(e*x)^(1+m)/e/(1+m)+1/6*I*b*exp(I*c)*(e*x)^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*
GAMMA(1/3+1/3*m,-I*d*x^3)/e-1/6*I*b*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*m)*GAMM
A(1/3+1/3*m,I*d*x^3)/e/exp(I*c)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {14, 3470, 2250}

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right) dx = \frac{a(ex)^{m+1}}{e(m+1)}+\frac{ibe^{ic}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},-idx^3\right)}{6e} - \frac{ibe^{-ic}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},idx^3\right)}{6e}$$

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*(e*x)^(1 + m))/(e*(1 + m)) + ((I/6)*b*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^3)
^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])/e - ((I/6)*b*(e*x)^(1 + m)*(I*d
*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, I*d*x^3])/(e*E^(I*c))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a(ex)^m + b(ex)^m \sin\left(c+dx^3\right)\right)\,dx \\
&= \frac{a(ex)^{1+m}}{e(1+m)} + b\int (ex)^m \sin\left(c+dx^3\right)\,dx \\
&= \frac{a(ex)^{1+m}}{e(1+m)} + \frac{1}{2}(ib)\int e^{-ic-idx^3}(ex)^m\,dx - \frac{1}{2}(ib)\int e^{ic+idx^3}(ex)^m\,dx \\
&= \frac{a(ex)^{1+m}}{e(1+m)} + \frac{ibe^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{6e} \\
&\quad - \frac{ibe^{-ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},idx^3\right)}{6e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.11

$$\int (ex)^m\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$
$$= \frac{x(ex)^m\left(d^2x^6\right)^{\frac{1}{3}(-1-m)}\left(6a\left(d^2x^6\right)^{\frac{1+m}{3}} - ib(1+m)\left(-idx^3\right)^{\frac{1+m}{3}}\Gamma\left(\frac{1+m}{3},idx^3\right)(\cos(c)-i\sin(c)) + ib(1+m)\right.}{6(1+m)}$$

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (x*(e*x)^m*(d^2*x^6)^((-1 - m)/3)*(6*a*(d^2*x^6)^((1 + m)/3) - I*b*(1 + m)*
((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*
b*(1 + m)*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3]*(Cos[c] + I*Si
n[c])))/(6*(1 + m))
```

Maple [F]

$$\int (ex)^m \left(a + b\sin\left(d\,x^3 + c\right)\right) dx$$

```
[In] int((e*x)^m*(a+b*sin(d*x^3+c)),x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^3+c)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)\,dx = \frac{6\,(ex)^m\,adx - (be^2m + be^2)e^{\left(-\frac{1}{3}\,(m-2)\log\left(\frac{i\,d}{e^3}\right) - i\,c\right)}\Gamma\left(\frac{1}{3}\,m + \frac{1}{3}, i\,dx^3\right) - (be^2m + be^2)e^{\left(-\frac{1}{3}\,(m-2)\log\left(-\frac{i\,d}{e^3}\right) + i\,c\right)}\Gamma(}{6\,(dm + d)}$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(e*x)^m*a*d*x - (b*e^2*m + b*e^2)*e^(-1/3*(m - 2)*log(I*d/e^3) - I*c
)*gamma(1/3*m + 1/3, I*d*x^3) - (b*e^2*m + b*e^2)*e^(-1/3*(m - 2)*log(-I*d/
e^3) + I*c)*gamma(1/3*m + 1/3, -I*d*x^3))/(d*m + d)
```

Sympy [F]

$$\int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)\,dx = \int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)\,dx$$

```
[In] integrate((e*x)**m*(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**3)), x)
```

Maxima [F]

$$\int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)\,dx = \int \left(b\sin\left(dx^3 + c\right) + a\right)(ex)^m\,dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] b*e^m*integrate(x^m*sin(d*x^3 + c), x) + (e*x)^(m + 1)*a/(e*(m + 1))
```

Giac [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int \left(b\sin\left(dx^3+c\right)+a\right)(ex)^m\,dx$$

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int (e\,x)^m\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)\,dx$$

```
[In] int((e*x)^m*(a + b*sin(c + d*x^3)),x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^3)), x)
```

3.101 $\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx = \operatorname{Int}\left(\frac{(ex)^m}{a+b\sin(c+dx^3)},x\right)$$

```
[Out] Unintegrable((e*x)^m/(a+b*sin(d*x^3+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx = \int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx$$

```
[In] Int[(e*x)^m/(a + b*Sin[c + d*x^3]),x]
[Out] Defer[Int][(e*x)^m/(a + b*Sin[c + d*x^3]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx=\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx$$

```
[In] Integrate[(e*x)^m/(a + b*Sin[c + d*x^3]),x]
[Out] Integrate[(e*x)^m/(a + b*Sin[c + d*x^3]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^m}{a+b\sin(d\,x^3+c)}dx$$

```
[In] int((e*x)^m/(a+b*sin(d*x^3+c)),x)
[Out] int((e*x)^m/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx=\int \frac{(ex)^m}{b\sin(dx^3+c)+a}\,dx$$

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral((e*x)^m/(b*sin(d*x^3 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx=\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx$$

```
[In] integrate((e*x)**m/(a+b*sin(d*x**3+c)),x)
[Out] Integral((e*x)**m/(a + b*sin(c + d*x**3)), x)
```

Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx = \int \frac{(ex)^m}{b\sin(dx^3+c)+a}\,dx$$

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate((e*x)^m/(b*sin(d*x^3 + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx = \int \frac{(ex)^m}{b\sin(dx^3+c)+a}\,dx$$

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate((e*x)^m/(b*sin(d*x^3 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 5.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx = \int \frac{(e\,x)^m}{a+b\,\sin(d\,x^3+c)}\,dx$$

```
[In] int((e*x)^m/(a + b*sin(c + d*x^3)),x)
[Out] int((e*x)^m/(a + b*sin(c + d*x^3)), x)
```

3.102 $\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx = \operatorname{Int}\left(\frac{(ex)^m}{(a+b\sin(c+dx^3))^2},x\right)$$

```
[Out] Unintegrable((e*x)^m/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx$$

```
[In] Int[(e*x)^m/(a + b*Sin[c + d*x^3])^2,x]
[Out] Defer[Int][(e*x)^m/(a + b*Sin[c + d*x^3])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx$$

```
[In] Integrate[(e*x)^m/(a + b*Sin[c + d*x^3])^2,x]
[Out] Integrate[(e*x)^m/(a + b*Sin[c + d*x^3])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^m}{(a+b\sin(d\,x^3+c))^2}dx$$

```
[In] int((e*x)^m/(a+b*sin(d*x^3+c))^2,x)
[Out] int((e*x)^m/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{(ex)^m}{(b\sin(dx^3+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
[Out] integral(-(e*x)^m/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2)
, x)
```

Sympy [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx$$

```
[In] integrate((e*x)**m/(a+b*sin(d*x**3+c))**2,x)
[Out] Integral((e*x)**m/(a + b*sin(c + d*x**3))**2, x)
```

Maxima [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 2505, normalized size of antiderivative = 125.25

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{(ex)^m}{(b\sin(dx^3+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
[Out] 1/3*(4*a*b*e^m*x^m*cos(d*x^3)*cos(c) + 2*b^2*e^m*x^m*cos(2*c)*sin(2*d*x^3)
+ 2*b^2*e^m*x^m*cos(2*d*x^3)*sin(2*c) - 4*a*b*e^m*x^m*sin(d*x^3)*sin(c) + 2
*(a*b*e^m*x^m*cos(2*d*x^3)*cos(2*c) - 2*a^2*e^m*x^m*cos(c)*sin(d*x^3) - a*b
*e^m*x^m*sin(2*d*x^3)*sin(2*c) - 2*a^2*e^m*x^m*cos(d*x^3)*sin(c) - a*b*e^m*
x^m)*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(
2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2
)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b
^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(c)*sin(d
*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^2*sin(d
*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^2
 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))
*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^2*cos(2*c) - 2*((a^3*b - a*b^3)*cos
(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^2*sin(d*x^3))*cos(2*d*x
^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(
c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)
*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^2*sin(2*c))*sin(2*
d*x^3))*integrate(2/3*((b^2*e^m*m*sin(2*c) - 2*b^2*e^m*sin(2*c))*x^m*cos(2*
d*x^3) + 2*(a*b*e^m*m*cos(c) - 2*a*b*e^m*cos(c))*x^m*cos(d*x^3) + (b^2*e^m*
m*cos(2*c) - 2*b^2*e^m*cos(2*c))*x^m*sin(2*d*x^3) - 2*(a*b*e^m*m*sin(c) - 2
*a*b*e^m*sin(c))*x^m*sin(d*x^3) - ((3*a*b*d*e^m*x^3*sin(2*c) - a*b*e^m*m*co
s(2*c) + 2*a*b*e^m*cos(2*c))*x^m*cos(2*d*x^3) + 2*(3*a^2*d*e^m*x^3*cos(c) +
 a^2*e^m*m*sin(c) - 2*a^2*e^m*sin(c))*x^m*cos(d*x^3) + (3*a*b*d*e^m*x^3*cos
```

```
(2*c) + a*b*e^m*m*sin(2*c) - 2*a*b*e^m*sin(2*c))*x^m*sin(2*d*x^3) - 2*(3*a^
2*d*e^m*x^3*sin(c) - a^2*e^m*m*cos(c) + 2*a^2*e^m*cos(c))*x^m*sin(d*x^3) +
(a*b*e^m*m - 2*a*b*e^m)*x^m)*cos(d*x^3 + c) - (3*a*b*d*e^m*x^3*x^m - (3*a*b
*d*e^m*x^3*cos(2*c) + a*b*e^m*m*sin(2*c) - 2*a*b*e^m*sin(2*c))*x^m*cos(2*d*
x^3) + 2*(3*a^2*d*e^m*x^3*sin(c) - a^2*e^m*m*cos(c) + 2*a^2*e^m*cos(c))*x^m
*cos(d*x^3) + (3*a*b*d*e^m*x^3*sin(2*c) - a*b*e^m*m*cos(2*c) + 2*a*b*e^m*co
s(2*c))*x^m*sin(2*d*x^3) + 2*(3*a^2*d*e^m*x^3*cos(c) + a^2*e^m*m*sin(c) - 2
*a^2*e^m*sin(c))*x^m*sin(d*x^3))*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)
^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^3*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*
cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^3*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*
cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^3*sin(2*d*x^3)^2 + 4*(a^3*b -
a*b^3)*d*x^3*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b
^2)*sin(c)^2)*d*x^3*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(d*x^3)*sin(c
) + (a^2*b^2 - b^4)*d*x^3 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b
- a*b^3)*cos(2*c)*sin(c))*d*x^3*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^3*cos(2*c)
 - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*
x^3*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3
*b - a*b^3)*sin(2*c)*sin(c))*d*x^3*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*s
in(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^3*sin(d*x^3) + (a^2*b^2 - b^
4)*d*x^3*sin(2*c))*sin(2*d*x^3)), x) + 2*(2*a^2*e^m*x^m*cos(d*x^3)*cos(c) +
 a*b*e^m*x^m*cos(2*c)*sin(2*d*x^3) + a*b*e^m*x^m*cos(2*d*x^3)*sin(2*c) - 2*
a^2*e^m*x^m*sin(d*x^3)*sin(c))*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2
 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*co
s(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*co
s(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4*(a^3*b - a*
b^3)*d*x^2*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2
)*sin(c)^2)*d*x^2*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*x^3)*sin(c)
+ (a^2*b^2 - b^4)*d*x^2 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b -
a*b^3)*cos(2*c)*sin(c))*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^2*cos(2*c) -
 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^
2*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b
 - a*b^3)*sin(2*c)*sin(c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin
(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^2*b^2 - b^4)
*d*x^2*sin(2*c))*sin(2*d*x^3))
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{(a + b\sin(c + dx^3))^2} \, dx = \int \frac{(ex)^m}{(b\sin(dx^3 + c) + a)^2} \, dx$$

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/(b*sin(d*x^3 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx=\int \frac{(e\,x)^m}{\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

```
[In] int((e*x)^m/(a + b*sin(c + d*x^3))^2,x)
[Out] int((e*x)^m/(a + b*sin(c + d*x^3))^2, x)
```

3.103 $\int x^2 \sin\left(a+\frac{b}{x}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int x^2 \sin\left(a+\frac{b}{x}\right)\,dx = \frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right)+\frac{1}{6}b^3\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right) - \frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)-\frac{1}{6}b^3\sin(a)\text{Si}\left(\frac{b}{x}\right)$$

```
[Out] 1/6*b^3*Ci(b/x)*cos(a)+1/6*b*x^2*cos(a+b/x)-1/6*b^3*Si(b/x)*sin(a)-1/6*b^2*
x*sin(a+b/x)+1/3*x^3*sin(a+b/x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3460, 3378, 3384, 3380, 3383}

$$\int x^2 \sin\left(a+\frac{b}{x}\right)\,dx = \frac{1}{6}b^3\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right)-\frac{1}{6}b^3\sin(a)\text{Si}\left(\frac{b}{x}\right) - \frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)+\frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right)$$

```
[In] Int[x^2*Sin[a + b/x],x]
```

```
[Out] (b*x^2*Cos[a + b/x])/6 + (b^3*Cos[a]*CosIntegral[b/x])/6 - (b^2*x*Sin[a + b
/x])/6 + (x^3*Sin[a + b/x])/3 - (b^3*Sin[a]*SinIntegral[b/x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sin(a+bx)}{x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right) - \frac{1}{3}b\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right) + \frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right) + \frac{1}{6}b^2\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right) - \frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right) \\
&\quad + \frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right) + \frac{1}{6}b^3\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= \frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right) - \frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right) + \frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right) + \frac{1}{6}\left(b^3\cos(a)\right)\text{Subst}\left(\int\frac{\cos(bx)}{x}\,dx, x, \frac{1}{x}\right) - \frac{1}{6}\left(b^3\sin(a)\right)\text{Subst}\left(\int\frac{\sin(bx)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right) + \frac{1}{6}b^3\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right) - \frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right) + \frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right) - \frac{1}{6}b^3\sin(a)\text{Si}\left(\frac{b}{x}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int x^2\sin\left(a+\frac{b}{x}\right)\,dx = \frac{1}{6}\left(b^3\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right) + x\left(bx\cos\left(a+\frac{b}{x}\right) - b^2\sin\left(a+\frac{b}{x}\right) + 2x^2\sin\left(a+\frac{b}{x}\right)\right) - b^3\sin(a)\text{Si}\left(\frac{b}{x}\right)\right)$$

```
[In] Integrate[x^2*Sin[a + b/x],x]
[Out] (b^3*Cos[a]*CosIntegral[b/x] + x*(b*x*Cos[a + b/x] - b^2*Sin[a + b/x] + 2*x
^2*Sin[a + b/x]) - b^3*Sin[a]*SinIntegral[b/x])/6
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-b^3\left(-\frac{\sin\left(a+\frac{b}{x}\right)x^3}{3b^3}-\frac{\cos\left(a+\frac{b}{x}\right)x^2}{6b^2}+\frac{\sin\left(a+\frac{b}{x}\right)x}{6b}+\frac{\mathrm{Si}\left(\frac{b}{x}\right)\sin(a)}{6}-\frac{\mathrm{Ci}\left(\frac{b}{x}\right)\cos(a)}{6}\right)$
default	$-b^3\left(-\frac{\sin\left(a+\frac{b}{x}\right)x^3}{3b^3}-\frac{\cos\left(a+\frac{b}{x}\right)x^2}{6b^2}+\frac{\sin\left(a+\frac{b}{x}\right)x}{6b}+\frac{\mathrm{Si}\left(\frac{b}{x}\right)\sin(a)}{6}-\frac{\mathrm{Ci}\left(\frac{b}{x}\right)\cos(a)}{6}\right)$
risch	$\frac{i\mathrm{e}^{-ia}\pi\ \mathrm{csgn}\left(\frac{b}{x}\right)b^3}{12}-\frac{i\mathrm{e}^{-ia}\,\mathrm{Si}\left(\frac{b}{x}\right)b^3}{6}-\frac{\mathrm{e}^{-ia}\,\mathrm{Ei}_1\left(-\frac{ib}{x}\right)b^3}{12}-\frac{\mathrm{e}^{ia}\,\mathrm{Ei}_1\left(-\frac{ib}{x}\right)b^3}{12}+\frac{x^2b\cos\left(\frac{ax+b}{x}\right)}{6}-\frac{\sin\left(\frac{ax+b}{x}\right)b^2}{6}$
parts	$-b\,x^2\,\mathrm{Ci}\left(\frac{b}{x}\right)\cos(a)+b\,x^2\,\mathrm{Si}\left(\frac{b}{x}\right)\sin(a)+x^3\sin\left(a+\frac{b}{x}\right)+2b\Big(-\cos(a)\,b^2\Big(-\frac{x^2\,\mathrm{Ci}\left(\frac{b}{x}\right)}{2b^2}$
meijerg	$-\frac{b^3\sqrt{\pi}\cos(a)\Big(-\frac{8x^2}{\sqrt{\pi}\,b^2}-\frac{4\left(2\gamma-\frac{11}{3}-2\ln(x)+2\ln(b)\right)}{3\sqrt{\pi}}+\frac{8x^2\left(-\frac{55b^2}{2x^2}+45\right)}{45\sqrt{\pi}\,b^2}+\frac{8\gamma}{3\sqrt{\pi}}+\frac{8\ln(2)}{3\sqrt{\pi}}+\frac{8\ln\left(\frac{b}{2x}\right)}{3\sqrt{\pi}}-\frac{8x^2\cos\left(\frac{b}{x}\right)}{3\sqrt{\pi}\,b^2}-\frac{16x^3\big(-}{}}{16}$

```
[In] int(x^2*sin(a+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] -b^3*(-1/3*sin(a+b/x)/b^3*x^3-1/6*cos(a+b/x)/b^2*x^2+1/6*sin(a+b/x)/b*x+1/6
*Si(b/x)*sin(a)-1/6*Ci(b/x)*cos(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int x^2\sin\left(a+\frac{b}{x}\right)\,dx=\frac{1}{6}\,b^3\cos(a)\,\mathrm{Ci}\left(\frac{b}{x}\right)-\frac{1}{6}\,b^3\sin(a)\,\mathrm{Si}\left(\frac{b}{x}\right)+\frac{1}{6}\,bx^2\cos\left(\frac{ax+b}{x}\right)-\frac{1}{6}\,(b^2x-2\,x^3)\sin\left(\frac{ax+b}{x}\right)$$

```
[In] integrate(x^2*sin(a+b/x),x, algorithm="fricas")
```

```
[Out] 1/6*b^3*cos(a)*cos_integral(b/x) - 1/6*b^3*sin(a)*sin_integral(b/x) + 1/6*b
*x^2*cos((a*x + b)/x) - 1/6*(b^2*x - 2*x^3)*sin((a*x + b)/x)
```

Sympy [F]

$$\int x^2 \sin\left(a+\frac{b}{x}\right)\,dx = \int x^2 \sin\left(a+\frac{b}{x}\right)\,dx$$

```
[In] integrate(x**2*sin(a+b/x),x)
[Out] Integral(x**2*sin(a + b/x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int x^2 \sin\left(a+\frac{b}{x}\right)\,dx$$
$$= \frac{1}{12}\left(\left(\mathrm{Ei}\left(\frac{i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\cos\left(a\right)+\left(i\,\mathrm{Ei}\left(\frac{i\,b}{x}\right)-i\,\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\sin\left(a\right)\right)b^3 + \frac{1}{6}\,bx^2\cos\left(\frac{ax+b}{x}\right)-\frac{1}{6}\,(b^2x-2\,x^3)\sin\left(\frac{ax+b}{x}\right)$$

```
[In] integrate(x^2*sin(a+b/x),x, algorithm="maxima")
[Out] 1/12*((Ei(I*b/x) + Ei(-I*b/x))*cos(a) + (I*Ei(I*b/x) - I*Ei(-I*b/x))*sin(a)
)*b^3 + 1/6*b*x^2*cos((a*x + b)/x) - 1/6*(b^2*x - 2*x^3)*sin((a*x + b)/x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(68) = 136.

Time = 0.30 (sec) , antiderivative size = 400, normalized size of antiderivative = 5.13

$$\int x^2 \sin\left(a+\frac{b}{x}\right)\,dx$$
$$= \frac{a^3b^4\cos\left(a\right)\mathrm{Ci}\left(-a+\frac{ax+b}{x}\right)+a^3b^4\sin\left(a\right)\mathrm{Si}\left(a-\frac{ax+b}{x}\right)-\frac{3\,(ax+b)a^2b^4\cos(a)\,\mathrm{Ci}\left(-a+\frac{ax+b}{x}\right)}{x}-\frac{3\,(ax+b)a^2b^4\sin(a)\,\mathrm{Si}\left(a-\right.}{x}}{}$$

```
[In] integrate(x^2*sin(a+b/x),x, algorithm="giac")
[Out] 1/6*(a^3*b^4*cos(a)*cos_integral(-a + (a*x + b)/x) + a^3*b^4*sin(a)*sin_int
egral(a - (a*x + b)/x) - 3*(a*x + b)*a^2*b^4*cos(a)*cos_integral(-a + (a*x
+ b)/x)/x - 3*(a*x + b)*a^2*b^4*sin(a)*sin_integral(a - (a*x + b)/x)/x + 3*
```

```
(a*x + b)^2*a*b^4*cos(a)*cos_integral(-a + (a*x + b)/x)/x^2 + a^2*b^4*sin((
a*x + b)/x) + 3*(a*x + b)^2*a*b^4*sin(a)*sin_integral(a - (a*x + b)/x)/x^2
+ a*b^4*cos((a*x + b)/x) - (a*x + b)^3*b^4*cos(a)*cos_integral(-a + (a*x +
b)/x)/x^3 - 2*(a*x + b)*a*b^4*sin((a*x + b)/x)/x - (a*x + b)^3*b^4*sin(a)*s
in_integral(a - (a*x + b)/x)/x^3 - (a*x + b)*b^4*cos((a*x + b)/x)/x - 2*b^4
*sin((a*x + b)/x) + (a*x + b)^2*b^4*sin((a*x + b)/x)/x^2)/((a^3 - 3*(a*x +
b)*a^2/x + 3*(a*x + b)^2*a/x^2 - (a*x + b)^3/x^3)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sin\left(a+\frac{b}{x}\right)\,dx = \int x^2\,\sin\left(a+\frac{b}{x}\right)\,dx$$

```
[In] int(x^2*sin(a + b/x),x)
[Out] int(x^2*sin(a + b/x), x)
```

3.104 $\int x \sin\left(a+\frac{b}{x}\right) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int x \sin\left(a+\frac{b}{x}\right) \, dx = \frac{1}{2}bx\cos\left(a+\frac{b}{x}\right)+\frac{1}{2}b^2\operatorname{CosIntegral}\left(\frac{b}{x}\right)\sin(a) +\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)+\frac{1}{2}b^2\cos(a)\operatorname{Si}\left(\frac{b}{x}\right)$$

```
[Out] 1/2*b*x*cos(a+b/x)+1/2*b^2*cos(a)*Si(b/x)+1/2*b^2*Ci(b/x)*sin(a)+1/2*x^2*si
n(a+b/x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3460, 3378, 3384, 3380, 3383}

$$\int x \sin\left(a+\frac{b}{x}\right) \, dx = \frac{1}{2}b^2\sin(a)\operatorname{CosIntegral}\left(\frac{b}{x}\right)+\frac{1}{2}b^2\cos(a)\operatorname{Si}\left(\frac{b}{x}\right) +\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\cos\left(a+\frac{b}{x}\right)$$

```
[In] Int[x*Sin[a + b/x],x]
```

```
[Out] (b*x*Cos[a + b/x])/2 + (b^2*CosIntegral[b/x]*Sin[a])/2 + (x^2*Sin[a + b/x])
/2 + (b^2*Cos[a]*SinIntegral[b/x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right) - \frac{1}{2}b\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}bx\cos\left(a+\frac{b}{x}\right) + \frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right) + \frac{1}{2}b^2\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}bx\cos\left(a+\frac{b}{x}\right) + \frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right) + \frac{1}{2}\left(b^2\cos(a)\right)\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&\quad + \frac{1}{2}\left(b^2\sin(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}bx\cos\left(a+\frac{b}{x}\right) + \frac{1}{2}b^2\,\text{CosIntegral}\left(\frac{b}{x}\right)\sin(a) + \frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right) + \frac{1}{2}b^2\cos(a)\text{Si}\left(\frac{b}{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x\sin\left(a+\frac{b}{x}\right)\,dx=\frac{1}{2}\left(b^2\,\text{CosIntegral}\left(\frac{b}{x}\right)\sin(a)+x\left(b\cos\left(a+\frac{b}{x}\right)+x\sin\left(a+\frac{b}{x}\right)\right)+b^2\cos(a)\text{Si}\left(\frac{b}{x}\right)\right)$$

```
[In] Integrate[x*Sin[a + b/x],x]
```

```
[Out] (b^2*CosIntegral[b/x]*Sin[a] + x*(b*Cos[a + b/x] + x*Sin[a + b/x]) + b^2*Co
s[a]*SinIntegral[b/x])/2
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-b^2\left(-\frac{\sin\left(a+\frac{b}{x}\right)x^2}{2b^2}-\frac{\cos\left(a+\frac{b}{x}\right)x}{2b}-\frac{\cos(a)\,\text{Si}\left(\frac{b}{x}\right)}{2}-\frac{\text{Ci}\left(\frac{b}{x}\right)\sin(a)}{2}\right)$
default	$-b^2\left(-\frac{\sin\left(a+\frac{b}{x}\right)x^2}{2b^2}-\frac{\cos\left(a+\frac{b}{x}\right)x}{2b}-\frac{\cos(a)\,\text{Si}\left(\frac{b}{x}\right)}{2}-\frac{\text{Ci}\left(\frac{b}{x}\right)\sin(a)}{2}\right)$
risch	$-\frac{\pi\,\text{csgn}\left(\frac{b}{x}\right)e^{-ia}b^2}{4}+\frac{\text{Si}\left(\frac{b}{x}\right)e^{-ia}b^2}{2}-\frac{ie^{-ia}\,\text{Ei}_1\left(-\frac{ib}{x}\right)b^2}{4}+\frac{ib^2\,\text{Ei}_1\left(-\frac{ib}{x}\right)e^{ia}}{4}+\frac{bx\cos\left(\frac{ax+b}{x}\right)}{2}+\frac{x^2\sin\left(\frac{ax+b}{x}\right)}{2}$ …
parts	$-bx\ \text{Ci}\left(\frac{b}{x}\right)\cos(a)+bx\ \text{Si}\left(\frac{b}{x}\right)\sin(a)+x^2\sin\left(a+\frac{b}{x}\right)+b\left(-\cos(a)\,b\left(-\frac{x\ \text{Ci}\left(\frac{b}{x}\right)}{b}-\cdots\right.\right.$ …
meijerg	$-\frac{b^2\sqrt{\pi}\,\cos(a)\left(-\frac{4x\cos\left(\frac{b}{x}\right)}{b\sqrt{\pi}}-\frac{4x^2\sin\left(\frac{b}{x}\right)}{b^2\sqrt{\pi}}-\frac{4\ \text{Si}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{8}-\frac{b^2\sqrt{\pi}\,\sin(a)\left(-\frac{4x^2}{\sqrt{\pi}\,b^2}-\frac{2\left(2\gamma-3-2\ln(x)+\ln\left(b^2\right)\right)}{\sqrt{\pi}}+\frac{4x^2\left(-\frac{9b^2}{2x^2}+\cdots\right)}{3\sqrt{\pi}\,b^2}\cdots\right)}{\cdots}$ …

```
[In] int(x*sin(a+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*(-1/2*sin(a+b/x)/b^2*x^2-1/2*cos(a+b/x)/b*x-1/2*cos(a)*Si(b/x)-1/2*Ci(
b/x)*sin(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int x\sin\left(a+\frac{b}{x}\right)\,dx=\frac{1}{2}\,b^2\operatorname{Ci}\left(\frac{b}{x}\right)\sin\left(a\right)+\frac{1}{2}\,b^2\cos\left(a\right)\operatorname{Si}\left(\frac{b}{x}\right)+\frac{1}{2}\,bx\cos\left(\frac{ax+b}{x}\right)+\frac{1}{2}\,x^2\sin\left(\frac{ax+b}{x}\right)$$

```
[In] integrate(x*sin(a+b/x),x, algorithm="fricas")
[Out] 1/2*b^2*cos_integral(b/x)*sin(a) + 1/2*b^2*cos(a)*sin_integral(b/x) + 1/2*b
*x*cos((a*x + b)/x) + 1/2*x^2*sin((a*x + b)/x)
```

Sympy [F]

$$\int x\sin\left(a+\frac{b}{x}\right)\,dx=\int x\sin\left(a+\frac{b}{x}\right)\,dx$$

```
[In] integrate(x*sin(a+b/x),x)
[Out] Integral(x*sin(a + b/x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.27

$$\int x\sin\left(a+\frac{b}{x}\right)\,dx=\frac{1}{4}\left(\left(-i\operatorname{Ei}\left(\frac{i\,b}{x}\right)+i\operatorname{Ei}\left(-\frac{i\,b}{x}\right)\right)\cos\left(a\right)+\left(\operatorname{Ei}\left(\frac{i\,b}{x}\right)+\operatorname{Ei}\left(-\frac{i\,b}{x}\right)\right)\sin\left(a\right)\right)b^2+\frac{1}{2}\,bx\cos\left(\frac{ax+b}{x}\right)+\frac{1}{2}\,x^2\sin\left(\frac{ax+b}{x}\right)$$

```
[In] integrate(x*sin(a+b/x),x, algorithm="maxima")
[Out] 1/4*((-I*Ei(I*b/x) + I*Ei(-I*b/x))*cos(a) + (Ei(I*b/x) + Ei(-I*b/x))*sin(a)
)*b^2 + 1/2*b*x*cos((a*x + b)/x) + 1/2*x^2*sin((a*x + b)/x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(52) = 104$.

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 4.18

$$\int x \sin\left(a + \frac{b}{x}\right) dx$$
$$= \frac{a^2b^3 \operatorname{Ci}\left(-a + \frac{ax+b}{x}\right) \sin(a) - a^2b^3 \cos(a) \operatorname{Si}\left(a - \frac{ax+b}{x}\right) - \frac{2(ax+b)ab^3 \operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)\sin(a)}{x} + \frac{2(ax+b)ab^3\cos(a)\operatorname{Si}\left(a-\frac{ax}{}\right.}{x}}{2\left(a^2 - \frac{2(ax+}{x}\right.}$$

```
[In] integrate(x*sin(a+b/x),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*b^3*cos_integral(-a + (a*x + b)/x)*sin(a) - a^2*b^3*cos(a)*sin_int
egral(a - (a*x + b)/x) - 2*(a*x + b)*a*b^3*cos_integral(-a + (a*x + b)/x)*s
in(a)/x + 2*(a*x + b)*a*b^3*cos(a)*sin_integral(a - (a*x + b)/x)/x - a*b^3*
cos((a*x + b)/x) + (a*x + b)^2*b^3*cos_integral(-a + (a*x + b)/x)*sin(a)/x^
2 - (a*x + b)^2*b^3*cos(a)*sin_integral(a - (a*x + b)/x)/x^2 + (a*x + b)*b^
3*cos((a*x + b)/x)/x + b^3*sin((a*x + b)/x))/((a^2 - 2*(a*x + b)*a/x + (a*x
 + b)^2/x^2)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x \sin\left(a + \frac{b}{x}\right) dx = \int x \sin\left(a + \frac{b}{x}\right) dx$$

```
[In] int(x*sin(a + b/x),x)
```

```
[Out] int(x*sin(a + b/x), x)
```

3.105 $\int \sin\left(a+\frac{b}{x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 32

$$\int \sin\left(a+\frac{b}{x}\right)\,dx=-b\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right)+x\sin\left(a+\frac{b}{x}\right)+b\sin(a)\text{Si}\left(\frac{b}{x}\right)$$

```
[Out] -b*Ci(b/x)*cos(a)+b*Si(b/x)*sin(a)+x*sin(a+b/x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3442, 3378, 3384, 3380, 3383}

$$\int \sin\left(a+\frac{b}{x}\right)\,dx=-b\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right)+b\sin(a)\text{Si}\left(\frac{b}{x}\right)+x\sin\left(a+\frac{b}{x}\right)$$

```
[In] Int[Sin[a + b/x],x]
[Out] -(b*Cos[a]*CosIntegral[b/x]) + x*Sin[a + b/x] + b*Sin[a]*SinIntegral[b/x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= x\sin\left(a+\frac{b}{x}\right) - b\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, \frac{1}{x}\right) \\
&= x\sin\left(a+\frac{b}{x}\right) - (b\cos(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{x}\right) \\
&\quad + (b\sin(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{x}\right) \\
&= -b\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right) + x\sin\left(a+\frac{b}{x}\right) + b\sin(a)\text{Si}\left(\frac{b}{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sin\left(a+\frac{b}{x}\right)\,dx = -b\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right) + x\sin\left(a+\frac{b}{x}\right) + b\sin(a)\text{Si}\left(\frac{b}{x}\right)$$

```
[In] Integrate[Sin[a + b/x],x]
[Out] -(b*Cos[a]*CosIntegral[b/x]) + x*Sin[a + b/x] + b*Sin[a]*SinIntegral[b/x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

method	result
derivativedivides	$-b\left(-\frac{\sin\left(a+\frac{b}{x}\right)x}{b}-\operatorname{Si}\left(\frac{b}{x}\right)\sin\left(a\right)+\operatorname{Ci}\left(\frac{b}{x}\right)\cos\left(a\right)\right)$
default	$-b\left(-\frac{\sin\left(a+\frac{b}{x}\right)x}{b}-\operatorname{Si}\left(\frac{b}{x}\right)\sin\left(a\right)+\operatorname{Ci}\left(\frac{b}{x}\right)\cos\left(a\right)\right)$
risch	$\frac{e^{ia}\operatorname{Ei}_1\left(-\frac{ib}{x}\right)b}{2}-\frac{i\pi\ \operatorname{csgn}\left(\frac{b}{x}\right)e^{-ia}b}{2}+i\operatorname{Si}\left(\frac{b}{x}\right)e^{-ia}b+\frac{\operatorname{Ei}_1\left(-\frac{ib}{x}\right)e^{-ia}b}{2}+x\sin\left(\frac{ax+b}{x}\right)$
meijerg	$-\frac{\sqrt{\pi}\ \cos(a)b\left(\frac{4\gamma-4-4\ln(x)+4\ln(b)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{b}{2x}\right)}{\sqrt{\pi}}-\frac{4x\sin\left(\frac{b}{x}\right)}{\sqrt{\pi}\,b}+\frac{4\ \operatorname{Ci}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{4}-\frac{\sin(a)\sqrt{\pi}\,\sqrt{b^2}\left(-\frac{4x\,b^2\,\mathrm{c}}{(b^2}\right.}{4}$

```
[In] int(sin(a+b/x),x,method=_RETURNVERBOSE)
[Out] -b*(-sin(a+b/x)/b*x-Si(b/x)*sin(a)+Ci(b/x)*cos(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int\sin\left(a+\frac{b}{x}\right)\,dx=-b\cos\left(a\right)\operatorname{Ci}\left(\frac{b}{x}\right)+b\sin\left(a\right)\operatorname{Si}\left(\frac{b}{x}\right)+x\sin\left(\frac{ax+b}{x}\right)$$

```
[In] integrate(sin(a+b/x),x, algorithm="fricas")
[Out] -b*cos(a)*cos_integral(b/x) + b*sin(a)*sin_integral(b/x) + x*sin((a*x + b)/
x)
```

Sympy [F]

$$\int\sin\left(a+\frac{b}{x}\right)\,dx=\int\sin\left(a+\frac{b}{x}\right)\,dx$$

```
[In] integrate(sin(a+b/x),x)
[Out] Integral(sin(a + b/x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.81

$$\int \sin\left(a+\frac{b}{x}\right)\,dx = -\frac{1}{2}\left(\left(\operatorname{Ei}\left(\frac{i\,b}{x}\right)+\operatorname{Ei}\left(-\frac{i\,b}{x}\right)\right)\cos\left(a\right)-\left(-i\operatorname{Ei}\left(\frac{i\,b}{x}\right)+i\operatorname{Ei}\left(-\frac{i\,b}{x}\right)\right)\sin\left(a\right)\right)b + x\sin\left(\frac{ax+b}{x}\right)$$

```
[In] integrate(sin(a+b/x),x, algorithm="maxima")
```

```
[Out] -1/2*((Ei(I*b/x) + Ei(-I*b/x))*cos(a) - (-I*Ei(I*b/x) + I*Ei(-I*b/x))*sin(a
))*b + x*sin((a*x + b)/x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(32) = 64.

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.12

$$\int \sin\left(a+\frac{b}{x}\right)\,dx = -\frac{ab^2\cos\left(a\right)\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)+ab^2\sin\left(a\right)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)-\frac{(ax+b)b^2\cos(a)\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)}{x}-\frac{(ax+b)b^2\sin(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)}{x}}{\left(a-\frac{ax+b}{x}\right)b}$$

```
[In] integrate(sin(a+b/x),x, algorithm="giac")
```

```
[Out] -(a*b^2*cos(a)*cos_integral(-a + (a*x + b)/x) + a*b^2*sin(a)*sin_integral(a
 - (a*x + b)/x) - (a*x + b)*b^2*cos(a)*cos_integral(-a + (a*x + b)/x)/x - (
a*x + b)*b^2*sin(a)*sin_integral(a - (a*x + b)/x)/x + b^2*sin((a*x + b)/x))
/((a - (a*x + b)/x)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+\frac{b}{x}\right)\,dx = \int \sin\left(a+\frac{b}{x}\right)\,dx$$

```
[In] int(sin(a + b/x),x)
```

```
[Out] int(sin(a + b/x), x)
```

3.106 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x} \, dx = -\operatorname{CosIntegral}\left(\frac{b}{x}\right)\sin(a) - \cos(a)\operatorname{Si}\left(\frac{b}{x}\right)$$

```
[Out] -cos(a)*Si(b/x)-Ci(b/x)*sin(a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3458, 3457, 3456}

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x} \, dx = \sin(a)\left(-\operatorname{CosIntegral}\left(\frac{b}{x}\right)\right) - \cos(a)\operatorname{Si}\left(\frac{b}{x}\right)$$

```
[In] Int[Sin[a + b/x]/x,x]
[Out] -(CosIntegral[b/x]*Sin[a]) - Cos[a]*SinIntegral[b/x]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cos(a)\int \frac{\sin\left(\frac{b}{x}\right)}{x}\,dx + \sin(a)\int \frac{\cos\left(\frac{b}{x}\right)}{x}\,dx\\ &= -\,\text{CosIntegral}\left(\frac{b}{x}\right)\sin(a) - \cos(a)\text{Si}\left(\frac{b}{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx = -\,\text{CosIntegral}\left(\frac{b}{x}\right)\sin(a) - \cos(a)\text{Si}\left(\frac{b}{x}\right)$$

```
[In] Integrate[Sin[a + b/x]/x,x]
[Out] -(CosIntegral[b/x]*Sin[a]) - Cos[a]*SinIntegral[b/x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\cos(a)\,\text{Si}\left(\frac{b}{x}\right)-\text{Ci}\left(\frac{b}{x}\right)\sin(a)$	22
default	$-\cos(a)\,\text{Si}\left(\frac{b}{x}\right)-\text{Ci}\left(\frac{b}{x}\right)\sin(a)$	22
risch	$-\frac{i e^{ia}\,\text{Ei}_1\left(-\frac{ib}{x}\right)}{2}+\frac{\pi\,\text{csgn}\left(\frac{b}{x}\right)\text{e}^{-ia}}{2}-\text{Si}\left(\frac{b}{x}\right)\text{e}^{-ia}+\frac{i\,\text{Ei}_1\left(-\frac{ib}{x}\right)\text{e}^{-ia}}{2}$	63
meijerg	$-\cos(a)\,\text{Si}\left(\frac{b}{x}\right)-\frac{\sqrt{\pi}\,\sin(a)\left(\frac{2\gamma-2\ln(x)+\ln\left(b^2\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{b}{2x}\right)}{\sqrt{\pi}}+\frac{2\,\text{Ci}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{2}$	72

```
[In] int(sin(a+b/x)/x,x,method=_RETURNVERBOSE)
[Out] -cos(a)*Si(b/x)-Ci(b/x)*sin(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx = -\operatorname{Ci}\left(\frac{b}{x}\right)\sin\left(a\right) - \cos\left(a\right)\operatorname{Si}\left(\frac{b}{x}\right)$$

```
[In] integrate(sin(a+b/x)/x,x, algorithm="fricas")
[Out] -cos_integral(b/x)*sin(a) - cos(a)*sin_integral(b/x)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx = -\sin\left(a\right)\operatorname{Ci}\left(\frac{b}{x}\right) - \cos\left(a\right)\operatorname{Si}\left(\frac{b}{x}\right)$$

```
[In] integrate(sin(a+b/x)/x,x)
[Out] -sin(a)*Ci(b/x) - cos(a)*Si(b/x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.05

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx = \frac{1}{2}\left(i\operatorname{Ei}\left(\frac{ib}{x}\right) - i\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\cos\left(a\right) - \frac{1}{2}\left(\operatorname{Ei}\left(\frac{ib}{x}\right) + \operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\sin\left(a\right)$$

```
[In] integrate(sin(a+b/x)/x,x, algorithm="maxima")
[Out] 1/2*(I*Ei(I*b/x) - I*Ei(-I*b/x))*cos(a) - 1/2*(Ei(I*b/x) + Ei(-I*b/x))*sin(
a)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx = -\frac{b\,\mathrm{Ci}\left(-a+\frac{ax+b}{x}\right)\sin\left(a\right) - b\cos\left(a\right)\mathrm{Si}\left(a-\frac{ax+b}{x}\right)}{b}$$

```
[In] integrate(sin(a+b/x)/x,x, algorithm="giac")
[Out] -(b*cos_integral(-a + (a*x + b)/x)*sin(a) - b*cos(a)*sin_integral(a - (a*x
+ b)/x))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx = -\sin\left(a\right)\,\mathrm{cosint}\left(\frac{b}{x}\right) - \cos\left(a\right)\,\mathrm{sinint}\left(\frac{b}{x}\right)$$

```
[In] int(sin(a + b/x)/x,x)
[Out] - sin(a)*cosint(b/x) - cos(a)*sinint(b/x)
```

3.107 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx=\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

```
[Out] cos(a+b/x)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3460, 2718}

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx=\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

```
[In] Int[Sin[a + b/x]/x^2,x]
```

```
[Out] Cos[a + b/x]/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
```

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{x}\right)\\ &= \frac{\cos\left(a+\frac{b}{x}\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx = \frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

```
[In] Integrate[Sin[a + b/x]/x^2,x]
[Out] Cos[a + b/x]/b
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$	13
default	$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$	13
risch	$\frac{\cos\left(\frac{ax+b}{x}\right)}{b}$	15
parallelrisch	$\frac{-1+\cos\left(\frac{ax+b}{x}\right)}{b}$	17
norman	$\frac{2}{b\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}$	23
meijerg	$-\frac{\sqrt{\pi}\,\cos(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{b}-\frac{\sin(a)\sin\left(\frac{b}{x}\right)}{b}$	40

```
[In] int(sin(a+b/x)/x^2,x,method=_RETURNVERBOSE)
[Out] cos(a+b/x)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx = \frac{\cos\left(\frac{ax+b}{x}\right)}{b}$$

```
[In] integrate(sin(a+b/x)/x^2,x, algorithm="fricas")
[Out] cos((a*x + b)/x)/b
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx = \begin{cases} \frac{\cos\left(a+\frac{b}{x}\right)}{b} & \text{for } b \neq 0 \\ -\frac{\sin(a)}{x} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(a+b/x)/x**2,x)
[Out] Piecewise((cos(a + b/x)/b, Ne(b, 0)), (-sin(a)/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx = \frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

```
[In] integrate(sin(a+b/x)/x^2,x, algorithm="maxima")
[Out] cos(a + b/x)/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx = \frac{\cos\left(\frac{ax+b}{x}\right)}{b}$$

```
[In] integrate(sin(a+b/x)/x^2,x, algorithm="giac")
[Out] cos((a*x + b)/x)/b
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx = \frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

```
[In] int(sin(a + b/x)/x^2,x)
[Out] cos(a + b/x)/b
```

3.108 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3} \, dx = \frac{\cos\left(a+\frac{b}{x}\right)}{bx} - \frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

```
[Out] cos(a+b/x)/b/x-sin(a+b/x)/b^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3460, 3377, 2717}

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3} \, dx = \frac{\cos\left(a+\frac{b}{x}\right)}{bx} - \frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

```
[In] Int[Sin[a + b/x]/x^3,x]
```

```
[Out] Cos[a + b/x]/(b*x) - Sin[a + b/x]/b^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \frac{1}{x}\right)\\ &= \frac{\cos\left(a+\frac{b}{x}\right)}{bx} - \frac{\text{Subst}\left(\int\cos(a+bx)\,dx, x, \frac{1}{x}\right)}{b}\\ &= \frac{\cos\left(a+\frac{b}{x}\right)}{bx} - \frac{\sin\left(a+\frac{b}{x}\right)}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{\cos\left(a+\frac{b}{x}\right)}{bx} - \frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

```
[In] Integrate[Sin[a + b/x]/x^3,x]
[Out] Cos[a + b/x]/(b*x) - Sin[a + b/x]/b^2
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

method	result	size
risch	$\frac{\cos\left(\frac{ax+b}{x}\right)}{bx}-\frac{\sin\left(\frac{ax+b}{x}\right)}{b^{2}}$	34
parallelrisch	$\frac{b\cos\left(\frac{ax+b}{x}\right)-x\sin\left(\frac{ax+b}{x}\right)}{b^{2}x}$	34
derivativedivides	$-\frac{\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)+a\cos\left(a+\frac{b}{x}\right)}{b^{2}}$	42
default	$-\frac{\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)+a\cos\left(a+\frac{b}{x}\right)}{b^{2}}$	42
norman	$\frac{\frac{x}{b}-\frac{2x^{2}\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^{2}}-\frac{x\left(\tan^{2}\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b}}{\left(1+\tan^{2}\left(\frac{a}{2}+\frac{b}{2x}\right)\right)x^{2}}$	66
meijerg	$-\frac{2\sqrt{\pi}\,\cos(a)\left(-\frac{b\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}+\frac{\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}}\right)}{b^{2}}-\frac{2\sqrt{\pi}\,\sin(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}}+\frac{b\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^{2}}$	81

```
[In] int(sin(a+b/x)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/b/x*cos((a*x+b)/x)-1/b^2*sin((a*x+b)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx=\frac{b\cos\left(\frac{ax+b}{x}\right)-x\sin\left(\frac{ax+b}{x}\right)}{b^2x}$$

```
[In] integrate(sin(a+b/x)/x^3,x, algorithm="fricas")
[Out] (b*cos((a*x + b)/x) - x*sin((a*x + b)/x))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx=\begin{cases}\frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\sin\left(a+\frac{b}{x}\right)}{b^2} & \text{for } b\neq 0\\ -\frac{\sin(a)}{2x^2} & \text{otherwise}\end{cases}$$

```
[In] integrate(sin(a+b/x)/x**3,x)
[Out] Piecewise((cos(a + b/x)/(b*x) - sin(a + b/x)/b**2, Ne(b, 0)), (-sin(a)/(2*x
**2), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int \frac{\sin\left(a + \frac{b}{x}\right)}{x^3}\,dx = -\frac{\left(i\,\Gamma\left(2, \frac{i\,b}{x}\right) - i\,\Gamma\left(2, -\frac{i\,b}{x}\right)\right)\cos\left(a\right) + \left(\Gamma\left(2, \frac{i\,b}{x}\right) + \Gamma\left(2, -\frac{i\,b}{x}\right)\right)\sin\left(a\right)}{2\,b^2}$$

```
[In] integrate(sin(a+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((I*gamma(2, I*b/x) - I*gamma(2, -I*b/x))*cos(a) + (gamma(2, I*b/x) +
gamma(2, -I*b/x))*sin(a))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \frac{\sin\left(a + \frac{b}{x}\right)}{x^3}\,dx = -\frac{a\cos\left(\frac{ax+b}{x}\right) - \frac{(ax+b)\cos\left(\frac{ax+b}{x}\right)}{x} + \sin\left(\frac{ax+b}{x}\right)}{b^2}$$

```
[In] integrate(sin(a+b/x)/x^3,x, algorithm="giac")
```

```
[Out] -(a*cos((a*x + b)/x) - (a*x + b)*cos((a*x + b)/x)/x + sin((a*x + b)/x))/b^2
```

Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a + \frac{b}{x}\right)}{x^3}\,dx = \frac{\cos\left(a + \frac{b}{x}\right)}{b\,x} - \frac{\sin\left(a + \frac{b}{x}\right)}{b^2}$$

```
[In] int(sin(a + b/x)/x^3,x)
```

```
[Out] cos(a + b/x)/(b*x) - sin(a + b/x)/b^2
```

3.109 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\frac{2\cos\left(a+\frac{b}{x}\right)}{b^3}+\frac{\cos\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x}$$

```
[Out] -2*cos(a+b/x)/b^3+cos(a+b/x)/b/x^2-2*sin(a+b/x)/b^2/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3460, 3377, 2718}

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\frac{2\cos\left(a+\frac{b}{x}\right)}{b^3}-\frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x}+\frac{\cos\left(a+\frac{b}{x}\right)}{bx^2}$$

```
[In] Int[Sin[a + b/x]/x^4,x]
[Out] (-2*Cos[a + b/x])/b^3 + Cos[a + b/x]/(b*x^2) - (2*Sin[a + b/x])/(b^2*x)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \frac{1}{x}\right)\\
&= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^2} - \frac{2\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \frac{1}{x}\right)}{b}\\
&= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^2} - \frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x} + \frac{2\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2}\\
&= -\frac{2\cos\left(a+\frac{b}{x}\right)}{b^3} + \frac{\cos\left(a+\frac{b}{x}\right)}{bx^2} - \frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{(b^2-2x^2)\cos\left(a+\frac{b}{x}\right) - 2bx\sin\left(a+\frac{b}{x}\right)}{b^3x^2}$$

```
[In] Integrate[Sin[a + b/x]/x^4,x]
[Out] ((b^2 - 2*x^2)*Cos[a + b/x] - 2*b*x*Sin[a + b/x])/(b^3*x^2)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result
risch	$\frac{(b^2-2x^2)\cos\left(\frac{ax+b}{x}\right)}{b^3x^2}-\frac{2\sin\left(\frac{ax+b}{x}\right)}{b^2x}$
parallelrisch	$\frac{(b^2-2x^2)\cos\left(\frac{ax+b}{x}\right)-2\sin\left(\frac{ax+b}{x}\right)bx+2x^2}{x^2b^3}$
norman	$\frac{\frac{x}{b}+\frac{4x^3\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b^3}-\frac{4x^2\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^2}-\frac{x\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)x^3}$
derivativedivides	$-\frac{-a^2\cos\left(a+\frac{b}{x}\right)-2a\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{b^3}$
default	$-\frac{-a^2\cos\left(a+\frac{b}{x}\right)-2a\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{b^3}$
meijerg	$-\frac{4\sqrt{\pi}\cos(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{b^2}{2x^2}+1\right)\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}}+\frac{b\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^3}-\frac{4\sqrt{\pi}\sin(a)\sqrt{b^2}\Bigg(\frac{\left(b^2\right)^{\frac{3}{2}}\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x\,b^2}-\frac{\left(b^2\right)^{\frac{3}{2}}\left(-\frac{3b^2}{2x^2}+3\right)\sin\left(\frac{b}{x}\right)}{6\sqrt{\pi}\,b^3}}{b^4}$

[In] int(sin(a+b/x)/x^4,x,method=_RETURNVERBOSE)

[Out] (b^2-2*x^2)/b^3/x^2*cos((a*x+b)/x)-2/b^2/x*sin((a*x+b)/x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx=-\frac{2\,bx\sin\left(\frac{ax+b}{x}\right)-(b^2-2\,x^2)\cos\left(\frac{ax+b}{x}\right)}{b^3x^2}$$

[In] integrate(sin(a+b/x)/x^4,x, algorithm="fricas")

[Out] -(2*b*x*sin((a*x + b)/x) - (b^2 - 2*x^2)*cos((a*x + b)/x))/(b^3*x^2)

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx=\begin{cases}\frac{\cos\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^3} & \text{for } b\neq 0\\ -\frac{\sin(a)}{3x^3} & \text{otherwise}\end{cases}$$

[In] integrate(sin(a+b/x)/x**4,x)

[Out] Piecewise((cos(a + b/x)/(b*x**2) - 2*sin(a + b/x)/(b**2*x) - 2*cos(a + b/x)/b**3, Ne(b, 0)), (-sin(a)/(3*x**3), True))

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\frac{\left(\Gamma\left(3, \frac{i\,b}{x}\right)+\Gamma\left(3, -\frac{i\,b}{x}\right)\right)\cos\left(a\right)-\left(i\,\Gamma\left(3, \frac{i\,b}{x}\right)-i\,\Gamma\left(3, -\frac{i\,b}{x}\right)\right)\sin\left(a\right)}{2\,b^3}$$

```
[In] integrate(sin(a+b/x)/x^4,x, algorithm="maxima")
[Out] -1/2*((gamma(3, I*b/x) + gamma(3, -I*b/x))*cos(a) - (I*gamma(3, I*b/x) - I*
gamma(3, -I*b/x))*sin(a))/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(45) = 90.

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.36

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx$$
$$= \frac{a^2\cos\left(\frac{ax+b}{x}\right)-\frac{2\,(ax+b)a\cos\left(\frac{ax+b}{x}\right)}{x}+2\,a\sin\left(\frac{ax+b}{x}\right)+\frac{(ax+b)^2\cos\left(\frac{ax+b}{x}\right)}{x^2}-\frac{2\,(ax+b)\sin\left(\frac{ax+b}{x}\right)}{x}-2\,\cos\left(\frac{ax+b}{x}\right)}{b^3}$$

```
[In] integrate(sin(a+b/x)/x^4,x, algorithm="giac")
[Out] (a^2*cos((a*x + b)/x) - 2*(a*x + b)*a*cos((a*x + b)/x)/x + 2*a*sin((a*x + b
)/x) + (a*x + b)^2*cos((a*x + b)/x)/x^2 - 2*(a*x + b)*sin((a*x + b)/x)/x -
2*cos((a*x + b)/x))/b^3
```

Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{b^2\,\cos\left(a+\frac{b}{x}\right)-2\,b\,x\,\sin\left(a+\frac{b}{x}\right)}{b^3\,x^2}-\frac{2\,\cos\left(a+\frac{b}{x}\right)}{b^3}$$

```
[In] int(sin(a + b/x)/x^4,x)
[Out] (b^2*cos(a + b/x) - 2*b*x*sin(a + b/x))/(b^3*x^2) - (2*cos(a + b/x))/b^3
```

3.110 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 61

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx=\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x}+\frac{6\sin\left(a+\frac{b}{x}\right)}{b^4}-\frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}$$

```
[Out] cos(a+b/x)/b/x^3-6*cos(a+b/x)/b^3/x+6*sin(a+b/x)/b^4-3*sin(a+b/x)/b^2/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3460, 3377, 2717}

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx=\frac{6\sin\left(a+\frac{b}{x}\right)}{b^4}-\frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x}-\frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}+\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}$$

```
[In] Int[Sin[a + b/x]/x^5,x]
```

```
[Out] Cos[a + b/x]/(b*x^3) - (6*Cos[a + b/x])/(b^3*x) + (6*Sin[a + b/x])/b^4 - (3
*Sin[a + b/x])/(b^2*x^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int x^3 \sin(a+bx)\,dx, x, \frac{1}{x}\right)\\ &= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{3\text{Subst}\left(\int x^2 \cos(a+bx)\,dx, x, \frac{1}{x}\right)}{b}\\ &= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2} + \frac{6\text{Subst}\left(\int x \sin(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2}\\ &= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2} + \frac{6\text{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{x}\right)}{b^3}\\ &= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} + \frac{6\sin\left(a+\frac{b}{x}\right)}{b^4} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx = \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} + \frac{6\sin\left(a+\frac{b}{x}\right)}{b^4} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}$$

```
[In] Integrate[Sin[a + b/x]/x^5,x]
```

```
[Out] Cos[a + b/x]/(b*x^3) - (6*Cos[a + b/x])/(b^3*x) + (6*Sin[a + b/x])/b^4 - (3
*Sin[a + b/x])/(b^2*x^2)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

method	result
risch	$\frac{(b^2-6x^2)\cos\left(\frac{ax+b}{x}\right)}{b^3x^3}-\frac{3(b^2-2x^2)\sin\left(\frac{ax+b}{x}\right)}{x^2b^4}$
parallelrisch	$\frac{6x^2\left(\tan^2\left(\frac{ax+b}{2x}\right)\right)b-\left(\tan^2\left(\frac{ax+b}{2x}\right)\right)b^3+12\tan\left(\frac{ax+b}{2x}\right)x^3-6x\tan\left(\frac{ax+b}{2x}\right)b^2-6b\,x^2+b^3}{x^3b^4\left(1+\tan^2\left(\frac{ax+b}{2x}\right)\right)}$
norman	$\frac{\frac{x}{b}-\frac{6x^3}{b^3}+\frac{12x^4\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^4}+\frac{6x^3\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b^3}-\frac{6x^2\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^2}-\frac{x\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)x^4}$
meijerg	$-\frac{8\sqrt{\pi}\,\cos(a)\left(\frac{b\left(-\frac{5b^2}{2x^2}+15\right)\cos\left(\frac{b}{x}\right)}{20\sqrt{\pi}\,x}-\frac{\left(-\frac{15b^2}{2x^2}+15\right)\sin\left(\frac{b}{x}\right)}{20\sqrt{\pi}}\right)}{b^4}-\frac{8\sqrt{\pi}\,\sin(a)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3b^2}{2x^2}+3\right)\cos\left(\frac{b}{x}\right)}{4\sqrt{\pi}}-\frac{b\left(-\frac{b^2}{2x^2}+3\right)}{4\sqrt{\pi}\,x}\right.}{b^4}$
derivativedivides	$-\frac{a^3\cos\left(a+\frac{b}{x}\right)+3a^2\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-3a\left(-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right.\right.}{b^4}$
default	$-\frac{a^3\cos\left(a+\frac{b}{x}\right)+3a^2\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-3a\left(-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right.\right.}{b^4}$

```
[In] int(sin(a+b/x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(b^2-6*x^2)/x^3*cos((a*x+b)/x)-3/x^2*(b^2-2*x^2)/b^4*sin((a*x+b)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx=\frac{(b^3-6\,bx^2)\cos\left(\frac{ax+b}{x}\right)-3\,(b^2x-2\,x^3)\sin\left(\frac{ax+b}{x}\right)}{b^4x^3}$$

```
[In] integrate(sin(a+b/x)/x^5,x, algorithm="fricas")
```

```
[Out] ((b^3 - 6*b*x^2)*cos((a*x + b)/x) - 3*(b^2*x - 2*x^3)*sin((a*x + b)/x))/(b^
4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx = \begin{cases} \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} + \frac{6\sin\left(a+\frac{b}{x}\right)}{b^4} & \text{for } b \neq 0 \\ -\frac{\sin(a)}{4x^4} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(a+b/x)/x**5,x)
[Out] Piecewise((cos(a + b/x)/(b*x**3) - 3*sin(a + b/x)/(b**2*x**2) - 6*cos(a + b
/x)/(b**3*x) + 6*sin(a + b/x)/b**4, Ne(b, 0)), (-sin(a)/(4*x**4), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx = \frac{\left(i\,\Gamma\left(4,\frac{i\,b}{x}\right) - i\,\Gamma\left(4,-\frac{i\,b}{x}\right)\right)\cos(a) + \left(\Gamma\left(4,\frac{i\,b}{x}\right) + \Gamma\left(4,-\frac{i\,b}{x}\right)\right)\sin(a)}{2\,b^4}$$

```
[In] integrate(sin(a+b/x)/x^5,x, algorithm="maxima")
[Out] 1/2*((I*gamma(4, I*b/x) - I*gamma(4, -I*b/x))*cos(a) + (gamma(4, I*b/x) + g
amma(4, -I*b/x))*sin(a))/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(61) = 122$.

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.13

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx = -\frac{a^3\cos\left(\frac{ax+b}{x}\right) - \frac{3\,(ax+b)a^2\cos\left(\frac{ax+b}{x}\right)}{x} + 3\,a^2\sin\left(\frac{ax+b}{x}\right) - 6\,a\cos\left(\frac{ax+b}{x}\right) + \frac{3\,(ax+b)^2a\cos\left(\frac{ax+b}{x}\right)}{x^2} - \frac{6\,(ax+b)a\sin\left(\frac{ax+b}{x}\right)}{x}}{b^4}$$

```
[In] integrate(sin(a+b/x)/x^5,x, algorithm="giac")
[Out] -(a^3*cos((a*x + b)/x) - 3*(a*x + b)*a^2*cos((a*x + b)/x)/x + 3*a^2*sin((a*
x + b)/x) - 6*a*cos((a*x + b)/x) + 3*(a*x + b)^2*a*cos((a*x + b)/x)/x^2 - 6
*(a*x + b)*a*sin((a*x + b)/x)/x - (a*x + b)^3*cos((a*x + b)/x)/x^3 + 6*(a*x
 + b)*cos((a*x + b)/x)/x + 3*(a*x + b)^2*sin((a*x + b)/x)/x^2 - 6*sin((a*x
+ b)/x))/b^4
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx=\frac{6\,\sin\left(a+\frac{b}{x}\right)}{b^4}-\frac{6\,b\,x^2\,\cos\left(a+\frac{b}{x}\right)-b^3\,\cos\left(a+\frac{b}{x}\right)+3\,b^2\,x\,\sin\left(a+\frac{b}{x}\right)}{b^4\,x^3}$$

```
[In] int(sin(a + b/x)/x^5,x)
```

```
[Out] (6*sin(a + b/x))/b^4 - (6*b*x^2*cos(a + b/x) - b^3*cos(a + b/x) + 3*b^2*x*s
in(a + b/x))/(b^4*x^3)
```

3.111 $\int x^2 \sin^2\left(a+\frac{b}{x}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 97

$$\int x^2 \sin^2\left(a+\frac{b}{x}\right) \, dx = \frac{x^3}{6}+\frac{1}{3}b^2 x \cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{2}{3}b^3 \operatorname{CosIntegral}\left(\frac{2b}{x}\right)\sin(2a)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)+\frac{2}{3}b^3\cos(2a)\operatorname{Si}\left(\frac{2b}{x}\right)$$

```
[Out] 1/6*x^3+1/3*b^2*x*cos(2*a+2*b/x)-1/6*x^3*cos(2*a+2*b/x)+2/3*b^3*cos(2*a)*Si
(2*b/x)+2/3*b^3*Ci(2*b/x)*sin(2*a)+1/6*b*x^2*sin(2*a+2*b/x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3506, 3461, 3378, 3384, 3380, 3383}

$$\int x^2 \sin^2\left(a+\frac{b}{x}\right) \, dx = \frac{2}{3}b^3\sin(2a)\operatorname{CosIntegral}\left(\frac{2b}{x}\right)+\frac{2}{3}b^3\cos(2a)\operatorname{Si}\left(\frac{2b}{x}\right)+\frac{1}{3}b^2 x\cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)+\frac{x^3}{6}$$

```
[In] Int[x^2*Sin[a + b/x]^2,x]
```

```
[Out] x^3/6 + (b^2*x*Cos[2*(a + b/x)])/3 - (x^3*Cos[2*(a + b/x)])/6 + (2*b^3*CosI
ntegral[(2*b)/x]*Sin[2*a])/3 + (b*x^2*Sin[2*(a + b/x)])/6 + (2*b^3*Cos[2*a]
*SinIntegral[(2*b)/x])/3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x^2}{2} - \frac{1}{2}x^2 \cos\left(2a + \frac{2b}{x}\right)\right) dx$$

$$= \frac{x^3}{6} - \frac{1}{2}\int x^2 \cos\left(2a + \frac{2b}{x}\right) dx$$

$$
\begin{aligned}
&= \frac{x^3}{6} + \frac{1}{2}\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x^4}\,dx, x, \frac{1}{x}\right)\\
&= \frac{x^3}{6} - \frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right) - \frac{1}{3}b\text{Subst}\left(\int \frac{\sin(2a+2bx)}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{x^3}{6} - \frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right) + \frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right) - \frac{1}{3}b^2\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{x^3}{6} + \frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right) - \frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)\\
&\quad + \frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right) + \frac{1}{3}\left(2b^3\right)\text{Subst}\left(\int \frac{\sin(2a+2bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= \frac{x^3}{6} + \frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right) - \frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)\\
&\quad + \frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right) + \frac{1}{3}\left(2b^3\cos(2a)\right)\text{Subst}\left(\int \frac{\sin(2bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&\quad + \frac{1}{3}\left(2b^3\sin(2a)\right)\text{Subst}\left(\int \frac{\cos(2bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= \frac{x^3}{6} + \frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right) - \frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)\\
&\quad + \frac{2}{3}b^3\,\text{CosIntegral}\left(\frac{2b}{x}\right)\sin(2a) + \frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right) + \frac{2}{3}b^3\cos(2a)\text{Si}\left(\frac{2b}{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

$$
\begin{aligned}
\int x^2\sin^2\left(a+\frac{b}{x}\right)\,dx = \frac{1}{6}\Bigg(&4b^3\,\text{CosIntegral}\left(\frac{2b}{x}\right)\sin(2a) + x\Bigg(x^2 + 2b^2\cos\left(2\left(a+\frac{b}{x}\right)\right)\\
&- x^2\cos\left(2\left(a+\frac{b}{x}\right)\right) + bx\sin\left(2\left(a+\frac{b}{x}\right)\right)\Bigg)\\
&+ 4b^3\cos(2a)\text{Si}\left(\frac{2b}{x}\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[x^2*Sin[a + b/x]^2,x]
```

```
[Out] (4*b^3*CosIntegral[(2*b)/x]*Sin[2*a] + x*(x^2 + 2*b^2*Cos[2*(a + b/x)] - x^
2*Cos[2*(a + b/x)] + b*x*Sin[2*(a + b/x)]) + 4*b^3*Cos[2*a]*SinIntegral[(2*
b)/x])/6
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

method	result
derivativedivides	$-b^3\left(-\frac{x^3}{6b^3}+\frac{\cos\left(2a+\frac{2b}{x}\right)x^3}{6b^3}-\frac{\sin\left(2a+\frac{2b}{x}\right)x^2}{6b^2}-\frac{\cos\left(2a+\frac{2b}{x}\right)x}{3b}-\frac{2\,\mathrm{Si}\left(\frac{2b}{x}\right)\cos(2a)}{3}-\frac{2\,\mathrm{Ci}\left(\frac{2b}{x}\right)\sin(2a)}{3}\right)$
default	$-b^3\left(-\frac{x^3}{6b^3}+\frac{\cos\left(2a+\frac{2b}{x}\right)x^3}{6b^3}-\frac{\sin\left(2a+\frac{2b}{x}\right)x^2}{6b^2}-\frac{\cos\left(2a+\frac{2b}{x}\right)x}{3b}-\frac{2\,\mathrm{Si}\left(\frac{2b}{x}\right)\cos(2a)}{3}-\frac{2\,\mathrm{Ci}\left(\frac{2b}{x}\right)\sin(2a)}{3}\right)$
risch	$-\frac{\mathrm{e}^{-2ia}\pi\,\mathrm{csgn}\left(\frac{b}{x}\right)b^3}{3}+\frac{2\,\mathrm{e}^{-2ia}\,\mathrm{Si}\left(\frac{2b}{x}\right)b^3}{3}-\frac{i\mathrm{e}^{-2ia}\,\mathrm{Ei}_1\left(-\frac{2ib}{x}\right)b^3}{3}+\frac{ib^3\,\mathrm{Ei}_1\left(-\frac{2ib}{x}\right)\mathrm{e}^{2ia}}{3}+\frac{x^3}{6}+\frac{\cos\left(\frac{2ax+2b}{x}\right)b^2}{3}$

```
[In] int(x^2*sin(a+b/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3*(-1/6/b^3*x^3+1/6*cos(2*a+2*b/x)/b^3*x^3-1/6*sin(2*a+2*b/x)/b^2*x^2-1/
3*cos(2*a+2*b/x)/b*x-2/3*Si(2*b/x)*cos(2*a)-2/3*Ci(2*b/x)*sin(2*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int x^2\sin^2\left(a+\frac{b}{x}\right)\,dx=\frac{2}{3}\,b^3\,\mathrm{Ci}\left(\frac{2\,b}{x}\right)\sin\left(2\,a\right)+\frac{1}{3}\,bx^2\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)+\frac{2}{3}\,b^3\cos\left(2\,a\right)\mathrm{Si}\left(\frac{2\,b}{x}\right)-\frac{1}{3}\,b^2x+\frac{1}{3}\,x^3+\frac{1}{3}\left(2\,b^2x-x^3\right)\cos\left(\frac{ax+b}{x}\right)^2$$

```
[In] integrate(x^2*sin(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] 2/3*b^3*cos_integral(2*b/x)*sin(2*a) + 1/3*b*x^2*cos((a*x + b)/x)*sin((a*x
+ b)/x) + 2/3*b^3*cos(2*a)*sin_integral(2*b/x) - 1/3*b^2*x + 1/3*x^3 + 1/3*
(2*b^2*x - x^3)*cos((a*x + b)/x)^2
```

Sympy [F]

$$\int x^2 \sin^2\left(a + \frac{b}{x}\right) dx = \int x^2 \sin^2\left(a + \frac{b}{x}\right) dx$$

```
[In] integrate(x**2*sin(a+b/x)**2,x)
[Out] Integral(x**2*sin(a + b/x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int x^2 \sin^2\left(a + \frac{b}{x}\right) dx$$
$$= -\frac{1}{3}\left(\left(i\,\mathrm{Ei}\left(\frac{2i\,b}{x}\right) - i\,\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos(2\,a) - \left(\mathrm{Ei}\left(\frac{2i\,b}{x}\right) + \mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin(2\,a)\right)b^3$$
$$+ \frac{1}{6}\,bx^2 \sin\left(\frac{2\,(ax+b)}{x}\right) + \frac{1}{6}\,x^3 + \frac{1}{6}\,(2\,b^2x - x^3)\cos\left(\frac{2\,(ax+b)}{x}\right)$$

```
[In] integrate(x^2*sin(a+b/x)^2,x, algorithm="maxima")
[Out] -1/3*((I*Ei(2*I*b/x) - I*Ei(-2*I*b/x))*cos(2*a) - (Ei(2*I*b/x) + Ei(-2*I*b/
x))*sin(2*a))*b^3 + 1/6*b*x^2*sin(2*(a*x + b)/x) + 1/6*x^3 + 1/6*(2*b^2*x -
 x^3)*cos(2*(a*x + b)/x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(88) = 176.

Time = 0.32 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.56

$$\int x^2 \sin^2\left(a + \frac{b}{x}\right) dx$$
$$= \frac{4\,a^3b^4\,\mathrm{Ci}\left(-2\,a + \frac{2\,(ax+b)}{x}\right)\sin(2\,a) - 4\,a^3b^4\cos(2\,a)\,\mathrm{Si}\left(2\,a - \frac{2\,(ax+b)}{x}\right) - \frac{12\,(ax+b)a^2b^4\,\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin(2\,a)}{x} + \cdots}{\cdots}$$

```
[In] integrate(x^2*sin(a+b/x)^2,x, algorithm="giac")
[Out] 1/6*(4*a^3*b^4*cos_integral(-2*a + 2*(a*x + b)/x)*sin(2*a) - 4*a^3*b^4*cos(
2*a)*sin_integral(2*a - 2*(a*x + b)/x) - 12*(a*x + b)*a^2*b^4*cos_integral(
```

```
-2*a + 2*(a*x + b)/x)*sin(2*a)/x + 12*(a*x + b)*a^2*b^4*cos(2*a)*sin_integr
al(2*a - 2*(a*x + b)/x)/x - 2*a^2*b^4*cos(2*(a*x + b)/x) + 12*(a*x + b)^2*a
*b^4*cos_integral(-2*a + 2*(a*x + b)/x)*sin(2*a)/x^2 - 12*(a*x + b)^2*a*b^4
*cos(2*a)*sin_integral(2*a - 2*(a*x + b)/x)/x^2 + 4*(a*x + b)*a*b^4*cos(2*(
a*x + b)/x)/x - 4*(a*x + b)^3*b^4*cos_integral(-2*a + 2*(a*x + b)/x)*sin(2*
a)/x^3 + a*b^4*sin(2*(a*x + b)/x) + 4*(a*x + b)^3*b^4*cos(2*a)*sin_integral
(2*a - 2*(a*x + b)/x)/x^3 + b^4*cos(2*(a*x + b)/x) - 2*(a*x + b)^2*b^4*cos(
2*(a*x + b)/x)/x^2 - (a*x + b)*b^4*sin(2*(a*x + b)/x)/x - b^4)/((a^3 - 3*(a
*x + b)*a^2/x + 3*(a*x + b)^2*a/x^2 - (a*x + b)^3/x^3)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sin^2\left(a+\frac{b}{x}\right) dx = \int x^2 \sin\left(a+\frac{b}{x}\right)^2 dx$$

```
[In] int(x^2*sin(a + b/x)^2,x)
[Out] int(x^2*sin(a + b/x)^2, x)
```

3.112 $\int x \sin^2\left(a+\frac{b}{x}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int x \sin^2\left(a+\frac{b}{x}\right) dx = -b^2\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right)+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)+b^2\sin(2a)\text{Si}\left(\frac{2b}{x}\right)$$

```
[Out] -b^2*Ci(2*b/x)*cos(2*a)+b^2*Si(2*b/x)*sin(2*a)+1/2*x^2*sin(a+b/x)^2+1/2*b*x
*sin(2*a+2*b/x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3474, 4669, 3454, 3442, 3378, 3384, 3380, 3383}

$$\int x \sin^2\left(a+\frac{b}{x}\right) dx = b^2(-\cos(2a))\,\text{CosIntegral}\left(\frac{2b}{x}\right)+b^2\sin(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)$$

```
[In] Int[x*Sin[a + b/x]^2,x]
```

```
[Out] -(b^2*Cos[2*a]*CosIntegral[(2*b)/x]) + (x^2*Sin[a + b/x]^2)/2 + (b*x*Sin[2*
(a + b/x)])/2 + b^2*Sin[2*a]*SinIntegral[(2*b)/x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3454

```
Int[((a_.) + (b_.)*Sin[u_])^(p_.), x_Symbol] :> Int[(a + b*Sin[ExpandToSum[
u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ
[u, x]
```

Rule 3474

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[x^(m +
1)*(Sin[a + b*x^n]^p/(m + 1)), x] - Dist[b*n*(p/(m + 1)), Int[Sin[a + b*x^n
]^(p - 1)*Cos[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4669

```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin[2
*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+b\int\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)\,dx\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}b\int\sin\left(2\left(a+\frac{b}{x}\right)\right)\,dx\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}b\int\sin\left(2a+\frac{2b}{x}\right)\,dx\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)-\frac{1}{2}b\text{Subst}\left(\int\frac{\sin(2a+2bx)}{x^2}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)-b^2\text{Subst}\left(\int\frac{\cos(2a+2bx)}{x}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)\\
&\quad-\left(b^2\cos(2a)\right)\text{Subst}\left(\int\frac{\cos(2bx)}{x}\,dx,x,\frac{1}{x}\right)\\
&\quad+\left(b^2\sin(2a)\right)\text{Subst}\left(\int\frac{\sin(2bx)}{x}\,dx,x,\frac{1}{x}\right)\\
&= -b^2\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right)+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)\\
&\quad+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)+b^2\sin(2a)\text{Si}\left(\frac{2b}{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int x\sin^2\left(a+\frac{b}{x}\right)\,dx &= -b^2\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right)\\
&\quad+\frac{1}{4}x\left(x-x\cos\left(2\left(a+\frac{b}{x}\right)\right)+2b\sin\left(2\left(a+\frac{b}{x}\right)\right)\right)\\
&\quad+b^2\sin(2a)\text{Si}\left(\frac{2b}{x}\right)
\end{aligned}
$$

```
[In] Integrate[x*Sin[a + b/x]^2,x]
[Out] -(b^2*Cos[2*a]*CosIntegral[(2*b)/x]) + (x*(x - x*Cos[2*(a + b/x)] + 2*b*Sin
[2*(a + b/x)]))/4 + b^2*Sin[2*a]*SinIntegral[(2*b)/x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-b^{2}\left(-\frac{x^{2}}{4b^{2}}+\frac{\cos\left(2a+\frac{2b}{x}\right)x^{2}}{4b^{2}}-\frac{\sin\left(2a+\frac{2b}{x}\right)x}{2b}-\operatorname{Si}\left(\frac{2b}{x}\right)\sin\left(2a\right)+\operatorname{Ci}\left(\frac{2b}{x}\right)\cos\left(2a\right)\right)$
default	$-b^{2}\left(-\frac{x^{2}}{4b^{2}}+\frac{\cos\left(2a+\frac{2b}{x}\right)x^{2}}{4b^{2}}-\frac{\sin\left(2a+\frac{2b}{x}\right)x}{2b}-\operatorname{Si}\left(\frac{2b}{x}\right)\sin\left(2a\right)+\operatorname{Ci}\left(\frac{2b}{x}\right)\cos\left(2a\right)\right)$
risch	$-\frac{i\pi\ \operatorname{csgn}\left(\frac{b}{x}\right)\mathrm{e}^{-2ia}b^{2}}{2}+i\operatorname{Si}\left(\frac{2b}{x}\right)\mathrm{e}^{-2ia}b^{2}+\frac{\operatorname{Ei}_{1}\left(-\frac{2ib}{x}\right)\mathrm{e}^{-2ia}b^{2}}{2}+\frac{\mathrm{e}^{2ia}\operatorname{Ei}_{1}\left(-\frac{2ib}{x}\right)b^{2}}{2}+\frac{x^{2}}{4}-\frac{x^{2}\cos\left(\frac{2ax+2}{x}\right.}{4}$

```
[In] int(x*sin(a+b/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*(-1/4/b^2*x^2+1/4*cos(2*a+2*b/x)/b^2*x^2-1/2*sin(2*a+2*b/x)/b*x-Si(2*b
/x)*sin(2*a)+Ci(2*b/x)*cos(2*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int x\sin^{2}\left(a+\frac{b}{x}\right)\,dx=-\frac{1}{2}\,x^{2}\cos\left(\frac{ax+b}{x}\right)^{2}-b^{2}\cos\left(2\,a\right)\operatorname{Ci}\left(\frac{2\,b}{x}\right)+bx\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)+b^{2}\sin\left(2\,a\right)\operatorname{Si}\left(\frac{2\,b}{x}\right)+\frac{1}{2}\,x^{2}$$

```
[In] integrate(x*sin(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x^2*cos((a*x + b)/x)^2 - b^2*cos(2*a)*cos_integral(2*b/x) + b*x*cos((a
*x + b)/x)*sin((a*x + b)/x) + b^2*sin(2*a)*sin_integral(2*b/x) + 1/2*x^2
```

Sympy [F]

$$\int x\sin^{2}\left(a+\frac{b}{x}\right)\,dx=\int x\sin^{2}\left(a+\frac{b}{x}\right)\,dx$$

```
[In] integrate(x*sin(a+b/x)**2,x)
```

```
[Out] Integral(x*sin(a + b/x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int x\sin^2\left(a+\frac{b}{x}\right)\,dx$$
$$=-\frac{1}{2}\left(\left(\mathrm{Ei}\left(\frac{2i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)+\left(i\,\mathrm{Ei}\left(\frac{2i\,b}{x}\right)-i\,\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)b^2$$
$$-\frac{1}{4}\,x^2\cos\left(\frac{2\,(ax+b)}{x}\right)+\frac{1}{2}\,bx\sin\left(\frac{2\,(ax+b)}{x}\right)+\frac{1}{4}\,x^2$$

```
[In] integrate(x*sin(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((Ei(2*I*b/x) + Ei(-2*I*b/x))*cos(2*a) + (I*Ei(2*I*b/x) - I*Ei(-2*I*b/
x))*sin(2*a))*b^2 - 1/4*x^2*cos(2*(a*x + b)/x) + 1/2*b*x*sin(2*(a*x + b)/x)
 + 1/4*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(62) = 124.

Time = 0.33 (sec) , antiderivative size = 283, normalized size of antiderivative = 4.35

$$\int x\sin^2\left(a+\frac{b}{x}\right)\,dx=$$
$$-\frac{4\,a^2b^3\cos\left(2\,a\right)\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)+4\,a^2b^3\sin\left(2\,a\right)\mathrm{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)-\frac{8\,(ax+b)ab^3\cos(2\,a)\,\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)}{x}}{}$$

```
[In] integrate(x*sin(a+b/x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(4*a^2*b^3*cos(2*a)*cos_integral(-2*a + 2*(a*x + b)/x) + 4*a^2*b^3*sin
(2*a)*sin_integral(2*a - 2*(a*x + b)/x) - 8*(a*x + b)*a*b^3*cos(2*a)*cos_in
tegral(-2*a + 2*(a*x + b)/x)/x - 8*(a*x + b)*a*b^3*sin(2*a)*sin_integral(2*
a - 2*(a*x + b)/x)/x + 4*(a*x + b)^2*b^3*cos(2*a)*cos_integral(-2*a + 2*(a*
x + b)/x)/x^2 + 2*a*b^3*sin(2*(a*x + b)/x) + 4*(a*x + b)^2*b^3*sin(2*a)*sin
_integral(2*a - 2*(a*x + b)/x)/x^2 + b^3*cos(2*(a*x + b)/x) - 2*(a*x + b)*b
^3*sin(2*(a*x + b)/x)/x - b^3)/((a^2 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2)*b
)
```

Mupad [F(-1)]

Timed out.

$$\int x\sin^2\left(a+\frac{b}{x}\right)\,dx=\int x\sin\left(a+\frac{b}{x}\right)^2\,dx$$

```
[In] int(x*sin(a + b/x)^2,x)
[Out] int(x*sin(a + b/x)^2, x)
```

3.113 $\int \sin^2\left(a+\frac{b}{x}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \sin^2\left(a+\frac{b}{x}\right) dx = -b\,\text{CosIntegral}\left(\frac{2b}{x}\right)\sin(2a) + x\sin^2\left(a+\frac{b}{x}\right) - b\cos(2a)\text{Si}\left(\frac{2b}{x}\right)$$

```
[Out] -b*cos(2*a)*Si(2*b/x)-b*Ci(2*b/x)*sin(2*a)+x*sin(a+b/x)^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3442, 3394, 12, 3384, 3380, 3383}

$$\int \sin^2\left(a+\frac{b}{x}\right) dx = -b\sin(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right) - b\cos(2a)\text{Si}\left(\frac{2b}{x}\right) + x\sin^2\left(a+\frac{b}{x}\right)$$

```
[In] Int[Sin[a + b/x]^2,x]
```

```
[Out] -(b*CosIntegral[(2*b)/x]*Sin[2*a]) + x*Sin[a + b/x]^2 - b*Cos[2*a]*SinInteg
ral[(2*b)/x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sin^2(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= x\sin^2\left(a+\frac{b}{x}\right) - (2b)\text{Subst}\left(\int \frac{\sin(2a+2bx)}{2x}\,dx, x, \frac{1}{x}\right)\\
&= x\sin^2\left(a+\frac{b}{x}\right) - b\text{Subst}\left(\int \frac{\sin(2a+2bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= x\sin^2\left(a+\frac{b}{x}\right) - (b\cos(2a))\text{Subst}\left(\int \frac{\sin(2bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&\quad - (b\sin(2a))\text{Subst}\left(\int \frac{\cos(2bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= -b\,\text{CosIntegral}\left(\frac{2b}{x}\right)\sin(2a) + x\sin^2\left(a+\frac{b}{x}\right) - b\cos(2a)\text{Si}\left(\frac{2b}{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \sin^2\left(a+\frac{b}{x}\right) dx = -b\,\text{CosIntegral}\left(\frac{2b}{x}\right)\sin(2a) + x\sin^2\left(a+\frac{b}{x}\right) - b\cos(2a)\text{Si}\left(\frac{2b}{x}\right)$$

```
[In] Integrate[Sin[a + b/x]^2,x]
[Out] -(b*CosIntegral[(2*b)/x]*Sin[2*a]) + x*Sin[a + b/x]^2 - b*Cos[2*a]*SinInteg
ral[(2*b)/x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$-b\left(-\frac{x}{2b}+\frac{\cos\left(2a+\frac{2b}{x}\right)x}{2b}+\operatorname{Si}\left(\frac{2b}{x}\right)\cos\left(2a\right)+\operatorname{Ci}\left(\frac{2b}{x}\right)\sin\left(2a\right)\right)$	52
default	$-b\left(-\frac{x}{2b}+\frac{\cos\left(2a+\frac{2b}{x}\right)x}{2b}+\operatorname{Si}\left(\frac{2b}{x}\right)\cos\left(2a\right)+\operatorname{Ci}\left(\frac{2b}{x}\right)\sin\left(2a\right)\right)$	52
risch	$\frac{e^{-2ia}\pi\ \operatorname{csgn}\left(\frac{b}{x}\right)b}{2}-e^{-2ia}\operatorname{Si}\left(\frac{2b}{x}\right)b+\frac{i\operatorname{Ei}_1\left(-\frac{2ib}{x}\right)e^{-2ia}b}{2}-\frac{ib\operatorname{Ei}_1\left(-\frac{2ib}{x}\right)e^{2ia}}{2}+\frac{x}{2}-\frac{x\cos\left(\frac{2ax+2b}{x}\right)}{2}$	85

```
[In] int(sin(a+b/x)^2,x,method=_RETURNVERBOSE)
[Out] -b*(-1/2*x/b+1/2*cos(2*a+2*b/x)/b*x+Si(2*b/x)*cos(2*a)+Ci(2*b/x)*sin(2*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \sin^2\left(a+\frac{b}{x}\right) dx = -x\cos\left(\frac{ax+b}{x}\right)^2 - b\,\text{Ci}\left(\frac{2\,b}{x}\right)\sin\left(2\,a\right) - b\cos\left(2\,a\right)\text{Si}\left(\frac{2\,b}{x}\right) + x$$

```
[In] integrate(sin(a+b/x)^2,x, algorithm="fricas")
[Out] -x*cos((a*x + b)/x)^2 - b*cos_integral(2*b/x)*sin(2*a) - b*cos(2*a)*sin_int
egral(2*b/x) + x
```

Sympy [F]

$$\int \sin^2\left(a+\frac{b}{x}\right)\,dx = \int \sin^2\left(a+\frac{b}{x}\right)\,dx$$

```
[In] integrate(sin(a+b/x)**2,x)
[Out] Integral(sin(a + b/x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \sin^2\left(a+\frac{b}{x}\right)\,dx = -\frac{1}{2}\left(\left(-i\,\mathrm{Ei}\left(\frac{2i\,b}{x}\right)+i\,\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)+\left(\mathrm{Ei}\left(\frac{2i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)b - \frac{1}{2}\,x\cos\left(\frac{2\,(ax+b)}{x}\right)+\frac{1}{2}\,x$$

```
[In] integrate(sin(a+b/x)^2,x, algorithm="maxima")
[Out] -1/2*((-I*Ei(2*I*b/x) + I*Ei(-2*I*b/x))*cos(2*a) + (Ei(2*I*b/x) + Ei(-2*I*b
/x))*sin(2*a))*b - 1/2*x*cos(2*(a*x + b)/x) + 1/2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(41) = 82.

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.73

$$\int \sin^2\left(a+\frac{b}{x}\right)\,dx = -\frac{2\,ab^2\,\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin\left(2\,a\right)-2\,ab^2\cos\left(2\,a\right)\mathrm{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)-\frac{2\,(ax+b)b^2\,\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin(2\,a)}{x}+}{2\left(a-\frac{ax+b}{x}\right)b}$$

```
[In] integrate(sin(a+b/x)^2,x, algorithm="giac")
[Out] -1/2*(2*a*b^2*cos_integral(-2*a + 2*(a*x + b)/x)*sin(2*a) - 2*a*b^2*cos(2*a
)*sin_integral(2*a - 2*(a*x + b)/x) - 2*(a*x + b)*b^2*cos_integral(-2*a + 2
*(a*x + b)/x)*sin(2*a)/x + 2*(a*x + b)*b^2*cos(2*a)*sin_integral(2*a - 2*(a
*x + b)/x)/x - b^2*cos(2*(a*x + b)/x) + b^2)/((a - (a*x + b)/x)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2\left(a + \frac{b}{x}\right) dx = \int \sin\left(a + \frac{b}{x}\right)^2 dx$$

```
[In] int(sin(a + b/x)^2,x)
[Out] int(sin(a + b/x)^2, x)
```

3.114 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx = \frac{1}{2}\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right)+\frac{\log(x)}{2}-\frac{1}{2}\sin(2a)\text{Si}\left(\frac{2b}{x}\right)$$

```
[Out] 1/2*Ci(2*b/x)*cos(2*a)+1/2*ln(x)-1/2*Si(2*b/x)*sin(2*a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3506, 3459, 3457, 3456}

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx = \frac{1}{2}\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right)-\frac{1}{2}\sin(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{\log(x)}{2}$$

```
[In] Int[Sin[a + b/x]^2/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[(2*b)/x])/2 + Log[x]/2 - (Sin[2*a]*SinIntegral[(2*b)/
x])/2
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2x} - \frac{\cos\left(2a + \frac{2b}{x}\right)}{2x}\right) dx \\
&= \frac{\log(x)}{2} - \frac{1}{2}\int \frac{\cos\left(2a + \frac{2b}{x}\right)}{x} dx \\
&= \frac{\log(x)}{2} - \frac{1}{2}\cos(2a)\int \frac{\cos\left(\frac{2b}{x}\right)}{x} dx + \frac{1}{2}\sin(2a)\int \frac{\sin\left(\frac{2b}{x}\right)}{x} dx \\
&= \frac{1}{2}\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right) + \frac{\log(x)}{2} - \frac{1}{2}\sin(2a)\text{Si}\left(\frac{2b}{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$
\int \frac{\sin^2\left(a + \frac{b}{x}\right)}{x} dx = \frac{1}{2}\left(\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right) + \log(x) - \sin(2a)\text{Si}\left(\frac{2b}{x}\right)\right)
$$

```
[In] Integrate[Sin[a + b/x]^2/x,x]
[Out] (Cos[2*a]*CosIntegral[(2*b)/x] + Log[x] - Sin[2*a]*SinIntegral[(2*b)/x])/2
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{b}{x}\right)}{2}-\frac{\mathrm{Si}\left(\frac{2b}{x}\right)\sin(2a)}{2}+\frac{\mathrm{Ci}\left(\frac{2b}{x}\right)\cos(2a)}{2}$	36
default	$-\frac{\ln\left(\frac{b}{x}\right)}{2}-\frac{\mathrm{Si}\left(\frac{2b}{x}\right)\sin(2a)}{2}+\frac{\mathrm{Ci}\left(\frac{2b}{x}\right)\cos(2a)}{2}$	36
risch	$\frac{i\pi\,\mathrm{csgn}\left(\frac{b}{x}\right)\mathrm{e}^{-2ia}}{4}-\frac{i\,\mathrm{Si}\left(\frac{2b}{x}\right)\mathrm{e}^{-2ia}}{2}-\frac{\mathrm{e}^{-2ia}\,\mathrm{Ei}_1\left(-\frac{2ib}{x}\right)}{4}-\frac{\mathrm{e}^{2ia}\,\mathrm{Ei}_1\left(-\frac{2ib}{x}\right)}{4}+\frac{\ln(x)}{2}$	68

```
[In] int(sin(a+b/x)^2/x,x,method=_RETURNVERBOSE)
[Out] -1/2*ln(b/x)-1/2*Si(2*b/x)*sin(2*a)+1/2*Ci(2*b/x)*cos(2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx=\frac{1}{2}\cos\left(2\,a\right)\mathrm{Ci}\left(\frac{2\,b}{x}\right)-\frac{1}{2}\sin\left(2\,a\right)\mathrm{Si}\left(\frac{2\,b}{x}\right)+\frac{1}{2}\log\left(x\right)$$

```
[In] integrate(sin(a+b/x)^2/x,x, algorithm="fricas")
[Out] 1/2*cos(2*a)*cos_integral(2*b/x) - 1/2*sin(2*a)*sin_integral(2*b/x) + 1/2*l
og(x)
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx=\frac{\log\left(x\right)}{2}-\frac{\sin\left(2a\right)\mathrm{Si}\left(\frac{2b}{x}\right)}{2}+\frac{\cos\left(2a\right)\mathrm{Ci}\left(\frac{2b}{x}\right)}{2}$$

```
[In] integrate(sin(a+b/x)**2/x,x)
[Out] log(x)/2 - sin(2*a)*Si(2*b/x)/2 + cos(2*a)*Ci(2*b/x)/2
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx = \frac{1}{4}\left(\mathrm{Ei}\left(\frac{2i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right) + \frac{1}{4}\left(i\,\mathrm{Ei}\left(\frac{2i\,b}{x}\right)-i\,\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)+\frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate(sin(a+b/x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*(Ei(2*I*b/x) + Ei(-2*I*b/x))*cos(2*a) + 1/4*(I*Ei(2*I*b/x) - I*Ei(-2*I*
b/x))*sin(2*a) + 1/2*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(31) = 62.

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.76

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx = \frac{b\cos\left(2\,a\right)\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)+b\sin\left(2\,a\right)\mathrm{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)-b\log\left(-a+\frac{ax+b}{x}\right)}{2\,b}$$

```
[In] integrate(sin(a+b/x)^2/x,x, algorithm="giac")
```

```
[Out] 1/2*(b*cos(2*a)*cos_integral(-2*a + 2*(a*x + b)/x) + b*sin(2*a)*sin_integra
l(2*a - 2*(a*x + b)/x) - b*log(-a + (a*x + b)/x))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx = \int \frac{{\sin\left(a+\frac{b}{x}\right)}^2}{x}\,dx$$

```
[In] int(sin(a + b/x)^2/x,x)
```

```
[Out] int(sin(a + b/x)^2/x, x)
```

3.115 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2} dx = -\frac{1}{2x} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2b}$$

```
[Out] -1/2/x+1/2*cos(a+b/x)*sin(a+b/x)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3460, 2715, 8}

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2} dx = \frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2b} - \frac{1}{2x}$$

```
[In] Int[Sin[a + b/x]^2/x^2,x]
```

```
[Out] -1/2*1/x + (Cos[a + b/x]*Sin[a + b/x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \sin^2(a+bx)\,dx, x, \frac{1}{x}\right)\\ &= \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2b} - \frac{1}{2}\text{Subst}\left(\int 1\,dx, x, \frac{1}{x}\right)\\ &= -\frac{1}{2x} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{a+\frac{b}{x}}{2b} + \frac{\sin\left(2\left(a+\frac{b}{x}\right)\right)}{4b}$$

```
[In] Integrate[Sin[a + b/x]^2/x^2,x]
[Out] -1/2*(a + b/x)/b + Sin[2*(a + b/x)]/(4*b)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{1}{2x}+\frac{\sin\left(\frac{2ax+2b}{x}\right)}{4b}$	23
parallelrisch	$\frac{-2b+x\sin\left(\frac{2ax+2b}{x}\right)}{4bx}$	28
derivativedivides	$-\frac{-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}}{b}$	34
default	$-\frac{-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}}{b}$	34
norman	$\frac{-\frac{1}{2}+\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}-\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)-\frac{\left(\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{2}-\frac{x\left(\tan^3\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)^2 x}$	89

```
[In] int(sin(a+b/x)^2/x^2,x,method=_RETURNVERBOSE)
[Out] -1/2/x+1/4/b*sin(2*(a*x+b)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx = \frac{x\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)-b}{2\,bx}$$

```
[In] integrate(sin(a+b/x)^2/x^2,x, algorithm="fricas")
[Out] 1/2*(x*cos((a*x + b)/x)*sin((a*x + b)/x) - b)/(b*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(20) = 40.

Time = 1.02 (sec) , antiderivative size = 262, normalized size of antiderivative = 8.45

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx = \begin{cases} -\frac{b\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)}{2bx\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx} - \frac{2b\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)}{2bx\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx} - \frac{b}{2bx\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx} \\ -\frac{\sin^2(a)}{x} \end{cases}$$

```
[In] integrate(sin(a+b/x)**2/x**2,x)
[Out] Piecewise((-b*tan(a/2 + b/(2*x))**4/(2*b*x*tan(a/2 + b/(2*x))**4 + 4*b*x*ta
n(a/2 + b/(2*x))**2 + 2*b*x) - 2*b*tan(a/2 + b/(2*x))**2/(2*b*x*tan(a/2 + b
/(2*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**2 + 2*b*x) - b/(2*b*x*tan(a/2 + b/(2
*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**2 + 2*b*x) - 2*x*tan(a/2 + b/(2*x))**3/
(2*b*x*tan(a/2 + b/(2*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**2 + 2*b*x) + 2*x*t
an(a/2 + b/(2*x))/(2*b*x*tan(a/2 + b/(2*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**
2 + 2*b*x), Ne(b, 0)), (-sin(a)**2/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx = \frac{x\sin\left(\frac{2\,(ax+b)}{x}\right)-2\,b}{4\,bx}$$

[In] `integrate(sin(a+b/x)^2/x^2,x, algorithm="maxima")`

[Out] `1/4*(x*sin(2*(a*x + b)/x) - 2*b)/(b*x)`

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\frac{2\,(ax+b)}{x}-\sin\left(\frac{2\,(ax+b)}{x}\right)}{4\,b}$$

[In] `integrate(sin(a+b/x)^2/x^2,x, algorithm="giac")`

[Out] `-1/4*(2*(a*x + b)/x - sin(2*(a*x + b)/x))/b`

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx = \frac{\sin\left(2\,a+\frac{2\,b}{x}\right)}{4\,b}-\frac{1}{2\,x}$$

[In] `int(sin(a + b/x)^2/x^2,x)`

[Out] `sin(2*a + (2*b)/x)/(4*b) - 1/(2*x)`

3.116 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3} dx = -\frac{1}{4x^2} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}$$

```
[Out] -1/4/x^2+1/2*cos(a+b/x)*sin(a+b/x)/b/x-1/4*sin(a+b/x)^2/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3460, 3391, 30}

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3} dx = -\frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2} + \frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx} - \frac{1}{4x^2}$$

```
[In] Int[Sin[a + b/x]^2/x^3,x]
```

```
[Out] -1/4*1/x^2 + (Cos[a + b/x]*Sin[a + b/x])/(2*b*x) - Sin[a + b/x]^2/(4*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
```

```
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x\sin^2(a+bx)\,dx, x, \frac{1}{x}\right) \\
&= \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2} - \frac{1}{2}\text{Subst}\left(\int x\,dx, x, \frac{1}{x}\right) \\
&= -\frac{1}{4x^2} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{x^2\cos\left(2\left(a+\frac{b}{x}\right)\right) - 2b\left(b - x\sin\left(2\left(a+\frac{b}{x}\right)\right)\right)}{8b^2x^2}$$

```
[In] Integrate[Sin[a + b/x]^2/x^3,x]
[Out] (x^2*Cos[2*(a + b/x)] - 2*b*(b - x*Sin[2*(a + b/x)]))/(8*b^2*x^2)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{1}{4x^2}+\frac{\cos\left(\frac{2ax+2b}{x}\right)}{8b^2}+\frac{\sin\left(\frac{2ax+2b}{x}\right)}{4bx}$	42
parallelrisch	$\frac{2bx\sin\left(\frac{2ax+2b}{x}\right)+x^2\cos\left(\frac{2ax+2b}{x}\right)-2b^2-x^2}{8x^2b^2}$	54
derivativedivides	$-\frac{\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\left(\sin^2\left(a+\frac{b}{x}\right)\right)}{4}-a\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)}{b^2}$	97
default	$-\frac{\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\left(\sin^2\left(a+\frac{b}{x}\right)\right)}{4}-a\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)}{b^2}$	97
norman	$\frac{-\frac{1}{4}+\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}-\frac{x^2\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b^2}-\frac{\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{2}-\frac{\left(\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{4}-\frac{x\left(\tan^3\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)^2x^2}$	110

```
[In] int(sin(a+b/x)^2/x^3,x,method=_RETURNVERBOSE)
[Out] -1/4/x^2+1/8/b^2*cos(2*(a*x+b)/x)+1/4/b/x*sin(2*(a*x+b)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx=\frac{2\,x^2\cos\left(\frac{ax+b}{x}\right)^2+4\,bx\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)-2\,b^2-x^2}{8\,b^2x^2}$$

```
[In] integrate(sin(a+b/x)^2/x^3,x, algorithm="fricas")
[Out] 1/8*(2*x^2*cos((a*x + b)/x)^2 + 4*b*x*cos((a*x + b)/x)*sin((a*x + b)/x) - 2
*b^2 - x^2)/(b^2*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(37) = 74.

Time = 1.30 (sec) , antiderivative size = 391, normalized size of antiderivative = 7.67

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx$$
$$=\begin{cases}-\frac{b^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2}-\frac{2b^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2}-\frac{b^2}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\,\mathrm{t}} \\ -\frac{\sin^2(a)}{2x^2}\end{cases}$$

```
[In] integrate(sin(a+b/x)**2/x**3,x)
```

```
[Out] Piecewise((-b**2*tan(a/2 + b/(2*x))**4/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 +
 8*b**2*x**2*tan(a/2 + b/(2*x))**2 + 4*b**2*x**2) - 2*b**2*tan(a/2 + b/(2*x
))**2/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b/(2*x))**
2 + 4*b**2*x**2) - b**2/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*ta
n(a/2 + b/(2*x))**2 + 4*b**2*x**2) - 4*b*x*tan(a/2 + b/(2*x))**3/(4*b**2*x*
*2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b/(2*x))**2 + 4*b**2*x**2)
 + 4*b*x*tan(a/2 + b/(2*x))/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**
2*tan(a/2 + b/(2*x))**2 + 4*b**2*x**2) - 4*x**2*tan(a/2 + b/(2*x))**2/(4*b*
*2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b/(2*x))**2 + 4*b**2*
x**2), Ne(b, 0)), (-sin(a)**2/(2*x**2), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{\left(\left(\Gamma\left(2,\frac{2i\,b}{x}\right)+\Gamma\left(2,-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)-\left(i\,\Gamma\left(2,\frac{2i\,b}{x}\right)-i\,\Gamma\left(2,-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)x^2-4\,b^2}{16\,b^2x^2}$$

```
[In] integrate(sin(a+b/x)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(((gamma(2, 2*I*b/x) + gamma(2, -2*I*b/x))*cos(2*a) - (I*gamma(2, 2*I*
b/x) - I*gamma(2, -2*I*b/x))*sin(2*a))*x^2 - 4*b^2)/(b^2*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\frac{2\,a\sin\left(\frac{2\,(ax+b)}{x}\right)-\frac{4\,(ax+b)a}{x}-\frac{2\,(ax+b)\sin\left(\frac{2\,(ax+b)}{x}\right)}{x}+\frac{2\,(ax+b)^2}{x^2}-\cos\left(\frac{2\,(ax+b)}{x}\right)}{8\,b^2}$$

```
[In] integrate(sin(a+b/x)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*a*sin(2*(a*x + b)/x) - 4*(a*x + b)*a/x - 2*(a*x + b)*sin(2*(a*x + b
)/x)/x + 2*(a*x + b)^2/x^2 - cos(2*(a*x + b)/x))/b^2
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx=\frac{\cos\left(2\,a+\frac{2\,b}{x}\right)}{8\,b^2}-\frac{1}{4\,x^2}+\frac{\sin\left(2\,a+\frac{2\,b}{x}\right)}{4\,b\,x}$$

```
[In] int(sin(a + b/x)^2/x^3,x)
[Out] cos(2*a + (2*b)/x)/(8*b^2) - 1/(4*x^2) + sin(2*a + (2*b)/x)/(4*b*x)
```

3.117 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 87

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\frac{1}{6x^3}+\frac{1}{4b^2x}-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^2}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}$$

```
[Out] -1/6/x^3+1/4/b^2/x-1/4*cos(a+b/x)*sin(a+b/x)/b^3+1/2*cos(a+b/x)*sin(a+b/x)/
b/x^2-1/2*sin(a+b/x)^2/b^2/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3460, 3392, 30, 2715, 8}

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{4b^3}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x} + \frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^2}+\frac{1}{4b^2x}-\frac{1}{6x^3}$$

```
[In] Int[Sin[a + b/x]^2/x^4,x]
```

```
[Out] -1/6*1/x^3 + 1/(4*b^2*x) - (Cos[a + b/x]*Sin[a + b/x])/(4*b^3) + (Cos[a + b
/x]*Sin[a + b/x])/(2*b*x^2) - Sin[a + b/x]^2/(2*b^2*x)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^2\sin^2(a+bx)\,dx, x, \frac{1}{x}\right)\\
&= \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^2} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}\\
&\quad - \frac{1}{2}\text{Subst}\left(\int x^2\,dx, x, \frac{1}{x}\right) + \frac{\text{Subst}\left(\int \sin^2(a+bx)\,dx, x, \frac{1}{x}\right)}{2b^2}\\
&= -\frac{1}{6x^3} - \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^2}\\
&\quad - \frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x} + \frac{\text{Subst}\left(\int 1\,dx, x, \frac{1}{x}\right)}{4b^2}\\
&= -\frac{1}{6x^3} + \frac{1}{4b^2x} - \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^2} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx=\frac{-4b^3+6bx^2\cos\left(2\left(a+\frac{b}{x}\right)\right)-3(-2b^2x+x^3)\sin\left(2\left(a+\frac{b}{x}\right)\right)}{24b^3x^3}$$

```
[In] Integrate[Sin[a + b/x]^2/x^4,x]
[Out] (-4*b^3 + 6*b*x^2*Cos[2*(a + b/x)] - 3*(-2*b^2*x + x^3)*Sin[2*(a + b/x)])/(
24*b^3*x^3)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{1}{6x^3}+\frac{\cos\left(\frac{2ax+2b}{x}\right)}{4b^2x}+\frac{(2b^2-x^2)\sin\left(\frac{2ax+2b}{x}\right)}{8b^3x^2}$
parallelrisch	$\frac{6x^2b\cos\left(\frac{2ax+2b}{x}\right)+6b^2x\sin\left(\frac{2ax+2b}{x}\right)-3x^3\sin\left(\frac{2ax+2b}{x}\right)-4b^3}{24x^3b^3}$
norman	$\frac{-\frac{1}{6}+\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}-\frac{\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{3}-\frac{\left(\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{6}+\frac{x^2}{4b^2}-\frac{x^3\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{2b^3}+\frac{x^3\left(\tan^3\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{2b^3}-\frac{3x^2\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{2b^2}+\frac{x^2}{}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)^2x^3}$
derivativedivides	$-\frac{a^2\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-2a\left(\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\left(\sin^2\left(a+\frac{b}{x}\right)\right)}{4}\right)+\left(a+\right.}{b^3}$
default	$-\frac{a^2\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-2a\left(\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\left(\sin^2\left(a+\frac{b}{x}\right)\right)}{4}\right)+\left(a+\right.}{b^3}$

```
[In] int(sin(a+b/x)^2/x^4,x,method=_RETURNVERBOSE)
[Out] -1/6/x^3+1/4/b^2/x*cos(2*(a*x+b)/x)+1/8*(2*b^2-x^2)/b^3/x^2*sin(2*(a*x+b)/x
)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx=\frac{6\,bx^2\cos\left(\frac{ax+b}{x}\right)^2-2\,b^3-3\,bx^2+3\,(2\,b^2x-x^3)\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)}{12\,b^3x^3}$$

```
[In] integrate(sin(a+b/x)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*x^2*cos((a*x + b)/x)^2 - 2*b^3 - 3*b*x^2 + 3*(2*b^2*x - x^3)*cos(
(a*x + b)/x)*sin((a*x + b)/x))/(b^3*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(68) = 136$.

Time = 1.77 (sec) , antiderivative size = 654, normalized size of antiderivative = 7.52

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx = \begin{cases} -\frac{2b^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+24b^3x^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+12b^3x^3} - \frac{4b^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+24b^3x^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+12b^3x^3} - \frac{}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)\cdots} \\ -\frac{\sin^2(a)}{3x^3} \end{cases}$$

```
[In] integrate(sin(a+b/x)**2/x**4,x)
```

```
[Out] Piecewise((-2*b**3*tan(a/2 + b/(2*x))**4/(12*b**3*x**3*tan(a/2 + b/(2*x))**
4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) - 4*b**3*tan(a/2 + b
/(2*x))**2/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(
2*x))**2 + 12*b**3*x**3) - 2*b**3/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*
b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) - 12*b**2*x*tan(a/2 + b/(2*
x))**3/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x)
)**2 + 12*b**3*x**3) + 12*b**2*x*tan(a/2 + b/(2*x))/(12*b**3*x**3*tan(a/2 +
 b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) + 3*b*x**
2*tan(a/2 + b/(2*x))**4/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*
tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) - 18*b*x**2*tan(a/2 + b/(2*x))**2/(12
*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*
b**3*x**3) + 3*b*x**2/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*ta
n(a/2 + b/(2*x))**2 + 12*b**3*x**3) + 6*x**3*tan(a/2 + b/(2*x))**3/(12*b**3
*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*
x**3) - 6*x**3*tan(a/2 + b/(2*x))/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*
b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3), Ne(b, 0)), (-sin(a)**2/(3*
x**3), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{3\left(\left(-i\,\Gamma\left(3,\frac{2i\,b}{x}\right)+i\,\Gamma\left(3,-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)-\left(\Gamma\left(3,\frac{2i\,b}{x}\right)+\Gamma\left(3,-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)x^3-16\,b^3}{96\,b^3x^3}$$

```
[In] integrate(sin(a+b/x)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(3*((-I*gamma(3, 2*I*b/x) + I*gamma(3, -2*I*b/x))*cos(2*a) - (gamma(3,
 2*I*b/x) + gamma(3, -2*I*b/x))*sin(2*a))*x^3 - 16*b^3)/(b^3*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.76

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx$$
$$= \frac{6\,a^2\sin\left(\frac{2\,(ax+b)}{x}\right) - \frac{12\,(ax+b)a^2}{x} - 6\,a\cos\left(\frac{2\,(ax+b)}{x}\right) - \frac{12\,(ax+b)a\sin\left(\frac{2\,(ax+b)}{x}\right)}{x} + \frac{12\,(ax+b)^2a}{x^2} + \frac{6\,(ax+b)\cos\left(\frac{2\,(ax+b)}{x}\right)}{x} +}{24\,b^3}$$

```
[In] integrate(sin(a+b/x)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(6*a^2*sin(2*(a*x + b)/x) - 12*(a*x + b)*a^2/x - 6*a*cos(2*(a*x + b)/x
) - 12*(a*x + b)*a*sin(2*(a*x + b)/x)/x + 12*(a*x + b)^2*a/x^2 + 6*(a*x + b
)*cos(2*(a*x + b)/x)/x + 6*(a*x + b)^2*sin(2*(a*x + b)/x)/x^2 - 4*(a*x + b)
^3/x^3 - 3*sin(2*(a*x + b)/x))/b^3
```

Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{\frac{b\,x^2\,\cos\left(2\,a+\frac{2\,b}{x}\right)}{4} - \frac{b^3}{6} + \frac{b^2\,x\,\sin\left(2\,a+\frac{2\,b}{x}\right)}{4}}{b^3\,x^3} - \frac{\sin\left(2\,a+\frac{2\,b}{x}\right)}{8\,b^3}$$

```
[In] int(sin(a + b/x)^2/x^4,x)
```

```
[Out] ((b*x^2*cos(2*a + (2*b)/x))/4 - b^3/6 + (b^2*x*sin(2*a + (2*b)/x))/4)/(b^3*
x^3) - sin(2*a + (2*b)/x)/(8*b^3)
```

3.118 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5} dx$

Optimal result

Integrand size = 14, antiderivative size = 107

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5} dx = -\frac{1}{8x^4} + \frac{3}{8b^2x^2} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^3} - \frac{3\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3x} + \frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4} - \frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}$$

```
[Out] -1/8/x^4+3/8/b^2/x^2+1/2*cos(a+b/x)*sin(a+b/x)/b/x^3-3/4*cos(a+b/x)*sin(a+b
/x)/b^3/x+3/8*sin(a+b/x)^2/b^4-3/4*sin(a+b/x)^2/b^2/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3460, 3392, 30, 3391}

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5} dx = \frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4} - \frac{3\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{4b^3x} - \frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2} + \frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^3} + \frac{3}{8b^2x^2} - \frac{1}{8x^4}$$

```
[In] Int[Sin[a + b/x]^2/x^5,x]
```

```
[Out] -1/8*1/x^4 + 3/(8*b^2*x^2) + (Cos[a + b/x]*Sin[a + b/x])/(2*b*x^3) - (3*Cos
[a + b/x]*Sin[a + b/x])/(4*b^3*x) + (3*Sin[a + b/x]^2)/(8*b^4) - (3*Sin[a +
 b/x]^2)/(4*b^2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^3\sin^2(a+bx)\,dx, x, \frac{1}{x}\right)\\
&= \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^3} - \frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}\\
&\quad -\frac{1}{2}\text{Subst}\left(\int x^3\,dx, x, \frac{1}{x}\right) + \frac{3\text{Subst}\left(\int x\sin^2(a+bx)\,dx, x, \frac{1}{x}\right)}{2b^2}\\
&= -\frac{1}{8x^4} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^3} - \frac{3\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3x}\\
&\quad + \frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4} - \frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2} + \frac{3\text{Subst}\left(\int x\,dx, x, \frac{1}{x}\right)}{4b^2}\\
&= -\frac{1}{8x^4} + \frac{3}{8b^2x^2} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^3}\\
&\quad - \frac{3\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3x} + \frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4} - \frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx = -\frac{3(-2b^2x^2+x^4)\cos\left(2\left(a+\frac{b}{x}\right)\right)+2b\left(b^3+(-2b^2x+3x^3)\sin\left(2\left(a+\frac{b}{x}\right)\right)\right)}{16b^4x^4}$$

```
[In] Integrate[Sin[a + b/x]^2/x^5,x]
```

```
[Out] -1/16*(3*(-2*b^2*x^2 + x^4)*Cos[2*(a + b/x)] + 2*b*(b^3 + (-2*b^2*x + 3*x^3
)*Sin[2*(a + b/x)]))/(b^4*x^4)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{1}{8x^4}+\frac{3(2b^2-x^2)\cos\left(\frac{2ax+2b}{x}\right)}{16x^2b^4}+\frac{(2b^2-3x^2)\sin\left(\frac{2ax+2b}{x}\right)}{8b^3x^3}$
parallelrisch	$\frac{(6x^2b^2-3x^4)\cos\left(\frac{2ax+2b}{x}\right)+(4b^3x-6b\,x^3)\sin\left(\frac{2ax+2b}{x}\right)-2b^4+3x^4}{16x^4b^4}$
norman	$\frac{-\frac{1}{8}+\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}-\frac{\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{4}-\frac{\left(\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{8}+\frac{3x^2}{8b^2}-\frac{3x^3\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{2b^3}+\frac{3x^3\left(\tan^3\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{2b^3}-\frac{9x^2\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{4b^2}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)^2x^4}$
derivativedivides	$-\frac{-a^3\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)+3a^2\left(\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\left(\sin^2\left(a+\frac{b}{x}\right)\right)}{4}\right)-}{}$
default	$-\frac{-a^3\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)+3a^2\left(\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\left(\sin^2\left(a+\frac{b}{x}\right)\right)}{4}\right)-}{}$

```
[In] int(sin(a+b/x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/x^4+3/16/x^2*(2*b^2-x^2)/b^4*cos(2*(a*x+b)/x)+1/8/b^3*(2*b^2-3*x^2)/x^
3*sin(2*(a*x+b)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx$$
$$= -\frac{2\,b^4+6\,b^2x^2-3\,x^4-6\,(2\,b^2x^2-x^4)\cos\left(\frac{ax+b}{x}\right)^2-4\,(2\,b^3x-3\,bx^3)\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)}{16\,b^4x^4}$$

```
[In] integrate(sin(a+b/x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(2*b^4 + 6*b^2*x^2 - 3*x^4 - 6*(2*b^2*x^2 - x^4)*cos((a*x + b)/x)^2 -
 4*(2*b^3*x - 3*b*x^3)*cos((a*x + b)/x)*sin((a*x + b)/x))/(b^4*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(92) = 184.

Time = 2.48 (sec) , antiderivative size = 726, normalized size of antiderivative = 6.79

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx$$
$$= \begin{cases} -\frac{b^4\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)}{8b^4x^4\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+16b^4x^4\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^4x^4} - \frac{2b^4\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)}{8b^4x^4\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+16b^4x^4\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^4x^4} - \frac{b^4}{8b^4x^4\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+16b^4x^4} \\ -\frac{\sin^2(a)}{4x^4} \end{cases}$$

```
[In] integrate(sin(a+b/x)**2/x**5,x)
```

```
[Out] Piecewise((-b**4*tan(a/2 + b/(2*x))**4/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 +
 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) - 2*b**4*tan(a/2 + b/(2*
x))**2/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))
**2 + 8*b**4*x**4) - b**4/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4
*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) - 8*b**3*x*tan(a/2 + b/(2*x))**3/(8*b
**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**
4*x**4) + 8*b**3*x*tan(a/2 + b/(2*x))/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 +
16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) + 3*b**2*x**2*tan(a/2 + b
/(2*x))**4/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2
*x))**2 + 8*b**4*x**4) - 18*b**2*x**2*tan(a/2 + b/(2*x))**2/(8*b**4*x**4*ta
n(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) + 3
*b**2*x**2/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2
*x))**2 + 8*b**4*x**4) + 12*b*x**3*tan(a/2 + b/(2*x))**3/(8*b**4*x**4*tan(a
/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) - 12*b
*x**3*tan(a/2 + b/(2*x))/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*
```

```
tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) + 12*x**4*tan(a/2 + b/(2*x))**2/(8*b**
4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*
x**4), Ne(b, 0)), (-sin(a)**2/(4*x**4), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx = -\frac{\left(\left(\Gamma\left(4,\frac{2ib}{x}\right)+\Gamma\left(4,-\frac{2ib}{x}\right)\right)\cos\left(2a\right)-\left(i\,\Gamma\left(4,\frac{2ib}{x}\right)-i\,\Gamma\left(4,-\frac{2ib}{x}\right)\right)\sin\left(2a\right)\right)x^4+8\,b^4}{64\,b^4x^4}$$

```
[In] integrate(sin(a+b/x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/64*(((gamma(4, 2*I*b/x) + gamma(4, -2*I*b/x))*cos(2*a) - (I*gamma(4, 2*I
*b/x) - I*gamma(4, -2*I*b/x))*sin(2*a))*x^4 + 8*b^4)/(b^4*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(95) = 190.

Time = 0.31 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.38

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx = -\frac{4\,a^3\sin\left(\frac{2\,(ax+b)}{x}\right)-\frac{8\,(ax+b)a^3}{x}-6\,a^2\cos\left(\frac{2\,(ax+b)}{x}\right)-\frac{12\,(ax+b)a^2\sin\left(\frac{2\,(ax+b)}{x}\right)}{x}+\frac{12\,(ax+b)^2a^2}{x^2}+\frac{12\,(ax+b)a\cos\left(\frac{2\,(a}{}\right.}{x}}{}$$

```
[In] integrate(sin(a+b/x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/16*(4*a^3*sin(2*(a*x + b)/x) - 8*(a*x + b)*a^3/x - 6*a^2*cos(2*(a*x + b)
/x) - 12*(a*x + b)*a^2*sin(2*(a*x + b)/x)/x + 12*(a*x + b)^2*a^2/x^2 + 12*(
a*x + b)*a*cos(2*(a*x + b)/x)/x - 6*a*sin(2*(a*x + b)/x) + 12*(a*x + b)^2*a
*sin(2*(a*x + b)/x)/x^2 - 8*(a*x + b)^3*a/x^3 - 6*(a*x + b)^2*cos(2*(a*x +
b)/x)/x^2 - 4*(a*x + b)^3*sin(2*(a*x + b)/x)/x^3 + 6*(a*x + b)*sin(2*(a*x +
 b)/x)/x + 2*(a*x + b)^4/x^4 + 3*cos(2*(a*x + b)/x))/b^4
```

Mupad [B] (verification not implemented)

Time = 6.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx = -\frac{3\,\cos\left(2\,a+\frac{2\,b}{x}\right)}{16\,b^4} - \frac{\frac{b^4}{8}-\frac{3\,b^2\,x^2\,\cos\left(2\,a+\frac{2\,b}{x}\right)}{8}+\frac{3\,b\,x^3\,\sin\left(2\,a+\frac{2\,b}{x}\right)}{8}-\frac{b^3\,x\,\sin\left(2\,a+\frac{2\,b}{x}\right)}{4}}{b^4\,x^4}$$

```
[In] int(sin(a + b/x)^2/x^5,x)
```

```
[Out] - (3*cos(2*a + (2*b)/x))/(16*b^4) - (b^4/8 - (3*b^2*x^2*cos(2*a + (2*b)/x))
/8 + (3*b*x^3*sin(2*a + (2*b)/x))/8 - (b^3*x*sin(2*a + (2*b)/x))/4)/(b^4*x^
4)
```

3.119 $\int \sin \left(a+\frac{b}{x^2}\right) dx$

Optimal result

Integrand size = 8, antiderivative size = 80

$$\int \sin \left(a+\frac{b}{x^2}\right) dx = -\sqrt{b}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) + \sqrt{b}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a) + x\sin\left(a+\frac{b}{x^2}\right)$$

```
[Out] x*sin(a+b/x^2)-cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*b^(1/2)*2^(1/2)*
Pi^(1/2)+FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)*b^(1/2)*2^(1/2)*Pi^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3440, 3468, 3435, 3433, 3432}

$$\int \sin \left(a+\frac{b}{x^2}\right) dx = \sqrt{2\pi}\left(-\sqrt{b}\right)\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) + \sqrt{2\pi}\sqrt{b}\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) + x\sin\left(a+\frac{b}{x^2}\right)$$

```
[In] Int[Sin[a + b/x^2],x]
```

```
[Out] -(Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]) + Sqrt[b]*Sqr
t[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a] + x*Sin[a + b/x^2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sin\left(a+bx^2\right)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= x\sin\left(a+\frac{b}{x^2}\right) - (2b)\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= x\sin\left(a+\frac{b}{x^2}\right) - (2b\cos(a))\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&\quad + (2b\sin(a))\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{x}\right)
\end{aligned}
$$

$$= -\sqrt{b}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) + \sqrt{b}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a) + x\sin\left(a + \frac{b}{x^2}\right)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \sin\left(a+\frac{b}{x^2}\right)\,dx = x\cos\left(\frac{b}{x^2}\right)\sin(a) - \sqrt{b}\sqrt{2\pi}\left(\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) - \text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)\right) + x\cos(a)\sin\left(\frac{b}{x^2}\right)$$

```
[In] Integrate[Sin[a + b/x^2],x]
```

```
[Out] x*Cos[b/x^2]*Sin[a] - Sqrt[b]*Sqrt[2*Pi]*(Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/P
i])/x] - FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a]) + x*Cos[a]*Sin[b/x^2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

method	result
derivativedivides	$x\sin\left(a+\frac{b}{x^2}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos\left(a\right)\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)-\sin\left(a\right)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)$
default	$x\sin\left(a+\frac{b}{x^2}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos\left(a\right)\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)-\sin\left(a\right)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)$
risch	$-\frac{\mathrm{e}^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{x}\right)}{2\sqrt{-ib}}-\frac{\mathrm{e}^{-ia}b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{ib}}{x}\right)}{2\sqrt{ib}}+x\sin\left(\frac{x^2a+b}{x^2}\right)$
meijerg	$-\frac{\sqrt{\pi}\,\cos(a)\sqrt{2}\,\sqrt{b}\left(-\frac{4\sqrt{2}\,x\sin\left(\frac{b}{x^2}\right)}{\sqrt{b}\,\sqrt{\pi}}+8\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{8}-\frac{\sqrt{\pi}\,\sin(a)\sqrt{2}\,\left(b^2\right)^{\frac{1}{4}}\left(-\frac{4x\sqrt{2}\,\cos\left(\frac{b}{x^2}\right)}{\sqrt{\pi}\,\left(b^2\right)^{\frac{1}{4}}}-\frac{8\sqrt{b}\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)}{\left(b^2\right)^{\frac{1}{4}}}\right)}{8}$

```
[In] int(sin(a+b/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x*sin(a+b/x^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi
^(1/2)/x)-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \sin\left(a+\frac{b}{x^2}\right)\,dx = -\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right) + \sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin(a) + x\sin\left(\frac{ax^2+b}{x^2}\right)$$

```
[In] integrate(sin(a+b/x^2),x, algorithm="fricas")
[Out] -sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x) + sqrt(2)*p
i*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x)*sin(a) + x*sin((a*x^2 + b)/x
^2)
```

Sympy [F]

$$\int \sin\left(a+\frac{b}{x^2}\right)\,dx = \int \sin\left(a+\frac{b}{x^2}\right)dx$$

```
[In] integrate(sin(a+b/x**2),x)
[Out] Integral(sin(a + b/x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.59

$$\int \sin\left(a+\frac{b}{x^2}\right)\,dx = \frac{\sqrt{2}\left(2\sqrt{2}bx^2\sqrt{\frac{1}{x^4}}\sin\left(\frac{ax^2+b}{x^2}\right) + \left(\left((i-1)\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{\frac{ib}{x^2}}\right)-1\right) - (i+1)\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{-\frac{ib}{x^2}}\right)-1\right)\right)\cos(a)\right.\right.}{4\,bx}$$

```
[In] integrate(sin(a+b/x^2),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*(2*sqrt(2)*b*x^2*sqrt(x^(-4))*sin((a*x^2 + b)/x^2) + (((I - 1)*
sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/x^2)) -
 1))*cos(a) + ((I + 1)*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - (I - 1)*sqrt(pi)
*(erf(sqrt(-I*b/x^2)) - 1))*sin(a))*b*(b^2/x^4)^(1/4))*sqrt(x^4)/(b*x)
```

Giac [F]

$$\int \sin\left(a+\frac{b}{x^2}\right)\,dx = \int \sin\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] integrate(sin(a+b/x^2),x, algorithm="giac")
[Out] integrate(sin(a + b/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+\frac{b}{x^2}\right)\,dx = \int \sin\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] int(sin(a + b/x^2),x)
[Out] int(sin(a + b/x^2), x)
```

3.120 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx = -\frac{1}{2}\,\text{CosIntegral}\left(\frac{b}{x^2}\right)\sin(a) - \frac{1}{2}\cos(a)\text{Si}\left(\frac{b}{x^2}\right)$$

```
[Out] -1/2*cos(a)*Si(b/x^2)-1/2*Ci(b/x^2)*sin(a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3458, 3457, 3456}

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx = -\frac{1}{2}\sin(a)\,\text{CosIntegral}\left(\frac{b}{x^2}\right) - \frac{1}{2}\cos(a)\text{Si}\left(\frac{b}{x^2}\right)$$

```
[In] Int[Sin[a + b/x^2]/x,x]
[Out] -1/2*(CosIntegral[b/x^2]*Sin[a]) - (Cos[a]*SinIntegral[b/x^2])/2
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cos(a)\int \frac{\sin\left(\frac{b}{x^2}\right)}{x}\,dx+\sin(a)\int \frac{\cos\left(\frac{b}{x^2}\right)}{x}\,dx\\ &= -\frac{1}{2}\,\text{CosIntegral}\left(\frac{b}{x^2}\right)\sin(a)-\frac{1}{2}\cos(a)\text{Si}\left(\frac{b}{x^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx=\frac{1}{2}\left(-\,\text{CosIntegral}\left(\frac{b}{x^2}\right)\sin(a)-\cos(a)\text{Si}\left(\frac{b}{x^2}\right)\right)$$

```
[In] Integrate[Sin[a + b/x^2]/x,x]
[Out] (-(CosIntegral[b/x^2]*Sin[a]) - Cos[a]*SinIntegral[b/x^2])/2
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\cos(a)\,\mathrm{Si}\left(\frac{b}{x^2}\right)}{2}-\frac{\mathrm{Ci}\left(\frac{b}{x^2}\right)\sin(a)}{2}$	22
default	$-\frac{\cos(a)\,\mathrm{Si}\left(\frac{b}{x^2}\right)}{2}-\frac{\mathrm{Ci}\left(\frac{b}{x^2}\right)\sin(a)}{2}$	22
risch	$-\frac{i\mathrm{e}^{ia}\,\mathrm{Ei}_1\left(-\frac{ib}{x^2}\right)}{4}+\frac{\mathrm{e}^{-ia}\pi\,\mathrm{csgn}\left(\frac{b}{x^2}\right)}{4}-\frac{\mathrm{e}^{-ia}\,\mathrm{Si}\left(\frac{b}{x^2}\right)}{2}+\frac{i\,\mathrm{Ei}_1\left(-\frac{ib}{x^2}\right)\mathrm{e}^{-ia}}{4}$	63
meijerg	$-\frac{\cos(a)\,\mathrm{Si}\left(\frac{b}{x^2}\right)}{2}-\frac{\sqrt{\pi}\,\sin(a)\left(\frac{2\gamma-4\ln(x)+\ln\left(b^2\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{b}{2x^2}\right)}{\sqrt{\pi}}+\frac{2\,\mathrm{Ci}\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{4}$	72

```
[In] int(sin(a+b/x^2)/x,x,method=_RETURNVERBOSE)
[Out] -1/2*cos(a)*Si(b/x^2)-1/2*Ci(b/x^2)*sin(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx = -\frac{1}{2}\,\mathrm{Ci}\left(\frac{b}{x^2}\right)\sin(a) - \frac{1}{2}\cos(a)\,\mathrm{Si}\left(\frac{b}{x^2}\right)$$

```
[In] integrate(sin(a+b/x^2)/x,x, algorithm="fricas")
[Out] -1/2*cos_integral(b/x^2)*sin(a) - 1/2*cos(a)*sin_integral(b/x^2)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx = \int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

```
[In] integrate(sin(a+b/x**2)/x,x)
[Out] Integral(sin(a + b/x**2)/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx = \frac{1}{4}\left(i\,\mathrm{Ei}\left(\frac{i\,b}{x^2}\right) - i\,\mathrm{Ei}\left(-\frac{i\,b}{x^2}\right)\right)\cos(a) - \frac{1}{4}\left(\mathrm{Ei}\left(\frac{i\,b}{x^2}\right) + \mathrm{Ei}\left(-\frac{i\,b}{x^2}\right)\right)\sin(a)$$

```
[In] integrate(sin(a+b/x^2)/x,x, algorithm="maxima")
[Out] 1/4*(I*Ei(I*b/x^2) - I*Ei(-I*b/x^2))*cos(a) - 1/4*(Ei(I*b/x^2) + Ei(-I*b/x^
2))*sin(a)
```

Giac [F]

$$\int \frac{\sin\left(a + \frac{b}{x^2}\right)}{x}\,dx = \int \frac{\sin\left(a + \frac{b}{x^2}\right)}{x}\,dx$$

```
[In] integrate(sin(a+b/x^2)/x,x, algorithm="giac")
[Out] integrate(sin(a + b/x^2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a + \frac{b}{x^2}\right)}{x}\,dx = -\frac{\sin(a)\ \mathrm{cosint}\left(\frac{b}{x^2}\right)}{2} - \frac{\cos(a)\ \mathrm{sinint}\left(\frac{b}{x^2}\right)}{2}$$

```
[In] int(sin(a + b/x^2)/x,x)
[Out] - (sin(a)*cosint(b/x^2))/2 - (cos(a)*sinint(b/x^2))/2
```

3.121 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{\sqrt{b}}$$

```
[Out] -1/2*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*2^(1/2)*Pi^(1/2)/b^(1/2)-1
/2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3464, 3434, 3433, 3432}

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}$$

```
[In] Int[Sin[a + b/x^2]/x^2,x]
```

```
[Out] -((Sqrt[Pi/2]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x])/Sqrt[b]) - (Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/Sqrt[b]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3464

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)], x_Symbol] :> Dist[2/n, Subst[I
nt[Sin[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
n/2 - 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \sin\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= -\left(\cos(a)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{x}\right)\right) - \sin(a)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$
\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\left(\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) + \text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)\right)}{\sqrt{b}}
$$

```
[In] Integrate[Sin[a + b/x^2]/x^2,x]
[Out] -((Sqrt[Pi/2]*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x] + FresnelC[(Sqrt[b]*
Sqrt[2/Pi])/x]*Sin[a]))/Sqrt[b])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(a\right)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)+\sin\left(a\right)\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{2\sqrt{b}}$	47
default	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(a\right)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)+\sin\left(a\right)\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{2\sqrt{b}}$	47
meijerg	$-\frac{\cos\left(a\right)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\sqrt{2}\,\sqrt{\pi}}{2\sqrt{b}}-\frac{\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\sin\left(a\right)\sqrt{2}\,\sqrt{\pi}}{2\sqrt{b}}$	56
risch	$\frac{i\,{e}^{i a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-i b}}{x}\right)}{4\sqrt{-i b}}-\frac{i\,{e}^{-i a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{i b}}{x}\right)}{4\sqrt{i b}}$	58

```
[In] int(sin(a+b/x^2)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/2*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)+
sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx=-\frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)+\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin\left(a\right)}{2\,b}$$

```
[In] integrate(sin(a+b/x^2)/x^2,x, algorithm="fricas")
[Out] -1/2*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x) + sqrt
(2)*pi*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x)*sin(a))/b
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx=\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

```
[In] integrate(sin(a+b/x**2)/x**2,x)
[Out] Integral(sin(a + b/x**2)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.31

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = -\frac{\sqrt{2}\sqrt{x^4}\left(\left((i+1)\,\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{ib}{x^2}}\right)-1\right)-(i-1)\,\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{ib}{x^2}}\right)-1\right)\right)\cos\left(a\right)+\left(-(i-1)\,\sqrt{\pi}\left(\operatorname{erf}\right.\right.\right.}{8\,bx}$$

```
[In] integrate(sin(a+b/x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(x^4)*(((I + 1)*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - (I - 1
)*sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt
(I*b/x^2)) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*sin(a))*(b^2/
x^4)^(1/4)/(b*x)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

```
[In] integrate(sin(a+b/x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x^2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = -\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}}{x\,\sqrt{\pi}}\right)\,\cos\left(a\right)}{2\,\sqrt{b}} - \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}}{x\,\sqrt{\pi}}\right)\,\sin\left(a\right)}{2\,\sqrt{b}}$$

```
[In] int(sin(a + b/x^2)/x^2,x)
```

```
[Out] - (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b^(1/2))/(x*pi^(1/2)))*cos(a))/(2*b^(
1/2)) - (2^(1/2)*pi^(1/2)*fresnelc((2^(1/2)*b^(1/2))/(x*pi^(1/2)))*sin(a))/
(2*b^(1/2))
```

3.122 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3} dx = \frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[Out] 1/2*cos(a+b/x^2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3460, 2718}

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3} dx = \frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[In] Int[Sin[a + b/x^2]/x^3,x]
```

```
[Out] Cos[a + b/x^2]/(2*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
```

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = \frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[In] Integrate[Sin[a + b/x^2]/x^3,x]
[Out] Cos[a + b/x^2]/(2*b)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$	14
default	$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$	14
risch	$\frac{\cos\left(\frac{x^2a+b}{x^2}\right)}{2b}$	18
parallelrisch	$\frac{-1+\cos\left(\frac{x^2a+b}{x^2}\right)}{2b}$	20
norman	$\frac{1}{b\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x^2}\right)\right)}$	22
meijerg	$-\frac{\sqrt{\pi}\,\cos(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{2b}-\frac{\sin(a)\sin\left(\frac{b}{x^2}\right)}{2b}$	40

```
[In] int(sin(a+b/x^2)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2*cos(a+b/x^2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx=\frac{\cos\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

```
[In] integrate(sin(a+b/x^2)/x^3,x, algorithm="fricas")
[Out] 1/2*cos((a*x^2 + b)/x^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx=\begin{cases}\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b} & \text{for } b\neq 0\\ -\frac{\sin(a)}{2x^2} & \text{otherwise}\end{cases}$$

```
[In] integrate(sin(a+b/x**2)/x**3,x)
[Out] Piecewise((cos(a + b/x**2)/(2*b), Ne(b, 0)), (-sin(a)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx=\frac{\cos\left(a+\frac{b}{x^2}\right)}{2\,b}$$

```
[In] integrate(sin(a+b/x^2)/x^3,x, algorithm="maxima")
[Out] 1/2*cos(a + b/x^2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = \frac{\cos\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

```
[In] integrate(sin(a+b/x^2)/x^3,x, algorithm="giac")
[Out] 1/2*cos((a*x^2 + b)/x^2)/b
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = \frac{\cos\left(a+\frac{b}{x^2}\right)}{2\,b}$$

```
[In] int(sin(a + b/x^2)/x^3,x)
[Out] cos(a + b/x^2)/(2*b)
```

3.123 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 97

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4} dx = \frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx} - \frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{2b^{3/2}}$$

```
[Out] 1/2*cos(a+b/x^2)/b/x-1/4*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*2^(1/2
)*Pi^(1/2)/b^(3/2)+1/4*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)*2^(1/2)*
Pi^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3490, 3466, 3435, 3433, 3432}

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4} dx = -\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}} + \frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}$$

```
[In] Int[Sin[a + b/x^2]/x^4,x]
```

```
[Out] Cos[a + b/x^2]/(2*b*x) - (Sqrt[Pi/2]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x
])/(2*b^(3/2)) + (Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/(2*b^
(3/2))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^2 \sin\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= \frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx} - \frac{\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)}{2b}\\
&= \frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx} - \frac{\cos(a)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{x}\right)}{2b} + \frac{\sin(a)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{x}\right)}{2b}\\
&= \frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx} - \frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \frac{2\sqrt{b}\cos\left(a+\frac{b}{x^2}\right)-\sqrt{2\pi}x\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+\sqrt{2\pi}x\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{4b^{3/2}x}$$

```
[In] Integrate[Sin[a + b/x^2]/x^4,x]
[Out] (2*Sqrt[b]*Cos[a + b/x^2] - Sqrt[2*Pi]*x*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi
])/x] + Sqrt[2*Pi]*x*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/(4*b^(3/2)*x)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\sqrt{2}\sqrt{\pi}\left(\cos(a)\,\mathrm{C}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}\,x}\right)-\sin(a)\,\mathrm{S}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{4b^{\frac{3}{2}}}$
default	$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\sqrt{2}\sqrt{\pi}\left(\cos(a)\,\mathrm{C}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}\,x}\right)-\sin(a)\,\mathrm{S}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{4b^{\frac{3}{2}}}$
risch	$-\frac{e^{ia}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{x}\right)}{8b\sqrt{-ib}}-\frac{e^{-ia}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{ib}}{x}\right)}{8b\sqrt{ib}}+\frac{\cos\left(\frac{x^2a+b}{x^2}\right)}{2bx}$
meijerg	$-\frac{\sqrt{\pi}\,\cos(a)\sqrt{2}\left(-\frac{\sqrt{2}\sqrt{b}\,\cos\left(\frac{b}{x^2}\right)}{2\sqrt{\pi}\,x}+\frac{\mathrm{C}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}\,x}\right)}{2}\right)}{2b^{\frac{3}{2}}}-\frac{\sqrt{\pi}\,\sin(a)\sqrt{2}\,(b^2)^{\frac{1}{4}}\left(\frac{\sqrt{2}\,(b^2)^{\frac{3}{4}}\sin\left(\frac{b}{x^2}\right)}{2\sqrt{\pi}\,xb}-\frac{(b^2)^{\frac{3}{4}}\,\mathrm{S}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}\,x}\right)}{2b^{\frac{3}{2}}}\right)}{2b^2}$

```
[In] int(sin(a+b/x^2)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/2*cos(a+b/x^2)/b/x-1/4/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*
2^(1/2)/Pi^(1/2)/x)-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = -\frac{\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)-\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin(a)-2\,b\cos\left(\frac{ax^2+b}{x^2}\right)}{4\,b^2x}$$

```
[In] integrate(sin(a+b/x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x) - sq
rt(2)*pi*x*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x)*sin(a) - 2*b*cos((a
*x^2 + b)/x^2))/(b^2*x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

```
[In] integrate(sin(a+b/x**2)/x**4,x)
```

```
[Out] Integral(sin(a + b/x**2)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = -\frac{\sqrt{2}(x^4)^{\frac{3}{2}}\left(\left((i-1)\,\Gamma\left(\frac{3}{2},\frac{ib}{x^2}\right)-(i+1)\,\Gamma\left(\frac{3}{2},-\frac{ib}{x^2}\right)\right)\cos(a)+\left((i+1)\,\Gamma\left(\frac{3}{2},\frac{ib}{x^2}\right)-(i-1)\,\Gamma\left(\frac{3}{2},-\frac{ib}{x^2}\right)\right)\sin(a\cdots}{8\,b^3x^3}$$

```
[In] integrate(sin(a+b/x^2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*(x^4)^(3/2)*(((I - 1)*gamma(3/2, I*b/x^2) - (I + 1)*gamma(3/2,
 -I*b/x^2))*cos(a) + ((I + 1)*gamma(3/2, I*b/x^2) - (I - 1)*gamma(3/2, -I*b
/x^2))*sin(a))*(b^2/x^4)^(3/4)/(b^3*x^3)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

```
[In] integrate(sin(a+b/x^2)/x^4,x, algorithm="giac")
[Out] integrate(sin(a + b/x^2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

```
[In] int(sin(a + b/x^2)/x^4,x)
[Out] int(sin(a + b/x^2)/x^4, x)
```

3.124 $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 8

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\sqrt{x}\right)$$

```
[Out] -2*cos(x^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3460, 2718}

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\sqrt{x}\right)$$

```
[In] Int[Sin[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Cos[Sqrt[x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)\\
&= -2\cos\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\sqrt{x}\right)$$

```
[In] Integrate[Sin[Sqrt[x]]/Sqrt[x],x]
[Out] -2*Cos[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-2\cos\left(\sqrt{x}\right)$	7
default	$-2\cos\left(\sqrt{x}\right)$	7
meijerg	$2\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\sqrt{x}\right)}{\sqrt{\pi}}\right)$	19

```
[In] int(sin(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*cos(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\,\cos\left(\sqrt{x}\right)$$

```
[In] integrate(sin(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] -2*cos(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\sqrt{x}\right)$$

```
[In] integrate(sin(x**(1/2))/x**(1/2),x)
[Out] -2*cos(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\,\cos\left(\sqrt{x}\right)$$

```
[In] integrate(sin(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] -2*cos(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\,\cos\left(\sqrt{x}\right)$$

```
[In] integrate(sin(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] -2*cos(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\,\cos\left(\sqrt{x}\right)$$

```
[In] int(sin(x^(1/2))/x^(1/2),x)
[Out] -2*cos(x^(1/2))
```

3.125 $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}} dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}} dx = -2\cos\left(\sqrt{x}\right) + \frac{2}{3}\cos^3\left(\sqrt{x}\right)$$

```
[Out] -2*cos(x^(1/2))+2/3*cos(x^(1/2))^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3460, 2713}

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}} dx = \frac{2}{3}\cos^3\left(\sqrt{x}\right) - 2\cos\left(\sqrt{x}\right)$$

```
[In] Int[Sin[Sqrt[x]]^3/Sqrt[x],x]
```

```
[Out] -2*Cos[Sqrt[x]] + (2*Cos[Sqrt[x]]^3)/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
```

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \sin^3(x)\,dx, x, \sqrt{x}\right)\\
&= -\left(2\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos\left(\sqrt{x}\right)\right)\right)\\
&= -2\cos\left(\sqrt{x}\right)+\frac{2}{3}\cos^3\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{3}{2}\cos\left(\sqrt{x}\right)+\frac{1}{6}\cos\left(3\sqrt{x}\right)$$

```
[In] Integrate[Sin[Sqrt[x]]^3/Sqrt[x],x]
[Out] (-3*Cos[Sqrt[x]])/2 + Cos[3*Sqrt[x]]/6
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{2\left(2+\sin^2\left(\sqrt{x}\right)\right)\cos\left(\sqrt{x}\right)}{3}$	15
default	$-\frac{2\left(2+\sin^2\left(\sqrt{x}\right)\right)\cos\left(\sqrt{x}\right)}{3}$	15

```
[In] int(sin(x^(1/2))^3/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(2+sin(x^(1/2))^2)*cos(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2}{3}\cos\left(\sqrt{x}\right)^3 - 2\cos\left(\sqrt{x}\right)$$

```
[In] integrate(sin(x^(1/2))^3/x^(1/2),x, algorithm="fricas")
[Out] 2/3*cos(sqrt(x))^3 - 2*cos(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sin^2\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right) - \frac{4\cos^3\left(\sqrt{x}\right)}{3}$$

```
[In] integrate(sin(x**(1/2))**3/x**(1/2),x)
[Out] -2*sin(sqrt(x))**2*cos(sqrt(x)) - 4*cos(sqrt(x))**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2}{3}\cos\left(\sqrt{x}\right)^3 - 2\cos\left(\sqrt{x}\right)$$

```
[In] integrate(sin(x^(1/2))^3/x^(1/2),x, algorithm="maxima")
[Out] 2/3*cos(sqrt(x))^3 - 2*cos(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2}{3}\cos\left(\sqrt{x}\right)^3 - 2\cos\left(\sqrt{x}\right)$$

```
[In] integrate(sin(x^(1/2))^3/x^(1/2),x, algorithm="giac")
[Out] 2/3*cos(sqrt(x))^3 - 2*cos(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\,\cos\left(\sqrt{x}\right)\,\left(\cos\left(\sqrt{x}\right)^2-3\right)}{3}$$

```
[In] int(sin(x^(1/2))^3/x^(1/2),x)
[Out] (2*cos(x^(1/2))*(cos(x^(1/2))^2 - 3))/3
```

3.126 $\int \sin \left(\sqrt{x}\right) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \sin \left(\sqrt{x}\right) \, dx = -2\sqrt{x}\cos \left(\sqrt{x}\right) + 2\sin \left(\sqrt{x}\right)$$

```
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3442, 3377, 2717}

$$\int \sin \left(\sqrt{x}\right) \, dx = 2\sin \left(\sqrt{x}\right) - 2\sqrt{x}\cos \left(\sqrt{x}\right)$$

```
[In] Int[Sin[Sqrt[x]],x]
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int x\sin(x)\,dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

```
[In] Integrate[Sin[Sqrt[x]],x]
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$2\sin\left(\sqrt{x}\right) - 2\cos\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\sin\left(\sqrt{x}\right) - 2\cos\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\left(-\frac{\sqrt{x}\,\cos(\sqrt{x})}{2\sqrt{\pi}} + \frac{\sin(\sqrt{x})}{2\sqrt{\pi}}\right)$	28

```
[In] int(sin(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\,\sqrt{x}\cos\left(\sqrt{x}\right) + 2\,\sin\left(\sqrt{x}\right)$$

```
[In] integrate(sin(x^(1/2)),x, algorithm="fricas")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

```
[In] integrate(sin(x**(1/2)),x)
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\,\sqrt{x}\cos\left(\sqrt{x}\right) + 2\,\sin\left(\sqrt{x}\right)$$

```
[In] integrate(sin(x^(1/2)),x, algorithm="maxima")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\,\sqrt{x}\cos\left(\sqrt{x}\right) + 2\,\sin\left(\sqrt{x}\right)$$

```
[In] integrate(sin(x^(1/2)),x, algorithm="giac")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\,dx = 2\,\sin\left(\sqrt{x}\right) - 2\,\sqrt{x}\,\cos\left(\sqrt{x}\right)$$

```
[In] int(sin(x^(1/2)),x)
[Out] 2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))
```

3.127 $\int \sin^2\left(\sqrt[3]{x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 69

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = -\frac{3\sqrt[3]{x}}{4} + \frac{x}{2} + \frac{3}{4}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right) - \frac{3}{2}x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right) + \frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)$$

```
[Out] -3/4*x^(1/3)+1/2*x+3/4*cos(x^(1/3))*sin(x^(1/3))-3/2*x^(2/3)*cos(x^(1/3))*s
in(x^(1/3))+3/2*x^(1/3)*sin(x^(1/3))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3442, 3392, 30, 2715, 8}

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = -\frac{3}{2}x^{2/3}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right) + \frac{x}{2} - \frac{3\sqrt[3]{x}}{4} + \frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right) + \frac{3}{4}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)$$

```
[In] Int[Sin[x^(1/3)]^2,x]
```

```
[Out] (-3*x^(1/3))/4 + x/2 + (3*Cos[x^(1/3)]*Sin[x^(1/3)])/4 - (3*x^(2/3)*Cos[x^(
1/3)]*Sin[x^(1/3)])/2 + (3*x^(1/3)*Sin[x^(1/3)]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2\sin^2(x)\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{3}{2}x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)+\frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)\\
&\quad +\frac{3}{2}\text{Subst}\left(\int x^2\,dx, x, \sqrt[3]{x}\right)-\frac{3}{2}\text{Subst}\left(\int \sin^2(x)\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{x}{2}+\frac{3}{4}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)\\
&\quad -\frac{3}{2}x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)+\frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)-\frac{3}{4}\text{Subst}\left(\int 1\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{3\sqrt[3]{x}}{4}+\frac{x}{2}+\frac{3}{4}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)-\frac{3}{2}x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)+\frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = \frac{1}{8}\left(4x - 6\sqrt[3]{x}\cos\left(2\sqrt[3]{x}\right) + \left(3 - 6x^{2/3}\right)\sin\left(2\sqrt[3]{x}\right)\right)$$

```
[In] Integrate[Sin[x^(1/3)]^2,x]
[Out] (4*x - 6*x^(1/3)*Cos[2*x^(1/3)] + (3 - 6*x^(2/3))*Sin[2*x^(1/3)])/8
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.28

method	result	size
meijerg	$\frac{3x^{\frac{5}{3}}\,{}_2F_3\left(1,\frac{5}{2};\frac{3}{2},2,\frac{7}{2};-x^{\frac{2}{3}}\right)}{5}$	19
derivativedivides	$3x^{\frac{2}{3}}\left(-\frac{\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{2}+\frac{x^{\frac{1}{3}}}{2}\right)-\frac{3x^{\frac{1}{3}}\left(\cos^2\left(x^{\frac{1}{3}}\right)\right)}{2}+\frac{3\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{4}+\frac{3x^{\frac{1}{3}}}{4}-x$	52
default	$3x^{\frac{2}{3}}\left(-\frac{\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{2}+\frac{x^{\frac{1}{3}}}{2}\right)-\frac{3x^{\frac{1}{3}}\left(\cos^2\left(x^{\frac{1}{3}}\right)\right)}{2}+\frac{3\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{4}+\frac{3x^{\frac{1}{3}}}{4}-x$	52

```
[In] int(sin(x^(1/3))^2,x,method=_RETURNVERBOSE)
[Out] 3/5*x^(5/3)*hypergeom([1,5/2],[3/2,2,7/2],-x^(2/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = -\frac{3}{4}\left(2\,x^{\frac{2}{3}}-1\right)\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)-\frac{3}{2}\,x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)^2+\frac{1}{2}\,x+\frac{3}{4}\,x^{\frac{1}{3}}$$

```
[In] integrate(sin(x^(1/3))^2,x, algorithm="fricas")
[Out] -3/4*(2*x^(2/3) - 1)*cos(x^(1/3))*sin(x^(1/3)) - 3/2*x^(1/3)*cos(x^(1/3))^2
 + 1/2*x + 3/4*x^(1/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(66) = 132.

Time = 0.45 (sec) , antiderivative size = 379, normalized size of antiderivative = 5.49

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = \frac{12x^{\frac{2}{3}}\tan^3\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} - \frac{12x^{\frac{2}{3}}\tan\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} - \frac{3\sqrt[3]{x}\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} + \frac{18\sqrt[3]{x}\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} - \frac{3\sqrt[3]{x}}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} + \frac{2x\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} + \frac{4x\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} + \frac{2x}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} - \frac{6\tan^3\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} + \frac{6\tan\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4}$$

```
[In] integrate(sin(x**(1/3))**2,x)
```

```
[Out] 12*x**(2/3)*tan(x**(1/3)/2)**3/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2
 + 4) - 12*x**(2/3)*tan(x**(1/3)/2)/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/
2)**2 + 4) - 3*x**(1/3)*tan(x**(1/3)/2)**4/(4*tan(x**(1/3)/2)**4 + 8*tan(x*
*(1/3)/2)**2 + 4) + 18*x**(1/3)*tan(x**(1/3)/2)**2/(4*tan(x**(1/3)/2)**4 +
8*tan(x**(1/3)/2)**2 + 4) - 3*x**(1/3)/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/
3)/2)**2 + 4) + 2*x*tan(x**(1/3)/2)**4/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/
3)/2)**2 + 4) + 4*x*tan(x**(1/3)/2)**2/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/
3)/2)**2 + 4) + 2*x/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2 + 4) - 6*t
an(x**(1/3)/2)**3/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2 + 4) + 6*tan
(x**(1/3)/2)/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2 + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.43

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = -\frac{3}{8}\left(2\,x^{\frac{2}{3}}-1\right)\sin\left(2\,x^{\frac{1}{3}}\right)-\frac{3}{4}\,x^{\frac{1}{3}}\cos\left(2\,x^{\frac{1}{3}}\right)+\frac{1}{2}\,x$$

```
[In] integrate(sin(x^(1/3))^2,x, algorithm="maxima")
[Out] -3/8*(2*x^(2/3) - 1)*sin(2*x^(1/3)) - 3/4*x^(1/3)*cos(2*x^(1/3)) + 1/2*x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.43

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = -\frac{3}{8}\left(2\,x^{\frac{2}{3}}-1\right)\sin\left(2\,x^{\frac{1}{3}}\right)-\frac{3}{4}\,x^{\frac{1}{3}}\cos\left(2\,x^{\frac{1}{3}}\right)+\frac{1}{2}\,x$$

```
[In] integrate(sin(x^(1/3))^2,x, algorithm="giac")
[Out] -3/8*(2*x^(2/3) - 1)*sin(2*x^(1/3)) - 3/4*x^(1/3)*cos(2*x^(1/3)) + 1/2*x
```

Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = \frac{x}{2}+\frac{3\,\sin\left(2\,x^{1/3}\right)}{8}-\frac{3\,x^{1/3}\,\cos\left(2\,x^{1/3}\right)}{4}-\frac{3\,x^{2/3}\,\sin\left(2\,x^{1/3}\right)}{4}$$

```
[In] int(sin(x^(1/3))^2,x)
[Out] x/2 + (3*sin(2*x^(1/3)))/8 - (3*x^(1/3)*cos(2*x^(1/3)))/4 - (3*x^(2/3)*sin(
2*x^(1/3)))/4
```

3.128 $\int \sin^3\left(\sqrt[3]{x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 87

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = \frac{14}{3}\cos\left(\sqrt[3]{x}\right) - 2x^{2/3}\cos\left(\sqrt[3]{x}\right) - \frac{2}{9}\cos^3\left(\sqrt[3]{x}\right) + 4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right) - x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right) + \frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)$$

```
[Out] 14/3*cos(x^(1/3))-2*x^(2/3)*cos(x^(1/3))-2/9*cos(x^(1/3))^3+4*x^(1/3)*sin(x
^(1/3))-x^(2/3)*cos(x^(1/3))*sin(x^(1/3))^2+2/3*x^(1/3)*sin(x^(1/3))^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3442, 3392, 3377, 2718, 2713}

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = -2x^{2/3}\cos\left(\sqrt[3]{x}\right) - x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right) + \frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right) + 4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right) - \frac{2}{9}\cos^3\left(\sqrt[3]{x}\right) + \frac{14}{3}\cos\left(\sqrt[3]{x}\right)$$

```
[In] Int[Sin[x^(1/3)]^3,x]
```

```
[Out] (14*Cos[x^(1/3)])/3 - 2*x^(2/3)*Cos[x^(1/3)] - (2*Cos[x^(1/3)]^3)/9 + 4*x^(
1/3)*Sin[x^(1/3)] - x^(2/3)*Cos[x^(1/3)]*Sin[x^(1/3)]^2 + (2*x^(1/3)*Sin[x^
(1/3)]^3)/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2\sin^3(x)\,dx, x, \sqrt[3]{x}\right)\\
&= -x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)\\
&\quad-\frac{2}{3}\text{Subst}\left(\int \sin^3(x)\,dx, x, \sqrt[3]{x}\right)+2\text{Subst}\left(\int x^2\sin(x)\,dx, x, \sqrt[3]{x}\right)\\
&= -2x^{2/3}\cos\left(\sqrt[3]{x}\right)-x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)\\
&\quad+\frac{2}{3}\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos\left(\sqrt[3]{x}\right)\right)+4\text{Subst}\left(\int x\cos(x)\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{2}{3}\cos\left(\sqrt[3]{x}\right)-2x^{2/3}\cos\left(\sqrt[3]{x}\right)-\frac{2}{9}\cos^3\left(\sqrt[3]{x}\right)+4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)\\
&\quad-x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)-4\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt[3]{x}\right)
\end{aligned}$$

$$= \frac{14}{3}\cos\left(\sqrt[3]{x}\right) - 2x^{2/3}\cos\left(\sqrt[3]{x}\right) - \frac{2}{9}\cos^3\left(\sqrt[3]{x}\right) + 4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right) - x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right) + \frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = \frac{1}{36}\left(-81\left(-2+x^{2/3}\right)\cos\left(\sqrt[3]{x}\right) + \left(-2+9x^{2/3}\right)\cos\left(3\sqrt[3]{x}\right) - 6\sqrt[3]{x}\left(-27\sin\left(\sqrt[3]{x}\right)+\sin\left(3\sqrt[3]{x}\right)\right)\right)$$

```
[In] Integrate[Sin[x^(1/3)]^3,x]
[Out] (-81*(-2 + x^(2/3))*Cos[x^(1/3)] + (-2 + 9*x^(2/3))*Cos[3*x^(1/3)] - 6*x^(1
/3)*(-27*Sin[x^(1/3)] + Sin[3*x^(1/3)]))/36
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-x^{\frac{2}{3}}\left(2+\sin^2\left(x^{\frac{1}{3}}\right)\right)\cos\left(x^{\frac{1}{3}}\right)+4\cos\left(x^{\frac{1}{3}}\right)+4x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)+\frac{2x^{\frac{1}{3}}\left(\sin^3\left(x^{\frac{1}{3}}\right)\right)}{3}+$ 2(2+sin²(x
default	$-x^{\frac{2}{3}}\left(2+\sin^2\left(x^{\frac{1}{3}}\right)\right)\cos\left(x^{\frac{1}{3}}\right)+4\cos\left(x^{\frac{1}{3}}\right)+4x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)+\frac{2x^{\frac{1}{3}}\left(\sin^3\left(x^{\frac{1}{3}}\right)\right)}{3}+$ 2(2+sin²(x

```
[In] int(sin(x^(1/3))^3,x,method=_RETURNVERBOSE)
[Out] -x^(2/3)*(2+sin(x^(1/3))^2)*cos(x^(1/3))+4*cos(x^(1/3))+4*x^(1/3)*sin(x^(1/
3))+2/3*x^(1/3)*sin(x^(1/3))^3+2/9*(2+sin(x^(1/3))^2)*cos(x^(1/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = \frac{1}{9}\left(9\,x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)^3 - \frac{1}{3}\left(9\,x^{\frac{2}{3}}-14\right)\cos\left(x^{\frac{1}{3}}\right) - \frac{2}{3}\left(x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)^2 - 7\,x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)$$

```
[In] integrate(sin(x^(1/3))^3,x, algorithm="fricas")
```

```
[Out] 1/9*(9*x^(2/3) - 2)*cos(x^(1/3))^3 - 1/3*(9*x^(2/3) - 14)*cos(x^(1/3)) - 2/
3*(x^(1/3)*cos(x^(1/3))^2 - 7*x^(1/3))*sin(x^(1/3))
```

Sympy [A] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = 3x^{\frac{2}{3}}\left(\frac{\cos^3\left(\sqrt[3]{x}\right)}{3} - \cos\left(\sqrt[3]{x}\right)\right) - 2\sqrt[3]{x}\left(-\frac{\sin^3\left(\sqrt[3]{x}\right)}{3} - 2\sin\left(\sqrt[3]{x}\right)\right) - \frac{2\cos^3\left(\sqrt[3]{x}\right)}{9} + \frac{14\cos\left(\sqrt[3]{x}\right)}{3}$$

```
[In] integrate(sin(x**(1/3))**3,x)
```

```
[Out] 3*x**(2/3)*(cos(x**(1/3))**3/3 - cos(x**(1/3))) - 2*x**(1/3)*(-sin(x**(1/3)
)**3/3 - 2*sin(x**(1/3))) - 2*cos(x**(1/3))**3/9 + 14*cos(x**(1/3))/3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.54

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = \frac{1}{36}\left(9\,x^{\frac{2}{3}} - 2\right)\cos\left(3\,x^{\frac{1}{3}}\right) - \frac{9}{4}\left(x^{\frac{2}{3}} - 2\right)\cos\left(x^{\frac{1}{3}}\right) - \frac{1}{6}\,x^{\frac{1}{3}}\sin\left(3\,x^{\frac{1}{3}}\right) + \frac{9}{2}\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

```
[In] integrate(sin(x^(1/3))^3,x, algorithm="maxima")
```

```
[Out] 1/36*(9*x^(2/3) - 2)*cos(3*x^(1/3)) - 9/4*(x^(2/3) - 2)*cos(x^(1/3)) - 1/6*
x^(1/3)*sin(3*x^(1/3)) + 9/2*x^(1/3)*sin(x^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.54

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = \frac{1}{36}\left(9\,x^{\frac{2}{3}} - 2\right)\cos\left(3\,x^{\frac{1}{3}}\right) - \frac{9}{4}\left(x^{\frac{2}{3}} - 2\right)\cos\left(x^{\frac{1}{3}}\right) - \frac{1}{6}\,x^{\frac{1}{3}}\sin\left(3\,x^{\frac{1}{3}}\right) + \frac{9}{2}\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

```
[In] integrate(sin(x^(1/3))^3,x, algorithm="giac")
```

```
[Out] 1/36*(9*x^(2/3) - 2)*cos(3*x^(1/3)) - 9/4*(x^(2/3) - 2)*cos(x^(1/3)) - 1/6*
x^(1/3)*sin(3*x^(1/3)) + 9/2*x^(1/3)*sin(x^(1/3))
```

Mupad [B] (verification not implemented)

Time = 6.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = \frac{14\cos\left(x^{1/3}\right)}{3} - 3\,x^{2/3}\,\cos\left(x^{1/3}\right) + \frac{14\,x^{1/3}\,\sin\left(x^{1/3}\right)}{3} - \frac{2\cos\left(x^{1/3}\right)^3}{9} + x^{2/3}\cos\left(x^{1/3}\right)^3 - \frac{2\,x^{1/3}\cos\left(x^{1/3}\right)^2\,\sin\left(x^{1/3}\right)}{3}$$

```
[In] int(sin(x^(1/3))^3,x)
```

```
[Out] (14*cos(x^(1/3)))/3 - 3*x^(2/3)*cos(x^(1/3)) + (14*x^(1/3)*sin(x^(1/3)))/3
- (2*cos(x^(1/3))^3)/9 + x^(2/3)*cos(x^(1/3))^3 - (2*x^(1/3)*cos(x^(1/3))^2
*sin(x^(1/3)))/3
```

3.129 $\int (ex)^m (b\sin(c+dx^n))^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (ex)^m (b\sin(c+dx^n))^p\,dx = \text{Int}((ex)^m (b\sin(c+dx^n))^p, x)$$

```
[Out] Unintegrable((e*x)^m*(b*sin(c+d*x^n))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (b\sin(c+dx^n))^p\,dx = \int (ex)^m (b\sin(c+dx^n))^p\,dx$$

```
[In] Int[(e*x)^m*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(b*Sin[c + d*x^n])^p, x]
```

Rubi steps

$$\text{integral} = \int (ex)^m (b\sin(c+dx^n))^p\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \int (ex)^m (b\sin(c+dx^n))^p \, dx$$

```
[In] Integrate[(e*x)^m*(b*Sin[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(b*Sin[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (ex)^m (b\sin(c+d\,x^n))^p dx$$

```
[In] int((e*x)^m*(b*sin(c+d*x^n))^p,x)
[Out] int((e*x)^m*(b*sin(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \int (ex)^m (b\sin(dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^m*(b*sin(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*sin(d*x^n + c))^p, x)
```

Sympy [N/A]

Not integrable

Time = 11.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \int (b\sin(c+dx^n))^p (ex)^m \, dx$$

```
[In] integrate((e*x)**m*(b*sin(c+d*x**n))**p,x)
[Out] Integral((b*sin(c + d*x**n))**p*(e*x)**m, x)
```

Maxima [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \int (ex)^m (b\sin(dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^m*(b*sin(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*sin(d*x^n + c))^p, x)
```

Giac [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \int (ex)^m (b\sin(dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^m*(b*sin(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*sin(d*x^n + c))^p, x)
```

Mupad [N/A]

Not integrable

Time = 6.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \int (b \, \sin(c+d\,x^n))^p \, (e\,x)^m \, dx$$

```
[In] int((b*sin(c + d*x^n))^p*(e*x)^m,x)
[Out] int((b*sin(c + d*x^n))^p*(e*x)^m, x)
```

3.130 $\int (ex)^m (a+b\sin(c+dx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a+b\sin(c+dx^n))^p \, dx = \text{Int}((ex)^m (a+b\sin(c+dx^n))^p, x)$$

```
[Out] Unintegrable((e*x)^m*(a+b*sin(c+d*x^n))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (a+b\sin(c+dx^n))^p \, dx = \int (ex)^m (a+b\sin(c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Sin[c + d*x^n])^p, x]
```

Rubi steps

$$\text{integral} = \int (ex)^m (a+b\sin(c+dx^n))^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\sin(c + dx^n))^p \, dx = \int (ex)^m (a + b\sin(c + dx^n))^p \, dx$$

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a + b\sin(c + d\,x^n))^p dx$$

```
[In] int((e*x)^m*(a+b*sin(c+d*x^n))^p,x)
[Out] int((e*x)^m*(a+b*sin(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\sin(c + dx^n))^p \, dx = \int (ex)^m (b\sin(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sin(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*sin(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 34.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a + b\sin(c + dx^n))^p \, dx = \int (ex)^m (a + b\sin(c + dx^n))^p \, dx$$

```
[In] integrate((e*x)**m*(a+b*sin(c+d*x**n))**p,x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\sin(c + dx^n))^p \, dx = \int (ex)^m (b\sin(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sin(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*sin(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 6.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\sin(c + dx^n))^p \, dx = \int (ex)^m (b\sin(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sin(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*sin(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 5.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\sin(c + dx^n))^p \, dx = \int (e\,x)^m \left(a + b\,\sin\left(c + d\,x^n\right)\right)^p \,dx$$

```
[In] int((e*x)^m*(a + b*sin(c + d*x^n))^p,x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^n))^p, x)
```

3.131 $\int (ex)^{-1+n} (b\sin(c+dx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int (ex)^{-1+n}(b\sin(c+dx^n))^p\,dx = \frac{x^{-n}(ex)^n\cos(c+dx^n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},\sin^2(c+dx^n)\right)(b\sin(c+dx^n))^{1+p}}{bden(1+p)\sqrt{\cos^2(c+dx^n)}}$$

```
[Out] (e*x)^n*cos(c+d*x^n)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],sin(c+d*x^n)^2)
*(b*sin(c+d*x^n))^(p+1)/b/d/e/n/(p+1)/(x^n)/(cos(c+d*x^n)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3462, 3460, 2722}

$$\int (ex)^{-1+n}(b\sin(c+dx^n))^p\,dx = \frac{x^{-n}(ex)^n\cos(c+dx^n)(b\sin(c+dx^n))^{p+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},\sin^2(dx^n+c)\right)}{bden(p+1)\sqrt{\cos^2(c+dx^n)}}$$

```
[In] Int[(e*x)^(-1 + n)*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*Cos[c + d*x^n]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[c
+ d*x^n]^2]*(b*Sin[c + d*x^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n*Sqrt[Cos[c + d
*x^n]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(x^{-n}(ex)^n)\int x^{-1+n}(b\sin(c+dx^n))^p\,dx}{e}\\ &= \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int (b\sin(c+dx))^p\,dx, x, x^n\right)}{en}\\ &= \frac{x^{-n}(ex)^n\cos(c+dx^n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},\sin^2(c+dx^n)\right)(b\sin(c+dx^n))^{1+p}}{bden(1+p)\sqrt{\cos^2(c+dx^n)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int (ex)^{-1+n}(b\sin(c+dx^n))^p\,dx$$
$$= \frac{x^{1-n}(ex)^{-1+n}\sqrt{\cos^2(c+dx^n)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},\sin^2(c+dx^n)\right)(b\sin(c+dx^n))^p\tan(c+d}{dn(1+p)}$$

```
[In] Integrate[(e*x)^(-1 + n)*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] (x^(1 - n)*(e*x)^(-1 + n)*Sqrt[Cos[c + d*x^n]^2]*Hypergeometric2F1[1/2, (1
+ p)/2, (3 + p)/2, Sin[c + d*x^n]^2]*(b*Sin[c + d*x^n])^p*Tan[c + d*x^n])/(
d*n*(1 + p))
```

Maple [F]

$$\int (ex)^{-1+n} (b\sin(c+d\,x^{n}))^{p} dx$$

```
[In] int((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1+n} (b\sin(c+dx^{n}))^{p} \, dx = \int (ex)^{n-1} (b\sin(dx^{n}+c))^{p} \, dx$$

```
[In] integrate((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(n - 1)*(b*sin(d*x^n + c))^p, x)
```

Sympy [F]

$$\int (ex)^{-1+n} (b\sin(c+dx^{n}))^{p} \, dx = \int (b\sin(c+dx^{n}))^{p} (ex)^{n-1} \, dx$$

```
[In] integrate((e*x)**(-1+n)*(b*sin(c+d*x**n))**p,x)
[Out] Integral((b*sin(c + d*x**n))**p*(e*x)**(n - 1), x)
```

Maxima [F]

$$\int (ex)^{-1+n} (b\sin(c+dx^{n}))^{p} \, dx = \int (ex)^{n-1} (b\sin(dx^{n}+c))^{p} \, dx$$

```
[In] integrate((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(n - 1)*(b*sin(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1+n} (b\sin(c+dx^n))^p \, dx = \int (ex)^{n-1} (b\sin(dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(n - 1)*(b*sin(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n} (b\sin(c+dx^n))^p \, dx = \int (b\,\sin(c+d\,x^n))^p\,(e\,x)^{n-1} \, dx$$

```
[In] int((b*sin(c + d*x^n))^p*(e*x)^(n - 1),x)
[Out] int((b*sin(c + d*x^n))^p*(e*x)^(n - 1), x)
```

3.132 $\int (ex)^{-1+2n} (b\sin(c+dx^n))^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (ex)^{-1+2n} (b\sin(c+dx^n))^p \, dx = \frac{x^{-2n}(ex)^{2n}\text{Int}(x^{-1+2n}(b\sin(c+dx^n))^p, x)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(b*sin(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^{-1+2n} (b\sin(c+dx^n))^p \, dx = \int (ex)^{-1+2n} (b\sin(c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^(-1 + 2*n)*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(b*Sin[c + d*x^n])^p, x])/(e*x^(2*n))
```

Rubi steps

$$\text{integral} = \frac{\left(x^{-2n}(ex)^{2n}\right)\int x^{-1+2n}(b\sin(c+dx^n))^p \, dx}{e}$$

Mathematica [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\sin(c+dx^n))^p\,dx=\int (ex)^{-1+2n}(b\sin(c+dx^n))^p\,dx$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(b*Sin[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^(-1 + 2*n)*(b*Sin[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (ex)^{-1+2n}(b\sin(c+d\,x^n))^p dx$$

```
[In] int((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\sin(c+dx^n))^p\,dx=\int (ex)^{2n-1}(b\sin(dx^n+c))^p\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(2*n - 1)*(b*sin(d*x^n + c))^p, x)
```

Sympy [N/A]

Not integrable

Time = 9.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (ex)^{-1+2n}(b\sin(c+dx^n))^p\,dx=\int (b\sin(c+dx^n))^p(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)**(-1+2*n)*(b*sin(c+d*x**n))**p,x)
[Out] Integral((b*sin(c + d*x**n))**p*(e*x)**(2*n - 1), x)
```

Maxima [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\sin(c+dx^n))^p\,dx=\int (ex)^{2\,n-1}(b\sin(dx^n+c))^p\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(2*n - 1)*(b*sin(d*x^n + c))^p, x)
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\sin(c+dx^n))^p\,dx=\int (ex)^{2\,n-1}(b\sin(dx^n+c))^p\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)*(b*sin(d*x^n + c))^p, x)
```

Mupad [N/A]

Not integrable

Time = 5.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\sin(c+dx^n))^p\,dx=\int (b\,\sin(c+d\,x^n))^p\,(e\,x)^{2\,n-1}\,dx$$

```
[In] int((b*sin(c + d*x^n))^p*(e*x)^(2*n - 1),x)
[Out] int((b*sin(c + d*x^n))^p*(e*x)^(2*n - 1), x)
```

3.133 $\int (ex)^{-1+n} (a+b\sin(c+dx^n))^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 132

$$\int (ex)^{-1+n} (a+b\sin(c+dx^n))^p \, dx =$$
$$-\frac{\sqrt{2}x^{-n}(ex)^n \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx^n)),\frac{b(1-\sin(c+dx^n))}{a+b}\right)\cos(c+dx^n)(a+b\sin(c+dx^n}{den\sqrt{1+\sin(c+dx^n)}}$$

```
[Out] -(e*x)^n*AppellF1(1/2,-p,1/2,3/2,b*(1-sin(c+d*x^n))/(a+b),1/2-1/2*sin(c+d*x
^n))*cos(c+d*x^n)*(a+b*sin(c+d*x^n))^p*2^(1/2)/d/e/n/(x^n)/(((a+b*sin(c+d*x
^n))/(a+b))^p)/(1+sin(c+d*x^n))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3462, 3460, 2744, 144, 143}

$$\int (ex)^{-1+n} (a+b\sin(c+dx^n))^p \, dx =$$
$$-\frac{\sqrt{2}x^{-n}(ex)^n\cos(c+dx^n)(a+b\sin(c+dx^n))^p\left(\frac{a+b\sin(c+dx^n)}{a+b}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-\sin(dx^n+}{den\sqrt{\sin(c+dx^n)+1}}$$

```
[In] Int[(e*x)^(-1 + n)*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] -((Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - Sin[c + d*x^n])/2, (b*(
1 - Sin[c + d*x^n]))/(a + b)]*Cos[c + d*x^n]*(a + b*Sin[c + d*x^n])^p)/(d*e
*n*x^n*Sqrt[1 + Sin[c + d*x^n]]*((a + b*Sin[c + d*x^n])/(a + b))^p))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{(x^{-n}(ex)^n)\int x^{-1+n}(a+b\sin(c+dx^n))^p\,dx}{e}$$

$$= \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int (a+b\sin(c+dx))^p\,dx, x, x^n\right)}{en}$$

$$= \frac{(x^{-n}(ex)^n\cos(c+dx^n))\,\text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(c+dx^n)\right)}{den\sqrt{1-\sin(c+dx^n)}\sqrt{1+\sin(c+dx^n)}}$$

$$= \frac{\left(x^{-n}(ex)^n\cos(c+dx^n)(a+b\sin(c+dx^n))^p\left(-\frac{a+b\sin(c+dx^n)}{-a-b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^p}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, s}{den\sqrt{1-\sin(c+dx^n)}\sqrt{1+\sin(c+dx^n)}}$$

$$= -\frac{\sqrt{2}x^{-n}(ex)^n\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx^n)),\frac{b(1-\sin(c+dx^n))}{a+b}\right)\cos(c+dx^n)(a+b\sin(}{den\sqrt{1+\sin(c+dx^n)}}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

$$\int (ex)^{-1+n}(a+b\sin(c+dx^n))^p\,dx$$

$$= \frac{x^{-n}(ex)^n\,\text{AppellF1}\left(1+p,\frac{1}{2},\frac{1}{2},2+p,\frac{a+b\sin(c+dx^n)}{a-b},\frac{a+b\sin(c+dx^n)}{a+b}\right)\sec(c+dx^n)\sqrt{-\frac{b(-1+\sin(c+dx^n))}{a+b}}\sqrt{\frac{b(1+\sin(}{-a}}}{bden(1+p)}$$

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b*Sin[c + d*x^n])/(a - b), (
a + b*Sin[c + d*x^n])/(a + b)]*Sec[c + d*x^n]*Sqrt[-((b*(-1 + Sin[c + d*x^n
]))/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x^n]))/(-a + b)]*(a + b*Sin[c + d*x^n]
)^(1 + p))/(b*d*e*n*(1 + p)*x^n)
```

Maple [F]

$$\int (ex)^{-1+n}(a+b\sin(c+d\,x^n))^p dx$$

```
[In] int((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1+n}(a+b\sin(c+dx^n))^p\,dx = \int (ex)^{n-1}(b\sin(dx^n+c)+a)^p\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Sympy [F]

$$\int (ex)^{-1+n}(a+b\sin(c+dx^n))^p\,dx = \int (ex)^{n-1}(a+b\sin(c+dx^n))^p\,dx$$

```
[In] integrate((e*x)**(-1+n)*(a+b*sin(c+d*x**n))**p,x)
[Out] Integral((e*x)**(n - 1)*(a + b*sin(c + d*x**n))**p, x)
```

Maxima [F]

$$\int (ex)^{-1+n}(a+b\sin(c+dx^n))^p\,dx = \int (ex)^{n-1}(b\sin(dx^n+c)+a)^p\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Giac [F]

$$\int (ex)^{-1+n}(a+b\sin(c+dx^n))^p\,dx = \int (ex)^{n-1}(b\sin(dx^n+c)+a)^p\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n} (a + b \sin (c + dx^n))^p \, dx = \int (e\,x)^{n-1} (a + b\,\sin (c + d\,x^n))^p \, dx$$

```
[In] int((e*x)^(n - 1)*(a + b*sin(c + d*x^n))^p,x)
[Out] int((e*x)^(n - 1)*(a + b*sin(c + d*x^n))^p, x)
```

3.134 $\int (ex)^{-1+2n} (a + b\sin(c + dx^n))^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (ex)^{-1+2n} (a + b\sin(c + dx^n))^p \, dx = \frac{x^{-2n}(ex)^{2n}\text{Int}(x^{-1+2n}(a + b\sin(c + dx^n))^p, x)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^{-1+2n} (a + b\sin(c + dx^n))^p \, dx = \int (ex)^{-1+2n} (a + b\sin(c + dx^n))^p \, dx$$

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p, x])/(e*x^(2*
n))
```

Rubi steps

$$\text{integral} = \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n}(a + b\sin(c + dx^n))^p \, dx}{e}$$

Mathematica [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}(a+b\sin(c+dx^n))^p\,dx=\int (ex)^{-1+2n}(a+b\sin(c+dx^n))^p\,dx$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (ex)^{-1+2n}(a+b\sin(c+d\,x^n))^p dx$$

```
[In] int((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}(a+b\sin(c+dx^n))^p\,dx=\int (ex)^{2\,n-1}(b\sin(dx^n+c)+a)^p\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(2*n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 26.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (ex)^{-1+2n}(a+b\sin(c+dx^n))^p\,dx=\int (ex)^{2n-1}(a+b\sin(c+dx^n))^p\,dx$$

```
[In] integrate((e*x)**(-1+2*n)*(a+b*sin(c+d*x**n))**p,x)
[Out] Integral((e*x)**(2*n - 1)*(a + b*sin(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}\left(a+b\sin\left(c+dx^n\right)\right)^p\,dx=\int (ex)^{2\,n-1}\left(b\sin\left(dx^n+c\right)+a\right)^p dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(2*n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 5.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}\left(a+b\sin\left(c+dx^n\right)\right)^p\,dx=\int (ex)^{2\,n-1}\left(b\sin\left(dx^n+c\right)+a\right)^p dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 6.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}\left(a+b\sin\left(c+dx^n\right)\right)^p\,dx=\int {\left(e\,x\right)}^{2\,n-1}\,{\left(a+b\,\sin\left(c+d\,x^n\right)\right)}^p\,dx$$

```
[In] int((e*x)^(2*n - 1)*(a + b*sin(c + d*x^n))^p,x)
[Out] int((e*x)^(2*n - 1)*(a + b*sin(c + d*x^n))^p, x)
```

3.135 $\int \frac{\sin(a+bx^n)}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\sin(a+bx^n)}{x} dx = \frac{\text{CosIntegral}(bx^n)\sin(a)}{n} + \frac{\cos(a)\text{Si}(bx^n)}{n}$$

```
[Out] cos(a)*Si(b*x^n)/n+Ci(b*x^n)*sin(a)/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3458, 3457, 3456}

$$\int \frac{\sin(a+bx^n)}{x} dx = \frac{\sin(a)\text{CosIntegral}(bx^n)}{n} + \frac{\cos(a)\text{Si}(bx^n)}{n}$$

```
[In] Int[Sin[a + b*x^n]/x,x]
```

```
[Out] (CosIntegral[b*x^n]*Sin[a])/n + (Cos[a]*SinIntegral[b*x^n])/n
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\text{integral} = \cos(a)\int \frac{\sin(bx^n)}{x}\,dx + \sin(a)\int \frac{\cos(bx^n)}{x}\,dx$$
$$= \frac{\text{CosIntegral}(bx^n)\sin(a)}{n} + \frac{\cos(a)\text{Si}(bx^n)}{n}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sin(a+bx^n)}{x}\,dx = \frac{\text{CosIntegral}(bx^n)\sin(a) + \cos(a)\text{Si}(bx^n)}{n}$$

```
[In] Integrate[Sin[a + b*x^n]/x,x]
[Out] (CosIntegral[b*x^n]*Sin[a] + Cos[a]*SinIntegral[b*x^n])/n
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\text{Si}(b\,x^n)\cos(a)+\text{Ci}(b\,x^n)\sin(a)}{n}$	24
default	$\frac{\text{Si}(b\,x^n)\cos(a)+\text{Ci}(b\,x^n)\sin(a)}{n}$	24
risch	$\frac{i e^{ia}\,\text{Ei}_1(-ib\,x^n)}{2n}-\frac{e^{-ia}\pi\,\text{csgn}(b\,x^n)}{2n}+\frac{e^{-ia}\,\text{Si}(b\,x^n)}{n}-\frac{i e^{-ia}\,\text{Ei}_1(-ib\,x^n)}{2n}$	74
meijerg	$\frac{\sqrt{\pi}\left(\frac{2\gamma+2n\ln(x)+\ln(b^2)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{b\,x^n}{2}\right)}{\sqrt{\pi}}+\frac{2\,\text{Ci}(b\,x^n)}{\sqrt{\pi}}\right)\sin(a)}{2n}+\frac{\cos(a)\,\text{Si}(b\,x^n)}{n}$	78

```
[In] int(sin(a+b*x^n)/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(Si(b*x^n)*cos(a)+Ci(b*x^n)*sin(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sin(a+bx^n)}{x}\,dx = \frac{\mathrm{Ci}\,(bx^n)\sin(a)+\cos(a)\,\mathrm{Si}\,(bx^n)}{n}$$

```
[In] integrate(sin(a+b*x^n)/x,x, algorithm="fricas")
[Out] (cos_integral(b*x^n)*sin(a) + cos(a)*sin_integral(b*x^n))/n
```

Sympy [F]

$$\int \frac{\sin(a+bx^n)}{x}\,dx = \int \frac{\sin(a+bx^n)}{x}\,dx$$

```
[In] integrate(sin(a+b*x**n)/x,x)
[Out] Integral(sin(a + b*x**n)/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.64

$$\int \frac{\sin(a+bx^n)}{x}\,dx = -\frac{\left(i\,\mathrm{Ei}(i\,bx^n)-i\,\mathrm{Ei}(-i\,bx^n)+i\,\mathrm{Ei}\left(i\,be^{\left(n\overline{\log(x)}\right)}\right)-i\,\mathrm{Ei}\left(-i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(a)-\left(\mathrm{Ei}(i\,bx^n)+\mathrm{Ei}(-i\,bx\right.}{4\,n}$$

```
[In] integrate(sin(a+b*x^n)/x,x, algorithm="maxima")
[Out] -1/4*((I*Ei(I*b*x^n) - I*Ei(-I*b*x^n) + I*Ei(I*b*e^(n*conjugate(log(x)))) -
 I*Ei(-I*b*e^(n*conjugate(log(x)))))*cos(a) - (Ei(I*b*x^n) + Ei(-I*b*x^n) +
 Ei(I*b*e^(n*conjugate(log(x)))) + Ei(-I*b*e^(n*conjugate(log(x)))))*sin(a)
)/n
```

Giac [F]

$$\int \frac{\sin(a+bx^n)}{x}\,dx=\int \frac{\sin(bx^n+a)}{x}\,dx$$

```
[In] integrate(sin(a+b*x^n)/x,x, algorithm="giac")
[Out] integrate(sin(b*x^n + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx^n)}{x}\,dx=\int \frac{\sin(a+b\,x^n)}{x}\,dx$$

```
[In] int(sin(a + b*x^n)/x,x)
[Out] int(sin(a + b*x^n)/x, x)
```

3.136 $\int \frac{\sin^2(a+bx^n)}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{\sin^2(a+bx^n)}{x} dx = -\frac{\cos(2a)\,\text{CosIntegral}(2bx^n)}{2n} + \frac{\log(x)}{2} + \frac{\sin(2a)\text{Si}(2bx^n)}{2n}$$

```
[Out] -1/2*Ci(2*b*x^n)*cos(2*a)/n+1/2*ln(x)+1/2*Si(2*b*x^n)*sin(2*a)/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3506, 3459, 3457, 3456}

$$\int \frac{\sin^2(a+bx^n)}{x} dx = -\frac{\cos(2a)\,\text{CosIntegral}(2bx^n)}{2n} + \frac{\sin(2a)\text{Si}(2bx^n)}{2n} + \frac{\log(x)}{2}$$

```
[In] Int[Sin[a + b*x^n]^2/x,x]
```

```
[Out] -1/2*(Cos[2*a]*CosIntegral[2*b*x^n])/n + Log[x]/2 + (Sin[2*a]*SinIntegral[2
*b*x^n])/(2*n)
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2x} - \frac{\cos(2a + 2bx^n)}{2x}\right) dx \\
&= \frac{\log(x)}{2} - \frac{1}{2}\int \frac{\cos(2a + 2bx^n)}{x}\, dx \\
&= \frac{\log(x)}{2} - \frac{1}{2}\cos(2a)\int \frac{\cos(2bx^n)}{x}\, dx + \frac{1}{2}\sin(2a)\int \frac{\sin(2bx^n)}{x}\, dx \\
&= -\frac{\cos(2a)\,\text{CosIntegral}(2bx^n)}{2n} + \frac{\log(x)}{2} + \frac{\sin(2a)\text{Si}(2bx^n)}{2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sin^2(a + bx^n)}{x}\, dx = \frac{-\cos(2a)\,\text{CosIntegral}(2bx^n) + n\log(x) + \sin(2a)\text{Si}(2bx^n)}{2n}$$

```
[In] Integrate[Sin[a + b*x^n]^2/x,x]
[Out] (-(Cos[2*a]*CosIntegral[2*b*x^n]) + n*Log[x] + Sin[2*a]*SinIntegral[2*b*x^n
])/(2*n)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\frac{\ln(b\,x^n)}{2}+\frac{\mathrm{Si}(2b\,x^n)\sin(2a)}{2}-\frac{\mathrm{Ci}(2b\,x^n)\cos(2a)}{2}}{n}$	40
default	$\frac{\frac{\ln(b\,x^n)}{2}+\frac{\mathrm{Si}(2b\,x^n)\sin(2a)}{2}-\frac{\mathrm{Ci}(2b\,x^n)\cos(2a)}{2}}{n}$	40
risch	$-\frac{i\mathrm{e}^{-2ia}\pi\ \mathrm{csgn}(b\,x^n)}{4n}+\frac{i\mathrm{e}^{-2ia}\,\mathrm{Si}(2b\,x^n)}{2n}+\frac{\mathrm{e}^{-2ia}\,\mathrm{Ei}_1(-2ib\,x^n)}{4n}+\frac{\mathrm{e}^{2ia}\,\mathrm{Ei}_1(-2ib\,x^n)}{4n}+\frac{\ln(x)}{2}$	80

```
[In] int(sin(a+b*x^n)^2/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(1/2*ln(b*x^n)+1/2*Si(2*b*x^n)*sin(2*a)-1/2*Ci(2*b*x^n)*cos(2*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2(a+bx^n)}{x}\,dx = -\frac{\cos(2a)\,\mathrm{Ci}(2bx^n)-n\log(x)-\sin(2a)\,\mathrm{Si}(2bx^n)}{2n}$$

```
[In] integrate(sin(a+b*x^n)^2/x,x, algorithm="fricas")
[Out] -1/2*(cos(2*a)*cos_integral(2*b*x^n) - n*log(x) - sin(2*a)*sin_integral(2*b
*x^n))/n
```

Sympy [F]

$$\int \frac{\sin^2(a+bx^n)}{x}\,dx = \int \frac{\sin^2(a+bx^n)}{x}\,dx$$

```
[In] integrate(sin(a+b*x**n)**2/x,x)
[Out] Integral(sin(a + b*x**n)**2/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.33

$$\int \frac{\sin^2(a+bx^n)}{x}\,dx = -\frac{\left(\mathrm{Ei}(2i\,bx^n)+\mathrm{Ei}(-2i\,bx^n)+\mathrm{Ei}\left(2i\,be^{\left(n\overline{\log(x)}\right)}\right)+\mathrm{Ei}\left(-2i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(2a)-4n\log(x)-\left(-i\,\mathrm{Ei}(\right.}{8n}$$

```
[In] integrate(sin(a+b*x^n)^2/x,x, algorithm="maxima")
```

```
[Out] -1/8*((Ei(2*I*b*x^n) + Ei(-2*I*b*x^n) + Ei(2*I*b*e^(n*conjugate(log(x)))) +
 Ei(-2*I*b*e^(n*conjugate(log(x)))))*cos(2*a) - 4*n*log(x) - (-I*Ei(2*I*b*x
^n) + I*Ei(-2*I*b*x^n) - I*Ei(2*I*b*e^(n*conjugate(log(x)))) + I*Ei(-2*I*b*
e^(n*conjugate(log(x)))))*sin(2*a))/n
```

Giac [F]

$$\int \frac{\sin^2(a+bx^n)}{x}\,dx = \int \frac{\sin(bx^n+a)^2}{x}\,dx$$

```
[In] integrate(sin(a+b*x^n)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx^n)}{x}\,dx = \int \frac{\sin(a+b\,x^n)^2}{x}\,dx$$

```
[In] int(sin(a + b*x^n)^2/x,x)
```

```
[Out] int(sin(a + b*x^n)^2/x, x)
```

3.137 $\int \frac{\sin^3(a+bx^n)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx = \frac{3\,\text{CosIntegral}(bx^n)\sin(a)}{4n} - \frac{\text{CosIntegral}(3bx^n)\sin(3a)}{4n} + \frac{3\cos(a)\text{Si}(bx^n)}{4n} - \frac{\cos(3a)\text{Si}(3bx^n)}{4n}$$

```
[Out] 3/4*cos(a)*Si(b*x^n)/n-1/4*cos(3*a)*Si(3*b*x^n)/n+3/4*Ci(b*x^n)*sin(a)/n-1/
4*Ci(3*b*x^n)*sin(3*a)/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3506, 3458, 3457, 3456}

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx = \frac{3\sin(a)\,\text{CosIntegral}(bx^n)}{4n} - \frac{\sin(3a)\,\text{CosIntegral}(3bx^n)}{4n} + \frac{3\cos(a)\text{Si}(bx^n)}{4n} - \frac{\cos(3a)\text{Si}(3bx^n)}{4n}$$

```
[In] Int[Sin[a + b*x^n]^3/x,x]
```

```
[Out] (3*CosIntegral[b*x^n]*Sin[a])/(4*n) - (CosIntegral[3*b*x^n]*Sin[3*a])/(4*n)
 + (3*Cos[a]*SinIntegral[b*x^n])/(4*n) - (Cos[3*a]*SinIntegral[3*b*x^n])/(4
*n)
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3\sin(a+bx^n)}{4x} - \frac{\sin(3a+3bx^n)}{4x}\right) dx \\
&= -\left(\frac{1}{4}\int \frac{\sin(3a+3bx^n)}{x}\,dx\right) + \frac{3}{4}\int \frac{\sin(a+bx^n)}{x}\,dx \\
&= \frac{1}{4}(3\cos(a))\int \frac{\sin(bx^n)}{x}\,dx - \frac{1}{4}\cos(3a)\int \frac{\sin(3bx^n)}{x}\,dx \\
&\quad + \frac{1}{4}(3\sin(a))\int \frac{\cos(bx^n)}{x}\,dx - \frac{1}{4}\sin(3a)\int \frac{\cos(3bx^n)}{x}\,dx \\
&= \frac{3\,\text{CosIntegral}(bx^n)\sin(a)}{4n} - \frac{\text{CosIntegral}(3bx^n)\sin(3a)}{4n} + \frac{3\cos(a)\text{Si}(bx^n)}{4n} - \frac{\cos(3a)\text{Si}(3bx^n)}{4n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx = \frac{3\,\text{CosIntegral}(bx^n)\sin(a) - \text{CosIntegral}(3bx^n)\sin(3a) + 3\cos(a)\text{Si}(bx^n) - \cos(3a)\text{Si}(3bx^n)}{4n}$$

```
[In] Integrate[Sin[a + b*x^n]^3/x,x]
[Out] (3*CosIntegral[b*x^n]*Sin[a] - CosIntegral[3*b*x^n]*Sin[3*a] + 3*Cos[a]*Sin
Integral[b*x^n] - Cos[3*a]*SinIntegral[3*b*x^n])/(4*n)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{-\frac{\operatorname{Si}(3b\,x^n)\cos(3a)}{4}-\frac{\operatorname{Ci}(3b\,x^n)\sin(3a)}{4}+\frac{3\operatorname{Si}(b\,x^n)\cos(a)}{4}+\frac{3\operatorname{Ci}(b\,x^n)\sin(a)}{4}}{n}$
default	$\frac{-\frac{\operatorname{Si}(3b\,x^n)\cos(3a)}{4}-\frac{\operatorname{Ci}(3b\,x^n)\sin(3a)}{4}+\frac{3\operatorname{Si}(b\,x^n)\cos(a)}{4}+\frac{3\operatorname{Ci}(b\,x^n)\sin(a)}{4}}{n}$
risch	$-\frac{i e^{3ia}\operatorname{Ei}_1(-3ib\,x^n)}{8n}+\frac{e^{-3ia}\pi\operatorname{csgn}(b\,x^n)}{8n}-\frac{e^{-3ia}\operatorname{Si}(3b\,x^n)}{4n}+\frac{i e^{-3ia}\operatorname{Ei}_1(-3ib\,x^n)}{8n}-\frac{3\,e^{-ia}\pi\operatorname{csgn}(b\,x^n)}{8n}+3\,e^{-}$

```
[In] int(sin(a+b*x^n)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-1/4*Si(3*b*x^n)*cos(3*a)-1/4*Ci(3*b*x^n)*sin(3*a)+3/4*Si(b*x^n)*cos(a
)+3/4*Ci(b*x^n)*sin(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx = -\frac{\operatorname{Ci}(3\,bx^n)\sin(3\,a) - 3\operatorname{Ci}(bx^n)\sin(a) + \cos(3\,a)\operatorname{Si}(3\,bx^n) - 3\cos(a)\operatorname{Si}(bx^n)}{4\,n}$$

```
[In] integrate(sin(a+b*x^n)^3/x,x, algorithm="fricas")
```

```
[Out] -1/4*(cos_integral(3*b*x^n)*sin(3*a) - 3*cos_integral(b*x^n)*sin(a) + cos(3
*a)*sin_integral(3*b*x^n) - 3*cos(a)*sin_integral(b*x^n))/n
```

Sympy [F]

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx = \int \frac{\sin^3(a+bx^n)}{x}\,dx$$

```
[In] integrate(sin(a+b*x**n)**3/x,x)
```

```
[Out] Integral(sin(a + b*x**n)**3/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.70

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx = \frac{\left(i\,\mathrm{Ei}(3i\,bx^n) - i\,\mathrm{Ei}(-3i\,bx^n) + i\,\mathrm{Ei}\left(3i\,be^{\left(n\overline{\log(x)}\right)}\right) - i\,\mathrm{Ei}\left(-3i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(3\,a) - 3\left(i\,\mathrm{Ei}(i\,bx^n) - \right.}{}$$

```
[In] integrate(sin(a+b*x^n)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*((I*Ei(3*I*b*x^n) - I*Ei(-3*I*b*x^n) + I*Ei(3*I*b*e^(n*conjugate(log(x
)))) - I*Ei(-3*I*b*e^(n*conjugate(log(x)))))*cos(3*a) - 3*(I*Ei(I*b*x^n) -
I*Ei(-I*b*x^n) + I*Ei(I*b*e^(n*conjugate(log(x)))) - I*Ei(-I*b*e^(n*conjuga
te(log(x)))))*cos(a) - (Ei(3*I*b*x^n) + Ei(-3*I*b*x^n) + Ei(3*I*b*e^(n*conj
ugate(log(x)))) + Ei(-3*I*b*e^(n*conjugate(log(x)))))*sin(3*a) + 3*(Ei(I*b*
x^n) + Ei(-I*b*x^n) + Ei(I*b*e^(n*conjugate(log(x)))) + Ei(-I*b*e^(n*conjug
ate(log(x)))))*sin(a))/n
```

Giac [F]

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx = \int \frac{\sin(bx^n+a)^3}{x}\,dx$$

```
[In] integrate(sin(a+b*x^n)^3/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx = \int \frac{\sin(a+b\,x^n)^3}{x}\,dx$$

```
[In] int(sin(a + b*x^n)^3/x,x)
```

```
[Out] int(sin(a + b*x^n)^3/x, x)
```

3.138 $\int \frac{\sin^4(a+bx^n)}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{\sin^4(a+bx^n)}{x} dx = -\frac{\cos(2a)\,\text{CosIntegral}(2bx^n)}{2n} + \frac{\cos(4a)\,\text{CosIntegral}(4bx^n)}{8n} + \frac{3\log(x)}{8} + \frac{\sin(2a)\text{Si}(2bx^n)}{2n} - \frac{\sin(4a)\text{Si}(4bx^n)}{8n}$$

```
[Out] -1/2*Ci(2*b*x^n)*cos(2*a)/n+1/8*Ci(4*b*x^n)*cos(4*a)/n+3/8*ln(x)+1/2*Si(2*b
*x^n)*sin(2*a)/n-1/8*Si(4*b*x^n)*sin(4*a)/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3506, 3459, 3457, 3456}

$$\int \frac{\sin^4(a+bx^n)}{x} dx = -\frac{\cos(2a)\,\text{CosIntegral}(2bx^n)}{2n} + \frac{\cos(4a)\,\text{CosIntegral}(4bx^n)}{8n} + \frac{\sin(2a)\text{Si}(2bx^n)}{2n} - \frac{\sin(4a)\text{Si}(4bx^n)}{8n} + \frac{3\log(x)}{8}$$

```
[In] Int[Sin[a + b*x^n]^4/x,x]
```

```
[Out] -1/2*(Cos[2*a]*CosIntegral[2*b*x^n])/n + (Cos[4*a]*CosIntegral[4*b*x^n])/(8
*n) + (3*Log[x])/8 + (Sin[2*a]*SinIntegral[2*b*x^n])/(2*n) - (Sin[4*a]*SinI
ntegral[4*b*x^n])/(8*n)
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{8x} - \frac{\cos(2a+2bx^n)}{2x} + \frac{\cos(4a+4bx^n)}{8x}\right) dx \\
&= \frac{3\log(x)}{8} + \frac{1}{8}\int \frac{\cos(4a+4bx^n)}{x}\,dx - \frac{1}{2}\int \frac{\cos(2a+2bx^n)}{x}\,dx \\
&= \frac{3\log(x)}{8} - \frac{1}{2}\cos(2a)\int \frac{\cos(2bx^n)}{x}\,dx + \frac{1}{8}\cos(4a)\int \frac{\cos(4bx^n)}{x}\,dx \\
&\quad + \frac{1}{2}\sin(2a)\int \frac{\sin(2bx^n)}{x}\,dx - \frac{1}{8}\sin(4a)\int \frac{\sin(4bx^n)}{x}\,dx \\
&= -\frac{\cos(2a)\,\text{CosIntegral}(2bx^n)}{2n} + \frac{\cos(4a)\,\text{CosIntegral}(4bx^n)}{8n} \\
&\quad + \frac{3\log(x)}{8} + \frac{\sin(2a)\text{Si}(2bx^n)}{2n} - \frac{\sin(4a)\text{Si}(4bx^n)}{8n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{\sin^4(a+bx^n)}{x}\,dx = \frac{3\log(x)}{8} + \frac{-4\cos(2a)\,\text{CosIntegral}(2bx^n) + \cos(4a)\,\text{CosIntegral}(4bx^n) + 4\sin(2a)\text{Si}(2bx^n) - \sin(4a)\text{Si}(4bx^n)}{8n}$$

```
[In] Integrate[Sin[a + b*x^n]^4/x,x]
```

```
[Out] (3*Log[x])/8 + (-4*Cos[2*a]*CosIntegral[2*b*x^n] + Cos[4*a]*CosIntegral[4*b
*x^n] + 4*Sin[2*a]*SinIntegral[2*b*x^n] - Sin[4*a]*SinIntegral[4*b*x^n])/(8
*n)
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{3\ln(b\,x^n)}{8}-\frac{\mathrm{Si}(4b\,x^n)\sin(4a)}{8}+\frac{\mathrm{Ci}(4b\,x^n)\cos(4a)}{8}+\frac{\mathrm{Si}(2b\,x^n)\sin(2a)}{2}-\frac{\mathrm{Ci}(2b\,x^n)\cos(2a)}{2}}{n}$
default	$\frac{\frac{3\ln(b\,x^n)}{8}-\frac{\mathrm{Si}(4b\,x^n)\sin(4a)}{8}+\frac{\mathrm{Ci}(4b\,x^n)\cos(4a)}{8}+\frac{\mathrm{Si}(2b\,x^n)\sin(2a)}{2}-\frac{\mathrm{Ci}(2b\,x^n)\cos(2a)}{2}}{n}$
risch	$\frac{i\mathrm{e}^{-4ia}\pi\ \mathrm{csgn}(b\,x^n)}{16n}-\frac{i\mathrm{e}^{-4ia}\,\mathrm{Si}(4b\,x^n)}{8n}-\frac{\mathrm{e}^{-4ia}\,\mathrm{Ei}_1(-4ib\,x^n)}{16n}-\frac{\mathrm{e}^{4ia}\,\mathrm{Ei}_1(-4ib\,x^n)}{16n}+\frac{3\ln(x)}{8}-\frac{i\mathrm{e}^{-2ia}\pi\ \mathrm{csgn}(b\,x^n)}{4n}$

```
[In] int(sin(a+b*x^n)^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(3/8*ln(b*x^n)-1/8*Si(4*b*x^n)*sin(4*a)+1/8*Ci(4*b*x^n)*cos(4*a)+1/2*Si
(2*b*x^n)*sin(2*a)-1/2*Ci(2*b*x^n)*cos(2*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{\sin^4(a+bx^n)}{x}\,dx = \frac{\cos(4\,a)\,\mathrm{Ci}\,(4\,bx^n)-4\,\cos(2\,a)\,\mathrm{Ci}\,(2\,bx^n)+3\,n\log(x)-\sin(4\,a)\,\mathrm{Si}\,(4\,bx^n)+4\,\sin(2\,a)\,\mathrm{Si}\,(2\,bx^n)}{8\,n}$$

```
[In] integrate(sin(a+b*x^n)^4/x,x, algorithm="fricas")
```

```
[Out] 1/8*(cos(4*a)*cos_integral(4*b*x^n) - 4*cos(2*a)*cos_integral(2*b*x^n) + 3*
n*log(x) - sin(4*a)*sin_integral(4*b*x^n) + 4*sin(2*a)*sin_integral(2*b*x^n
))/n
```

Sympy [F]

$$\int \frac{\sin^4(a+bx^n)}{x}\,dx = \int \frac{\sin^4(a+bx^n)}{x}\,dx$$

```
[In] integrate(sin(a+b*x**n)**4/x,x)
```

```
[Out] Integral(sin(a + b*x**n)**4/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.39

$$\int \frac{\sin^4(a+bx^n)}{x}\,dx$$
$$= \frac{\left(\mathrm{Ei}(4i\,bx^n)+\mathrm{Ei}(-4i\,bx^n)+\mathrm{Ei}\left(4i\,be^{\left(n\overline{\log(x)}\right)}\right)+\mathrm{Ei}\left(-4i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(4\,a)-4\left(\mathrm{Ei}(2i\,bx^n)+\mathrm{Ei}(-2\right.}{}$$

```
[In] integrate(sin(a+b*x^n)^4/x,x, algorithm="maxima")
```

```
[Out] 1/32*((Ei(4*I*b*x^n) + Ei(-4*I*b*x^n) + Ei(4*I*b*e^(n*conjugate(log(x))) +
 Ei(-4*I*b*e^(n*conjugate(log(x)))))*cos(4*a) - 4*(Ei(2*I*b*x^n) + Ei(-2*I*
b*x^n) + Ei(2*I*b*e^(n*conjugate(log(x))) + Ei(-2*I*b*e^(n*conjugate(log(x
)))))*cos(2*a) + 12*n*log(x) + (I*Ei(4*I*b*x^n) - I*Ei(-4*I*b*x^n) + I*Ei(4
*I*b*e^(n*conjugate(log(x))) - I*Ei(-4*I*b*e^(n*conjugate(log(x)))))*sin(4
*a) - 4*(I*Ei(2*I*b*x^n) - I*Ei(-2*I*b*x^n) + I*Ei(2*I*b*e^(n*conjugate(log
(x))) - I*Ei(-2*I*b*e^(n*conjugate(log(x)))))*sin(2*a))/n
```

Giac [F]

$$\int \frac{\sin^4(a+bx^n)}{x}\,dx = \int \frac{\sin(bx^n+a)^4}{x}\,dx$$

```
[In] integrate(sin(a+b*x^n)^4/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^4/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx^n)}{x}\,dx = \int \frac{\sin(a+b\,x^n)^4}{x}\,dx$$

```
[In] int(sin(a + b*x^n)^4/x,x)
```

```
[Out] int(sin(a + b*x^n)^4/x, x)
```

3.139 $\int \sin(a+bx^n)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 87

$$\int \sin(a+bx^n)\,dx = \frac{ie^{ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-ibx^n\right)}{2n} - \frac{ie^{-ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ibx^n\right)}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x*GAMMA(1/n,-I*b*x^n)/n/((-I*b*x^n)^(1/n))-1/2*I*x*GAMMA(1/n
,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3446, 2239}

$$\int \sin(a+bx^n)\,dx = \frac{ie^{ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-ibx^n\right)}{2n} - \frac{ie^{-ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ibx^n\right)}{2n}$$

```
[In] Int[Sin[a + b*x^n],x]
```

```
[Out] ((I/2)*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(n*((-I)*b*x^n)^n^(-1)) - ((I/2
)*x*Gamma[n^(-1), I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}i\int e^{-ia-ibx^n}\,dx - \frac{1}{2}i\int e^{ia+ibx^n}\,dx\\ &= \frac{ie^{ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-ibx^n\right)}{2n} - \frac{ie^{-ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ibx^n\right)}{2n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \sin\left(a+bx^n\right)\,dx$$
$$= \frac{ix(b^2x^{2n})^{-1/n}\left(-(-ibx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},-ibx^n\right)(\cos(a)+i\sin(a))\right)}{2n}$$

```
[In] Integrate[Sin[a + b*x^n],x]
[Out] ((I/2)*x*(-(((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n]*(Cos[a] - I*Sin[a]))
 + (I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x^n]*(Cos[a] + I*Sin[a])))/(n*(b^2
*x^(2*n))^n^(-1))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

method	result	size
meijerg	$x\,{}_1F_2\left(\frac{1}{2n};\frac{1}{2},1+\frac{1}{2n};-\frac{x^{2n}b^2}{4}\right)\sin(a)+\frac{b\,x^{1+n}\,{}_1F_2\left(\frac{1}{2}+\frac{1}{2n};\frac{3}{2},\frac{3}{2}+\frac{1}{2n};-\frac{x^{2n}b^2}{4}\right)\cos(a)}{1+n}$	74

```
[In] int(sin(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] x*hypergeom([1/2/n],[1/2,1+1/2/n],-1/4*x^(2*n)*b^2)*sin(a)+b/(1+n)*x^(1+n)*
hypergeom([1/2+1/2/n],[3/2,3/2+1/2/n],-1/4*x^(2*n)*b^2)*cos(a)
```

Fricas [F]

$$\int \sin(a+bx^n)\,dx = \int \sin(bx^n+a)\,dx$$

[In] integrate(sin(a+b*x^n),x, algorithm="fricas")

[Out] integral(sin(b*x^n + a), x)

Sympy [F]

$$\int \sin(a+bx^n)\,dx = \int \sin(a+bx^n)\,dx$$

[In] integrate(sin(a+b*x**n),x)

[Out] Integral(sin(a + b*x**n), x)

Maxima [F]

$$\int \sin(a+bx^n)\,dx = \int \sin(bx^n+a)\,dx$$

[In] integrate(sin(a+b*x^n),x, algorithm="maxima")

[Out] integrate(sin(b*x^n + a), x)

Giac [F]

$$\int \sin(a+bx^n)\,dx = \int \sin(bx^n+a)\,dx$$

[In] integrate(sin(a+b*x^n),x, algorithm="giac")

[Out] integrate(sin(b*x^n + a), x)

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx^n)\,dx = \int \sin\left(a+b\,x^n\right)\,dx$$

```
[In] int(sin(a + b*x^n),x)
[Out] int(sin(a + b*x^n), x)
```

3.140 $\int \sin^2(a+bx^n)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 100

$$\int \sin^2(a+bx^n)\,dx=\frac{x}{2}+\frac{2^{-2-\frac{1}{n}}e^{2ia}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n}+\frac{2^{-2-\frac{1}{n}}e^{-2ia}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}$$

```
[Out] 1/2*x+2^(-2-1/n)*exp(2*I*a)*x*GAMMA(1/n,-2*I*b*x^n)/n/((-I*b*x^n)^(1/n))+2^
(-2-1/n)*x*GAMMA(1/n,2*I*b*x^n)/exp(2*I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3448, 3447, 2239}

$$\int \sin^2(a+bx^n)\,dx=\frac{e^{2ia}2^{-\frac{1}{n}-2}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}2^{-\frac{1}{n}-2}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}+\frac{x}{2}$$

```
[In] Int[Sin[a + b*x^n]^2,x]
```

```
[Out] x/2 + (2^(-2 - n^(-1))*E^((2*I)*a)*x*Gamma[n^(-1), (-2*I)*b*x^n])/(n*((-I)*
b*x^n)^n^(-1)) + (2^(-2 - n^(-1))*x*Gamma[n^(-1), (2*I)*b*x^n])/(E^((2*I)*a
)*n*(I*b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
```

```
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3448

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2} - \frac{1}{2}\cos\left(2a + 2bx^n\right)\right) dx \\
&= \frac{x}{2} - \frac{1}{2}\int \cos\left(2a + 2bx^n\right) dx \\
&= \frac{x}{2} - \frac{1}{4}\int e^{-2ia-2ibx^n}\,dx - \frac{1}{4}\int e^{2ia+2ibx^n}\,dx \\
&= \frac{x}{2} + \frac{2^{-2-\frac{1}{n}}e^{2ia}x\left(-ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, -2ibx^n\right)}{n} + \frac{2^{-2-\frac{1}{n}}e^{-2ia}x\left(ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, 2ibx^n\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \sin^2\left(a + bx^n\right) dx = \frac{x\left(2n + 2^{-1/n}e^{2ia}\left(-ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, -2ibx^n\right) + 2^{-1/n}e^{-2ia}\left(ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, 2ibx^n\right)\right)}{4n}$$

```
[In] Integrate[Sin[a + b*x^n]^2,x]
```

```
[Out] (x*(2*n + (E^((2*I)*a)*Gamma[n^(-1), (-2*I)*b*x^n])/(2^n^(-1)*((-I)*b*x^n)^
n^(-1)) + Gamma[n^(-1), (2*I)*b*x^n]/(2^n^(-1)*E^((2*I)*a)*(I*b*x^n)^n^(-1)
)))/(4*n)
```

Maple [F]

$$\int \left(\sin^2\left(a+b\,x^n\right)\right)dx$$

```
[In] int(sin(a+b*x^n)^2,x)
[Out] int(sin(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \sin^2\left(a+bx^n\right)\,dx=\int \sin\left(bx^n+a\right)^2\,dx$$

```
[In] integrate(sin(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral(-cos(b*x^n + a)^2 + 1, x)
```

Sympy [F]

$$\int \sin^2\left(a+bx^n\right)\,dx=\int \sin^2\left(a+bx^n\right)dx$$

```
[In] integrate(sin(a+b*x**n)**2,x)
[Out] Integral(sin(a + b*x**n)**2, x)
```

Maxima [F]

$$\int \sin^2\left(a+bx^n\right)\,dx=\int \sin\left(bx^n+a\right)^2\,dx$$

```
[In] integrate(sin(a+b*x^n)^2,x, algorithm="maxima")
[Out] 1/2*x - 1/2*integrate(cos(2*b*x^n + 2*a), x)
```

Giac [F]

$$\int \sin^2(a+bx^n)\,dx = \int \sin(bx^n+a)^2\,dx$$

```
[In] integrate(sin(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx^n)\,dx = \int \sin(a+b\,x^n)^2\,dx$$

```
[In] int(sin(a + b*x^n)^2,x)
```

```
[Out] int(sin(a + b*x^n)^2, x)
```

3.141 $\int \sin^3(a+bx^n)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 187

$$\int \sin^3(a+bx^n)\,dx = \frac{3ie^{ia}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n} - \frac{3ie^{-ia}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{8n} - \frac{i3^{-1/n}e^{3ia}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3ibx^n\right)}{8n} + \frac{i3^{-1/n}e^{-3ia}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},3ibx^n\right)}{8n}$$

```
[Out] 3/8*I*exp(I*a)*x*GAMMA(1/n,-I*b*x^n)/n/((-I*b*x^n)^(1/n))-3/8*I*x*GAMMA(1/n
,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^(1/n))-1/8*I*exp(3*I*a)*x*GAMMA(1/n,-3*I*b*
x^n)/(3^(1/n))/n/((-I*b*x^n)^(1/n))+1/8*I*x*GAMMA(1/n,3*I*b*x^n)/(3^(1/n))/
exp(3*I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3448, 3446, 2239}

$$\int \sin^3(a+bx^n)\,dx = \frac{3ie^{ia}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n} - \frac{ie^{3ia}3^{-1/n}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3ibx^n\right)}{8n} - \frac{3ie^{-ia}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{8n} + \frac{ie^{-3ia}3^{-1/n}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},3ibx^n\right)}{8n}$$

```
[In] Int[Sin[a + b*x^n]^3,x]
```

```
[Out] (((3*I)/8)*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(n*((-I)*b*x^n)^n^(-1)) - (
((3*I)/8)*x*Gamma[n^(-1), I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^n^(-1)) - ((I/8)*E
```

```
^((3*I)*a)*x*Gamma[n^(-1), (-3*I)*b*x^n])/(3^n^(-1)*n*((-I)*b*x^n)^n^(-1))
+ ((I/8)*x*Gamma[n^(-1), (3*I)*b*x^n])/(3^n^(-1)*E^((3*I)*a)*n*(I*b*x^n)^n^
(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3448

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{4}\sin\left(a+bx^n\right)-\frac{1}{4}\sin\left(3a+3bx^n\right)\right)\,dx\\
&= -\left(\frac{1}{4}\int \sin\left(3a+3bx^n\right)\,dx\right)+\frac{3}{4}\int \sin\left(a+bx^n\right)\,dx\\
&= -\left(\frac{1}{8}i\int e^{-3ia-3ibx^n}\,dx\right)+\frac{1}{8}i\int e^{3ia+3ibx^n}\,dx+\frac{3}{8}i\int e^{-ia-ibx^n}\,dx-\frac{3}{8}i\int e^{ia+ibx^n}\,dx\\
&= \frac{3ie^{ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{3ie^{-ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ibx^n\right)}{8n}\\
&\quad -\frac{i3^{-1/n}e^{3ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-3ibx^n\right)}{8n}+\frac{i3^{-1/n}e^{-3ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},3ibx^n\right)}{8n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

$$\int \sin^3\left(a+bx^n\right)\,dx$$
$$=\frac{i3^{-1/n}e^{-3ia}x\left(b^2x^{2n}\right)^{-1/n}\left(3^{1+\frac{1}{n}}e^{4ia}(ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-ibx^n\right)-3^{1+\frac{1}{n}}e^{2ia}(-ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},ibx^n\right)-e^{6ia}(ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},\right.\right.}{8n}$$

```
[In] Integrate[Sin[a + b*x^n]^3,x]
```

```
[Out] ((I/8)*x*(3^(1 + n^(-1))*E^((4*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*
x^n] - 3^(1 + n^(-1))*E^((2*I)*a)*((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n
] - E^((6*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-3*I)*b*x^n] + ((-I)*b*x^n)
^n^(-1)*Gamma[n^(-1), (3*I)*b*x^n]))/(3^n^(-1)*E^((3*I)*a)*n*(b^2*x^(2*n))^
n^(-1))
```

Maple [F]

$$\int \left(\sin^3\left(a+b\,x^n\right)\right)dx$$

```
[In] int(sin(a+b*x^n)^3,x)
```

```
[Out] int(sin(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \sin^3\left(a+bx^n\right)\,dx=\int \sin\left(bx^n+a\right)^3\,dx$$

```
[In] integrate(sin(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x^n + a)^2 - 1)*sin(b*x^n + a), x)
```

Sympy [F]

$$\int \sin^3\left(a+bx^n\right)\,dx=\int \sin^3\left(a+bx^n\right)dx$$

```
[In] integrate(sin(a+b*x**n)**3,x)
```

```
[Out] Integral(sin(a + b*x**n)**3, x)
```

Maxima [F]

$$\int \sin^3(a+bx^n)\,dx = \int \sin(bx^n+a)^3\,dx$$

```
[In] integrate(sin(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate(sin(b*x^n + a)^3, x)
```

Giac [F]

$$\int \sin^3(a+bx^n)\,dx = \int \sin(bx^n+a)^3\,dx$$

```
[In] integrate(sin(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(sin(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx^n)\,dx = \int \sin(a+b\,x^n)^3\,dx$$

```
[In] int(sin(a + b*x^n)^3,x)
[Out] int(sin(a + b*x^n)^3, x)
```

3.142 $\int x^m \sin(a+bx^n)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 109

$$\int x^m \sin(a+bx^n)\,dx = \frac{ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{2n} - \frac{ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x^(1+m)*GAMMA((1+m)/n,-I*b*x^n)/n/((-I*b*x^n)^((1+m)/n))-1/2
*I*x^(1+m)*GAMMA((1+m)/n,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3504, 2250}

$$\int x^m \sin(a+bx^n)\,dx = \frac{ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-ibx^n\right)}{2n} - \frac{ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},ibx^n\right)}{2n}$$

```
[In] Int[x^m*Sin[a + b*x^n],x]
```

```
[Out] ((I/2)*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(n*((-I)*b*x^n)^((1
+ m)/n)) - ((I/2)*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(E^(I*a)*n*(I*b*x^n)
^((1 + m)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}i\int e^{-ia-ibx^n}x^m\,dx - \frac{1}{2}i\int e^{ia+ibx^n}x^m\,dx$$
$$= \frac{ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{2n} - \frac{ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},ibx^n\right)}{2n}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.08

$$\int x^m\sin\left(a+bx^n\right)\,dx$$
$$= \frac{ix^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}}\left(-(-ibx^n)^{\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-ibx^n\right)(\cos(a)+i\right.}{2n}$$

```
[In] Integrate[x^m*Sin[a + b*x^n],x]
```

```
[Out] ((I/2)*x^(1 + m)*(-(((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, I*b*x^n]*(Cos
[a] - I*Sin[a])) + (I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-I)*b*x^n]*(Cos[
a] + I*Sin[a])))/(n*(b^2*x^(2*n))^((1 + m)/n))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

method	result	size
meijerg	$\frac{x^{1+m}\,{}_1F_2\left(\frac{m}{2n}+\frac{1}{2n};\frac{1}{2},1+\frac{m}{2n}+\frac{1}{2n};-\frac{x^{2n}b^2}{4}\right)\sin(a)}{1+m}+\frac{b\,x^{n+m+1}\,{}_1F_2\left(\frac{1}{2}+\frac{m}{2n}+\frac{1}{2n};\frac{3}{2},\frac{3}{2}+\frac{m}{2n}+\frac{1}{2n};-\frac{x^{2n}b^2}{4}\right)\cos(a)}{n+m+1}$	110

```
[In] int(x^m*sin(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+m)*x^(1+m)*hypergeom([1/2/n*m+1/2/n],[1/2,1+1/2/n*m+1/2/n],-1/4*x^(2*n
)*b^2)*sin(a)+b/(n+m+1)*x^(n+m+1)*hypergeom([1/2+1/2/n*m+1/2/n],[3/2,3/2+1/
2/n*m+1/2/n],-1/4*x^(2*n)*b^2)*cos(a)
```

Fricas [F]

$$\int x^m \sin(a+bx^n)\,dx = \int x^m \sin(bx^n+a)\,dx$$

```
[In] integrate(x^m*sin(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^m*sin(b*x^n + a), x)
```

Sympy [F]

$$\int x^m \sin(a+bx^n)\,dx = \int x^m \sin(a+bx^n)\,dx$$

```
[In] integrate(x**m*sin(a+b*x**n),x)
```

```
[Out] Integral(x**m*sin(a + b*x**n), x)
```

Maxima [F]

$$\int x^m \sin(a+bx^n)\,dx = \int x^m \sin(bx^n+a)\,dx$$

```
[In] integrate(x^m*sin(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sin(b*x^n + a), x)
```

Giac [F]

$$\int x^m \sin(a+bx^n)\,dx = \int x^m \sin(bx^n+a)\,dx$$

```
[In] integrate(x^m*sin(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*sin(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sin(a+bx^n)\,dx = \int x^m \sin(a+b\,x^n)\,dx$$

```
[In] int(x^m*sin(a + b*x^n),x)
[Out] int(x^m*sin(a + b*x^n), x)
```

3.143 $\int x^m \sin^2(a+bx^n)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 139

$$\int x^m \sin^2(a+bx^n)\,dx = \frac{x^{1+m}}{2(1+m)} + \frac{2^{-\frac{1+m+2n}{n}} e^{2ia} x^{1+m} (-ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -2ibx^n\right)}{n} + \frac{2^{-\frac{1+m+2n}{n}} e^{-2ia} x^{1+m} (ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 2ibx^n\right)}{n}$$

```
[Out] 1/2*x^(1+m)/(1+m)+exp(2*I*a)*x^(1+m)*GAMMA((1+m)/n,-2*I*b*x^n)/(2^((1+m+2*n
)/n))/n/((-I*b*x^n)^((1+m)/n))+x^(1+m)*GAMMA((1+m)/n,2*I*b*x^n)/(2^((1+m+2*
n)/n))/exp(2*I*a)/n/((I*b*x^n)^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3506, 3505, 2250}

$$\int x^m \sin^2(a+bx^n)\,dx = \frac{e^{2ia} 2^{-\frac{m+2n+1}{n}} x^{m+1} (-ibx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, -2ibx^n\right)}{n} + \frac{e^{-2ia} 2^{-\frac{m+2n+1}{n}} x^{m+1} (ibx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, 2ibx^n\right)}{n} + \frac{x^{m+1}}{2(m+1)}$$

```
[In] Int[x^m*Sin[a + b*x^n]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) + (E^((2*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x^
n])/(2^((1 + m + 2*n)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + (x^(1 + m)*Gamma[(1
+ m)/n, (2*I)*b*x^n])/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*n*(I*b*x^n)^((1 + m)
/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^m}{2}-\frac{1}{2}x^m\cos\left(2a+2bx^n\right)\right)\,dx\\
&= \frac{x^{1+m}}{2(1+m)}-\frac{1}{2}\int x^m\cos\left(2a+2bx^n\right)\,dx\\
&= \frac{x^{1+m}}{2(1+m)}-\frac{1}{4}\int e^{-2ia-2ibx^n}x^m\,dx-\frac{1}{4}\int e^{2ia+2ibx^n}x^m\,dx\\
&= \frac{x^{1+m}}{2(1+m)}+\frac{2^{-\frac{1+m+2n}{n}}e^{2ia}x^{1+m}\left(-ibx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2ibx^n\right)}{n}\\
&\quad+\frac{2^{-\frac{1+m+2n}{n}}e^{-2ia}x^{1+m}\left(ibx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},2ibx^n\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93

$$\int x^m\sin^2\left(a+bx^n\right)\,dx$$

$$=\frac{x^{1+m}\left(2n+2^{-\frac{1+m}{n}}e^{2ia}(1+m)\left(-ibx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2ibx^n\right)+2^{-\frac{1+m}{n}}e^{-2ia}(1+m)\left(ibx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},2ib\right.\right.}{4(1+m)n}$$

```
[In] Integrate[x^m*Sin[a + b*x^n]^2,x]
```

```
[Out] (x^(1 + m)*(2*n + (E^((2*I)*a)*(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x^n])/(2^(
(1 + m)/n)*((-I)*b*x^n)^((1 + m)/n)) + ((1 + m)*Gamma[(1 + m)/n, (2*I)*b*x^
n])/(2^((1 + m)/n)*E^((2*I)*a)*(I*b*x^n)^((1 + m)/n))))/(4*(1 + m)*n)
```

Maple [F]

$$\int x^m \left(\sin^2\left(a+b\,x^n\right)\right)dx$$

```
[In] int(x^m*sin(a+b*x^n)^2,x)
[Out] int(x^m*sin(a+b*x^n)^2,x)
```

Fricas [F]

$$\int x^m \sin^2\left(a+bx^n\right)\,dx = \int x^m \sin\left(bx^n+a\right)^2\,dx$$

```
[In] integrate(x^m*sin(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral(-x^m*cos(b*x^n + a)^2 + x^m, x)
```

Sympy [F]

$$\int x^m \sin^2\left(a+bx^n\right)\,dx = \int x^m \sin^2\left(a+bx^n\right)dx$$

```
[In] integrate(x**m*sin(a+b*x**n)**2,x)
[Out] Integral(x**m*sin(a + b*x**n)**2, x)
```

Maxima [F]

$$\int x^m \sin^2\left(a+bx^n\right)\,dx = \int x^m \sin\left(bx^n+a\right)^2\,dx$$

```
[In] integrate(x^m*sin(a+b*x^n)^2,x, algorithm="maxima")
[Out] 1/2*(x*x^m - (m + 1)*integrate(x^m*cos(2*b*x^n + 2*a), x))/(m + 1)
```

Giac [F]

$$\int x^m \sin^2(a+bx^n)\,dx=\int x^m \sin\left(bx^n+a\right)^2\,dx$$

```
[In] integrate(x^m*sin(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x^m*sin(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sin^2(a+bx^n)\,dx=\int x^m \sin\left(a+b\,x^n\right)^2\,dx$$

```
[In] int(x^m*sin(a + b*x^n)^2,x)
[Out] int(x^m*sin(a + b*x^n)^2, x)
```

3.144 $\int x^m \sin^3(a+bx^n)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 237

$$\int x^m \sin^3(a+bx^n)\,dx = \frac{3ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{8n} - \frac{3ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)}{8n} - \frac{i3^{-\frac{1+m}{n}}e^{3ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3ibx^n\right)}{8n} + \frac{i3^{-\frac{1+m}{n}}e^{-3ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},3ibx^n\right)}{8n}$$

```
[Out] 3/8*I*exp(I*a)*x^(1+m)*GAMMA((1+m)/n,-I*b*x^n)/n/((-I*b*x^n)^((1+m)/n))-3/8
*I*x^(1+m)*GAMMA((1+m)/n,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))-1/8*I*ex
p(3*I*a)*x^(1+m)*GAMMA((1+m)/n,-3*I*b*x^n)/(3^((1+m)/n))/n/((-I*b*x^n)^((1+
m)/n))+1/8*I*x^(1+m)*GAMMA((1+m)/n,3*I*b*x^n)/(3^((1+m)/n))/exp(3*I*a)/n/((
I*b*x^n)^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used

= {3506, 3504, 2250}

$$\int x^m \sin^3\left(a+bx^n\right)\,dx = \frac{3ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},-ibx^n\right)}{8n} - \frac{3ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},ibx^n\right)}{8n} - \frac{ie^{3ia}3^{-\frac{m+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},-3ibx^n\right)}{8n} + \frac{ie^{-3ia}3^{-\frac{m+1}{n}}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},3ibx^n\right)}{8n}$$

```
[In] Int[x^m*Sin[a + b*x^n]^3,x]
```

```
[Out] (((3*I)/8)*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(n*((-I)*b*x^n)^
((1 + m)/n)) - (((3*I)/8)*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(E^(I*a)*n*(
I*b*x^n)^((1 + m)/n)) - ((I/8)*E^((3*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-3*I
)*b*x^n])/(3^((1 + m)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + ((I/8)*x^(1 + m)*Gam
ma[(1 + m)/n, (3*I)*b*x^n])/(3^((1 + m)/n)*E^((3*I)*a)*n*(I*b*x^n)^((1 + m)
/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{3}{4}x^m\sin\left(a+bx^n\right)-\frac{1}{4}x^m\sin\left(3a+3bx^n\right)\right)\,dx$$

$$= -\left(\frac{1}{4}\int x^m\sin\left(3a+3bx^n\right)\,dx\right)+\frac{3}{4}\int x^m\sin\left(a+bx^n\right)\,dx$$

$$= -\left(\frac{1}{8}i\int e^{-3ia-3ibx^n}x^m\,dx\right)+\frac{1}{8}i\int e^{3ia+3ibx^n}x^m\,dx$$
$$+\frac{3}{8}i\int e^{-ia-ibx^n}x^m\,dx-\frac{3}{8}i\int e^{ia+ibx^n}x^m\,dx$$
$$=\frac{3ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{8n}-\frac{3ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)}{8n}$$
$$-\frac{i3^{-\frac{1+m}{n}}e^{3ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3ibx^n\right)}{8n}$$
$$+\frac{i3^{-\frac{1+m}{n}}e^{-3ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},3ibx^n\right)}{8n}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.95

$$\int x^m\sin^3\left(a+bx^n\right)\,dx$$
$$=\frac{i3^{-\frac{1+m}{n}}e^{-3ia}x^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}}\left(3^{\frac{1+m+n}{n}}e^{4ia}(ibx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)-3^{\frac{1+m+n}{n}}e^{2ia}(-ibx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)\right.}{8n}$$

```
[In] Integrate[x^m*Sin[a + b*x^n]^3,x]
```

```
[Out] ((I/8)*x^(1 + m)*(3^((1 + m + n)/n)*E^((4*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamma
[(1 + m)/n, (-I)*b*x^n] - 3^((1 + m + n)/n)*E^((2*I)*a)*((-I)*b*x^n)^((1 +
m)/n)*Gamma[(1 + m)/n, I*b*x^n] - E^((6*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamma[(
1 + m)/n, (-3*I)*b*x^n] + ((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (3*I)*b
*x^n]))/(3^((1 + m)/n)*E^((3*I)*a)*n*(b^2*x^(2*n))^((1 + m)/n))
```

Maple [F]

$$\int x^m\left(\sin^3\left(a+b\,x^n\right)\right)dx$$

```
[In] int(x^m*sin(a+b*x^n)^3,x)
```

```
[Out] int(x^m*sin(a+b*x^n)^3,x)
```

Fricas [F]

$$\int x^m \sin^3(a+bx^n)\, dx = \int x^m \sin(bx^n+a)^3\, dx$$

```
[In] integrate(x^m*sin(a+b*x^n)^3,x, algorithm="fricas")
[Out] integral(-(x^m*cos(b*x^n + a)^2 - x^m)*sin(b*x^n + a), x)
```

Sympy [F]

$$\int x^m \sin^3(a+bx^n)\, dx = \int x^m \sin^3(a+bx^n)\, dx$$

```
[In] integrate(x**m*sin(a+b*x**n)**3,x)
[Out] Integral(x**m*sin(a + b*x**n)**3, x)
```

Maxima [F]

$$\int x^m \sin^3(a+bx^n)\, dx = \int x^m \sin(bx^n+a)^3\, dx$$

```
[In] integrate(x^m*sin(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate(x^m*sin(b*x^n + a)^3, x)
```

Giac [F]

$$\int x^m \sin^3(a+bx^n)\, dx = \int x^m \sin(bx^n+a)^3\, dx$$

```
[In] integrate(x^m*sin(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(x^m*sin(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sin^3(a+bx^n)\,dx = \int x^m \sin(a+b\,x^n)^3\,dx$$

```
[In] int(x^m*sin(a + b*x^n)^3,x)
[Out] int(x^m*sin(a + b*x^n)^3, x)
```

3.145 $\int x^{-1+2n} \sin(a + bx^n) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int x^{-1+2n} \sin(a + bx^n) \, dx = -\frac{x^n \cos(a + bx^n)}{bn} + \frac{\sin(a + bx^n)}{b^2 n}$$

```
[Out] -x^n*cos(a+b*x^n)/b/n+sin(a+b*x^n)/b^2/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3460, 3377, 2717}

$$\int x^{-1+2n} \sin(a + bx^n) \, dx = \frac{\sin(a + bx^n)}{b^2 n} - \frac{x^n \cos(a + bx^n)}{bn}$$

```
[In] Int[x^(-1 + 2*n)*Sin[a + b*x^n],x]
```

```
[Out] -((x^n*Cos[a + b*x^n])/(b*n)) + Sin[a + b*x^n]/(b^2*n)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x\sin(a+bx)\,dx,x,x^n\right)}{n}\\ &= -\frac{x^n\cos\left(a+bx^n\right)}{bn}+\frac{\text{Subst}\left(\int \cos(a+bx)\,dx,x,x^n\right)}{bn}\\ &= -\frac{x^n\cos\left(a+bx^n\right)}{bn}+\frac{\sin\left(a+bx^n\right)}{b^2n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int x^{-1+2n}\sin\left(a+bx^n\right)\,dx=\frac{-bx^n\cos\left(a+bx^n\right)+\sin\left(a+bx^n\right)}{b^2n}$$

```
[In] Integrate[x^(-1 + 2*n)*Sin[a + b*x^n],x]
[Out] (-(b*x^n*Cos[a + b*x^n]) + Sin[a + b*x^n])/(b^2*n)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{x^{n}\cos(a+b\,x^{n})}{bn}+\frac{\sin(a+b\,x^{n})}{b^{2}n}$	36
default	$\frac{\sin(a+b\,x^{n})-(a+b\,x^{n})\cos(a+b\,x^{n})+a\cos(a+b\,x^{n})}{n\,b^{2}}$	44
meijerg	$\frac{2\sqrt{\pi}\,G_{1,3}^{1,1}\left(\frac{x^{2n}b^{2}}{4}\,\middle\vert\,\begin{matrix}1\\1,\frac{3}{2},0\end{matrix}\right)\sin(a)}{b^{2}n}+\frac{2\sqrt{\pi}\left(-\frac{x^{n}b\cos(b\,x^{n})}{2\sqrt{\pi}}+\frac{\sin(b\,x^{n})}{2\sqrt{\pi}}\right)\cos(a)}{b^{2}n}$	76

```
[In] int(x^(-1+2*n)*sin(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] -x^n*cos(a+b*x^n)/b/n+sin(a+b*x^n)/b^2/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int x^{-1+2n}\sin\left(a+bx^n\right)\,dx = -\frac{bx^n\cos\left(bx^n+a\right)-\sin\left(bx^n+a\right)}{b^2n}$$

```
[In] integrate(x^(-1+2*n)*sin(a+b*x^n),x, algorithm="fricas")
[Out] -(b*x^n*cos(b*x^n + a) - sin(b*x^n + a))/(b^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(27) = 54$.

Time = 3.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int x^{-1+2n}\sin\left(a+bx^n\right)\,dx = \begin{cases} \log\left(x\right)\sin\left(a\right) & \text{for}\ b=0\wedge n=0 \\ \frac{xx^{2n-1}\sin\left(a\right)}{2n} & \text{for}\ b=0 \\ \log\left(x\right)\sin\left(a+b\right) & \text{for}\ n=0 \\ -\frac{x^n\cos\left(a+bx^n\right)}{bn}+\frac{\sin\left(a+bx^n\right)}{b^2n} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(-1+2*n)*sin(a+b*x**n),x)
[Out] Piecewise((log(x)*sin(a), Eq(b, 0) & Eq(n, 0)), (x*x**(2*n - 1)*sin(a)/(2*n
), Eq(b, 0)), (log(x)*sin(a + b), Eq(n, 0)), (-x**n*cos(a + b*x**n)/(b*n) +
 sin(a + b*x**n)/(b**2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int x^{-1+2n}\sin\left(a+bx^n\right)\,dx = -\frac{bx^n\cos\left(bx^n+a\right)-\sin\left(bx^n+a\right)}{b^2n}$$

```
[In] integrate(x^(-1+2*n)*sin(a+b*x^n),x, algorithm="maxima")
[Out] -(b*x^n*cos(b*x^n + a) - sin(b*x^n + a))/(b^2*n)
```

Giac [F]

$$\int x^{-1+2n} \sin(a+bx^n) \, dx = \int x^{2n-1} \sin(bx^n+a) \, dx$$

```
[In] integrate(x^(-1+2*n)*sin(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)*sin(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n} \sin(a+bx^n) \, dx = \int x^{2n-1} \sin(a+b\,x^n) \, dx$$

```
[In] int(x^(2*n - 1)*sin(a + b*x^n),x)
[Out] int(x^(2*n - 1)*sin(a + b*x^n), x)
```

3.146 $\int x^{-1+2n} \cos(a+bx^n)\, dx$

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int x^{-1+2n} \cos(a+bx^n)\, dx = \frac{\cos(a+bx^n)}{b^2n} + \frac{x^n \sin(a+bx^n)}{bn}$$

```
[Out] cos(a+b*x^n)/b^2/n+x^n*sin(a+b*x^n)/b/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3461, 3377, 2718}

$$\int x^{-1+2n} \cos(a+bx^n)\, dx = \frac{\cos(a+bx^n)}{b^2n} + \frac{x^n \sin(a+bx^n)}{bn}$$

```
[In] Int[x^(-1 + 2*n)*Cos[a + b*x^n],x]
```

```
[Out] Cos[a + b*x^n]/(b^2*n) + (x^n*Sin[a + b*x^n])/(b*n)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x\cos(a+bx)\,dx,x,x^n\right)}{n}\\ &= \frac{x^n\sin\left(a+bx^n\right)}{bn}-\frac{\text{Subst}\left(\int \sin(a+bx)\,dx,x,x^n\right)}{bn}\\ &= \frac{\cos\left(a+bx^n\right)}{b^2n}+\frac{x^n\sin\left(a+bx^n\right)}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x^{-1+2n}\cos\left(a+bx^n\right)\,dx=\frac{\cos\left(a+bx^n\right)+bx^n\sin\left(a+bx^n\right)}{b^2n}$$

```
[In] Integrate[x^(-1 + 2*n)*Cos[a + b*x^n],x]
[Out] (Cos[a + b*x^n] + b*x^n*Sin[a + b*x^n])/(b^2*n)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{\cos(a+b\,x^n)}{b^2n}+\frac{x^n\sin(a+b\,x^n)}{bn}$	35
default	$\frac{\cos(a+b\,x^n)+(a+b\,x^n)\sin(a+b\,x^n)-a\sin(a+b\,x^n)}{n\,b^2}$	44
meijerg	$\frac{2\sqrt{\pi}\,G_{1,3}^{1,1}\left(\frac{x^{2n}b^2}{4}\,\middle\vert\,\begin{matrix}1\\1,\frac{3}{2},0\end{matrix}\right)\cos(a)}{b^2n}-\frac{2\sqrt{\pi}\left(-\frac{x^n b\cos(b\,x^n)}{2\sqrt{\pi}}+\frac{\sin(b\,x^n)}{2\sqrt{\pi}}\right)\sin(a)}{b^2n}$	76

```
[In] int(x^(-1+2*n)*cos(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] cos(a+b*x^n)/b^2/n+x^n*sin(a+b*x^n)/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x^{-1+2n}\cos\left(a+bx^n\right)\,dx=\frac{bx^n\sin\left(bx^n+a\right)+\cos\left(bx^n+a\right)}{b^2n}$$

```
[In] integrate(x^(-1+2*n)*cos(a+b*x^n),x, algorithm="fricas")
[Out] (b*x^n*sin(b*x^n + a) + cos(b*x^n + a))/(b^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(27) = 54.

Time = 3.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int x^{-1+2n}\cos\left(a+bx^n\right)\,dx=\begin{cases}\log\left(x\right)\cos\left(a\right) & \text{for } b=0\wedge n=0\\ \frac{xx^{2n-1}\cos\left(a\right)}{2n} & \text{for } b=0\\ \log\left(x\right)\cos\left(a+b\right) & \text{for } n=0\\ \frac{x^n\sin\left(a+bx^n\right)}{bn}+\frac{\cos\left(a+bx^n\right)}{b^2n} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**(-1+2*n)*cos(a+b*x**n),x)
[Out] Piecewise((log(x)*cos(a), Eq(b, 0) & Eq(n, 0)), (x*x**(2*n - 1)*cos(a)/(2*n
), Eq(b, 0)), (log(x)*cos(a + b), Eq(n, 0)), (x**n*sin(a + b*x**n)/(b*n) +
cos(a + b*x**n)/(b**2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x^{-1+2n}\cos\left(a+bx^n\right)\,dx=\frac{bx^n\sin\left(bx^n+a\right)+\cos\left(bx^n+a\right)}{b^2n}$$

```
[In] integrate(x^(-1+2*n)*cos(a+b*x^n),x, algorithm="maxima")
[Out] (b*x^n*sin(b*x^n + a) + cos(b*x^n + a))/(b^2*n)
```

Giac [F]

$$\int x^{-1+2n}\cos\left(a+bx^n\right)\,dx=\int x^{2\,n-1}\cos\left(bx^n+a\right)\,dx$$

```
[In] integrate(x^(-1+2*n)*cos(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)*cos(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\cos\left(a+bx^n\right)\,dx=\int x^{2\,n-1}\,\cos\left(a+b\,x^n\right)\,dx$$

```
[In] int(x^(2*n - 1)*cos(a + b*x^n),x)
[Out] int(x^(2*n - 1)*cos(a + b*x^n), x)
```

3.147 $\int x^{-1-n} \sin(a + bx^n) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int x^{-1-n} \sin(a + bx^n) \, dx = \frac{b \cos(a) \operatorname{CosIntegral}(bx^n)}{n} - \frac{x^{-n} \sin(a + bx^n)}{n} - \frac{b \sin(a) \operatorname{Si}(bx^n)}{n}$$

```
[Out] b*Ci(b*x^n)*cos(a)/n-b*Si(b*x^n)*sin(a)/n-sin(a+b*x^n)/n/(x^n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3460, 3378, 3384, 3380, 3383}

$$\int x^{-1-n} \sin(a + bx^n) \, dx = \frac{b \cos(a) \operatorname{CosIntegral}(bx^n)}{n} - \frac{b \sin(a) \operatorname{Si}(bx^n)}{n} - \frac{x^{-n} \sin(a + bx^n)}{n}$$

```
[In] Int[x^(-1 - n)*Sin[a + b*x^n],x]
```

```
[Out] (b*Cos[a]*CosIntegral[b*x^n])/n - Sin[a + b*x^n]/(n*x^n) - (b*Sin[a]*SinInt
egral[b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, x^n\right)}{n}\\
&= -\frac{x^{-n}\sin(a+bx^n)}{n} + \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, x^n\right)}{n}\\
&= -\frac{x^{-n}\sin(a+bx^n)}{n} + \frac{(b\cos(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, x^n\right)}{n}\\
&\quad - \frac{(b\sin(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{b\cos(a)\,\text{CosIntegral}(bx^n)}{n} - \frac{x^{-n}\sin(a+bx^n)}{n} - \frac{b\sin(a)\text{Si}(bx^n)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int x^{-1-n}\sin\left(a+bx^n\right)\,dx = \frac{x^{-n}(bx^n\cos(a)\,\text{CosIntegral}\left(bx^n\right)-\sin\left(a+bx^n\right)-bx^n\sin(a)\text{Si}(bx^n))}{n}$$

```
[In] Integrate[x^(-1 - n)*Sin[a + b*x^n],x]
[Out] (b*x^n*Cos[a]*CosIntegral[b*x^n] - Sin[a + b*x^n] - b*x^n*Sin[a]*SinIntegra
l[b*x^n])/(n*x^n)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{b\left(-\frac{\sin(a+b\,x^n)\,x^{-n}}{b}-\text{Si}(b\,x^n)\sin(a)+\text{Ci}(b\,x^n)\cos(a)\right)}{n}$	44
risch	$-\frac{b\,e^{ia}\,\text{Ei}_1(-ib\,x^n)}{2n}+\frac{ib\,e^{-ia}\pi\ \text{csgn}(b\,x^n)}{2n}-\frac{ib\,e^{-ia}\,\text{Si}(b\,x^n)}{n}-\frac{b\,e^{-ia}\,\text{Ei}_1(-ib\,x^n)}{2n}-\frac{\sin(a+b\,x^n)x^{-n}}{n}$	97

```
[In] int(x^(-n-1)*sin(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] 1/n*b*(-sin(a+b*x^n)/b/(x^n)-Si(b*x^n)*sin(a)+Ci(b*x^n)*cos(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int x^{-1-n}\sin\left(a+bx^n\right)\,dx = \frac{bx^n\cos\left(a\right)\text{Ci}\left(bx^n\right)-bx^n\sin\left(a\right)\text{Si}\left(bx^n\right)-\sin\left(bx^n+a\right)}{nx^n}$$

```
[In] integrate(x^(-1-n)*sin(a+b*x^n),x, algorithm="fricas")
[Out] (b*x^n*cos(a)*cos_integral(b*x^n) - b*x^n*sin(a)*sin_integral(b*x^n) - sin(
b*x^n + a))/(n*x^n)
```

Sympy [F]

$$\int x^{-1-n}\sin\left(a+bx^{n}\right)\,dx=\int x^{-n-1}\sin\left(a+bx^{n}\right)dx$$

```
[In] integrate(x**(-1-n)*sin(a+b*x**n),x)
[Out] Integral(x**(-n - 1)*sin(a + b*x**n), x)
```

Maxima [F]

$$\int x^{-1-n}\sin\left(a+bx^{n}\right)\,dx=\int x^{-n-1}\sin\left(bx^{n}+a\right)\,dx$$

```
[In] integrate(x^(-1-n)*sin(a+b*x^n),x, algorithm="maxima")
[Out] integrate(x^(-n - 1)*sin(b*x^n + a), x)
```

Giac [F]

$$\int x^{-1-n}\sin\left(a+bx^{n}\right)\,dx=\int x^{-n-1}\sin\left(bx^{n}+a\right)\,dx$$

```
[In] integrate(x^(-1-n)*sin(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^(-n - 1)*sin(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\sin\left(a+bx^{n}\right)\,dx=\int\frac{\sin\left(a+b\,x^{n}\right)}{x^{n+1}}\,dx$$

```
[In] int(sin(a + b*x^n)/x^(n + 1),x)
[Out] int(sin(a + b*x^n)/x^(n + 1), x)
```

3.148 $\int x^{-1-n}\sin^2(a+bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int x^{-1-n}\sin^2(a+bx^n)\,dx = -\frac{x^{-n}}{2n}+\frac{x^{-n}\cos(2(a+bx^n))}{2n}+\frac{b\,\text{CosIntegral}(2bx^n)\sin(2a)}{n}+\frac{b\cos(2a)\text{Si}(2bx^n)}{n}$$

```
[Out] -1/2/n/(x^n)+1/2*cos(2*a+2*b*x^n)/n/(x^n)+b*cos(2*a)*Si(2*b*x^n)/n+b*Ci(2*b
*x^n)*sin(2*a)/n
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3506, 3461, 3378, 3384, 3380, 3383}

$$\int x^{-1-n}\sin^2(a+bx^n)\,dx = \frac{b\sin(2a)\,\text{CosIntegral}(2bx^n)}{n}+\frac{b\cos(2a)\text{Si}(2bx^n)}{n}+\frac{x^{-n}\cos(2(a+bx^n))}{2n}-\frac{x^{-n}}{2n}$$

```
[In] Int[x^(-1 - n)*Sin[a + b*x^n]^2,x]
```

```
[Out] -1/2*1/(n*x^n) + Cos[2*(a + b*x^n)]/(2*n*x^n) + (b*CosIntegral[2*b*x^n]*Sin
[2*a])/n + (b*Cos[2*a]*SinIntegral[2*b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^{-1-n}}{2} - \frac{1}{2}x^{-1-n}\cos(2a+2bx^n)\right) dx \\
&= -\frac{x^{-n}}{2n} - \frac{1}{2}\int x^{-1-n}\cos(2a+2bx^n)\, dx \\
&= -\frac{x^{-n}}{2n} - \frac{\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x^2}\, dx, x, x^n\right)}{2n} \\
&= -\frac{x^{-n}}{2n} + \frac{x^{-n}\cos(2(a+bx^n))}{2n} + \frac{b\text{Subst}\left(\int \frac{\sin(2a+2bx)}{x}\, dx, x, x^n\right)}{n}
\end{aligned}$$

$$= -\frac{x^{-n}}{2n} + \frac{x^{-n}\cos\left(2(a+bx^n)\right)}{2n} + \frac{(b\cos(2a))\text{Subst}\left(\int \frac{\sin(2bx)}{x}\,dx, x, x^n\right)}{n} + \frac{(b\sin(2a))\text{Subst}\left(\int \frac{\cos(2bx)}{x}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-n}}{2n} + \frac{x^{-n}\cos\left(2(a+bx^n)\right)}{2n} + \frac{b\,\text{CosIntegral}\left(2bx^n\right)\sin(2a)}{n} + \frac{b\cos(2a)\text{Si}(2bx^n)}{n}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int x^{-1-n}\sin^2\left(a+bx^n\right)\,dx = \frac{x^{-n}\left(-1+\cos\left(2(a+bx^n)\right)+2bx^n\,\text{CosIntegral}\left(2bx^n\right)\sin(2a)+2bx^n\cos(2a)\text{Si}(2bx^n)\right)}{2n}$$

```
[In] Integrate[x^(-1 - n)*Sin[a + b*x^n]^2,x]
[Out] (-1 + Cos[2*(a + b*x^n)] + 2*b*x^n*CosIntegral[2*b*x^n]*Sin[2*a] + 2*b*x^n*
Cos[2*a]*SinIntegral[2*b*x^n])/(2*n*x^n)
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{x^{-n}}{2n}-\frac{b\left(-\frac{\cos\left(2a+2b\,x^{n}\right)x^{-n}}{2b}-\text{Si}\left(2b\,x^{n}\right)\cos\left(2a\right)-\text{Ci}\left(2b\,x^{n}\right)\sin\left(2a\right)\right)}{n}$	66
risch	$-\frac{\left(ib\,e^{-2ia}\,\text{Ei}_{1}\left(-2ib\,x^{n}\right)x^{n}-ib\,e^{2ia}\,\text{Ei}_{1}\left(-2ib\,x^{n}\right)x^{n}+b\,e^{-2ia}\pi\,\,\text{csgn}\left(b\,x^{n}\right)x^{n}-2b\,e^{-2ia}\,\text{Si}\left(2b\,x^{n}\right)x^{n}-\cos\left(2a+2b\,x^{n}\right)+1\right)x^{-n}}{2n}$	103

```
[In] int(x^(-n-1)*sin(a+b*x^n)^2,x,method=_RETURNVERBOSE)
[Out] -1/2/n/(x^n)-1/n*b*(-1/2*cos(2*a+2*b*x^n)/b/(x^n)-Si(2*b*x^n)*cos(2*a)-Ci(2
*b*x^n)*sin(2*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int x^{-1-n}\sin^2(a+bx^n)\,dx = \frac{bx^n \operatorname{Ci}(2bx^n)\sin(2a) + bx^n\cos(2a)\operatorname{Si}(2bx^n) + \cos(bx^n+a)^2 - 1}{nx^n}$$

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^2,x, algorithm="fricas")
[Out] (b*x^n*cos_integral(2*b*x^n)*sin(2*a) + b*x^n*cos(2*a)*sin_integral(2*b*x^n
) + cos(b*x^n + a)^2 - 1)/(n*x^n)
```

Sympy [F]

$$\int x^{-1-n}\sin^2(a+bx^n)\,dx = \int x^{-n-1}\sin^2(a+bx^n)\,dx$$

```
[In] integrate(x**(-1-n)*sin(a+b*x**n)**2,x)
[Out] Integral(x**(-n - 1)*sin(a + b*x**n)**2, x)
```

Maxima [F]

$$\int x^{-1-n}\sin^2(a+bx^n)\,dx = \int x^{-n-1}\sin(bx^n+a)^2\,dx$$

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^2,x, algorithm="maxima")
[Out] -1/2*(n*x^n*integrate(cos(2*b*x^n + 2*a)/(x*x^n), x) + 1)/(n*x^n)
```

Giac [F]

$$\int x^{-1-n}\sin^2(a+bx^n)\,dx = \int x^{-n-1}\sin(bx^n+a)^2\,dx$$

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x^(-n - 1)*sin(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n} \sin^2(a + bx^n) \, dx = \int \frac{\sin(a + b\,x^n)^2}{x^{n+1}} \, dx$$

```
[In] int(sin(a + b*x^n)^2/x^(n + 1),x)
[Out] int(sin(a + b*x^n)^2/x^(n + 1), x)
```

3.149 $\int x^{-1-n}\sin^3(a+bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int x^{-1-n}\sin^3(a+bx^n)\,dx=\frac{3b\cos(a)\,\text{CosIntegral}(bx^n)}{4n}-\frac{3b\cos(3a)\,\text{CosIntegral}(3bx^n)}{4n}-\frac{3x^{-n}\sin(a+bx^n)}{4n}+\frac{x^{-n}\sin(3(a+bx^n))}{4n}-\frac{3b\sin(a)\text{Si}(bx^n)}{4n}+\frac{3b\sin(3a)\text{Si}(3bx^n)}{4n}$$

```
[Out] 3/4*b*Ci(b*x^n)*cos(a)/n-3/4*b*Ci(3*b*x^n)*cos(3*a)/n-3/4*b*Si(b*x^n)*sin(a
)/n+3/4*b*Si(3*b*x^n)*sin(3*a)/n-3/4*sin(a+b*x^n)/n/(x^n)+1/4*sin(3*a+3*b*x
^n)/n/(x^n)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3506, 3460, 3378, 3384, 3380, 3383}

$$\int x^{-1-n}\sin^3(a+bx^n)\,dx=\frac{3b\cos(a)\,\text{CosIntegral}(bx^n)}{4n}-\frac{3b\cos(3a)\,\text{CosIntegral}(3bx^n)}{4n}-\frac{3b\sin(a)\text{Si}(bx^n)}{4n}+\frac{3b\sin(3a)\text{Si}(3bx^n)}{4n}-\frac{3x^{-n}\sin(a+bx^n)}{4n}+\frac{x^{-n}\sin(3(a+bx^n))}{4n}$$

```
[In] Int[x^(-1 - n)*Sin[a + b*x^n]^3,x]
```

```
[Out] (3*b*Cos[a]*CosIntegral[b*x^n])/(4*n) - (3*b*Cos[3*a]*CosIntegral[3*b*x^n])
/(4*n) - (3*Sin[a + b*x^n])/(4*n*x^n) + Sin[3*(a + b*x^n)]/(4*n*x^n) - (3*b
```

```
*Sin[a]*SinIntegral[b*x^n])/(4*n) + (3*b*Sin[3*a]*SinIntegral[3*b*x^n])/(4*
n)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{3}{4}x^{-1-n}\sin\left(a+bx^n\right) - \frac{1}{4}x^{-1-n}\sin\left(3a+3bx^n\right)\right)\,dx$$

$$= -\left(\frac{1}{4}\int x^{-1-n}\sin\left(3a+3bx^n\right)\,dx\right)+\frac{3}{4}\int x^{-1-n}\sin\left(a+bx^n\right)\,dx$$

$$= -\frac{\text{Subst}\left(\int\frac{\sin(3a+3bx)}{x^2}\,dx,x,x^n\right)}{4n}+\frac{3\text{Subst}\left(\int\frac{\sin(a+bx)}{x^2}\,dx,x,x^n\right)}{4n}$$

$$= -\frac{3x^{-n}\sin\left(a+bx^n\right)}{4n}+\frac{x^{-n}\sin\left(3(a+bx^n)\right)}{4n} + \frac{(3b)\text{Subst}\left(\int\frac{\cos(a+bx)}{x}\,dx,x,x^n\right)}{4n}-\frac{(3b)\text{Subst}\left(\int\frac{\cos(3a+3bx)}{x}\,dx,x,x^n\right)}{4n}$$

$$= -\frac{3x^{-n}\sin\left(a+bx^n\right)}{4n}+\frac{x^{-n}\sin\left(3(a+bx^n)\right)}{4n} + \frac{(3b\cos(a))\text{Subst}\left(\int\frac{\cos(bx)}{x}\,dx,x,x^n\right)}{4n}-\frac{(3b\cos(3a))\text{Subst}\left(\int\frac{\cos(3bx)}{x}\,dx,x,x^n\right)}{4n} - \frac{(3b\sin(a))\text{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,x^n\right)}{4n}+\frac{(3b\sin(3a))\text{Subst}\left(\int\frac{\sin(3bx)}{x}\,dx,x,x^n\right)}{4n}$$

$$= \frac{3b\cos(a)\,\text{CosIntegral}\left(bx^n\right)}{4n}-\frac{3b\cos(3a)\,\text{CosIntegral}\left(3bx^n\right)}{4n}-\frac{3x^{-n}\sin\left(a+bx^n\right)}{4n} + \frac{x^{-n}\sin\left(3(a+bx^n)\right)}{4n}-\frac{3b\sin(a)\text{Si}(bx^n)}{4n}+\frac{3b\sin(3a)\text{Si}(3bx^n)}{4n}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int x^{-1-n}\sin^3\left(a+bx^n\right)\,dx = \frac{x^{-n}(3bx^n\cos(a)\,\text{CosIntegral}\left(bx^n\right)-3bx^n\cos(3a)\,\text{CosIntegral}\left(3bx^n\right)-3\sin\left(a+bx^n\right)+\sin\left(3(a+bx^n)\right)-}{4n}$$

```
[In] Integrate[x^(-1 - n)*Sin[a + b*x^n]^3,x]
```

```
[Out] (3*b*x^n*Cos[a]*CosIntegral[b*x^n] - 3*b*x^n*Cos[3*a]*CosIntegral[3*b*x^n]
- 3*Sin[a + b*x^n] + Sin[3*(a + b*x^n)] - 3*b*x^n*Sin[a]*SinIntegral[b*x^n]
 + 3*b*x^n*Sin[3*a]*SinIntegral[3*b*x^n])/(4*n*x^n)
```

Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

method	result
default	$\frac{3b\left(-\frac{\sin(a+b\,x^{n})x^{-n}}{b}-\mathrm{Si}(b\,x^{n})\sin(a)+\mathrm{Ci}(b\,x^{n})\cos(a)\right)}{4n}-\frac{3b\left(-\frac{\sin(3a+3b\,x^{n})x^{-n}}{3b}-\mathrm{Si}(3b\,x^{n})\sin(3a)+\mathrm{Ci}(3b\,x^{n})\cos(3a)\right)}{4n}$
risch	$-\frac{(-3ib\,\mathrm{e}^{-ia}\pi\ \mathrm{csgn}(b\,x^{n})x^{n}+3ib\,\mathrm{e}^{-3ia}\pi\ \mathrm{csgn}(b\,x^{n})x^{n}+6ib\,\mathrm{e}^{-ia}\ \mathrm{Si}(b\,x^{n})x^{n}-6ib\,\mathrm{e}^{-3ia}\ \mathrm{Si}(3b\,x^{n})x^{n}+3b\,\mathrm{e}^{-ia}\ \mathrm{Ei}_{1}(-ib\,x^{n})x^{n}+3b\,\mathrm{e}^{ia}\ \mathrm{I}}{8n}$

```
[In] int(x^(-n-1)*sin(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/n*b*(-sin(a+b*x^n)/b/(x^n)-Si(b*x^n)*sin(a)+Ci(b*x^n)*cos(a))-3/4/n*b*(
-1/3*sin(3*a+3*b*x^n)/b/(x^n)-Si(3*b*x^n)*sin(3*a)+Ci(3*b*x^n)*cos(3*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int x^{-1-n}\sin^3\left(a+bx^n\right)\,dx =$$
$$-\frac{3\,bx^n\cos\left(3\,a\right)\mathrm{Ci}\left(3\,bx^n\right)-3\,bx^n\cos\left(a\right)\mathrm{Ci}\left(bx^n\right)-3\,bx^n\sin\left(3\,a\right)\mathrm{Si}\left(3\,bx^n\right)+3\,bx^n\sin\left(a\right)\mathrm{Si}\left(bx^n\right)-4\,(\mathrm{c}}{4\,nx^n}$$

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*b*x^n*cos(3*a)*cos_integral(3*b*x^n) - 3*b*x^n*cos(a)*cos_integral(
b*x^n) - 3*b*x^n*sin(3*a)*sin_integral(3*b*x^n) + 3*b*x^n*sin(a)*sin_integr
al(b*x^n) - 4*(cos(b*x^n + a)^2 - 1)*sin(b*x^n + a))/(n*x^n)
```

Sympy [F]

$$\int x^{-1-n}\sin^3\left(a+bx^n\right)\,dx=\int x^{-n-1}\sin^3\left(a+bx^n\right)dx$$

```
[In] integrate(x**(-1-n)*sin(a+b*x**n)**3,x)
```

```
[Out] Integral(x**(-n - 1)*sin(a + b*x**n)**3, x)
```

Maxima [F]

$$\int x^{-1-n}\sin^3(a+bx^n)\,dx=\int x^{-n-1}\sin(bx^n+a)^3\,dx$$

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate(x^(-n - 1)*sin(b*x^n + a)^3, x)
```

Giac [F]

$$\int x^{-1-n}\sin^3(a+bx^n)\,dx=\int x^{-n-1}\sin(bx^n+a)^3\,dx$$

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(x^(-n - 1)*sin(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\sin^3(a+bx^n)\,dx=\int\frac{\sin(a+b\,x^n)^3}{x^{n+1}}\,dx$$

```
[In] int(sin(a + b*x^n)^3/x^(n + 1),x)
[Out] int(sin(a + b*x^n)^3/x^(n + 1), x)
```

3.150 $\int x^{-1-2n} \sin(a+bx^n)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int x^{-1-2n} \sin(a+bx^n)\,dx = -\frac{bx^{-n}\cos(a+bx^n)}{2n} - \frac{b^2\,\text{CosIntegral}(bx^n)\sin(a)}{2n} - \frac{x^{-2n}\sin(a+bx^n)}{2n} - \frac{b^2\cos(a)\text{Si}(bx^n)}{2n}$$

```
[Out] -1/2*b*cos(a+b*x^n)/n/(x^n)-1/2*b^2*cos(a)*Si(b*x^n)/n-1/2*b^2*Ci(b*x^n)*si
n(a)/n-1/2*sin(a+b*x^n)/n/(x^(2*n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3460, 3378, 3384, 3380, 3383}

$$\int x^{-1-2n} \sin(a+bx^n)\,dx = -\frac{b^2\sin(a)\,\text{CosIntegral}(bx^n)}{2n} - \frac{b^2\cos(a)\text{Si}(bx^n)}{2n} - \frac{x^{-2n}\sin(a+bx^n)}{2n} - \frac{bx^{-n}\cos(a+bx^n)}{2n}$$

```
[In] Int[x^(-1 - 2*n)*Sin[a + b*x^n],x]
```

```
[Out] -1/2*(b*Cos[a + b*x^n])/(n*x^n) - (b^2*CosIntegral[b*x^n]*Sin[a])/(2*n) - S
in[a + b*x^n]/(2*n*x^(2*n)) - (b^2*Cos[a]*SinIntegral[b*x^n])/(2*n)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

]

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, x^n\right)}{n}\\
&= -\frac{x^{-2n}\sin(a+bx^n)}{2n} + \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, x^n\right)}{2n}\\
&= -\frac{bx^{-n}\cos(a+bx^n)}{2n} - \frac{x^{-2n}\sin(a+bx^n)}{2n} - \frac{b^2\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, x^n\right)}{2n}\\
&= -\frac{bx^{-n}\cos(a+bx^n)}{2n} - \frac{x^{-2n}\sin(a+bx^n)}{2n}\\
&\quad - \frac{(b^2\cos(a))\,\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, x^n\right)}{2n} - \frac{(b^2\sin(a))\,\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, x^n\right)}{2n}\\
&= -\frac{bx^{-n}\cos(a+bx^n)}{2n} - \frac{b^2\,\text{CosIntegral}(bx^n)\sin(a)}{2n} - \frac{x^{-2n}\sin(a+bx^n)}{2n} - \frac{b^2\cos(a)\text{Si}(bx^n)}{2n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int x^{-1-2n}\sin\left(a+bx^n\right)\,dx = -\frac{x^{-2n}\left(bx^n\cos\left(a+bx^n\right)+b^2x^{2n}\,\text{CosIntegral}\left(bx^n\right)\sin(a)+\sin\left(a+bx^n\right)+b^2x^{2n}\cos(a)\text{Si}(bx^n)\right)}{2n}$$

```
[In] Integrate[x^(-1 - 2*n)*Sin[a + b*x^n],x]
```

```
[Out] -1/2*(b*x^n*Cos[a + b*x^n] + b^2*x^(2*n)*CosIntegral[b*x^n]*Sin[a] + Sin[a
+ b*x^n] + b^2*x^(2*n)*Cos[a]*SinIntegral[b*x^n])/(n*x^(2*n))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result
default	$\frac{b^2\left(-\frac{\sin(a+b\,x^n)x^{-2n}}{2b^2}-\frac{\cos(a+b\,x^n)x^{-n}}{2b}-\frac{\text{Si}(b\,x^n)\cos(a)}{2}-\frac{\text{Ci}(b\,x^n)\sin(a)}{2}\right)}{n}$
risch	$-\frac{\left(-b^2\mathrm{e}^{-ia}\pi\ \text{csgn}(b\,x^n)x^{2n}-ib^2\mathrm{e}^{-ia}\,\text{Ei}_1(-ib\,x^n)x^{2n}+ib^2\mathrm{e}^{ia}\,\text{Ei}_1(-ib\,x^n)x^{2n}+2b^2\mathrm{e}^{-ia}\,\text{Si}(b\,x^n)x^{2n}+2x^n\cos(a+b\,x^n)b+2\sin(a+b\,x\right.}{4n}$
meijerg	$\frac{b^2\sqrt{\pi}\left(-\frac{x^{2\left(\frac{-1-2n}{2n}+\frac{1}{2n}\right)n}2^{-\frac{-1-2n}{n}-\frac{1}{n}}}{\sqrt{\pi}\,b^2}+\frac{(-1)^{-\frac{-1-2n}{2n}-\frac{1}{2n}}\left(-\Psi\left(1-\frac{-1-2n}{2n}-\frac{1}{2n}\right)-\Psi\left(\frac{1}{2}-\frac{-1-2n}{2n}-\frac{1}{2n}\right)+2n\ln(x)-2\ln(2)+\ln\left(b^2\right)\right)\sqrt{2}\,2}{2\sqrt{\pi}\,\Gamma\left(-\frac{-1-2n}{n}-\frac{1}{n}\right)}\right.}{}$

```
[In] int(x^(-1-2*n)*sin(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*b^2*(-1/2*sin(a+b*x^n)/b^2/(x^n)^2-1/2*cos(a+b*x^n)/b/(x^n)-1/2*Si(b*x^
n)*cos(a)-1/2*Ci(b*x^n)*sin(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int x^{-1-2n}\sin\left(a+bx^n\right)\,dx = -\frac{b^2x^{2\,n}\,\text{Ci}\left(bx^n\right)\sin\left(a\right)+b^2x^{2\,n}\cos\left(a\right)\text{Si}\left(bx^n\right)+bx^n\cos\left(bx^n+a\right)+\sin\left(bx^n+a\right)}{2\,nx^{2\,n}}$$

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^(2*n)*cos_integral(b*x^n)*sin(a) + b^2*x^(2*n)*cos(a)*sin_integ
ral(b*x^n) + b*x^n*cos(b*x^n + a) + sin(b*x^n + a))/(n*x^(2*n))
```

Sympy [F]

$$\int x^{-1-2n}\sin\left(a+bx^{n}\right)\,dx=\int x^{-2n-1}\sin\left(a+bx^{n}\right)dx$$

```
[In] integrate(x**(-1-2*n)*sin(a+b*x**n),x)
[Out] Integral(x**(-2*n - 1)*sin(a + b*x**n), x)
```

Maxima [F]

$$\int x^{-1-2n}\sin\left(a+bx^{n}\right)\,dx=\int x^{-2\,n-1}\sin\left(bx^{n}+a\right)\,dx$$

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n),x, algorithm="maxima")
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a), x)
```

Giac [F]

$$\int x^{-1-2n}\sin\left(a+bx^{n}\right)\,dx=\int x^{-2\,n-1}\sin\left(bx^{n}+a\right)\,dx$$

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\sin\left(a+bx^{n}\right)\,dx=\int\frac{\sin\left(a+b\,x^{n}\right)}{x^{2\,n+1}}\,dx$$

```
[In] int(sin(a + b*x^n)/x^(2*n + 1),x)
[Out] int(sin(a + b*x^n)/x^(2*n + 1), x)
```

3.151 $\int x^{-1-2n} \sin^2(a+bx^n) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int x^{-1-2n} \sin^2(a+bx^n) \, dx = -\frac{x^{-2n}}{4n} + \frac{x^{-2n}\cos(2(a+bx^n))}{4n} + \frac{b^2\cos(2a)\operatorname{CosIntegral}(2bx^n)}{n} - \frac{bx^{-n}\sin(2(a+bx^n))}{2n} - \frac{b^2\sin(2a)\operatorname{Si}(2bx^n)}{n}$$

```
[Out] -1/4/n/(x^(2*n))+b^2*Ci(2*b*x^n)*cos(2*a)/n+1/4*cos(2*a+2*b*x^n)/n/(x^(2*n)
)-b^2*Si(2*b*x^n)*sin(2*a)/n-1/2*b*sin(2*a+2*b*x^n)/n/(x^n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3506, 3461, 3378, 3384, 3380, 3383}

$$\int x^{-1-2n} \sin^2(a+bx^n) \, dx = \frac{b^2\cos(2a)\operatorname{CosIntegral}(2bx^n)}{n} - \frac{b^2\sin(2a)\operatorname{Si}(2bx^n)}{n} - \frac{bx^{-n}\sin(2(a+bx^n))}{2n} + \frac{x^{-2n}\cos(2(a+bx^n))}{4n} - \frac{x^{-2n}}{4n}$$

```
[In] Int[x^(-1 - 2*n)*Sin[a + b*x^n]^2,x]
```

```
[Out] -1/4*1/(n*x^(2*n)) + Cos[2*(a + b*x^n)]/(4*n*x^(2*n)) + (b^2*Cos[2*a]*CosIn
tegral[2*b*x^n])/n - (b*Sin[2*(a + b*x^n)])/(2*n*x^n) - (b^2*Sin[2*a]*SinIn
tegral[2*b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}x^{-1-2n} - \frac{1}{2}x^{-1-2n}\cos\left(2a+2bx^n\right)\right) dx \\
&= -\frac{x^{-2n}}{4n} - \frac{1}{2}\int x^{-1-2n}\cos\left(2a+2bx^n\right) dx \\
&= -\frac{x^{-2n}}{4n} - \frac{\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x^3}\,dx, x, x^n\right)}{2n}
\end{aligned}$$

$$= -\frac{x^{-2n}}{4n} + \frac{x^{-2n}\cos(2(a+bx^n))}{4n} + \frac{b\text{Subst}\left(\int \frac{\sin(2a+2bx)}{x^2}\,dx, x, x^n\right)}{2n}$$

$$= -\frac{x^{-2n}}{4n} + \frac{x^{-2n}\cos(2(a+bx^n))}{4n} - \frac{bx^{-n}\sin(2(a+bx^n))}{2n} + \frac{b^2\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-2n}}{4n} + \frac{x^{-2n}\cos(2(a+bx^n))}{4n} - \frac{bx^{-n}\sin(2(a+bx^n))}{2n} + \frac{(b^2\cos(2a))\,\text{Subst}\left(\int \frac{\cos(2bx)}{x}\,dx, x, x^n\right)}{n} - \frac{(b^2\sin(2a))\,\text{Subst}\left(\int \frac{\sin(2bx)}{x}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-2n}}{4n} + \frac{x^{-2n}\cos(2(a+bx^n))}{4n} + \frac{b^2\cos(2a)\,\text{CosIntegral}(2bx^n)}{n} - \frac{bx^{-n}\sin(2(a+bx^n))}{2n} - \frac{b^2\sin(2a)\text{Si}(2bx^n)}{n}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int x^{-1-2n}\sin^2(a+bx^n)\,dx = \frac{x^{-2n}(-1+\cos(2(a+bx^n))+4b^2x^{2n}\cos(2a)\,\text{CosIntegral}(2bx^n)-2bx^n\sin(2(a+bx^n))-4b^2x^{2n}\sin(2a)\text{S}}{4n}$$

```
[In] Integrate[x^(-1 - 2*n)*Sin[a + b*x^n]^2,x]
```

```
[Out] (-1 + Cos[2*(a + b*x^n)] + 4*b^2*x^(2*n)*Cos[2*a]*CosIntegral[2*b*x^n] - 2*
b*x^n*Sin[2*(a + b*x^n)] - 4*b^2*x^(2*n)*Sin[2*a]*SinIntegral[2*b*x^n])/(4*
n*x^(2*n))
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

method	result
default	$-\frac{x^{-2n}}{4n}-\frac{2b^2\left(-\frac{\cos(2a+2b\,x^n)x^{-2n}}{8b^2}+\frac{\sin(2a+2b\,x^n)x^{-n}}{4b}+\frac{\text{Si}(2b\,x^n)\sin(2a)}{2}-\frac{\text{Ci}(2b\,x^n)\cos(2a)}{2}\right)}{n}$
risch	$-\frac{(-2ib^2e^{-2ia}\pi\ \text{csgn}(b\,x^n)x^{2n}+4ib^2e^{-2ia}\,\text{Si}(2b\,x^n)x^{2n}+2b^2e^{2ia}\,\text{Ei}_1(-2ib\,x^n)x^{2n}+2b^2e^{-2ia}\,\text{Ei}_1(-2ib\,x^n)x^{2n}+2b\sin(2a+2b\,x^n)x^{}}{4n}$

```
[In] int(x^(-1-2*n)*sin(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(x^n)^2/n-2/n*b^2*(-1/8*cos(2*a+2*b*x^n)/b^2/(x^n)^2+1/4*sin(2*a+2*b*x
^n)/b/(x^n)+1/2*Si(2*b*x^n)*sin(2*a)-1/2*Ci(2*b*x^n)*cos(2*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int x^{-1-2n}\sin^2\left(a+bx^n\right)\,dx$$
$$=\frac{2\,b^2x^{2\,n}\cos\left(2\,a\right)\operatorname{Ci}\left(2\,bx^n\right)-2\,b^2x^{2\,n}\sin\left(2\,a\right)\operatorname{Si}\left(2\,bx^n\right)-2\,bx^n\cos\left(bx^n+a\right)\sin\left(bx^n+a\right)+\cos\left(bx^n+a\right)^2}{2\,nx^{2\,n}}$$

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^(2*n)*cos(2*a)*cos_integral(2*b*x^n) - 2*b^2*x^(2*n)*sin(2*a)*
sin_integral(2*b*x^n) - 2*b*x^n*cos(b*x^n + a)*sin(b*x^n + a) + cos(b*x^n +
 a)^2 - 1)/(n*x^(2*n))
```

Sympy [F]

$$\int x^{-1-2n}\sin^2\left(a+bx^n\right)\,dx=\int x^{-2n-1}\sin^2\left(a+bx^n\right)dx$$

```
[In] integrate(x**(-1-2*n)*sin(a+b*x**n)**2,x)
```

```
[Out] Integral(x**(-2*n - 1)*sin(a + b*x**n)**2, x)
```

Maxima [F]

$$\int x^{-1-2n}\sin^2\left(a+bx^n\right)\,dx=\int x^{-2\,n-1}\sin\left(bx^n+a\right)^2\,dx$$

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*n*x^(2*n)*integrate(cos(2*b*x^n + 2*a)/(x*x^(2*n)), x) + 1)/(n*x^(2
*n))
```

Giac [F]

$$\int x^{-1-2n}\sin^2(a+bx^n)\,dx = \int x^{-2\,n-1}\sin\left(bx^n+a\right)^2\,dx$$

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\sin^2(a+bx^n)\,dx = \int \frac{\sin\left(a+b\,x^n\right)^2}{x^{2\,n+1}}\,dx$$

```
[In] int(sin(a + b*x^n)^2/x^(2*n + 1),x)
[Out] int(sin(a + b*x^n)^2/x^(2*n + 1), x)
```

3.152 $\int x^{-1-2n}\sin^3(a+bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 165

$$\int x^{-1-2n}\sin^3(a+bx^n)\,dx = -\frac{3bx^{-n}\cos(a+bx^n)}{8n}+\frac{3bx^{-n}\cos(3(a+bx^n))}{8n} - \frac{3b^2\,\text{CosIntegral}(bx^n)\sin(a)}{8n}+\frac{9b^2\,\text{CosIntegral}(3bx^n)\sin(3a)}{8n} - \frac{3x^{-2n}\sin(a+bx^n)}{8n}+\frac{x^{-2n}\sin(3(a+bx^n))}{8n} - \frac{3b^2\cos(a)\text{Si}(bx^n)}{8n}+\frac{9b^2\cos(3a)\text{Si}(3bx^n)}{8n}$$

```
[Out] -3/8*b*cos(a+b*x^n)/n/(x^n)+3/8*b*cos(3*a+3*b*x^n)/n/(x^n)-3/8*b^2*cos(a)*S
i(b*x^n)/n+9/8*b^2*cos(3*a)*Si(3*b*x^n)/n-3/8*b^2*Ci(b*x^n)*sin(a)/n+9/8*b^
2*Ci(3*b*x^n)*sin(3*a)/n-3/8*sin(a+b*x^n)/n/(x^(2*n))+1/8*sin(3*a+3*b*x^n)/
n/(x^(2*n))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3506, 3460, 3378, 3384, 3380, 3383}

$$\int x^{-1-2n}\sin^3(a+bx^n)\,dx = -\frac{3b^2\sin(a)\,\text{CosIntegral}(bx^n)}{8n}+\frac{9b^2\sin(3a)\,\text{CosIntegral}(3bx^n)}{8n} - \frac{3b^2\cos(a)\text{Si}(bx^n)}{8n}+\frac{9b^2\cos(3a)\text{Si}(3bx^n)}{8n} - \frac{3x^{-2n}\sin(a+bx^n)}{8n}+\frac{x^{-2n}\sin(3(a+bx^n))}{8n} - \frac{3bx^{-n}\cos(a+bx^n)}{8n}+\frac{3bx^{-n}\cos(3(a+bx^n))}{8n}$$

```
[In] Int[x^(-1 - 2*n)*Sin[a + b*x^n]^3,x]
```

```
[Out] (-3*b*Cos[a + b*x^n])/(8*n*x^n) + (3*b*Cos[3*(a + b*x^n)])/(8*n*x^n) - (3*b
^2*CosIntegral[b*x^n]*Sin[a])/(8*n) + (9*b^2*CosIntegral[3*b*x^n]*Sin[3*a])
/(8*n) - (3*Sin[a + b*x^n])/(8*n*x^(2*n)) + Sin[3*(a + b*x^n)]/(8*n*x^(2*n)
) - (3*b^2*Cos[a]*SinIntegral[b*x^n])/(8*n) + (9*b^2*Cos[3*a]*SinIntegral[3
*b*x^n])/(8*n)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{3}{4}x^{-1-2n}\sin\left(a+bx^n\right) - \frac{1}{4}x^{-1-2n}\sin\left(3a+3bx^n\right)\right) dx$$

$$= -\left(\frac{1}{4}\int x^{-1-2n}\sin\left(3a+3bx^n\right) dx\right) + \frac{3}{4}\int x^{-1-2n}\sin\left(a+bx^n\right) dx$$

$$= -\frac{\text{Subst}\left(\int \frac{\sin(3a+3bx)}{x^3} dx, x, x^n\right)}{4n} + \frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3} dx, x, x^n\right)}{4n}$$

$$= -\frac{3x^{-2n}\sin\left(a+bx^n\right)}{8n} + \frac{x^{-2n}\sin\left(3(a+bx^n)\right)}{8n} + \frac{(3b)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2} dx, x, x^n\right)}{8n} - \frac{(3b)\text{Subst}\left(\int \frac{\cos(3a+3bx)}{x^2} dx, x, x^n\right)}{8n}$$

$$= -\frac{3bx^{-n}\cos\left(a+bx^n\right)}{8n} + \frac{3bx^{-n}\cos\left(3(a+bx^n)\right)}{8n} - \frac{3x^{-2n}\sin\left(a+bx^n\right)}{8n} + \frac{x^{-2n}\sin\left(3(a+bx^n)\right)}{8n} - \frac{(3b^2)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x} dx, x, x^n\right)}{8n} + \frac{(9b^2)\,\text{Subst}\left(\int \frac{\sin(3a+3bx)}{x} dx, x, x^n\right)}{8n}$$

$$= -\frac{3bx^{-n}\cos\left(a+bx^n\right)}{8n} + \frac{3bx^{-n}\cos\left(3(a+bx^n)\right)}{8n} - \frac{3x^{-2n}\sin\left(a+bx^n\right)}{8n} + \frac{x^{-2n}\sin\left(3(a+bx^n)\right)}{8n} - \frac{(3b^2\cos(a))\,\text{Subst}\left(\int \frac{\sin(bx)}{x} dx, x, x^n\right)}{8n} + \frac{(9b^2\cos(3a))\,\text{Subst}\left(\int \frac{\sin(3bx)}{x} dx, x, x^n\right)}{8n} - \frac{(3b^2\sin(a))\,\text{Subst}\left(\int \frac{\cos(bx)}{x} dx, x, x^n\right)}{8n} + \frac{(9b^2\sin(3a))\,\text{Subst}\left(\int \frac{\cos(3bx)}{x} dx, x, x^n\right)}{8n}$$

$$= -\frac{3bx^{-n}\cos\left(a+bx^n\right)}{8n} + \frac{3bx^{-n}\cos\left(3(a+bx^n)\right)}{8n} - \frac{3b^2\,\text{CosIntegral}\left(bx^n\right)\sin(a)}{8n} + \frac{9b^2\,\text{CosIntegral}\left(3bx^n\right)\sin(3a)}{8n} - \frac{3x^{-2n}\sin\left(a+bx^n\right)}{8n} + \frac{x^{-2n}\sin\left(3(a+bx^n)\right)}{8n} - \frac{3b^2\cos(a)\text{Si}(bx^n)}{8n} + \frac{9b^2\cos(3a)\text{Si}(3bx^n)}{8n}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.85

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx$$
$$=\frac{x^{-2n}(-3bx^n\cos\left(a+bx^n\right)+3bx^n\cos\left(3(a+bx^n)\right)-3b^2x^{2n}\,\text{CosIntegral}\left(bx^n\right)\sin(a)+9b^2x^{2n}\,\text{CosIntegral}}{8n}$$

```
[In] Integrate[x^(-1 - 2*n)*Sin[a + b*x^n]^3,x]
```

```
[Out] (-3*b*x^n*Cos[a + b*x^n] + 3*b*x^n*Cos[3*(a + b*x^n)] - 3*b^2*x^(2*n)*CosIn
tegral[b*x^n]*Sin[a] + 9*b^2*x^(2*n)*CosIntegral[3*b*x^n]*Sin[3*a] - 3*Sin[
a + b*x^n] + Sin[3*(a + b*x^n)] - 3*b^2*x^(2*n)*Cos[a]*SinIntegral[b*x^n] +
 9*b^2*x^(2*n)*Cos[3*a]*SinIntegral[3*b*x^n])/(8*n*x^(2*n))
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

method	result
default	$\frac{3b^2\left(-\frac{\sin(a+b\,x^n)x^{-2n}}{2b^2}-\frac{\cos(a+b\,x^n)x^{-n}}{2b}-\frac{\mathrm{Si}(b\,x^n)\cos(a)}{2}-\frac{\mathrm{Ci}(b\,x^n)\sin(a)}{2}\right)}{4n}-\frac{9b^2\left(-\frac{\sin(3a+3b\,x^n)x^{-2n}}{18b^2}-\frac{\cos(3a+3b\,x^n)x^{-n}}{6b}-\mathrm{Si}(3b\right.}{4n}$
risch	$-\frac{(-9ib^2\mathrm{e}^{3ia}\,\mathrm{Ei}_1(-3ib\,x^n)x^{2n}+9ib^2\mathrm{e}^{-3ia}\,\mathrm{Ei}_1(-3ib\,x^n)x^{2n}+3ib^2\mathrm{e}^{ia}\,\mathrm{Ei}_1(-ib\,x^n)x^{2n}-3ib^2\mathrm{e}^{-ia}\,\mathrm{Ei}_1(-ib\,x^n)x^{2n}+9b^2\mathrm{e}^{-3ia}\pi\,\mathrm{csgn}(b}{}$

```
[In] int(x^(-1-2*n)*sin(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/n*b^2*(-1/2*sin(a+b*x^n)/b^2/(x^n)^2-1/2*cos(a+b*x^n)/b/(x^n)-1/2*Si(b*
x^n)*cos(a)-1/2*Ci(b*x^n)*sin(a))-9/4/n*b^2*(-1/18*sin(3*a+3*b*x^n)/b^2/(x^
n)^2-1/6*cos(3*a+3*b*x^n)/b/(x^n)-1/2*Si(3*b*x^n)*cos(3*a)-1/2*Ci(3*b*x^n)*
sin(3*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.87

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx$$
$$=\frac{12\,bx^n\cos\left(bx^n+a\right)^3+9\,b^2x^{2\,n}\,\mathrm{Ci}\left(3\,bx^n\right)\sin\left(3\,a\right)-3\,b^2x^{2\,n}\,\mathrm{Ci}\left(bx^n\right)\sin\left(a\right)+9\,b^2x^{2\,n}\cos\left(3\,a\right)\mathrm{Si}\left(3\,bx^n\right)-}{8\,nx^{2\,n}}$$

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(12*b*x^n*cos(b*x^n + a)^3 + 9*b^2*x^(2*n)*cos_integral(3*b*x^n)*sin(3*
a) - 3*b^2*x^(2*n)*cos_integral(b*x^n)*sin(a) + 9*b^2*x^(2*n)*cos(3*a)*sin_
integral(3*b*x^n) - 3*b^2*x^(2*n)*cos(a)*sin_integral(b*x^n) - 12*b*x^n*cos
(b*x^n + a) + 4*(cos(b*x^n + a)^2 - 1)*sin(b*x^n + a))/(n*x^(2*n))
```

Sympy [F]

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx=\int x^{-2n-1}\sin^3\left(a+bx^n\right)dx$$

```
[In] integrate(x**(-1-2*n)*sin(a+b*x**n)**3,x)
[Out] Integral(x**(-2*n - 1)*sin(a + b*x**n)**3, x)
```

Maxima [F]

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx=\int x^{-2n-1}\sin\left(bx^n+a\right)^3\,dx$$

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a)^3, x)
```

Giac [F]

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx=\int x^{-2n-1}\sin\left(bx^n+a\right)^3\,dx$$

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx=\int \frac{\sin\left(a+b\,x^n\right)^3}{x^{2\,n+1}}\,dx$$

```
[In] int(sin(a + b*x^n)^3/x^(2*n + 1),x)
[Out] int(sin(a + b*x^n)^3/x^(2*n + 1), x)
```

3.153 $\int (e+fx)^3 \sin\left(b(c+dx)^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 223

$$\begin{aligned}\int (e+fx)^3 \sin\left(b(c+dx)^2\right)\,dx =& -\frac{3f(de-cf)^2\cos\left(b(c+dx)^2\right)}{2bd^4} \\ &-\frac{3f^2(de-cf)(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^4} \\ &-\frac{f^3(c+dx)^2\cos\left(b(c+dx)^2\right)}{2bd^4} \\ &+\frac{3f^2(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4} \\ &+\frac{(de-cf)^3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4} \\ &+\frac{f^3\sin\left(b(c+dx)^2\right)}{2b^2d^4}\end{aligned}$$

```
[Out] -3/2*f*(-c*f+d*e)^2*cos(b*(d*x+c)^2)/b/d^4-3/2*f^2*(-c*f+d*e)*(d*x+c)*cos(b
*(d*x+c)^2)/b/d^4-1/2*f^3*(d*x+c)^2*cos(b*(d*x+c)^2)/b/d^4+1/2*f^3*sin(b*(d
*x+c)^2)/b^2/d^4+3/4*f^2*(-c*f+d*e)*FresnelC((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/
2))*2^(1/2)*Pi^(1/2)/b^(3/2)/d^4+1/2*(-c*f+d*e)^3*FresnelS((d*x+c)*b^(1/2)*
2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^4/b^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3514, 3432, 3460, 2718, 3466, 3433, 3377, 2717}

$$\int (e+fx)^3 \sin \left(b(c+dx)^2\right) \, dx = \frac{3\sqrt{\frac{\pi}{2}} f^2 (de-cf) \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4} + \frac{f^3 \sin \left(b(c+dx)^2\right)}{2b^2d^4} - \frac{3f^2(c+dx)(de-cf)\cos \left(b(c+dx)^2\right)}{2bd^4} + \frac{\sqrt{\frac{\pi}{2}}(de-cf)^3 \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4} - \frac{3f(de-cf)^2\cos \left(b(c+dx)^2\right)}{2bd^4} - \frac{f^3(c+dx)^2\cos \left(b(c+dx)^2\right)}{2bd^4}$$

```
[In] Int[(e + f*x)^3*Sin[b*(c + d*x)^2],x]
```

```
[Out] (-3*f*(d*e - c*f)^2*Cos[b*(c + d*x)^2])/(2*b*d^4) - (3*f^2*(d*e - c*f)*(c +
 d*x)*Cos[b*(c + d*x)^2])/(2*b*d^4) - (f^3*(c + d*x)^2*Cos[b*(c + d*x)^2])/
(2*b*d^4) + (3*f^2*(d*e - c*f)*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c +
d*x)])/(2*b^(3/2)*d^4) + ((d*e - c*f)^3*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/
Pi]*(c + d*x)])/(Sqrt[b]*d^4) + (f^3*Sin[b*(c + d*x)^2])/(2*b^2*d^4)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$=\frac{\text{Subst}\left(\int \left(d^3e^3\left(1-\frac{cf(3d^2e^2-3cdef+c^2f^2)}{d^3e^3}\right)\sin\left(bx^2\right)+3d^2e^2f\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)x\sin\left(bx^2\right)+3def^2\left(1-\frac{cf}{de}\right)\right.}{d^4}$$

$$=\frac{f^3\text{Subst}\left(\int x^3\sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^4}+\frac{\left(3f^2(de-cf)\right)\text{Subst}\left(\int x^2\sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^4}+\frac{\left(3f(de-cf)^2\right)\text{Subst}\left(\int x\sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^4}+\frac{(de-cf)^3\text{Subst}\left(\int\sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^4}$$

$$= -\frac{3f^2(de-cf)(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^4} + \frac{(de-cf)^3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4} + \frac{f^3\text{Subst}\left(\int x\sin(bx)\,dx,x,(c+dx)^2\right)}{2d^4} + \frac{(3f^2(de-cf))\,\text{Subst}\left(\int\cos\left(bx^2\right)\,dx,x,c+dx\right)}{2bd^4} + \frac{(3f(de-cf)^2)\,\text{Subst}\left(\int\sin(bx)\,dx,x,(c+dx)^2\right)}{2d^4}$$

$$= -\frac{3f(de-cf)^2\cos\left(b(c+dx)^2\right)}{2bd^4} - \frac{3f^2(de-cf)(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^4} - \frac{f^3(c+dx)^2\cos\left(b(c+dx)^2\right)}{2bd^4} + \frac{3f^2(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4} + \frac{(de-cf)^3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4} + \frac{f^3\text{Subst}\left(\int\cos(bx)\,dx,x,(c+dx)^2\right)}{2bd^4}$$

$$= -\frac{3f(de-cf)^2\cos\left(b(c+dx)^2\right)}{2bd^4} - \frac{3f^2(de-cf)(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^4} - \frac{f^3(c+dx)^2\cos\left(b(c+dx)^2\right)}{2bd^4} + \frac{3f^2(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4} + \frac{(de-cf)^3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4} + \frac{f^3\sin\left(b(c+dx)^2\right)}{2b^2d^4}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int (e+fx)^3\sin\left(b(c+dx)^2\right)\,dx = \frac{-4bf(c^2f^2-cdf(3e+fx)+d^2(3e^2+3efx+f^2x^2))\cos\left(b(c+dx)^2\right)-6\sqrt{b}f^2(-de+cf)\sqrt{2\pi}\,\text{FresnelC}\left(\cdots\right.}{8b^2d^4}$$

```
[In] Integrate[(e + f*x)^3*Sin[b*(c + d*x)^2],x]
```

```
[Out] (-4*b*f*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Cos
[b*(c + d*x)^2] - 6*Sqrt[b]*f^2*(-(d*e) + c*f)*Sqrt[2*Pi]*FresnelC[Sqrt[b]*
Sqrt[2/Pi]*(c + d*x)] + 4*b^(3/2)*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelS[Sqrt[b]
*Sqrt[2/Pi]*(c + d*x)] + 4*f^3*Sin[b*(c + d*x)^2])/(8*b^2*d^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(194) = 388$.

Time = 1.14 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.63

method	result
default	$-\frac{f^3 x^2 \cos(d^2x^2b+2cdxb+c^2b)}{2b\,d^2} - \frac{f^3 c\left(-\frac{x\cos\left(d^2x^2b+2cdxb+c^2b\right)}{2b\,d^2} - \frac{c\left(-\frac{\cos\left(d^2x^2b+2cdxb+c^2b\right)}{2b\,d^2} - \frac{c\sqrt{2}\,\sqrt{\pi}\ \mathrm{S}\left(\frac{\sqrt{2}\left(b\,d^2x+cdb\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)}{2d\sqrt{b\,d^2}}\right)}{d} + \cdots\right)}{d}$
risch	$\frac{i\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)\sqrt{\pi}\,e^3}{4\sqrt{-ib}\,d} - \frac{if^3c^3\sqrt{\pi}\ \mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^4\sqrt{-ib}} + \frac{3f^3c\sqrt{\pi}\ \mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{8d^4b\sqrt{-ib}} - \frac{3ie^2fc\sqrt{\pi}\ \mathrm{erf}\left(-d\sqrt{-\cdots}\right.}{4d^2\sqrt{-ib}\cdots}$
parts	Expression too large to display

```
[In] int((f*x+e)^3*sin(b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f^3/b/d^2*x^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-f^3*c/d*(-1/2/b/d^2*x*cos
(b*d^2*x^2+2*b*c*d*x+b*c^2)-c/d*(-1/2/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-
1/2*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1
/2)*(b*d^2*x+b*c*d)))+1/4/b/d^2*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelC(2^(
1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d)))+f^3/b/d^2*(1/2/b/d^2*sin(b*d^
2*x^2+2*b*c*d*x+b*c^2)-1/2*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelC(2^(1
/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d)))-3/2*e*f^2/b/d^2*x*cos(b*d^2*x^
2+2*b*c*d*x+b*c^2)-3*e*f^2*c/d*(-1/2/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-1
/2*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/
2)*(b*d^2*x+b*c*d)))+3/4*e*f^2/b/d^2*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*Fresnel
C(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))-3/2*e^2*f/b/d^2*cos(b*d^2
*x^2+2*b*c*d*x+b*c^2)-3/2*e^2*f*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelS
(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))+1/2*2^(1/2)*Pi^(1/2)/(b*d^
2)^(1/2)*e^3*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.14

$$\int (e+fx)^3 \sin\left(b(c+dx)^2\right)\,dx$$

$$= \frac{2\,df^3\sin\left(bd^2x^2+2\,bcdx+bc^2\right)+3\,\sqrt{2}\pi(def^2-cf^3)\sqrt{\frac{bd^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)+2\,\sqrt{2}\pi(bd^3e^3-3\,bcd^2e^2f\cdots}{\cdots}$$

```
[In] integrate((f*x+e)^3*sin(b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*f^3*sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2) + 3*sqrt(2)*pi*(d*e*f^2 - c
*f^3)*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) + 2*sq
rt(2)*pi*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt(b
*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^3*f^3*x^2
 + 3*b*d^3*e^2*f - 3*b*c*d^2*e*f^2 + b*c^2*d*f^3 + (3*b*d^3*e*f^2 - b*c*d^2
*f^3)*x)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2))/(b^2*d^5)
```

Sympy [F]

$$\int (e+fx)^3 \sin\left(b(c+dx)^2\right)\,dx = \int (e+fx)^3 \sin\left(bc^2+2bcdx+bd^2x^2\right)dx$$

```
[In] integrate((f*x+e)**3*sin(b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**3*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 974, normalized size of antiderivative = 4.37

$$\int (e+fx)^3 \sin\left(b(c+dx)^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^3*sin(b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*sqrt(pi)*e^3*((I + 1)*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I - 1
)*erf((I*b*d*x + I*b*c)/sqrt(-I*b)))/(sqrt(b)*d) - 3/8*(2*d*x*(e^(I*b*d^2*x
^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - s
qrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*(-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b
*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqr
t(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*c + 2*c*(e^(I*b*d^2*x^2 + 2*
I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*e^2*f/(b*
d^3*x + b*c*d^2) + 3/8*(4*b*c*d*x*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)
+ e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + 4*b*c^2*(e^(I*b*d^2*x^2 + 2*I
*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - sqrt(b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^
2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*
d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c^2 - (I - 1)*sqrt(2)*gamma(3/2,
I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I + 1)*sqrt(2)*gamma(3/2, -I*b*d^2*
x^2 - 2*I*b*c*d*x - I*b*c^2)))*e*f^2/(b^2*d^4*x + b^2*c*d^3) - 1/8*(6*b*c^3
*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x -
```

```
 I*b*c^2)) + 2*(3*b*c^2*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*
d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - I*gamma(2, I*b*d^2*x^2 + 2*I*b*c*d*x +
I*b*c^2) + I*gamma(2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*d*x + 2*c*(-I*
gamma(2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*gamma(2, -I*b*d^2*x^2 - 2
*I*b*c*d*x - I*b*c^2)) - ((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2
+ 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^
2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c^3 - 3*((I - 1)*sqrt(2)*gamma(3/2,
 I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - (I + 1)*sqrt(2)*gamma(3/2, -I*b*d^2
*x^2 - 2*I*b*c*d*x - I*b*c^2))*c)*sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2))*f^3/
(b^2*d^5*x + b^2*c*d^4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.27

$$\int (e+fx)^3\sin \left(b(c+dx)^2\right)\,dx$$

$$= \frac{\frac{\sqrt{2}\sqrt{\pi}\left(2\,bd^3e^3-6\,bcd^2e^2f+6\,bc^2def^2-2\,bc^3f^3+3i\,def^2-3i\,cf^3\right)\operatorname{erf}\left(-\frac{1}{2}i\,\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}-\frac{2i\left(-i\,bd^2f^3\left(x+\frac{c}{d}\right)^2-3\,bd^2ef^2(i\,x\cdots\right.}{\cdots}}{8\,d^3} + \frac{\frac{\sqrt{2}\sqrt{\pi}\left(2\,bd^3e^3-6\,bcd^2e^2f+6\,bc^2def^2-2\,bc^3f^3-3i\,def^2+3i\,cf^3\right)\operatorname{erf}\left(\frac{1}{2}i\,\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}-\frac{2i\left(-i\,bd^2f^3\left(x+\frac{c}{d}\right)^2-3\,bd^2ef^2\cdots\right.}{\cdots}}{8\,d^3}$$

```
[In] integrate((f*x+e)^3*sin(b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*sqrt(pi)*(2*b*d^3*e^3 - 6*b*c*d^2*e^2*f + 6*b*c^2*d*e*f^2 - 2*
b*c^3*f^3 + 3*I*d*e*f^2 - 3*I*c*f^3)*erf(-1/2*I*sqrt(2)*sqrt(b*d^2)*(I*b*d^
2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b)
 - 2*I*(-I*b*d^2*f^3*(x + c/d)^2 - 3*b*d^2*e*f^2*(I*x + I*c/d) - 3*b*c*d*f^
3*(-I*x - I*c/d) - 3*I*b*d^2*e^2*f + 6*I*b*c*d*e*f^2 - 3*I*b*c^2*f^3 + f^3)
*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)/(b^2*d))/d^3 + 1/8*(sqrt(2)*sqrt(p
i)*(2*b*d^3*e^3 - 6*b*c*d^2*e^2*f + 6*b*c^2*d*e*f^2 - 2*b*c^3*f^3 - 3*I*d*e
*f^2 + 3*I*c*f^3)*erf(1/2*I*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1
)*(x + c/d))/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*I*(-I*b*d^2*f
^3*(x + c/d)^2 - 3*b*d^2*e*f^2*(I*x + I*c/d) - 3*b*c*d*f^3*(-I*x - I*c/d) -
 3*I*b*d^2*e^2*f + 6*I*b*c*d*e*f^2 - 3*I*b*c^2*f^3 - f^3)*e^(-I*b*d^2*x^2 -
 2*I*b*c*d*x - I*b*c^2)/(b^2*d))/d^3
```

Mupad [B] (verification not implemented)

Time = 6.31 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.04

$$\int (e+fx)^3\sin\left(b(c+dx)^2\right)\,dx$$
$$=\frac{f^3\,\sin\left(b\,(c+d\,x)^2\right)}{2\,b^2\,d^4}-\frac{\cos\left(b\,(c+d\,x)^2\right)\,\left(c^2\,f^3-3\,c\,d\,e\,f^2+3\,d^2\,e^2\,f\right)}{2\,b\,d^4}-\frac{f^3\,x^2\,\cos\left(b\,(c+d\,x)^2\right)}{2\,b\,d^2}+\frac{x\,\cos\left(b\,(c+d\,x)^2\right)\,\left(c\,f^3-3\,d\,e\,f^2\right)}{2\,b\,d^3}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}\,(c+d\,x)}{\sqrt{\pi}}\right)\,\left(c^3\,f^3-3\,c^2\,d\,e\,f^2+3\,c\,d^2\,e^2\,f-d^3\,e^3\right)}{2\,\sqrt{b}\,d^4}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}\,(c+d\,x)}{\sqrt{\pi}}\right)\,\left(3\,c\,f^3-3\,d\,e\,f^2\right)}{4\,b^{3/2}\,d^4}$$

```
[In] int(sin(b*(c + d*x)^2)*(e + f*x)^3,x)
```

```
[Out] (f^3*sin(b*(c + d*x)^2))/(2*b^2*d^4) - (cos(b*(c + d*x)^2)*(c^2*f^3 + 3*d^2
*e^2*f - 3*c*d*e*f^2))/(2*b*d^4) - (f^3*x^2*cos(b*(c + d*x)^2))/(2*b*d^2) +
 (x*cos(b*(c + d*x)^2)*(c*f^3 - 3*d*e*f^2))/(2*b*d^3) - (2^(1/2)*pi^(1/2)*f
resnels((2^(1/2)*b^(1/2)*(c + d*x))/pi^(1/2))*(c^3*f^3 - d^3*e^3 + 3*c*d^2*
e^2*f - 3*c^2*d*e*f^2))/(2*b^(1/2)*d^4) - (2^(1/2)*pi^(1/2)*fresnelc((2^(1/
2)*b^(1/2)*(c + d*x))/pi^(1/2))*(3*c*f^3 - 3*d*e*f^2))/(4*b^(3/2)*d^4)
```

3.154 $\int (e+fx)^2 \sin \left(b(c+dx)^2\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\int (e+fx)^2 \sin \left(b(c+dx)^2\right) \, dx = -\frac{f(de-cf)\cos\left(b(c+dx)^2\right)}{bd^3} - \frac{f^2(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^3} + \frac{f^2\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3} + \frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3}$$

```
[Out] -f*(-c*f+d*e)*cos(b*(d*x+c)^2)/b/d^3-1/2*f^2*(d*x+c)*cos(b*(d*x+c)^2)/b/d^3
+1/4*f^2*FresnelC((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2
)/d^3+1/2*(-c*f+d*e)^2*FresnelS((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*P
i^(1/2)/d^3/b^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3514, 3432, 3460, 2718, 3466, 3433}

$$\int (e+fx)^2 \sin \left(b(c+dx)^2\right) \, dx = \frac{\sqrt{\frac{\pi}{2}}f^2\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3} + \frac{\sqrt{\frac{\pi}{2}}(de-cf)^2\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3} - \frac{f(de-cf)\cos\left(b(c+dx)^2\right)}{bd^3} - \frac{f^2(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^3}$$

```
[In] Int[(e + f*x)^2*Sin[b*(c + d*x)^2],x]
```

```
[Out] -((f*(d*e - c*f)*Cos[b*(c + d*x)^2])/(b*d^3)) - (f^2*(c + d*x)*Cos[b*(c + d
*x)^2])/(2*b*d^3) + (f^2*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])
/(2*b^(3/2)*d^3) + ((d*e - c*f)^2*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)])/(Sqrt[b]*d^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(bx^2\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(bx^2\right)+f^2x^2\sin\left(bx^2\right)\right)\,dx,x,c+dx\right)}{d^3}$$

$$= \frac{f^2\text{Subst}\left(\int x^2\sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^3} + \frac{(2f(de-cf))\text{Subst}\left(\int x\sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^3} + \frac{(de-cf)^2\text{Subst}\left(\int \sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^3}$$

$$= -\frac{f^2(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^3} + \frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3} + \frac{f^2\text{Subst}\left(\int \cos\left(bx^2\right)\,dx,x,c+dx\right)}{2bd^3} + \frac{(f(de-cf))\text{Subst}\left(\int \sin(bx)\,dx,x,(c+dx)^2\right)}{d^3}$$

$$= -\frac{f(de-cf)\cos\left(b(c+dx)^2\right)}{bd^3} - \frac{f^2(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^3} + \frac{f^2\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3} + \frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.78

$$\int (e+fx)^2\sin\left(b(c+dx)^2\right)\,dx$$

$$= \frac{-2\sqrt{b}f(2de-cf+dfx)\cos\left(b(c+dx)^2\right)+f^2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)+2b(de-cf)^2\sqrt{2\pi}\,\text{Fresn}}{4b^{3/2}d^3}$$

```
[In] Integrate[(e + f*x)^2*Sin[b*(c + d*x)^2],x]
```

```
[Out] (-2*Sqrt[b]*f*(2*d*e - c*f + d*f*x)*Cos[b*(c + d*x)^2] + f^2*Sqrt[2*Pi]*Fre
snelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)] + 2*b*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelS
[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(4*b^(3/2)*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(127) = 254$.

Time = 0.62 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.94

method	result
default	$-\frac{f^2 x\cos(d^2x^2b+2cdxb+c^2b)}{2b\,d^2}-\frac{f^2c\left(-\frac{\cos(d^2x^2b+2cdxb+c^2b)}{2b\,d^2}-\frac{c\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,(b\,d^2x+cdb)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)}{2d\sqrt{b\,d^2}}\right)}{d}+\frac{f^2\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,(b\,d^2x+cdb)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)}{4b\,d^2\sqrt{b\,d^2}}$
risch	$\frac{i\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)\sqrt{\pi}\,e^2}{4\sqrt{-ib}\,d}+\frac{if^2c^2\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^3\sqrt{-ib}}-\frac{f^2\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{8b\,d^3\sqrt{-ib}}-\frac{iefc\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\right.}{2d^2\sqrt{-ib}}$
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,(b\,d^2x+cdb)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)x^2f^2}{2\sqrt{b\,d^2}}+\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,(b\,d^2x+cdb)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)fex}{\sqrt{b\,d^2}}+\frac{\sqrt{2}\,\sqrt{\pi}\,e^2\,\mathrm{S}\left(\frac{\sqrt{2}\,(b\,d^2x+cdb)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)}{2\sqrt{b\,d^2}}-\frac{\pi f\left(\frac{\mathrm{S}\left(\frac{\sqrt{2}\,b\,d^2x}{\sqrt{\pi}\,\sqrt{b\,d^2}}+\frac{\sqrt{2}\,cd}{\sqrt{\pi}\,\sqrt{b}}\right.}{}\right.}{}$

```
[In] int((f*x+e)^2*sin(b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f^2/b/d^2*x*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-f^2*c/d*(-1/2/b/d^2*cos(b*d
^2*x^2+2*b*c*d*x+b*c^2)-1/2*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelS(2^(
1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d)))+1/4*f^2/b/d^2*2^(1/2)*Pi^(1/2
)/(b*d^2)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))-e*
f/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-e*f*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/
2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))+1/2*2^(1/2)*Pi^
(1/2)/(b*d^2)^(1/2)*e^2*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*
c*d))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08

$$\int (e+fx)^2\sin\left(b(c+dx)^2\right)\,dx$$

$$=\frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}f^2\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)+2\sqrt{2}\pi(bd^2e^2-2\,bcdef+bc^2f^2)\sqrt{\frac{bd^2}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)-2\left(bd^2f^2x+2\,b\right.}{4\,b^2d^4}$$

```
[In] integrate((f*x+e)^2*sin(b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(b*d^2/pi)*f^2*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x
+ c)/d) + 2*sqrt(2)*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt(b*d^2/pi)
*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^2*f^2*x + 2*b*d^2
*e*f - b*c*d*f^2)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2))/(b^2*d^4)
```

Sympy [F]

$$\int (e+fx)^2\sin\left(b(c+dx)^2\right)\,dx=\int (e+fx)^2\sin\left(bc^2+2bcdx+bd^2x^2\right)dx$$

```
[In] integrate((f*x+e)**2*sin(b*(d*x+c)**2),x)
[Out] Integral((e + f*x)**2*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 564, normalized size of antiderivative = 3.76

$$\int (e+fx)^2\sin\left(b(c+dx)^2\right)\,dx=\frac{\sqrt{2}\sqrt{\pi}e^2\left((i+1)\ \mathrm{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{i\,b}}\right)+(i-1)\ \mathrm{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{-i\,b}}\right)\right)}{8\sqrt{b}d}$$
$$-\frac{\left(2\,dx\left(e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}+e^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}\right)-\sqrt{bd^2x^2+2\,bcdx+bc^2}\left(-(i+1)\ \sqrt{2}\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{i\,b\right.\right.\right.}}{}$$
$$+\frac{\left(4\,bcdx\left(e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}+e^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}\right)+4\,bc^2\left(e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}+e^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}\right.\right.}{}$$

```
[In] integrate((f*x+e)^2*sin(b*(d*x+c)^2),x, algorithm="maxima")
[Out] 1/8*sqrt(2)*sqrt(pi)*e^2*((I + 1)*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I - 1
)*erf((I*b*d*x + I*b*c)/sqrt(-I*b)))/(sqrt(b)*d) - 1/4*(2*d*x*(e^(I*b*d^2*x
^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - s
qrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*(-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b
*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqr
t(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*c + 2*c*(e^(I*b*d^2*x^2 + 2*
I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*e*f/(b*d^
3*x + b*c*d^2) + 1/8*(4*b*c*d*x*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) +
e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + 4*b*c^2*(e^(I*b*d^2*x^2 + 2*I*b
*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - sqrt(b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2
+ 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^
2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c^2 - (I - 1)*sqrt(2)*gamma(3/2, I*
b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I + 1)*sqrt(2)*gamma(3/2, -I*b*d^2*x^
2 - 2*I*b*c*d*x - I*b*c^2)))*f^2/(b^2*d^4*x + b^2*c*d^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.22

$$\int (e+fx)^2\sin\left(b(c+dx)^2\right)\,dx = -\frac{\frac{i\sqrt{2}\sqrt{\pi}(2i\,bd^2e^2-4i\,bcdef+2i\,bc^2f^2-f^2)\operatorname{erf}\left(-\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}+\frac{2\left(df^2\left(x+\frac{c}{d}\right)+2\,def-2\,cf^2\right)e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}}{bd}}{8\,d^2} - \frac{-\frac{i\sqrt{2}\sqrt{\pi}(-2i\,bd^2e^2+4i\,bcdef-2i\,bc^2f^2-f^2)\operatorname{erf}\left(\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}+\frac{2\left(df^2\left(x+\frac{c}{d}\right)+2\,def-2\,cf^2\right)e^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}}{bd}}{8\,d^2}$$

```
[In] integrate((f*x+e)^2*sin(b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(I*sqrt(2)*sqrt(pi)*(2*I*b*d^2*e^2 - 4*I*b*c*d*e*f + 2*I*b*c^2*f^2 - f
^2)*erf(-1/2*I*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(
sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) + 2*(d*f^2*(x + c/d) + 2*d*e*f -
 2*c*f^2)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)/(b*d))/d^2 - 1/8*(-I*sqrt
(2)*sqrt(pi)*(-2*I*b*d^2*e^2 + 4*I*b*c*d*e*f - 2*I*b*c^2*f^2 - f^2)*erf(1/2
*I*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)
*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b) + 2*(d*f^2*(x + c/d) + 2*d*e*f - 2*c*f^2)*
e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)/(b*d))/d^2
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

$$\int (e+fx)^2\sin\left(b(c+dx)^2\right)\,dx = \frac{\cos\left(b\,(c+d\,x)^2\right)\,\left(c\,f^2-2\,d\,e\,f\right)}{2\,b\,d^3} - \frac{f^2\,x\,\cos\left(b\,(c+d\,x)^2\right)}{2\,b\,d^2} + \frac{\sqrt{2}\,f^2\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}\,(c+d\,x)}{\sqrt{\pi}}\right)}{4\,b^{3/2}\,d^3} + \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}\,(c+d\,x)}{\sqrt{\pi}}\right)\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2\right)}{2\,\sqrt{b}\,d^3}$$

```
[In] int(sin(b*(c + d*x)^2)*(e + f*x)^2,x)
```

```
[Out] (cos(b*(c + d*x)^2)*(c*f^2 - 2*d*e*f))/(2*b*d^3) - (f^2*x*cos(b*(c + d*x)^2
))/(2*b*d^2) + (2^(1/2)*f^2*pi^(1/2)*fresnelc((2^(1/2)*b^(1/2)*(c + d*x))/p
i^(1/2)))/(4*b^(3/2)*d^3) + (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b^(1/2)*(c
+ d*x))/pi^(1/2))*(c^2*f^2 + d^2*e^2 - 2*c*d*e*f))/(2*b^(1/2)*d^3)
```

3.155 $\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 69

$$\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx = -\frac{f\cos\left(b(c+dx)^2\right)}{2bd^2} + \frac{(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2}$$

```
[Out] -1/2*f*cos(b*(d*x+c)^2)/b/d^2+1/2*(-c*f+d*e)*FresnelS((d*x+c)*b^(1/2)*2^(1/
2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^2/b^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3514, 3432, 3460, 2718}

$$\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx = \frac{\sqrt{\frac{\pi}{2}}(de-cf)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2} - \frac{f\cos\left(b(c+dx)^2\right)}{2bd^2}$$

```
[In] Int[(e + f*x)*Sin[b*(c + d*x)^2],x]
```

```
[Out] -1/2*(f*Cos[b*(c + d*x)^2])/(b*d^2) + ((d*e - c*f)*Sqrt[Pi/2]*FresnelS[Sqrt
[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(bx^2\right)+fx\sin\left(bx^2\right)\right)\,dx,x,c+dx\right)}{d^2}\\
&= \frac{f\text{Subst}\left(\int x\sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^2}\\
&= \frac{(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2}+\frac{f\text{Subst}\left(\int \sin(bx)\,dx,x,(c+dx)^2\right)}{2d^2}\\
&= -\frac{f\cos\left(b(c+dx)^2\right)}{2bd^2}+\frac{(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx = \frac{-f\cos\left(b(c+dx)^2\right)+\sqrt{b}(de-cf)\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2bd^2}$$

```
[In] Integrate[(e + f*x)*Sin[b*(c + d*x)^2],x]
[Out] (-(f*Cos[b*(c + d*x)^2]) + Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*
Sqrt[2/Pi]*(c + d*x)])/(2*b*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(58) = 116.

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

method	result
default	$-\frac{f\cos(d^2x^2b+2cdxb+c^2b)}{2b\,d^2}-\frac{fc\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\left(b\,d^2x+cdb\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)}{2d\sqrt{b\,d^2}}+\frac{\sqrt{2}\,\sqrt{\pi}\,e\,\mathrm{S}\left(\frac{\sqrt{2}\,\left(b\,d^2x+cdb\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)}{2\sqrt{b\,d^2}}$
risch	$\frac{i\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)\sqrt{\pi}\,e}{4\sqrt{-ib}\,d}-\frac{ifc\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^2\sqrt{-ib}}+\frac{ie\sqrt{\pi}\,\mathrm{erf}\left(d\sqrt{ib}\,x+\frac{ibc}{\sqrt{ib}}\right)}{4d\sqrt{ib}}-\frac{ifc\sqrt{\pi}\,\mathrm{erf}\left(d\sqrt{ib}\,x+\frac{ibc}{\sqrt{ib}}\right)}{4d^2\sqrt{ib}}-\frac{f\,c}{}$
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\left(b\,d^2x+cdb\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)fx}{2\sqrt{b\,d^2}}+\frac{\sqrt{2}\,\sqrt{\pi}\,e\,\mathrm{S}\left(\frac{\sqrt{2}\,\left(b\,d^2x+cdb\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)}{2\sqrt{b\,d^2}}-\frac{\pi f\left(\mathrm{S}\left(\frac{\sqrt{2}\,b\,d^2x}{\sqrt{\pi}\,\sqrt{b\,d^2}}+\frac{\sqrt{2}\,cdb}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)\left(\frac{\sqrt{2}\,b\,d^2x}{\sqrt{\pi}\,\sqrt{b\,d^2}}+\frac{\sqrt{2}\,cdb}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)+\frac{\cos\left(\frac{\pi}{}\right)}{}\right.}{2b\,d^2}$

```
[In] int((f*x+e)*sin(b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-1/2*f*c/d*2^(1/2)*Pi^(1/2)/(b*d
^2)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))+1/2*2^(1
/2)*Pi^(1/2)/(b*d^2)^(1/2)*e*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2
*x+b*c*d))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx = \frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}(de-cf)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)-df\cos\left(bd^2x^2+2\,bcdx+bc^2\right)}{2\,bd^3}$$

```
[In] integrate((f*x+e)*sin(b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*sqrt(b*d^2/pi)*(d*e - c*f)*fresnel_sin(sqrt(2)*sqrt(b*d^2/p
i)*(d*x + c)/d) - d*f*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2))/(b*d^3)
```

Sympy [F]

$$\int (e+fx)\sin \left(b(c+dx)^2\right)\, dx = \int (e+fx)\sin \left(bc^2+2bcdx+bd^2x^2\right) dx$$

```
[In] integrate((f*x+e)*sin(b*(d*x+c)**2),x)
[Out] Integral((e + f*x)*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.93

$$\int (e+fx)\sin \left(b(c+dx)^2\right)\, dx = \frac{\sqrt{2}\sqrt{\pi}e\left((i+1)\ \mathrm{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{i\,b}}\right)+(i-1)\ \mathrm{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{-i\,b}}\right)\right)}{8\sqrt{b}d}$$
$$-\frac{\left(2\,dx\left(e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}+e^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}\right)-\sqrt{bd^2x^2+2\,bcdx+bc^2}\left(-(i+1)\ \sqrt{2}\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{i\,bd^2}\right.\right.\right.\right.}{}$$

```
[In] integrate((f*x+e)*sin(b*(d*x+c)^2),x, algorithm="maxima")
[Out] 1/8*sqrt(2)*sqrt(pi)*e*((I + 1)*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I - 1)*
erf((I*b*d*x + I*b*c)/sqrt(-I*b)))/(sqrt(b)*d) - 1/8*(2*d*x*(e^(I*b*d^2*x^2
 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - sqr
t(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*(-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d
^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(
-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*c + 2*c*(e^(I*b*d^2*x^2 + 2*I*
b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*f/(b*d^3*x
+ b*c*d^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.41

$$\int (e+fx)\sin \left(b(c+dx)^2\right)\, dx$$
$$= -\frac{-\frac{i\sqrt{2}\sqrt{\pi}(-i\,de+i\,cf)\ \mathrm{erf}\left(-\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}}{bd}}{4\,d}$$
$$-\frac{\frac{i\sqrt{2}\sqrt{\pi}(i\,de-i\,cf)\ \mathrm{erf}\left(\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}}{bd}}{4\,d}$$

```
[In] integrate((f*x+e)*sin(b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*(-I*sqrt(2)*sqrt(pi)*(-I*d*e + I*c*f)*erf(-1/2*I*sqrt(2)*sqrt(b*d^2)*(
I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) +
 1)) + f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)/(b*d))/d - 1/4*(I*sqrt(2)*
sqrt(pi)*(I*d*e - I*c*f)*erf(1/2*I*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d
^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(-I*b*
d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)/(b*d))/d
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx=-\frac{f\,\cos\left(b\,{\left(c+d\,x\right)}^2\right)}{2\,b\,d^2}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}\,\left(c+d\,x\right)}{\sqrt{\pi}}\right)\,\left(c\,f-d\,e\right)}{2\,\sqrt{b}\,d^2}$$

```
[In] int(sin(b*(c + d*x)^2)*(e + f*x),x)
```

```
[Out] - (f*cos(b*(c + d*x)^2))/(2*b*d^2) - (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b^
(1/2)*(c + d*x))/pi^(1/2))*(c*f - d*e))/(2*b^(1/2)*d^2)
```

3.156 $\int \sin \left(b(c+dx)^2\right) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \sin \left(b(c+dx)^2\right) \, dx = \frac{\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

```
[Out] 1/2*FresnelS((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3432}

$$\int \sin \left(b(c+dx)^2\right) \, dx = \frac{\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

```
[In] Int[Sin[b*(c + d*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \sin \left(b(c+dx)^2\right) \, dx = \frac{\sqrt{\frac{\pi}{2}} \text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

```
[In] Integrate[Sin[b*(c + d*x)^2],x]
[Out] (Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\left(b\,d^{2} x+c d b\right)}{\sqrt{\pi}\,\sqrt{b\,d^{2}}}\right)}{2 \sqrt{b\,d^{2}}}$	42
risch	$\frac{i\sqrt{\pi}\,\mathrm{erf}\left(d\sqrt{i b}\,x+\frac{i b c}{\sqrt{i b}}\right)}{4 d\sqrt{i b}}+\frac{i\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-i b}\,x+\frac{i b c}{\sqrt{-i b}}\right)}{4 d\sqrt{-i b}}$	77

```
[In] int(sin(b*(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*
(b*d^2*x+b*c*d))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \sin \left(b(c+dx)^2\right) \, dx = \frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)}{2\,bd^2}$$

```
[In] integrate(sin(b*(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*pi*sqrt(b*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/
d)/(b*d^2)
```

Sympy [F]

$$\int \sin \left(b(c+dx)^2\right) \, dx = \int \sin \left(b(c+dx)^2\right) dx$$

```
[In] integrate(sin(b*(d*x+c)**2),x)
```

```
[Out] Integral(sin(b*(c + d*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \sin \left(b(c+dx)^2\right) \, dx = \frac{\sqrt{2}\sqrt{\pi}\left((i+1) \operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{i\,b}}\right) + (i-1) \operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{-i\,b}}\right)\right)}{8\sqrt{b}d}$$

```
[In] integrate(sin(b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*sqrt(pi)*((I + 1)*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I - 1)*er
f((I*b*d*x + I*b*c)/sqrt(-I*b)))/(sqrt(b)*d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.67

$$\int \sin \left(b(c+dx)^2\right) \, dx = \frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{4\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)} + \frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}\left(\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{4\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}$$

```
[In] integrate(sin(b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(pi)*erf(-1/2*I*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4)
+ 1)*(x + c/d))/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 1/4*sqrt(2)*sqr
t(pi)*erf(1/2*I*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))
/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1))
```

Mupad [B] (verification not implemented)

Time = 6.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \sin \left(b(c+dx)^2\right)\, dx = \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,b\,d\,\sqrt{\frac{1}{b\,d^2}}\,(c+d\,x)}{\sqrt{\pi}}\right)\,\sqrt{\frac{1}{b\,d^2}}}{2}$$

```
[In] int(sin(b*(c + d*x)^2),x)
[Out] (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b*d*(1/(b*d^2))^(1/2)*(c + d*x))/pi^(1/
2))*(1/(b*d^2))^(1/2))/2
```

3.157 $\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx = \operatorname{Int}\left(\frac{\sin\left(b(c+dx)^2\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(b*(d*x+c)^2)/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx = \int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$$

```
[In] Int[Sin[b*(c + d*x)^2]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[b*(c + d*x)^2]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin(b(c+dx)^2)}{e+fx}\,dx=\int \frac{\sin(b(c+dx)^2)}{e+fx}\,dx$$

```
[In] Integrate[Sin[b*(c + d*x)^2]/(e + f*x),x]
[Out] Integrate[Sin[b*(c + d*x)^2]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(b(dx+c)^2\right)}{fx+e}dx$$

```
[In] int(sin(b*(d*x+c)^2)/(f*x+e),x)
[Out] int(sin(b*(d*x+c)^2)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sin(b(c+dx)^2)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^2b\right)}{fx+e}\,dx$$

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
[Out] integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2)/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{\sin(b(c+dx)^2)}{e+fx}\,dx=\int \frac{\sin(bc^2+2bcdx+bd^2x^2)}{e+fx}\,dx$$

```
[In] integrate(sin(b*(d*x+c)**2)/(f*x+e),x)
[Out] Integral(sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin(b(c+dx)^2)}{e+fx}\,dx = \int \frac{\sin\left((dx+c)^2b\right)}{fx+e}\,dx$$

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^2*b)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin(b(c+dx)^2)}{e+fx}\,dx = \int \frac{\sin\left((dx+c)^2b\right)}{fx+e}\,dx$$

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^2*b)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 6.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin(b(c+dx)^2)}{e+fx}\,dx = \int \frac{\sin\left(b\,(c+d\,x)^2\right)}{e+f\,x}\,dx$$

```
[In] int(sin(b*(c + d*x)^2)/(e + f*x),x)
[Out] int(sin(b*(c + d*x)^2)/(e + f*x), x)
```

3.158 $\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx = \text{Int}\left(\frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(b*(d*x+c)^2)/(f*x+e)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx$$

```
[In] Int[Sin[b*(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[b*(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin(b(c+dx)^2)}{(e+fx)^2}\,dx = \int \frac{\sin(b(c+dx)^2)}{(e+fx)^2}\,dx$$

```
[In] Integrate[Sin[b*(c + d*x)^2]/(e + f*x)^2,x]
[Out] Integrate[Sin[b*(c + d*x)^2]/(e + f*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(b(dx+c)^2\right)}{(fx+e)^2}dx$$

```
[In] int(sin(b*(d*x+c)^2)/(f*x+e)^2,x)
[Out] int(sin(b*(d*x+c)^2)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{\sin(b(c+dx)^2)}{(e+fx)^2}\,dx = \int \frac{\sin\left((dx+c)^2b\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sin(b(c+dx)^2)}{(e+fx)^2}\,dx = \int \frac{\sin(bc^2+2bcdx+bd^2x^2)}{(e+fx)^2}\,dx$$

```
[In] integrate(sin(b*(d*x+c)**2)/(f*x+e)**2,x)
[Out] Integral(sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin(b(c+dx)^2)}{(e+fx)^2}\,dx = \int \frac{\sin\left((dx+c)^2b\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^2*b)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin(b(c+dx)^2)}{(e+fx)^2}\,dx = \int \frac{\sin\left((dx+c)^2b\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin((d*x + c)^2*b)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin(b(c+dx)^2)}{(e+fx)^2}\,dx = \int \frac{\sin\left(b\,(c+d\,x)^2\right)}{(e+f\,x)^2}\,dx$$

```
[In] int(sin(b*(c + d*x)^2)/(e + f*x)^2,x)
[Out] int(sin(b*(c + d*x)^2)/(e + f*x)^2, x)
```

3.159 $\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 337

$$\begin{aligned}\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right) dx = {} & \frac{2bf^2(de-cf)(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{d^4} \\ & + \frac{bf^3(c+dx)^2\cos\left(\frac{b}{(c+dx)^2}\right)}{4d^4} \\ & - \frac{3bf(de-cf)^2\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^4} \\ & - \frac{\sqrt{b}(de-cf)^3\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^4} \\ & + \frac{2b^{3/2}f^2(de-cf)\sqrt{2\pi}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^4} \\ & + \frac{(de-cf)^3(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4} \\ & + \frac{3f(de-cf)^2(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^4} \\ & + \frac{f^2(de-cf)(c+dx)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4} \\ & + \frac{f^3(c+dx)^4\sin\left(\frac{b}{(c+dx)^2}\right)}{4d^4} + \frac{b^2f^3\operatorname{Si}\left(\frac{b}{(c+dx)^2}\right)}{4d^4}\end{aligned}$$

```
[Out] -3/2*b*f*(-c*f+d*e)^2*Ci(b/(d*x+c)^2)/d^4+2*b*f^2*(-c*f+d*e)*(d*x+c)*cos(b/
(d*x+c)^2)/d^4+1/4*b*f^3*(d*x+c)^2*cos(b/(d*x+c)^2)/d^4+1/4*b^2*f^3*Si(b/(d
```

```
*x+c)^2)/d^4+(-c*f+d*e)^3*(d*x+c)*sin(b/(d*x+c)^2)/d^4+3/2*f*(-c*f+d*e)^2*(
d*x+c)^2*sin(b/(d*x+c)^2)/d^4+f^2*(-c*f+d*e)*(d*x+c)^3*sin(b/(d*x+c)^2)/d^4
+1/4*f^3*(d*x+c)^4*sin(b/(d*x+c)^2)/d^4+2*b^(3/2)*f^2*(-c*f+d*e)*FresnelS(b
^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*2^(1/2)*Pi^(1/2)/d^4-(-c*f+d*e)^3*FresnelC
(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3514, 3440, 3468, 3433, 3460, 3378, 3383, 3490, 3469, 3432, 3380}

$$\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right) dx = \frac{2\sqrt{2\pi}b^{3/2}f^2(de-cf)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^4} + \frac{b^2f^3\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{4d^4} - \frac{3bf(de-cf)^2\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^4} + \frac{f^2(c+dx)^3(de-cf)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{2bf^2(c+dx)(de-cf)\cos\left(\frac{b}{(c+dx)^2}\right)}{d^4} - \frac{\sqrt{2\pi}\sqrt{b}(de-cf)^3\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^4} + \frac{3f(c+dx)^2(de-cf)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^4} + \frac{(c+dx)(de-cf)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{f^3(c+dx)^4\sin\left(\frac{b}{(c+dx)^2}\right)}{4d^4} + \frac{bf^3(c+dx)^2\cos\left(\frac{b}{(c+dx)^2}\right)}{4d^4}$$

```
[In] Int[(e + f*x)^3*Sin[b/(c + d*x)^2],x]
```

```
[Out] (2*b*f^2*(d*e - c*f)*(c + d*x)*Cos[b/(c + d*x)^2])/d^4 + (b*f^3*(c + d*x)^2
*Cos[b/(c + d*x)^2])/(4*d^4) - (3*b*f*(d*e - c*f)^2*CosIntegral[b/(c + d*x)
^2])/(2*d^4) - (Sqrt[b]*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/P
i])/(c + d*x)])/d^4 + (2*b^(3/2)*f^2*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi])/(c + d*x)])/d^4 + ((d*e - c*f)^3*(c + d*x)*Sin[b/(c + d*x)^2
])/d^4 + (3*f*(d*e - c*f)^2*(c + d*x)^2*Sin[b/(c + d*x)^2])/(2*d^4) + (f^2*
(d*e - c*f)*(c + d*x)^3*Sin[b/(c + d*x)^2])/d^4 + (f^3*(c + d*x)^4*Sin[b/(c
 + d*x)^2])/(4*d^4) + (b^2*f^3*SinIntegral[b/(c + d*x)^2])/(4*d^4)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
```

```
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{\text{Subst}\left(\int \left(d^3e^3\left(1-\frac{cf(3d^2e^2-3cdef+c^2f^2)}{d^3e^3}\right)\sin\left(\frac{b}{x^2}\right)+3d^2e^2f\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)x\sin\left(\frac{b}{x^2}\right)+3def^2\left(1-\frac{cf}{de}\right)x^2 \cdots\right.\right.}{d^4}$$

$$= \frac{f^3\text{Subst}\left(\int x^3\sin\left(\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^4} + \frac{(3f^2(de-cf))\,\text{Subst}\left(\int x^2\sin\left(\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^4} + \frac{(3f(de-cf)^2)\,\text{Subst}\left(\int x\sin\left(\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^4} + \frac{(de-cf)^3\text{Subst}\left(\int \sin\left(\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^4}$$

$$= -\frac{f^3\text{Subst}\left(\int \frac{\sin(bx)}{x^3}\,dx,x,\frac{1}{(c+dx)^2}\right)}{2d^4} - \frac{(3f^2(de-cf))\,\text{Subst}\left(\int \frac{\sin(bx^2)}{x^4}\,dx,x,\frac{1}{c+dx}\right)}{d^4} - \frac{(3f(de-cf)^2)\,\text{Subst}\left(\int \frac{\sin(bx)}{x^2}\,dx,x,\frac{1}{(c+dx)^2}\right)}{2d^4} - \frac{(de-cf)^3\text{Subst}\left(\int \frac{\sin(bx^2)}{x^2}\,dx,x,\frac{1}{c+dx}\right)}{d^4}$$

$$= \frac{(de-cf)^3(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{3f(de-cf)^2(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^4} + \frac{f^2(de-cf)(c+dx)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{f^3(c+dx)^4\sin\left(\frac{b}{(c+dx)^2}\right)}{4d^4} - \frac{(bf^3)\operatorname{Subst}\left(\int \frac{\cos(bx)}{x^2}\,dx, x, \frac{1}{(c+dx)^2}\right)}{4d^4} - \frac{(2bf^2(de-cf))\operatorname{Subst}\left(\int \frac{\cos(bx^2)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d^4} - \frac{(3bf(de-cf)^2)\operatorname{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{(c+dx)^2}\right)}{2d^4} - \frac{(2b(de-cf)^3)\operatorname{Subst}\left(\int \cos(bx^2)\,dx, x, \frac{1}{c+dx}\right)}{d^4}$$

$$= \frac{2bf^2(de-cf)(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{bf^3(c+dx)^2\cos\left(\frac{b}{(c+dx)^2}\right)}{4d^4} - \frac{3bf(de-cf)^2\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^4} - \frac{\sqrt{b}(de-cf)^3\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^4} + \frac{(de-cf)^3(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{3f(de-cf)^2(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^4} + \frac{f^2(de-cf)(c+dx)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{f^3(c+dx)^4\sin\left(\frac{b}{(c+dx)^2}\right)}{4d^4} + \frac{(b^2f^3)\operatorname{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{(c+dx)^2}\right)}{4d^4} + \frac{(4b^2f^2(de-cf))\operatorname{Subst}\left(\int \sin(bx^2)\,dx, x, \frac{1}{c+dx}\right)}{d^4}$$

$$= \frac{2bf^2(de-cf)(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{bf^3(c+dx)^2\cos\left(\frac{b}{(c+dx)^2}\right)}{4d^4} - \frac{3bf(de-cf)^2\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^4} - \frac{\sqrt{b}(de-cf)^3\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^4} + \frac{2b^{3/2}f^2(de-cf)\sqrt{2\pi}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^4} + \frac{(de-cf)^3(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{3f(de-cf)^2(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^4} + \frac{f^2(de-cf)(c+dx)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{f^3(c+dx)^4\sin\left(\frac{b}{(c+dx)^2}\right)}{4d^4} + \frac{b^2f^3\operatorname{Si}\left(\frac{b}{(c+dx)^2}\right)}{4d^4}$$

Mathematica [A] (verified)

Time = 6.43 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.31

$$\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right) dx$$
$$= \frac{8bcdef^2\cos\left(\frac{b}{(c+dx)^2}\right) - 7bc^2f^3\cos\left(\frac{b}{(c+dx)^2}\right) + 8bd^2ef^2x\cos\left(\frac{b}{(c+dx)^2}\right) - 6bcdf^3x\cos\left(\frac{b}{(c+dx)^2}\right) + bd^2f^3x^2}{}$$

```
[In] Integrate[(e + f*x)^3*Sin[b/(c + d*x)^2],x]
```

```
[Out] (8*b*c*d*e*f^2*Cos[b/(c + d*x)^2] - 7*b*c^2*f^3*Cos[b/(c + d*x)^2] + 8*b*d^
2*e*f^2*x*Cos[b/(c + d*x)^2] - 6*b*c*d*f^3*x*Cos[b/(c + d*x)^2] + b*d^2*f^3
*x^2*Cos[b/(c + d*x)^2] - 6*b*f*(d*e - c*f)^2*CosIntegral[b/(c + d*x)^2] -
4*Sqrt[b]*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]
 + 8*b^(3/2)*d*e*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] -
8*b^(3/2)*c*f^3*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 4*c*d
^3*e^3*Sin[b/(c + d*x)^2] - 6*c^2*d^2*e^2*f*Sin[b/(c + d*x)^2] + 4*c^3*d*e*
f^2*Sin[b/(c + d*x)^2] - c^4*f^3*Sin[b/(c + d*x)^2] + 4*d^4*e^3*x*Sin[b/(c
+ d*x)^2] + 6*d^4*e^2*f*x^2*Sin[b/(c + d*x)^2] + 4*d^4*e*f^2*x^3*Sin[b/(c +
 d*x)^2] + d^4*f^3*x^4*Sin[b/(c + d*x)^2] + b^2*f^3*SinIntegral[b/(c + d*x)
^2])/(4*d^4)
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-(cf-de)^3(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+(cf-de)^3\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)+\frac{3f(cf-de)^2(dx+c)^2\sin\left(\frac{b}{(dx+c)^2}\right)}{2}-\frac{3f(cf-de)^2b}{}}{}$
default	$\frac{-(cf-de)^3(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+(cf-de)^3\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)+\frac{3f(cf-de)^2(dx+c)^2\sin\left(\frac{b}{(dx+c)^2}\right)}{2}-\frac{3f(cf-de)^2b}{}}{}$
risch	$\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)c^3f^3}{2d^4\sqrt{-ib}}-\frac{3b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)c^2e\,f^2}{2d^3\sqrt{-ib}}+\frac{3b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)c\,e^2f}{2d^2\sqrt{-ib}}-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)e^3}{2d\sqrt{-ib}}+\frac{3b\,\mathrm{Ei}_1\left(-\frac{i}{(dx+}\right.}{4d^4}$
parts	Expression too large to display

```
[In] int((f*x+e)^3*sin(b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-(c*f-d*e)^3*(d*x+c)*sin(b/(d*x+c)^2)+(c*f-d*e)^3*b^(1/2)*2^(1/2)*Pi
^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))+3/2*f*(c*f-d*e)^2*(d*x+c)
^2*sin(b/(d*x+c)^2)-3/2*f*(c*f-d*e)^2*b*Ci(b/(d*x+c)^2)-f^2*(c*f-d*e)*(d*x+
```

```
c)^3*sin(b/(d*x+c)^2)+2*f^2*(c*f-d*e)*b*(-(d*x+c)*cos(b/(d*x+c)^2)-b^(1/2)*
2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)))+1/4*f^3*(d*x+c
)^4*sin(b/(d*x+c)^2)-1/2*f^3*b*(-1/2*(d*x+c)^2*cos(b/(d*x+c)^2)-1/2*b*Si(b/
(d*x+c)^2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.18

$$\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right) dx$$
$$= \frac{b^2 f^3 \operatorname{Si}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right) - 4\,\sqrt{2}\pi(d^4e^3 - 3\,cd^3e^2f + 3\,c^2d^2ef^2 - c^3df^3)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right) + 8\,\sqrt{2}\pi(bd^2ef^2 -}{}$$

```
[In] integrate((f*x+e)^3*sin(b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*f^3*sin_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) - 4*sqrt(2)*pi*(d^4*
e^3 - 3*c*d^3*e^2*f + 3*c^2*d^2*e*f^2 - c^3*d*f^3)*sqrt(b/(pi*d^2))*fresnel
_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) + 8*sqrt(2)*pi*(b*d^2*e*f^2 - b*
c*d*f^3)*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c))
 + (b*d^2*f^3*x^2 + 8*b*c*d*e*f^2 - 7*b*c^2*f^3 + 2*(4*b*d^2*e*f^2 - 3*b*c*
d*f^3)*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - 6*(b*d^2*e^2*f - 2*b*c*d*e*f^2
 + b*c^2*f^3)*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) + (d^4*f^3*x^4 + 4*
d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x + 4*c*d^3*e^3 - 6*c^2*d^2*e^2
*f + 4*c^3*d*e*f^2 - c^4*f^3)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d^4
```

Sympy [F]

$$\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right) dx = \int (e+fx)^3 \sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right) dx$$

```
[In] integrate((f*x+e)**3*sin(b/(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**3*sin(b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int (fx+e)^3 \sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

```
[In] integrate((f*x+e)^3*sin(b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*d^3*integrate(1/4*((4*b*c^3*d*e*f^2 - 3*b*c^4*f^3 - 6*(b*d^4*e^2*f
- 2*b*c*d^3*e*f^2 + b*c^2*d^2*f^3)*x^2 - 4*(b*d^4*e^3 - 3*b*c^2*d^2*e*f^2 +
 2*b*c^3*d*f^3)*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b^2*d^2*f^3*x^2 + 2*
(4*b^2*d^2*e*f^2 - 3*b^2*c*d*f^3)*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/(d^6
*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3), x) + 4*d^3*integrate(1/4*((4*b
*c^3*d*e*f^2 - 3*b*c^4*f^3 - 6*(b*d^4*e^2*f - 2*b*c*d^3*e*f^2 + b*c^2*d^2*f
^3)*x^2 - 4*(b*d^4*e^3 - 3*b*c^2*d^2*e*f^2 + 2*b*c^3*d*f^3)*x)*cos(b/(d^2*x
^2 + 2*c*d*x + c^2)) + (b^2*d^2*f^3*x^2 + 2*(4*b^2*d^2*e*f^2 - 3*b^2*c*d*f^
3)*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4
*x + c^3*d^3)*cos(b/(d^2*x^2 + 2*c*d*x + c^2))^2 + (d^6*x^3 + 3*c*d^5*x^2 +
 3*c^2*d^4*x + c^3*d^3)*sin(b/(d^2*x^2 + 2*c*d*x + c^2))^2), x) - (b*d*f^3*
x^2 + 2*(4*b*d*e*f^2 - 3*b*c*f^3)*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - (d^
3*f^3*x^4 + 4*d^3*e*f^2*x^3 + 6*d^3*e^2*f*x^2 + 4*d^3*e^3*x)*sin(b/(d^2*x^2
 + 2*c*d*x + c^2)))/d^3
```

Giac [F]

$$\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int (fx+e)^3 \sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

```
[In] integrate((f*x+e)^3*sin(b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(b/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int \sin\left(\frac{b}{(c+d\,x)^2}\right)\,(e+f\,x)^3\,dx$$

```
[In] int(sin(b/(c + d*x)^2)*(e + f*x)^3,x)
```

```
[Out] int(sin(b/(c + d*x)^2)*(e + f*x)^3, x)
```

3.160 $\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 233

$$\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right) dx = \frac{2bf^2(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{3d^3} - \frac{bf(de-cf)\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3} - \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3} + \frac{2b^{3/2}f^2\sqrt{2\pi}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3} + \frac{(de-cf)^2(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{3d^3}$$

```
[Out] -b*f*(-c*f+d*e)*Ci(b/(d*x+c)^2)/d^3+2/3*b*f^2*(d*x+c)*cos(b/(d*x+c)^2)/d^3+
(-c*f+d*e)^2*(d*x+c)*sin(b/(d*x+c)^2)/d^3+f*(-c*f+d*e)*(d*x+c)^2*sin(b/(d*x
+c)^2)/d^3+1/3*f^2*(d*x+c)^3*sin(b/(d*x+c)^2)/d^3+2/3*b^(3/2)*f^2*FresnelS(
b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*2^(1/2)*Pi^(1/2)/d^3-(-c*f+d*e)^2*Fresnel
C(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3514, 3440, 3468, 3433, 3460, 3378, 3383, 3490, 3469, 3432}

$$\int (e+fx)^2\sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\frac{2\sqrt{2\pi}b^{3/2}f^2\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3}-\frac{bf(de-cf)\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-\frac{\sqrt{2\pi}\sqrt{b}(de-cf)^2\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3}+\frac{f(c+dx)^2(de-cf)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{(c+dx)(de-cf)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{f^2(c+dx)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{3d^3}+\frac{2bf^2(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{3d^3}$$

```
[In] Int[(e + f*x)^2*Sin[b/(c + d*x)^2],x]
```

```
[Out] (2*b*f^2*(c + d*x)*Cos[b/(c + d*x)^2])/(3*d^3) - (b*f*(d*e - c*f)*CosIntegr
al[b/(c + d*x)^2])/d^3 - (Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelC[(Sqrt[b
]*Sqrt[2/Pi])/(c + d*x)])/d^3 + (2*b^(3/2)*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]
*Sqrt[2/Pi])/(c + d*x)])/(3*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin[b/(c + d*x)
^2])/d^3 + (f*(d*e - c*f)*(c + d*x)^2*Sin[b/(c + d*x)^2])/d^3 + (f^2*(c + d
*x)^3*Sin[b/(c + d*x)^2])/(3*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
```

```
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(\frac{b}{x^2}\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(\frac{b}{x^2}\right)+f^2x^2\sin\left(\frac{b}{x^2}\right)\right)\,dx,x,c+dx\right)}{d^3}$$

$$= \frac{f^2\text{Subst}\left(\int x^2\sin\left(\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^3} + \frac{(2f(de-cf))\text{Subst}\left(\int x\sin\left(\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^3} + \frac{(de-cf)^2\text{Subst}\left(\int \sin\left(\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^3}$$

$$= -\frac{f^2\text{Subst}\left(\int \frac{\sin(bx^2)}{x^4}\,dx,x,\frac{1}{c+dx}\right)}{d^3} - \frac{(f(de-cf))\text{Subst}\left(\int \frac{\sin(bx)}{x^2}\,dx,x,\frac{1}{(c+dx)^2}\right)}{d^3} - \frac{(de-cf)^2\text{Subst}\left(\int \frac{\sin(bx^2)}{x^2}\,dx,x,\frac{1}{c+dx}\right)}{d^3}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{3d^3} - \frac{(2bf^2)\,\text{Subst}\left(\int \frac{\cos(bx^2)}{x^2}\,dx,x,\frac{1}{c+dx}\right)}{3d^3} - \frac{(bf(de-cf))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,\frac{1}{(c+dx)^2}\right)}{d^3} - \frac{(2b(de-cf)^2)\,\text{Subst}\left(\int \cos(bx^2)\,dx,x,\frac{1}{c+dx}\right)}{d^3}$$

$$= \frac{2bf^2(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{3d^3} - \frac{bf(de-cf)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3} - \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3} + \frac{(de-cf)^2(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{3d^3} + \frac{(4b^2f^2)\,\text{Subst}\left(\int \sin(bx^2)\,dx,x,\frac{1}{c+dx}\right)}{3d^3}$$

$$= \frac{2bf^2(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{3d^3} - \frac{bf(de-cf)\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3} - \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3} + \frac{2b^{3/2}f^2\sqrt{2\pi}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3} + \frac{(de-cf)^2(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{3d^3}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.14

$$\int (e+fx)^2\sin\left(\frac{b}{(c+dx)^2}\right)\,dx$$

$$= \frac{2bcf^2\cos\left(\frac{b}{(c+dx)^2}\right) + 2bdf^2x\cos\left(\frac{b}{(c+dx)^2}\right) + 3bf(-de+cf)\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^2}\right) - 3\sqrt{b}(de-cf)^2\sqrt{2\pi}\operatorname{Fr}}{}$$

```
[In] Integrate[(e + f*x)^2*Sin[b/(c + d*x)^2],x]
```

```
[Out] (2*b*c*f^2*Cos[b/(c + d*x)^2] + 2*b*d*f^2*x*Cos[b/(c + d*x)^2] + 3*b*f*(-(d
*e) + c*f)*CosIntegral[b/(c + d*x)^2] - 3*Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*
FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 2*b^(3/2)*f^2*Sqrt[2*Pi]*Fresnel
S[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 3*c*d^2*e^2*Sin[b/(c + d*x)^2] - 3*c^2*
d*e*f*Sin[b/(c + d*x)^2] + c^3*f^2*Sin[b/(c + d*x)^2] + 3*d^3*e^2*x*Sin[b/(
c + d*x)^2] + 3*d^3*e*f*x^2*Sin[b/(c + d*x)^2] + d^3*f^2*x^3*Sin[b/(c + d*x
)^2])/(3*d^3)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{-(cf-de)^2(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+(cf-de)^2\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)+f(cf-de)(dx+c)^2\sin\left(\frac{b}{(dx+c)^2}\right)-f(cf-de)b}{d^3}$
default	$-\frac{-(cf-de)^2(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+(cf-de)^2\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)+f(cf-de)(dx+c)^2\sin\left(\frac{b}{(dx+c)^2}\right)-f(cf-de)b}{d^3}$
risch	$-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)c^2f^2}{2d^3\sqrt{-ib}}+\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)cef}{d^2\sqrt{-ib}}-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)e^2}{2d\sqrt{-ib}}-\frac{b\,\mathrm{Ei}_1\left(-\frac{ib}{(dx+c)^2}\right)c\,f^2}{2d^3}+\frac{b\,\mathrm{Ei}_1\left(-\frac{ib}{(dx+c)^2}\right.}{2d^2}$
parts	$-\frac{\sqrt{\pi}\,\sqrt{b}\,\sqrt{2}\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)f^2x^2}{d}+\sin\left(\frac{b}{(dx+c)^2}\right)f^2x^3-\frac{2\sqrt{\pi}\,\sqrt{b}\,\sqrt{2}\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)efx}{d}+\frac{\sin\left(\frac{b}{(dx+c)^2}\right)c\,f}{d}$

```
[In] int((f*x+e)^2*sin(b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^3*(-(c*f-d*e)^2*(d*x+c)*sin(b/(d*x+c)^2)+(c*f-d*e)^2*b^(1/2)*2^(1/2)*P
i^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))+f*(c*f-d*e)*(d*x+c)^2*si
n(b/(d*x+c)^2)-f*(c*f-d*e)*b*Ci(b/(d*x+c)^2)-1/3*f^2*(d*x+c)^3*sin(b/(d*x+c
)^2)+2/3*f^2*b*(-(d*x+c)*cos(b/(d*x+c)^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS
(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.12

$$\int (e+fx)^2\sin\left(\frac{b}{(c+dx)^2}\right)\,dx$$
$$=\frac{2\sqrt{2}\pi bdf^2\sqrt{\frac{b}{\pi d^2}}\,\mathrm{S}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-3\sqrt{2}\pi(d^3e^2-2\,cd^2ef+c^2df^2)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)+2\left(bdf^2x+bcf^2\right)\mathrm{co}}{}$$

```
[In] integrate((f*x+e)^2*sin(b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(2)*pi*b*d*f^2*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi
*d^2))/(d*x + c)) - 3*sqrt(2)*pi*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sqrt(b
/(pi*d^2))*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) + 2*(b*d*f^2*x
 + b*c*f^2)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(b*d*e*f - b*c*f^2)*cos_in
tegral(b/(d^2*x^2 + 2*c*d*x + c^2)) + (d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*
e^2*x + 3*c*d^2*e^2 - 3*c^2*d*e*f + c^3*f^2)*sin(b/(d^2*x^2 + 2*c*d*x + c^2
)))/d^3
```

Sympy [F]

$$\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right) dx = \int (e+fx)^2 \sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right) dx$$

```
[In] integrate((f*x+e)**2*sin(b/(d*x+c)**2),x)
[Out] Integral((e + f*x)**2*sin(b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right) dx = \int (fx+e)^2 \sin\left(\frac{b}{(dx+c)^2}\right) dx$$

```
[In] integrate((f*x+e)^2*sin(b/(d*x+c)^2),x, algorithm="maxima")
[Out] 1/3*(2*b*f^2*x*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*d^2*integrate(1/3*(2*b^
2*d*f^2*x*sin(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 - 3*(b*d^3*e*f - b*
c*d^2*f^2)*x^2 - 3*(b*d^3*e^2 - b*c^2*d*f^2)*x)*cos(b/(d^2*x^2 + 2*c*d*x +
c^2)))/(d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2), x) - 3*d^2*integrat
e(1/3*(2*b^2*d*f^2*x*sin(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 - 3*(b*d
^3*e*f - b*c*d^2*f^2)*x^2 - 3*(b*d^3*e^2 - b*c^2*d*f^2)*x)*cos(b/(d^2*x^2 +
 2*c*d*x + c^2)))/((d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2)*cos(b/(d
^2*x^2 + 2*c*d*x + c^2))^2 + (d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2
)*sin(b/(d^2*x^2 + 2*c*d*x + c^2))^2), x) + (d^2*f^2*x^3 + 3*d^2*e*f*x^2 +
3*d^2*e^2*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d^2
```

Giac [F]

$$\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right) dx = \int (fx+e)^2 \sin\left(\frac{b}{(dx+c)^2}\right) dx$$

```
[In] integrate((f*x+e)^2*sin(b/(d*x+c)^2),x, algorithm="giac")
[Out] integrate((f*x + e)^2*sin(b/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int \sin\left(\frac{b}{(c+d\,x)^2}\right)\,(e+f\,x)^2\,dx$$

```
[In] int(sin(b/(c + d*x)^2)*(e + f*x)^2,x)
[Out] int(sin(b/(c + d*x)^2)*(e + f*x)^2, x)
```

3.161 $\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 120

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx = -\frac{bf\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2} - \frac{\sqrt{b}(de-cf)\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2} + \frac{(de-cf)(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^2}$$

```
[Out] -1/2*b*f*Ci(b/(d*x+c)^2)/d^2+(-c*f+d*e)*(d*x+c)*sin(b/(d*x+c)^2)/d^2+1/2*f*
(d*x+c)^2*sin(b/(d*x+c)^2)/d^2-(-c*f+d*e)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)
/(d*x+c))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used

= {3514, 3440, 3468, 3433, 3460, 3378, 3383}

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx = -\frac{bf\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2} - \frac{\sqrt{2\pi}\sqrt{b}(de-cf)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2} + \frac{(c+dx)(de-cf)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^2}$$

```
[In] Int[(e + f*x)*Sin[b/(c + d*x)^2],x]
```

```
[Out] -1/2*(b*f*CosIntegral[b/(c + d*x)^2])/d^2 - (Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]
*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d^2 + ((d*e - c*f)*(c + d*x)*Sin
[b/(c + d*x)^2])/d^2 + (f*(c + d*x)^2*Sin[b/(c + d*x)^2])/(2*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(\frac{b}{x^2}\right)+fx\sin\left(\frac{b}{x^2}\right)\right)\,dx,x,c+dx\right)}{d^2}\\
&= \frac{f\text{Subst}\left(\int x\sin\left(\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\text{Subst}\left(\int \sin\left(\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^2}\\
&= -\frac{f\text{Subst}\left(\int \frac{\sin(bx)}{x^2}\,dx,x,\frac{1}{(c+dx)^2}\right)}{2d^2}-\frac{(de-cf)\text{Subst}\left(\int \frac{\sin(bx^2)}{x^2}\,dx,x,\frac{1}{c+dx}\right)}{d^2}\\
&= \frac{(de-cf)(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^2}\\
&\quad -\frac{(bf)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,\frac{1}{(c+dx)^2}\right)}{2d^2}-\frac{(2b(de-cf))\text{Subst}\left(\int \cos\left(bx^2\right)\,dx,x,\frac{1}{c+dx}\right)}{d^2}\\
&= -\frac{bf\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{\sqrt{b}(de-cf)\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2}\\
&\quad +\frac{(de-cf)(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx =$$
$$-\frac{bf\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)+2\sqrt{b}(de-cf)\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+(c+dx)(-2de+cf-dfx)\sin\left(\frac{b}{(c+}\right.}{2d^2}$$

```
[In] Integrate[(e + f*x)*Sin[b/(c + d*x)^2],x]
```

```
[Out] -1/2*(b*f*CosIntegral[b/(c + d*x)^2] + 2*Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*Fre
snelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + (c + d*x)*(-2*d*e + c*f - d*f*x)*Si
n[b/(c + d*x)^2])/d^2
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-(cf-de)(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+(cf-de)\sqrt{b}\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)+\frac{f(dx+c)^2\sin\left(\frac{b}{(dx+c)^2}\right)}{2}-\frac{fb\,\mathrm{Ci}\left(\frac{b}{(dx+c)^2}\right)}{2}}{d^2}$
default	$\frac{-(cf-de)(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+(cf-de)\sqrt{b}\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)+\frac{f(dx+c)^2\sin\left(\frac{b}{(dx+c)^2}\right)}{2}-\frac{fb\,\mathrm{Ci}\left(\frac{b}{(dx+c)^2}\right)}{2}}{d^2}$
risch	$\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)cf}{2d^2\sqrt{-ib}}-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)e}{2d\sqrt{-ib}}+\frac{b\,\mathrm{Ei}_1\left(-\frac{ib}{(dx+c)^2}\right)f}{4d^2}+\frac{b\,\mathrm{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)\sqrt{\pi}\,cf}{2d^2\sqrt{ib}}-\frac{b\,\mathrm{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)\sqrt{\pi}\,e}{2d\sqrt{ib}}+\frac{b\,\mathrm{E}}{}$
parts	$-\frac{\sqrt{b}\sqrt{2}\,\mathrm{C}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)\sqrt{\pi}\,fx}{d}-\frac{\sqrt{b}\sqrt{2}\,\mathrm{C}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)\sqrt{\pi}\,e}{d}+\sin\left(\frac{b}{(dx+c)^2}\right)f\,x^2+\frac{\sin\left(\frac{b}{(dx+c)^2}\right)cfx}{d}+\mathrm{si}$

```
[In] int((f*x+e)*sin(b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(-(c*f-d*e)*(d*x+c)*sin(b/(d*x+c)^2)+(c*f-d*e)*b^(1/2)*2^(1/2)*Pi^(1/
2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))+1/2*f*(d*x+c)^2*sin(b/(d*x+c)
^2)-1/2*f*b*Ci(b/(d*x+c)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx = -\frac{2\sqrt{2}\pi(d^2e-cdf)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)+bf\,\mathrm{Ci}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)-(d^2fx^2+2\,d^2ex+2\,cde-c^2f)\sin\left(\frac{b}{d^2x^2+ \right.}{2\,d^2}$$

```
[In] integrate((f*x+e)*sin(b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(2)*pi*(d^2*e - c*d*f)*sqrt(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*s
qrt(b/(pi*d^2))/(d*x + c)) + b*f*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2))
- (d^2*f*x^2 + 2*d^2*e*x + 2*c*d*e - c^2*f)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)
))/d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int (e+fx)\sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(b/(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)*sin(b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int (fx+e)\sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*e*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)) + integrate(1/2*(b*d*f
*x^2 + 2*b*d*e*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 +
 3*c^2*d*x + c^3), x) + integrate(1/2*(b*d*f*x^2 + 2*b*d*e*x)*cos(b/(d^2*x^
2 + 2*c*d*x + c^2))/((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b/(d^2*x
^2 + 2*c*d*x + c^2))^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin(b/(d
^2*x^2 + 2*c*d*x + c^2))^2), x)
```

Giac [F]

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\int (fx+e)\sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(b/(d*x+c)^2),x, algorithm="giac")
[Out] integrate((f*x + e)*sin(b/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\int \sin\left(\frac{b}{(c+d\,x)^2}\right)\,(e+f\,x)\,dx$$

```
[In] int(sin(b/(c + d*x)^2)*(e + f*x),x)
[Out] int(sin(b/(c + d*x)^2)*(e + f*x), x)
```

3.162 $\int \sin \left(\frac{b}{(c+dx)^2}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \sin \left(\frac{b}{(c+dx)^2}\right) dx = -\frac{\sqrt{b}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d}$$

```
[Out] (d*x+c)*sin(b/(d*x+c)^2)/d-FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*b^(1/
2)*2^(1/2)*Pi^(1/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3440, 3468, 3433}

$$\int \sin \left(\frac{b}{(c+dx)^2}\right) dx = \frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d} - \frac{\sqrt{2\pi}\sqrt{b}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d}$$

```
[In] Int[Sin[b/(c + d*x)^2],x]
```

```
[Out] -((Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d) + ((c +
d*x)*Sin[b/(c + d*x)^2])/d
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sin(bx^2)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d} \\
&= \frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d} - \frac{(2b)\text{Subst}\left(\int \cos(bx^2)\,dx, x, \frac{1}{c+dx}\right)}{d} \\
&= -\frac{\sqrt{b}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \sin\left(\frac{b}{(c+dx)^2}\right)\,dx = -\frac{\sqrt{b}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d}$$

```
[In] Integrate[Sin[b/(c + d*x)^2],x]
```

```
[Out] -((Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d) + ((c +
d*x)*Sin[b/(c + d*x)^2])/d
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$-\frac{-(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)}{d}$	52
default	$-\frac{-(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)}{d}$	52
risch	$-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)}{2d\sqrt{ib}}-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{2d\sqrt{-ib}}-\frac{(-dx-c)\sin\left(\frac{b}{(dx+c)^2}\right)}{d}$	85

```
[In] int(sin(b/(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] -1/d*(-(d*x+c)*sin(b/(d*x+c)^2)+b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2
^(1/2)/Pi^(1/2)/(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \sin\left(\frac{b}{(c+dx)^2}\right)\,dx=-\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d^2}}\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-(dx+c)\sin\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{d}$$

```
[In] integrate(sin(b/(d*x+c)^2),x, algorithm="fricas")
[Out] -(sqrt(2)*pi*d*sqrt(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x
 + c)) - (d*x + c)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]

$$\int \sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\int \sin\left(\frac{b}{(c+dx)^2}\right)\,dx$$

```
[In] integrate(sin(b/(d*x+c)**2),x)
[Out] Integral(sin(b/(c + d*x)**2), x)
```

Maxima [F]

$$\int \sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int \sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

```
[In] integrate(sin(b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*d*integrate(x*cos(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3
*c^2*d*x + c^3), x) + b*d*integrate(x*cos(b/(d^2*x^2 + 2*c*d*x + c^2))/((d^
3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b/(d^2*x^2 + 2*c*d*x + c^2))^2 +
 (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin(b/(d^2*x^2 + 2*c*d*x + c^2))
^2), x) + x*sin(b/(d^2*x^2 + 2*c*d*x + c^2))
```

Giac [F]

$$\int \sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int \sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

```
[In] integrate(sin(b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate(sin(b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 6.94 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \frac{\sin\left(\frac{b}{(c+d\,x)^2}\right)\,(c+d\,x)}{d} - \frac{\sqrt{2}\,\sqrt{b}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}}{\sqrt{\pi}\,(c+d\,x)}\right)}{d}$$

```
[In] int(sin(b/(c + d*x)^2),x)
```

```
[Out] (sin(b/(c + d*x)^2)*(c + d*x))/d - (2^(1/2)*b^(1/2)*pi^(1/2)*fresnelc((2^(1
/2)*b^(1/2))/(pi^(1/2)*(c + d*x))))/d
```

3.163 $\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx} dx = \operatorname{Int}\left(\frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}, x\right)$$

```
[Out] Unintegrable(sin(b/(d*x+c)^2)/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx} dx = \int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx} dx$$

```
[In] Int[Sin[b/(c + d*x)^2]/(e + f*x),x]
[Out] Defer[Int][Sin[b/(c + d*x)^2]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx} dx$$

Mathematica [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

```
[In] Integrate[Sin[b/(c + d*x)^2]/(e + f*x),x]
[Out] Integrate[Sin[b/(c + d*x)^2]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{fx+e}dx$$

```
[In] int(sin(b/(d*x+c)^2)/(f*x+e),x)
[Out] int(sin(b/(d*x+c)^2)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
[Out] integral(sin(b/(d^2*x^2 + 2*c*d*x + c^2))/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 18.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right)}{e+fx}\,dx$$

```
[In] integrate(sin(b/(d*x+c)**2)/(f*x+e),x)
[Out] Integral(sin(b/(c**2 + 2*c*d*x + d**2*x**2))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin(b/(d*x + c)^2)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e),x, algorithm="giac")
[Out] integrate(sin(b/(d*x + c)^2)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 7.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(\frac{b}{(c+d\,x)^2}\right)}{e+f\,x}\,dx$$

```
[In] int(sin(b/(c + d*x)^2)/(e + f*x),x)
[Out] int(sin(b/(c + d*x)^2)/(e + f*x), x)
```

3.164 $\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2} \, dx = \text{Int}\left(\frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}, x\right)$$

```
[Out] Unintegrable(sin(b/(d*x+c)^2)/(f*x+e)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2} \, dx = \int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2} \, dx$$

```
[In] Int[Sin[b/(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[b/(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2} \, dx$$

Mathematica [N/A]

Not integrable

Time = 16.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

```
[In] Integrate[Sin[b/(c + d*x)^2]/(e + f*x)^2,x]
[Out] Integrate[Sin[b/(c + d*x)^2]/(e + f*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}dx$$

```
[In] int(sin(b/(d*x+c)^2)/(f*x+e)^2,x)
[Out] int(sin(b/(d*x+c)^2)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sin(b/(d^2*x^2 + 2*c*d*x + c^2))/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b/(d*x+c)**2)/(f*x+e)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin(b/(d*x + c)^2)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin(b/(d*x + c)^2)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 13.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{b}{(c+d\,x)^2}\right)}{\left(e+f\,x\right)^2}\,dx$$

```
[In] int(sin(b/(c + d*x)^2)/(e + f*x)^2,x)
[Out] int(sin(b/(c + d*x)^2)/(e + f*x)^2, x)
```

3.165 $\int (e+fx)^3 \sin \left(a+b(c+dx)^2\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 341

$$\begin{aligned}\int (e+fx)^3 \sin \left(a+b(c+dx)^2\right) \, dx = & -\frac{3f(de-cf)^2 \cos \left(a+b(c+dx)^2\right)}{2bd^4} \\ & -\frac{3f^2(de-cf)(c+dx) \cos \left(a+b(c+dx)^2\right)}{2bd^4} \\ & -\frac{f^3(c+dx)^2 \cos \left(a+b(c+dx)^2\right)}{2bd^4} \\ & +\frac{3f^2(de-cf)\sqrt{\frac{\pi}{2}} \cos(a) \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4} \\ & +\frac{(de-cf)^3\sqrt{\frac{\pi}{2}} \cos(a) \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4} \\ & +\frac{(de-cf)^3\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right) \sin(a)}{\sqrt{b}d^4} \\ & -\frac{3f^2(de-cf)\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right) \sin(a)}{2b^{3/2}d^4} \\ & +\frac{f^3 \sin \left(a+b(c+dx)^2\right)}{2b^2d^4}\end{aligned}$$

```
[Out] -3/2*f*(-c*f+d*e)^2*cos(a+b*(d*x+c)^2)/b/d^4-3/2*f^2*(-c*f+d*e)*(d*x+c)*cos
(a+b*(d*x+c)^2)/b/d^4-1/2*f^3*(d*x+c)^2*cos(a+b*(d*x+c)^2)/b/d^4+1/2*f^3*si
n(a+b*(d*x+c)^2)/b^2/d^4+3/4*f^2*(-c*f+d*e)*cos(a)*FresnelC((d*x+c)*b^(1/2)
*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/d^4-3/4*f^2*(-c*f+d*e)*FresnelS
((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(3/2)/d^4+1/2*
(-c*f+d*e)^3*cos(a)*FresnelS((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(
1/2)/d^4/b^(1/2)+1/2*(-c*f+d*e)^3*FresnelC((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2)
)*sin(a)*2^(1/2)*Pi^(1/2)/d^4/b^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3514, 3434, 3433, 3432, 3460, 2718, 3466, 3435, 3377, 2717}

$$\int (e+fx)^3\sin\left(a+b(c+dx)^2\right)\,dx=\frac{3\sqrt{\frac{\pi}{2}}f^2\cos(a)(de-cf)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4}-\frac{3\sqrt{\frac{\pi}{2}}f^2\sin(a)(de-cf)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4}+\frac{f^3\sin\left(a+b(c+dx)^2\right)}{2b^2d^4}-\frac{3f^2(c+dx)(de-cf)\cos\left(a+b(c+dx)^2\right)}{2bd^4}+\frac{\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)^3\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)^3\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4}-\frac{3f(de-cf)^2\cos\left(a+b(c+dx)^2\right)}{2bd^4}-\frac{f^3(c+dx)^2\cos\left(a+b(c+dx)^2\right)}{2bd^4}$$

```
[In] Int[(e + f*x)^3*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (-3*f*(d*e - c*f)^2*Cos[a + b*(c + d*x)^2])/(2*b*d^4) - (3*f^2*(d*e - c*f)*
(c + d*x)*Cos[a + b*(c + d*x)^2])/(2*b*d^4) - (f^3*(c + d*x)^2*Cos[a + b*(c
 + d*x)^2])/(2*b*d^4) + (3*f^2*(d*e - c*f)*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[
b]*Sqrt[2/Pi]*(c + d*x)])/(2*b^(3/2)*d^4) + ((d*e - c*f)^3*Sqrt[Pi/2]*Cos[a
]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^4) + ((d*e - c*f)^3*Sq
rt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d^4) - (3*
f^2*(d*e - c*f)*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(
2*b^(3/2)*d^4) + (f^3*Sin[a + b*(c + d*x)^2])/(2*b^2*d^4)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
```

```
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{\text{Subst}\left(\int \left(d^3e^3\left(1-\frac{cf(3d^2e^2-3cdef+c^2f^2)}{d^3e^3}\right)\sin\left(a+bx^2\right)+3d^2e^2f\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)x\sin\left(a+bx^2\right)+3def^2(\right.\right.}{d^4}$$

$$= \frac{f^3\text{Subst}\left(\int x^3\sin\left(a+bx^2\right)\,dx,x,c+dx\right)}{d^4} + \frac{(3f^2(de-cf))\,\text{Subst}\left(\int x^2\sin\left(a+bx^2\right)\,dx,x,c+dx\right)}{d^4} + \frac{(3f(de-cf)^2)\,\text{Subst}\left(\int x\sin\left(a+bx^2\right)\,dx,x,c+dx\right)}{d^4} + \frac{(de-cf)^3\text{Subst}\left(\int \sin\left(a+bx^2\right)\,dx,x,c+dx\right)}{d^4}$$

$$= -\frac{3f^2(de-cf)(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^4} + \frac{f^3\text{Subst}\left(\int x\sin(a+bx)\,dx,x,(c+dx)^2\right)}{2d^4} + \frac{(3f^2(de-cf))\,\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx,x,c+dx\right)}{2bd^4} + \frac{(3f(de-cf)^2)\,\text{Subst}\left(\int \sin(a+bx)\,dx,x,(c+dx)^2\right)}{2d^4} + \frac{((de-cf)^3\cos(a))\,\text{Subst}\left(\int \sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^4} + \frac{((de-cf)^3\sin(a))\,\text{Subst}\left(\int \cos\left(bx^2\right)\,dx,x,c+dx\right)}{d^4}$$

$$= -\frac{3f(de-cf)^2\cos(a+b(c+dx)^2)}{2bd^4} - \frac{3f^2(de-cf)(c+dx)\cos(a+b(c+dx)^2)}{2bd^4} - \frac{f^3(c+dx)^2\cos(a+b(c+dx)^2)}{2bd^4} + \frac{(de-cf)^3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4} + \frac{(de-cf)^3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d^4} + \frac{f^3\text{Subst}\left(\int\cos(a+bx)\,dx,x,(c+dx)^2\right)}{2bd^4} + \frac{(3f^2(de-cf)\cos(a))\,\text{Subst}\left(\int\cos(bx^2)\,dx,x,c+dx\right)}{2bd^4} - \frac{(3f^2(de-cf)\sin(a))\,\text{Subst}\left(\int\sin(bx^2)\,dx,x,c+dx\right)}{2bd^4}$$

$$= -\frac{3f(de-cf)^2\cos(a+b(c+dx)^2)}{2bd^4} - \frac{3f^2(de-cf)(c+dx)\cos(a+b(c+dx)^2)}{2bd^4} - \frac{f^3(c+dx)^2\cos(a+b(c+dx)^2)}{2bd^4} + \frac{3f^2(de-cf)\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4} + \frac{(de-cf)^3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4} + \frac{(de-cf)^3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d^4} - \frac{3f^2(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{2b^{3/2}d^4} + \frac{f^3\sin(a+b(c+dx)^2)}{2b^2d^4}$$

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.64

$$\int(e+fx)^3\sin\left(a+b(c+dx)^2\right)\,dx = \frac{-4bf(c^2f^2-cdf(3e+fx)+d^2(3e^2+3efx+f^2x^2))\cos(a+b(c+dx)^2)+2\sqrt{b}(de-cf)\sqrt{2\pi}\,\text{FresnelS}\left(\right.}{}$$

```
[In] Integrate[(e + f*x)^3*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (-4*b*f*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Cos
[a + b*(c + d*x)^2] + 2*Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqr
```

```
t[2/Pi]*(c + d*x)]*(2*b*(d*e - c*f)^2*Cos[a] - 3*f^2*Sin[a]) + 2*Sqrt[b]*(d
*e - c*f)*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*(3*f^2*Cos[a] +
 2*b*(d*e - c*f)^2*Sin[a]) + 4*f^3*Sin[a + b*(c + d*x)^2])/(8*b^2*d^4)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.02

method	result
risch	$\frac{i\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)\sqrt{\pi}\,e^{3}\mathrm{e}^{ia}}{4\sqrt{-ib}\,d}-\frac{if^{3}\mathrm{e}^{ia}c^{3}\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^{4}\sqrt{-ib}}+\frac{3f^{3}\mathrm{e}^{ia}c\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{8d^{4}b\sqrt{-ib}}-\frac{3if\,e^{2}\mathrm{e}^{ia}c\sqrt{}}{}$
default	Expression too large to display
parts	Expression too large to display

```
[In] int((f*x+e)^3*sin(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))/(-I*b)^(1/2)/d*Pi^(1/2)*e^3
*exp(I*a)-1/4*I*f^3*exp(I*a)*c^3/d^4*Pi^(1/2)/(-I*b)^(1/2)*erf(-d*(-I*b)^(1
/2)*x+I*b*c/(-I*b)^(1/2))+3/8*f^3*exp(I*a)*c/d^4/b*Pi^(1/2)/(-I*b)^(1/2)*er
f(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))-3/4*I*f*e^2*exp(I*a)*c/d^2*Pi^(1/2)
/(-I*b)^(1/2)*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))+3/4*I*f^2*e*exp(I*a
)*c^2/d^3*Pi^(1/2)/(-I*b)^(1/2)*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))-3
/8*f^2*e*exp(I*a)/b/d^3*Pi^(1/2)/(-I*b)^(1/2)*erf(-d*(-I*b)^(1/2)*x+I*b*c/(
-I*b)^(1/2))+1/4*I*exp(-I*a)*e^3*Pi^(1/2)/d/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x
+I*b*c/(I*b)^(1/2))-1/4*I*f^3*exp(-I*a)*c^3/d^4*Pi^(1/2)/(I*b)^(1/2)*erf(d*
(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2))-3/8*f^3*exp(-I*a)*c/d^4/b*Pi^(1/2)/(I*b)^(
1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2))-3/4*I*f*e^2*exp(-I*a)*c/d^2*Pi^
(1/2)/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2))+3/4*I*f^2*e*exp(-I
*a)*c^2/d^3*Pi^(1/2)/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2))+3/8
*f^2*e*exp(-I*a)/b/d^3*Pi^(1/2)/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)
^(1/2))-1/2*f/b*(d^2*f^2*x^2-c*d*f^2*x+3*d^2*e*f*x+c^2*f^2-3*c*d*e*f+3*d^2*
e^2)/d^4*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2*f^3/b^2/d^4*sin(b*d^2*x^2+2*b
*c*d*x+b*c^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.96

$$\int (e+fx)^3\sin\left(a+b(c+dx)^2\right)\,dx$$

$$=\frac{2\,df^{3}\sin\left(bd^{2}x^{2}+2\,bcdx+bc^{2}+a\right)+\sqrt{2}(3\,\pi(def^{2}-cf^{3})\cos\left(a\right)+2\,\pi(bd^{3}e^{3}-3\,bcd^{2}e^{2}f+3\,bc^{2}def^{2}-b}{}$$

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*f^3*sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a) + sqrt(2)*(3*pi*(d*e*f^
2 - c*f^3)*cos(a) + 2*pi*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b
*c^3*f^3)*sin(a))*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x +
c)/d) + sqrt(2)*(2*pi*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^
3*f^3)*cos(a) - 3*pi*(d*e*f^2 - c*f^3)*sin(a))*sqrt(b*d^2/pi)*fresnel_sin(s
qrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^3*f^3*x^2 + 3*b*d^3*e^2*f - 3*b
*c*d^2*e*f^2 + b*c^2*d*f^3 + (3*b*d^3*e*f^2 - b*c*d^2*f^3)*x)*cos(b*d^2*x^2
 + 2*b*c*d*x + b*c^2 + a))/(b^2*d^5)
```

Sympy [F]

$$\int (e+fx)^3 \sin \left(a+b(c+dx)^2\right) \, dx = \int (e+fx)^3 \sin \left(a+bc^2+2bcdx+bd^2x^2\right) dx$$

```
[In] integrate((f*x+e)**3*sin(a+b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**3*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 1824, normalized size of antiderivative = 5.35

$$\int (e+fx)^3 \sin \left(a+b(c+dx)^2\right) \, dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf((I*b*d*x + I*
b*c)/sqrt(I*b)) + (-(I - 1)*cos(a) + (I + 1)*sin(a))*erf((I*b*d*x + I*b*c)/
sqrt(-I*b)))*e^3/(sqrt(b)*d) - 3/8*(2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*
c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^
2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*si
n(a))*d*x - sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*((-(I + 1)*sqrt(2)*sqrt(pi)
*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqr
t(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + ((I -
 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) -
 (I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))
- 1))*sin(a))*c + 2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2
*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x +
I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*c)*e^2*f/(b*
d^3*x + b*c*d^2) + 3/8*(4*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I
```

```
*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*
d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c*d*
x + 4*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c
*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e
^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c^2 - sqrt(b*d^2*x^2 + 2
*b*c*d*x + b*c^2)*(((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*
b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2
- 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + ((I - 1)*sqrt(2)*sqrt(pi)*(erf(sqr
t(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - (I + 1)*sqrt(2)*sqrt(pi)*(er
f(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*b*c^2 + (-(I -
1)*sqrt(2)*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I + 1)*sqrt(2
)*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) + (-(I + 1)*sqrt
(2)*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I - 1)*sqrt(2)*gamma
(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a)))*e*f^2/(b^2*d^4*x + b^
2*c*d^3) - 1/8*(6*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x
^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*
b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c^3 + 2*(3*(
(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x -
I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*
d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c^2 - (I*gamma(2, I*b*d^2*x^2 +
 2*I*b*c*d*x + I*b*c^2) - I*gamma(2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))
*cos(a) - (gamma(2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(2, -I*b*d^
2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*d*x - 2*((I*gamma(2, I*b*d^2*x^2 +
2*I*b*c*d*x + I*b*c^2) - I*gamma(2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*
cos(a) + (gamma(2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(2, -I*b*d^2
*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*c - (((-(I + 1)*sqrt(2)*sqrt(pi)*(er
f(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi
)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + ((I - 1)*
sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - (I
+ 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1)
)*sin(a))*b*c^3 + 3*((-(I - 1)*sqrt(2)*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x
 + I*b*c^2) + (I + 1)*sqrt(2)*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c
^2))*cos(a) + (-(I + 1)*sqrt(2)*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*
c^2) + (I - 1)*sqrt(2)*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*si
n(a))*c)*sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2))*f^3/(b^2*d^5*x + b^2*c*d^4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.53

$$\int (e+fx)^3 \sin \left(a+b(c+dx)^2\right)\,dx$$
$$= \frac{\frac{\sqrt{2}\sqrt{\pi}\left(2\,bd^3e^3-6\,bcd^2e^2f+6\,bc^2def^2-2\,bc^3f^3+3i\,def^2-3i\,cf^3\right)\operatorname{erf}\left(-\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(i\,a)}}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b} - \frac{2i\left(-i\,bd^2f^3\left(x+\frac{c}{d}\right)^2-3\,bd^2ef^2\left(\cdots\right.\right.}{\cdots}}{8\,d^3}$$
$$+ \frac{\frac{\sqrt{2}\sqrt{\pi}\left(2\,bd^3e^3-6\,bcd^2e^2f+6\,bc^2def^2-2\,bc^3f^3-3i\,def^2+3i\,cf^3\right)\operatorname{erf}\left(\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-i\,a)}}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b} - \frac{2i\left(-i\,bd^2f^3\left(x+\frac{c}{d}\right)^2-3\,bd^2\cdots\right.}{\cdots}}{8\,d^3}$$

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*sqrt(pi)*(2*b*d^3*e^3 - 6*b*c*d^2*e^2*f + 6*b*c^2*d*e*f^2 - 2*
b*c^3*f^3 + 3*I*d*e*f^2 - 3*I*c*f^3)*erf(-1/2*I*sqrt(2)*sqrt(b*d^2)*(I*b*d^
2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4)
 + 1)*b) - 2*I*(-I*b*d^2*f^3*(x + c/d)^2 - 3*b*d^2*e*f^2*(I*x + I*c/d) - 3*
b*c*d*f^3*(-I*x - I*c/d) - 3*I*b*d^2*e^2*f + 6*I*b*c*d*e*f^2 - 3*I*b*c^2*f^
3 + f^3)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a)/(b^2*d))/d^3 + 1/8*(
sqrt(2)*sqrt(pi)*(2*b*d^3*e^3 - 6*b*c*d^2*e^2*f + 6*b*c^2*d*e*f^2 - 2*b*c^3
*f^3 - 3*I*d*e*f^2 + 3*I*c*f^3)*erf(1/2*I*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqr
t(b^2*d^4) + 1)*(x + c/d))*e^(-I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) +
1)*b) - 2*I*(-I*b*d^2*f^3*(x + c/d)^2 - 3*b*d^2*e*f^2*(I*x + I*c/d) - 3*b*c
*d*f^3*(-I*x - I*c/d) - 3*I*b*d^2*e^2*f + 6*I*b*c*d*e*f^2 - 3*I*b*c^2*f^3 -
 f^3)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a)/(b^2*d))/d^3
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \sin \left(a+b(c+dx)^2\right)\,dx = \int \sin \left(a+b\,{\left(c+d\,x\right)}^2\right)\,{\left(e+f\,x\right)}^3\,dx$$

```
[In] int(sin(a + b*(c + d*x)^2)*(e + f*x)^3,x)
```

```
[Out] int(sin(a + b*(c + d*x)^2)*(e + f*x)^3, x)
```

3.166 $\int (e+fx)^2 \sin\left(a+b(c+dx)^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 256

$$\begin{aligned}\int (e+fx)^2 \sin\left(a+b(c+dx)^2\right)\,dx = &-\frac{f(de-cf)\cos\left(a+b(c+dx)^2\right)}{bd^3}\\ &-\frac{f^2(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^3}\\ &+\frac{f^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3}\\ &+\frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3}\\ &+\frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d^3}\\ &-\frac{f^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{2b^{3/2}d^3}\end{aligned}$$

```
[Out] -f*(-c*f+d*e)*cos(a+b*(d*x+c)^2)/b/d^3-1/2*f^2*(d*x+c)*cos(a+b*(d*x+c)^2)/b
/d^3+1/4*f^2*cos(a)*FresnelC((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/d^3-1/4*f^2*FresnelS((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*
2^(1/2)*Pi^(1/2)/b^(3/2)/d^3+1/2*(-c*f+d*e)^2*cos(a)*FresnelS((d*x+c)*b^(1/
2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^3/b^(1/2)+1/2*(-c*f+d*e)^2*FresnelC
((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/d^3/b^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3514, 3434, 3433, 3432, 3460, 2718, 3466, 3435}

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^2\right) dx = \frac{\sqrt{\frac{\pi}{2}} f^2 \cos(a) \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3} - \frac{\sqrt{\frac{\pi}{2}} f^2 \sin(a) \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3} + \frac{\sqrt{\frac{\pi}{2}} \sin(a)(de-cf)^2 \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3} + \frac{\sqrt{\frac{\pi}{2}} \cos(a)(de-cf)^2 \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3} - \frac{f(de-cf)\cos\left(a+b(c+dx)^2\right)}{bd^3} - \frac{f^2(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^3}$$

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] -((f*(d*e - c*f)*Cos[a + b*(c + d*x)^2])/(b*d^3)) - (f^2*(c + d*x)*Cos[a + 
b*(c + d*x)^2])/(2*b*d^3) + (f^2*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/
Pi]*(c + d*x)])/(2*b^(3/2)*d^3) + ((d*e - c*f)^2*Sqrt[Pi/2]*Cos[a]*FresnelS
[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^3) + ((d*e - c*f)^2*Sqrt[Pi/2]*F
resnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d^3) - (f^2*Sqrt[Pi/
2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(2*b^(3/2)*d^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(a+bx^2\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(a+bx^2\right)+f^2x^2\sin\left(a+bx^2\right)\right)\,dx,x,$$

$$d^3$$

$$= \frac{f^2\text{Subst}\left(\int x^2\sin\left(a+bx^2\right)\,dx,x,c+dx\right)}{d^3} + \frac{(2f(de-cf))\text{Subst}\left(\int x\sin\left(a+bx^2\right)\,dx,x,c+dx\right)}{d^3} + \frac{(de-cf)^2\text{Subst}\left(\int \sin\left(a+bx^2\right)\,dx,x,c+dx\right)}{d^3}$$

$$= -\frac{f^2(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^3} + \frac{f^2\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, c+dx\right)}{2bd^3} + \frac{(f(de-cf))\text{Subst}\left(\int \sin(a+bx)\,dx, x, (c+dx)^2\right)}{d^3} + \frac{\left((de-cf)^2\cos(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, c+dx\right)}{d^3} + \frac{\left((de-cf)^2\sin(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, c+dx\right)}{d^3}$$

$$= -\frac{f(de-cf)\cos\left(a+b(c+dx)^2\right)}{bd^3} - \frac{f^2(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^3} + \frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3} + \frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d^3} + \frac{\left(f^2\cos(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, c+dx\right)}{2bd^3} - \frac{\left(f^2\sin(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, c+dx\right)}{2bd^3}$$

$$= -\frac{f(de-cf)\cos\left(a+b(c+dx)^2\right)}{bd^3} - \frac{f^2(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^3} + \frac{f^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3} + \frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3} + \frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d^3} - \frac{f^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{2b^{3/2}d^3}$$

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.59

$$\int (e+fx)^2\sin\left(a+b(c+dx)^2\right)\,dx$$
$$=\frac{-4\sqrt{b}f(2de-cf+dfx)\cos\left(a+b(c+dx)^2\right)+2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\left(2b(de-cf)^2\cos(a)-\right.}{8b^{3/2}d^3}$$

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (-4*Sqrt[b]*f*(2*d*e - c*f + d*f*x)*Cos[a + b*(c + d*x)^2] + 2*Sqrt[2*Pi]*F
resnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*(2*b*(d*e - c*f)^2*Cos[a] - f^2*Sin[a
]) + 2*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*(f^2*Cos[a] + 2*b*
(d*e - c*f)^2*Sin[a]))/(8*b^(3/2)*d^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.71

method	result
risch	$\frac{i\,\text{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)\sqrt{\pi}\,e^2 e^{ia}}{4\sqrt{-ib}\,d}+\frac{if^2 e^{ia}c^2\sqrt{\pi}\,\text{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^3\sqrt{-ib}}-\frac{f^2 e^{ia}\sqrt{\pi}\,\text{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{8b\,d^3\sqrt{-ib}}-\frac{ife\,e^{ia}c\sqrt{\pi}\,\text{erf}}{2d}$
default	$-\frac{f^2x\cos\left(d^2x^2b+2cdxb+c^2b+a\right)}{2b\,d^2}-\frac{f^2c\left(-\frac{\cos\left(d^2x^2b+2cdxb+c^2b+a\right)}{2b\,d^2}-\frac{c\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{b^2c^2d^2-\left(c^2b+a\right)b\,d^2}{b\,d^2}\right)\text{S}\left(\frac{\sqrt{2}\left(b\,d^2x+cdb\right)}{\sqrt{\pi}\sqrt{b\,d^2}}\right)-\sin\right.}{2d\sqrt{b\,d^2}}\right.}{d}$
parts	Expression too large to display

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))/(-I*b)^(1/2)/d*Pi^(1/2)*e^2
*exp(I*a)+1/4*I*f^2*exp(I*a)*c^2/d^3*Pi^(1/2)/(-I*b)^(1/2)*erf(-d*(-I*b)^(1
/2)*x+I*b*c/(-I*b)^(1/2))-1/8*f^2*exp(I*a)/b/d^3*Pi^(1/2)/(-I*b)^(1/2)*erf(
-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))-1/2*I*f*e*exp(I*a)*c/d^2*Pi^(1/2)/(-I
*b)^(1/2)*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))+1/4*I*exp(-I*a)*e^2*Pi^
(1/2)/d/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2))+1/4*I*f^2*exp(-I
*a)*c^2/d^3*Pi^(1/2)/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2))+1/8
*f^2*exp(-I*a)/b/d^3*Pi^(1/2)/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(
1/2))-1/2*I*f*e*exp(-I*a)*c/d^2*Pi^(1/2)/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*
b*c/(I*b)^(1/2))+2*(1/2*I*f^2*(1/2*I*x/d^2/b-1/2*I/b/d^3*c)-1/2*e*f/d^2/b)*
cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.81

$$\int (e+fx)^2\sin\left(a+b(c+dx)^2\right)\,dx$$
$$=\frac{\sqrt{2}(\pi f^2\cos{(a)}+2\,\pi(bd^2e^2-2\,bcdef+bc^2f^2)\sin{(a)})\sqrt{\frac{bd^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)-\sqrt{2}(\pi f^2\sin{(a)}-2\,\pi(bd^2e}{}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(pi*f^2*cos(a) + 2*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*si
n(a))*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - sqrt
(2)*(pi*f^2*sin(a) - 2*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cos(a))*sqr
t(b*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^2*f^2*
x + 2*b*d^2*e*f - b*c*d*f^2)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(b^2*d
^4)
```

Sympy [F]

$$\int (e+fx)^2\sin\left(a+b(c+dx)^2\right)\,dx=\int (e+fx)^2\sin\left(a+bc^2+2bcdx+bd^2x^2\right)dx$$

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 1038, normalized size of antiderivative = 4.05

$$\int (e+fx)^2\sin\left(a+b(c+dx)^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf((I*b*d*x + I*
b*c)/sqrt(I*b)) + (-(I - 1)*cos(a) + (I + 1)*sin(a))*erf((I*b*d*x + I*b*c)/
sqrt(-I*b)))*e^2/(sqrt(b)*d) - 1/4*(2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*
c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^
2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*si
```

```
n(a))*d*x - sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*((-(I + 1)*sqrt(2)*sqrt(pi)
*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqr
t(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + ((I -
 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) -
 (I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))
- 1))*sin(a))*c + 2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2
*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x +
I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*c)*e*f/(b*d^
3*x + b*c*d^2) + 1/8*(4*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b
*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*
x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c*d*x
+ 4*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d
*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(
-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c^2 - sqrt(b*d^2*x^2 + 2*b
*c*d*x + b*c^2)*(((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*
c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 -
2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + ((I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(
I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - (I + 1)*sqrt(2)*sqrt(pi)*(erf(
sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*b*c^2 + (-(I - 1)
*sqrt(2)*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I + 1)*sqrt(2)*
gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) + (-(I + 1)*sqrt(2
)*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I - 1)*sqrt(2)*gamma(3
/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a)))*f^2/(b^2*d^4*x + b^2*c*
d^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.36

$$\int (e+fx)^2\sin \left(a+b(c+dx)^2\right)\,dx = -\frac{\frac{i\sqrt{2}\sqrt{\pi}\left(2i\,bd^2e^2-4i\,bcdef+2i\,bc^2f^2-f^2\right)\operatorname{erf}\left(-\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(i\,a)}}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}+\frac{2\left(df^2\left(x+\frac{c}{d}\right)+2\,def-2\,cf^2\right)e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc\cdots\right.}}{bd}}{8\,d^2} - \frac{-\frac{i\sqrt{2}\sqrt{\pi}\left(-2i\,bd^2e^2+4i\,bcdef-2i\,bc^2f^2-f^2\right)\operatorname{erf}\left(\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-i\,a)}}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}+\frac{2\left(df^2\left(x+\frac{c}{d}\right)+2\,def-2\,cf^2\right)e^{\left(-i\,bd^2x^2-2i\,b\cdots\right.}}{bd}}{8\,d^2}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(I*sqrt(2)*sqrt(pi)*(2*I*b*d^2*e^2 - 4*I*b*c*d*e*f + 2*I*b*c^2*f^2 - f
^2)*erf(-1/2*I*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e
```

```
^(I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) + 2*(d*f^2*(x + c/d) + 2
*d*e*f - 2*c*f^2)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a)/(b*d))/d^2
- 1/8*(-I*sqrt(2)*sqrt(pi)*(-2*I*b*d^2*e^2 + 4*I*b*c*d*e*f - 2*I*b*c^2*f^2
- f^2)*erf(1/2*I*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d)
)*e^(-I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b) + 2*(d*f^2*(x + c/d
) + 2*d*e*f - 2*c*f^2)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a)/(b*d)
)/d^2
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^2\right) \, dx = \int \sin \left(a+b\,(c+d\,x)^2\right)\,(e+f\,x)^2 \,dx$$

```
[In] int(sin(a + b*(c + d*x)^2)*(e + f*x)^2,x)
```

```
[Out] int(sin(a + b*(c + d*x)^2)*(e + f*x)^2, x)
```

3.167 $\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 122

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx = -\frac{f\cos\left(a+b(c+dx)^2\right)}{2bd^2} + \frac{(de-cf)\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2} + \frac{(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d^2}$$

```
[Out] -1/2*f*cos(a+b*(d*x+c)^2)/b/d^2+1/2*(-c*f+d*e)*cos(a)*FresnelS((d*x+c)*b^(1
/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^2/b^(1/2)+1/2*(-c*f+d*e)*FresnelC(
(d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/d^2/b^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3514, 3434, 3433, 3432, 3460, 2718}

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx = \frac{\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2} + \frac{\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2} - \frac{f\cos\left(a+b(c+dx)^2\right)}{2bd^2}$$

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] -1/2*(f*Cos[a + b*(c + d*x)^2])/(b*d^2) + ((d*e - c*f)*Sqrt[Pi/2]*Cos[a]*Fr
esnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^2) + ((d*e - c*f)*Sqrt[Pi/
2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(a+bx^2\right)+fx\sin\left(a+bx^2\right)\right)\,dx,x,c+dx\right)}{d^2}$$

$$= \frac{f\mathrm{Subst}\left(\int x\sin\left(a+bx^2\right)\,dx,x,c+dx\right)}{d^2} + \frac{(de-cf)\mathrm{Subst}\left(\int \sin\left(a+bx^2\right)\,dx,x,c+dx\right)}{d^2}$$

$$= \frac{f\mathrm{Subst}\left(\int \sin(a+bx)\,dx,x,(c+dx)^2\right)}{2d^2} + \frac{((de-cf)\cos(a))\mathrm{Subst}\left(\int \sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^2} + \frac{((de-cf)\sin(a))\mathrm{Subst}\left(\int \cos\left(bx^2\right)\,dx,x,c+dx\right)}{d^2}$$

$$= -\frac{f\cos\left(a+b(c+dx)^2\right)}{2bd^2} + \frac{(de-cf)\sqrt{\frac{\pi}{2}}\cos(a)\,\mathrm{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2} + \frac{(de-cf)\sqrt{\frac{\pi}{2}}\,\mathrm{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d^2}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx$$

$$= \frac{-f\cos\left(a+b(c+dx)^2\right)+\sqrt{b}(de-cf)\sqrt{2\pi}\cos(a)\,\mathrm{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)+\sqrt{b}(de-cf)\sqrt{2\pi}\,\mathrm{Fresne}}{2bd^2}$$

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (-(f*Cos[a + b*(c + d*x)^2]) + Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*Fresne
lS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)] + Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*FresnelC[
Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(2*b*d^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.71

method	result
risch	$\frac{i\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)\sqrt{\pi}\,e\,\mathrm{e}^{ia}}{4\sqrt{-ib}\,d}-\frac{if\,\mathrm{e}^{ia}c\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^2\sqrt{-ib}}+\frac{i\mathrm{e}^{-ia}e\sqrt{\pi}\,\mathrm{erf}\left(d\sqrt{ib}\,x+\frac{ibc}{\sqrt{ib}}\right)}{4d\sqrt{ib}}-\frac{if\,\mathrm{e}^{-ia}c\sqrt{\pi}\,\mathrm{erf}\left(d\sqrt{ib}\,x\right.}{4d^2\sqrt{ib}}$
default	$-\frac{f\cos\left(d^2x^2b+2cdxb+c^2b+a\right)}{2b\,d^2}-\frac{fc\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{b^2c^2d^2-\left(c^2b+a\right)b\,d^2}{b\,d^2}\right)\mathrm{S}\left(\frac{\sqrt{2}\left(b\,d^2x+cdb\right)}{\sqrt{\pi}\sqrt{b\,d^2}}\right)-\sin\left(\frac{b^2c^2d^2-\left(c^2b+a\right)b\,d^2}{b\,d^2}\right)\mathrm{C}\left(\frac{\sqrt{2}\left(b\right.}{\sqrt{\pi}}\right.\right.}{2d\sqrt{b\,d^2}}$
parts	$\frac{\sqrt{2}\sqrt{\pi}\cos\left(\frac{b^2c^2d^2-\left(c^2b+a\right)b\,d^2}{b\,d^2}\right)\mathrm{S}\left(\frac{\sqrt{2}\left(b\,d^2x+cdb\right)}{\sqrt{\pi}\sqrt{b\,d^2}}\right)fx}{2\sqrt{b\,d^2}}-\frac{\sqrt{2}\sqrt{\pi}\sin\left(\frac{b^2c^2d^2-\left(c^2b+a\right)b\,d^2}{b\,d^2}\right)\mathrm{C}\left(\frac{\sqrt{2}\left(b\,d^2x+cdb\right)}{\sqrt{\pi}\sqrt{b\,d^2}}\right)fx}{2\sqrt{b\,d^2}}+\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{c}}{}$

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))/(-I*b)^(1/2)/d*Pi^(1/2)*e*e
xp(I*a)-1/4*I*f*exp(I*a)*c/d^2*Pi^(1/2)/(-I*b)^(1/2)*erf(-d*(-I*b)^(1/2)*x+
I*b*c/(-I*b)^(1/2))+1/4*I*exp(-I*a)*e*Pi^(1/2)/d/(I*b)^(1/2)*erf(d*(I*b)^(1
/2)*x+I*b*c/(I*b)^(1/2))-1/4*I*f*exp(-I*a)*c/d^2*Pi^(1/2)/(I*b)^(1/2)*erf(d
*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2))-1/2*f/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2
+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx$$
$$=\frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}(de-cf)\cos(a)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)+\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}(de-cf)\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)\sin(a)-df\cos\left(bd^2x\right.}{2\,bd^3}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*sqrt(b*d^2/pi)*(d*e - c*f)*cos(a)*fresnel_sin(sqrt(2)*sqrt(
b*d^2/pi)*(d*x + c)/d) + sqrt(2)*pi*sqrt(b*d^2/pi)*(d*e - c*f)*fresnel_cos(
sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d)*sin(a) - d*f*cos(b*d^2*x^2 + 2*b*c*d*x
+ b*c^2 + a))/(b*d^3)
```

Sympy [F]

$$\int (e+fx)\sin \left(a+b(c+dx)^2\right)\,dx=\int (e+fx)\sin \left(a+bc^2+2bcdx+bd^2x^2\right)dx$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**2),x)
[Out] Integral((e + f*x)*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.96

$$\int (e+fx)\sin \left(a+b(c+dx)^2\right)\,dx=$$

$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\cos(a)+(i-1)\sin(a)\right)\operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{i\,b}}\right)+\left(-(i-1)\cos(a)+(i+1)\sin(a)\right)\operatorname{erf}\left(\frac{i\,bd}{\sqrt{}}\right.\cdots}{8\sqrt{b}d}$$

$$-\frac{\left(2\left(\left(e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}+e^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}\right)\cos(a)-\left(-i\,e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}+i\,e^{\left(-i\,bd^2x^2-2i\,bcdx-\cdots\right.}\right.\right.\cdots}{\cdots}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^2),x, algorithm="maxima")
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf((I*b*d*x + I*
b*c)/sqrt(I*b)) + (-(I - 1)*cos(a) + (I + 1)*sin(a))*erf((I*b*d*x + I*b*c)/
sqrt(-I*b)))*e/(sqrt(b)*d) - 1/8*(2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^
2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2
+ 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(
a))*d*x - sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*((-(I + 1)*sqrt(2)*sqrt(pi)*(
erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(
pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + ((I - 1
)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - (
I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
1))*sin(a))*c + 2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x
^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*
b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*c)*f/(b*d^3*x
+ b*c*d^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.04

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx$$
$$= -\frac{-\frac{i\sqrt{2}\sqrt{\pi}(-i\,de+i\,cf)\,\mathrm{erf}\left(-\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(i\,a)}}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2+i\,a\right)}}{bd}}{4\,d}$$
$$-\frac{\frac{i\sqrt{2}\sqrt{\pi}(i\,de-i\,cf)\,\mathrm{erf}\left(\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-i\,a)}}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2-i\,a\right)}}{bd}}{4\,d}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*(-I*sqrt(2)*sqrt(pi)*(-I*d*e + I*c*f)*erf(-1/2*I*sqrt(2)*sqrt(b*d^2)*(
I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^
2*d^4) + 1)) + f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a)/(b*d))/d - 1
/4*(I*sqrt(2)*sqrt(pi)*(I*d*e - I*c*f)*erf(1/2*I*sqrt(2)*sqrt(b*d^2)*(-I*b*
d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(-I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*
d^4) + 1)) + f*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a)/(b*d))/d
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx = \int \sin\left(a+b\,(c+d\,x)^2\right)\,(e+f\,x)\,dx$$

```
[In] int(sin(a + b*(c + d*x)^2)*(e + f*x),x)
```

```
[Out] int(sin(a + b*(c + d*x)^2)*(e + f*x), x)
```

3.168 $\int \sin \left(a+b(c+dx)^2\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \sin \left(a+b(c+dx)^2\right) \, dx=\frac{\sqrt{\frac{\pi}{2}} \cos(a) \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right) \sin(a)}{\sqrt{b}d}$$

```
[Out] 1/2*cos(a)*FresnelS((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d/b^
(1/2)+1/2*FresnelC((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2
)/d/b^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3434, 3433, 3432}

$$\int \sin \left(a+b(c+dx)^2\right) \, dx=\frac{\sqrt{\frac{\pi}{2}} \sin(a) \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{2}} \cos(a) \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

```
[In] Int[Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d) + (S
qrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\text{integral} = \cos(a)\int \sin\left(b(c+dx)^2\right)\,dx + \sin(a)\int \cos\left(b(c+dx)^2\right)\,dx$$

$$= \frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d} + \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \sin\left(a+b(c+dx)^2\right)\,dx = \frac{\sqrt{\frac{\pi}{2}}\left(\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)+\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)\right)}{\sqrt{b}d}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^2],x]
[Out] (Sqrt[Pi/2]*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)] + FresnelC[Sqrt[
b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a]))/(Sqrt[b]*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

method	result	size
risch	$\frac{i\sqrt{\pi}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(d\sqrt{ib}\,x+\frac{ibc}{\sqrt{ib}}\right)}{4d\sqrt{ib}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d\sqrt{-ib}}$	87
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{b^2c^2d^2-\left(c^2b+a\right)b\,d^2}{b\,d^2}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\left(b\,d^2x+cdb\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)-\sin\left(\frac{b^2c^2d^2-\left(c^2b+a\right)b\,d^2}{b\,d^2}\right)\mathrm{C}\left(\frac{\sqrt{2}\,\left(b\,d^2x+cdb\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)\right)}{2\sqrt{b\,d^2}}$	136

```
[In] int(sin(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*exp(-I*a)/d/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2
))+1/4*I*Pi^(1/2)*exp(I*a)/d/(-I*b)^(1/2)*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b
)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

$$\int \sin\left(a+b(c+dx)^2\right)\,dx = \frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}\cos\left(a\right)\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)+\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)\sin\left(a\right)}{2\,bd^2}$$

```
[In] integrate(sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*sqrt(b*d^2/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d
*x + c)/d) + sqrt(2)*pi*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(
d*x + c)/d)*sin(a))/(b*d^2)
```

Sympy [F]

$$\int \sin\left(a+b(c+dx)^2\right)\,dx = \int \sin\left(a+b(c+dx)^2\right)dx$$

```
[In] integrate(sin(a+b*(d*x+c)**2),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \sin \left(a+b(c+dx)^2\right)\,dx =$$

$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\cos(a)+(i-1)\sin(a)\right)\operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{i\,b}}\right)+\left(-(i-1)\cos(a)+(i+1)\sin(a)\right)\operatorname{erf}\left(\frac{i\,bdx-}{\sqrt{-}}\right.}{8\sqrt{b}d}$$

```
[In] integrate(sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf((I*b*d*x + I*
b*c)/sqrt(I*b)) + (-(I - 1)*cos(a) + (I + 1)*sin(a))*erf((I*b*d*x + I*b*c)/
sqrt(-I*b)))/(sqrt(b)*d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.82

$$\int \sin \left(a+b(c+dx)^2\right)\,dx = \frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(i\,a)}}{4\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)} + \frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}i\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-i\,a)}}{4\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}$$

```
[In] integrate(sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(pi)*erf(-1/2*I*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4)
+ 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 1/4*sqr
t(2)*sqrt(pi)*erf(1/2*I*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x
 + c/d))*e^(-I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \sin \left(a+b(c+dx)^2\right) \, dx = \frac{\sqrt{2}\,\sqrt{\pi}\,\cos \left(a\right)\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{b\,d^2}}\,\left(b\,x\,d^2+b\,c\,d\right)}{\sqrt{\pi}}\right)\,\sqrt{\frac{1}{b\,d^2}}}{2} + \frac{\sqrt{2}\,\sqrt{\pi}\,\sin \left(a\right)\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{b\,d^2}}\,\left(b\,x\,d^2+b\,c\,d\right)}{\sqrt{\pi}}\right)\,\sqrt{\frac{1}{b\,d^2}}}{2}$$

```
[In] int(sin(a + b*(c + d*x)^2),x)
```

```
[Out] (2^(1/2)*pi^(1/2)*cos(a)*fresnels((2^(1/2)*(1/(b*d^2))^(1/2)*(b*c*d + b*d^2
*x))/pi^(1/2))*(1/(b*d^2))^(1/2))/2 + (2^(1/2)*pi^(1/2)*sin(a)*fresnelc((2^
(1/2)*(1/(b*d^2))^(1/2)*(b*c*d + b*d^2*x))/pi^(1/2))*(1/(b*d^2))^(1/2))/2
```

3.169 $\int \frac{\sin(a+b(c+dx)^2)}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin(a+b(c+dx)^2)}{e+fx}\,dx = \operatorname{Int}\left(\frac{\sin(a+b(c+dx)^2)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^2)/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin(a+b(c+dx)^2)}{e+fx}\,dx = \int \frac{\sin(a+b(c+dx)^2)}{e+fx}\,dx$$

```
[In] Int[Sin[a + b*(c + d*x)^2]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^2]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin(a+b(c+dx)^2)}{e+fx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+b(c+dx)^2)}{e+fx}\,dx = \int \frac{\sin(a+b(c+dx)^2)}{e+fx}\,dx$$

```
[In] Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x),x]
[Out] Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(dx+c)^2\right)}{fx+e}dx$$

```
[In] int(sin(a+b*(d*x+c)^2)/(f*x+e),x)
[Out] int(sin(a+b*(d*x+c)^2)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\sin(a+b(c+dx)^2)}{e+fx}\,dx = \int \frac{\sin\left((dx+c)^2b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
[Out] integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{\sin(a+b(c+dx)^2)}{e+fx}\,dx = \int \frac{\sin(a+bc^2+2bcdx+bd^2x^2)}{e+fx}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**2)/(f*x+e),x)
[Out] Integral(sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin (a + b(c + dx)^2)}{e + fx} \, dx = \int \frac{\sin \left((dx + c)^2 b + a \right)}{fx + e} \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^2*b + a)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin (a + b(c + dx)^2)}{e + fx} \, dx = \int \frac{\sin \left((dx + c)^2 b + a \right)}{fx + e} \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^2*b + a)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 6.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin (a + b(c + dx)^2)}{e + fx} \, dx = \int \frac{\sin \left(a + b \, (c + d \, x)^2 \right)}{e + f \, x} \, dx$$

```
[In] int(sin(a + b*(c + d*x)^2)/(e + f*x),x)
[Out] int(sin(a + b*(c + d*x)^2)/(e + f*x), x)
```

3.170 $\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx = \operatorname{Int}\left(\frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx$$

```
[In] Int[Sin[a + b*(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin (a+b(c+dx)^2)}{(e+fx)^2}\,dx=\int \frac{\sin (a+b(c+dx)^2)}{(e+fx)^2}\,dx$$

```
[In] Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(dx+c)^{2}\right)}{\left(fx+e\right)^{2}}dx$$

```
[In] int(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x)
[Out] int(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{\sin (a+b(c+dx)^2)}{(e+fx)^2}\,dx=\int \frac{\sin \left((dx+c)^{2}b+a\right)}{\left(fx+e\right)^{2}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(f^2*x^2 + 2*e*f*x + e^2),
x)
```

Sympy [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+bc^2+2bcdx+bd^2x^2\right)}{\left(e+fx\right)^2}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**2)/(f*x+e)**2,x)
[Out] Integral(sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left((dx+c)^2b+a\right)}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^2*b + a)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 6.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left((dx+c)^2b+a\right)}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin((d*x + c)^2*b + a)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin (a+b(c+dx)^2)}{(e+fx)^2} \, dx = \int \frac{\sin \left(a+b\,(c+d\,x)^2\right)}{(e+f\,x)^2} \,dx$$

```
[In] int(sin(a + b*(c + d*x)^2)/(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^2)/(e + f*x)^2, x)
```

3.171 $\int (e+fx)^3 \sin\left(a+b(c+dx)^3\right)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 434

$$
\begin{aligned}
\int (e+fx)^3 \sin\left(a+b(c+dx)^3\right)\, dx = & -\frac{f^2(de-cf)\cos\left(a+b(c+dx)^3\right)}{bd^4} \\
& -\frac{f^3(c+dx)\cos\left(a+b(c+dx)^3\right)}{3bd^4} \\
& -\frac{e^{ia}f^3(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{18bd^4\sqrt[3]{-ib(c+dx)^3}} \\
& +\frac{ie^{ia}(de-cf)^3(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^4\sqrt[3]{-ib(c+dx)^3}} \\
& -\frac{e^{-ia}f^3(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{18bd^4\sqrt[3]{ib(c+dx)^3}} \\
& -\frac{ie^{-ia}(de-cf)^3(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^4\sqrt[3]{ib(c+dx)^3}} \\
& +\frac{ie^{ia}f(de-cf)^2(c+dx)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{2d^4\left(-ib(c+dx)^3\right)^{2/3}} \\
& -\frac{ie^{-ia}f(de-cf)^2(c+dx)^2\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{2d^4\left(ib(c+dx)^3\right)^{2/3}}
\end{aligned}
$$

```
[Out] -f^2*(-c*f+d*e)*cos(a+b*(d*x+c)^3)/b/d^4-1/3*f^3*(d*x+c)*cos(a+b*(d*x+c)^3)
/b/d^4-1/18*exp(I*a)*f^3*(d*x+c)*GAMMA(1/3,-I*b*(d*x+c)^3)/b/d^4/(-I*b*(d*x
+c)^3)^(1/3)+1/6*I*exp(I*a)*(-c*f+d*e)^3*(d*x+c)*GAMMA(1/3,-I*b*(d*x+c)^3)/
d^4/(-I*b*(d*x+c)^3)^(1/3)-1/18*f^3*(d*x+c)*GAMMA(1/3,I*b*(d*x+c)^3)/b/d^4/
exp(I*a)/(I*b*(d*x+c)^3)^(1/3)-1/6*I*(-c*f+d*e)^3*(d*x+c)*GAMMA(1/3,I*b*(d*
x+c)^3)/d^4/exp(I*a)/(I*b*(d*x+c)^3)^(1/3)+1/2*I*exp(I*a)*f*(-c*f+d*e)^2*(d
*x+c)^2*GAMMA(2/3,-I*b*(d*x+c)^3)/d^4/(-I*b*(d*x+c)^3)^(2/3)-1/2*I*f*(-c*f+
d*e)^2*(d*x+c)^2*GAMMA(2/3,I*b*(d*x+c)^3)/d^4/exp(I*a)/(I*b*(d*x+c)^3)^(2/3
)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3514, 3436, 2239, 3470, 2250, 3460, 2718, 3466, 3437}

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^3\right) dx = -\frac{f^2(de-cf)\cos\left(a+b(c+dx)^3\right)}{bd^4} + \frac{ie^{ia}f(c+dx)^2(de-cf)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{2d^4\left(-ib(c+dx)^3\right)^{2/3}} - \frac{ie^{-ia}f(c+dx)^2(de-cf)^2\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{2d^4\left(ib(c+dx)^3\right)^{2/3}} + \frac{ie^{ia}(c+dx)(de-cf)^3\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^4\sqrt[3]{-ib(c+dx)^3}} - \frac{ie^{-ia}(c+dx)(de-cf)^3\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^4\sqrt[3]{ib(c+dx)^3}} - \frac{f^3(c+dx)\cos\left(a+b(c+dx)^3\right)}{3bd^4} - \frac{e^{ia}f^3(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{18bd^4\sqrt[3]{-ib(c+dx)^3}} - \frac{e^{-ia}f^3(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{18bd^4\sqrt[3]{ib(c+dx)^3}}$$

```
[In] Int[(e + f*x)^3*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] -((f^2*(d*e - c*f)*Cos[a + b*(c + d*x)^3])/(b*d^4)) - (f^3*(c + d*x)*Cos[a
+ b*(c + d*x)^3])/(3*b*d^4) - (E^(I*a)*f^3*(c + d*x)*Gamma[1/3, (-I)*b*(c +
 d*x)^3])/(18*b*d^4*((-I)*b*(c + d*x)^3)^(1/3)) + ((I/6)*E^(I*a)*(d*e - c*f
)^3*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d^4*((-I)*b*(c + d*x)^3)^(1/
3)) - (f^3*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(18*b*d^4*E^(I*a)*(I*b*(c
 + d*x)^3)^(1/3)) - ((I/6)*(d*e - c*f)^3*(c + d*x)*Gamma[1/3, I*b*(c + d*x)
^3])/(d^4*E^(I*a)*(I*b*(c + d*x)^3)^(1/3)) + ((I/2)*E^(I*a)*f*(d*e - c*f)^2
*(c + d*x)^2*Gamma[2/3, (-I)*b*(c + d*x)^3])/(d^4*((-I)*b*(c + d*x)^3)^(2/3
)) - ((I/2)*f*(d*e - c*f)^2*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(d^4*E
^(I*a)*(I*b*(c + d*x)^3)^(2/3))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
```

```
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{\text{Subst}\left(\int \left(d^3e^3\left(1-\frac{cf(3d^2e^2-3cdef+c^2f^2)}{d^3e^3}\right)\sin\left(a+bx^3\right)+3d^2e^2f\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)x\sin\left(a+bx^3\right)+3def^2(1\right.}{d^4}$$

$$= \frac{f^3\text{Subst}\left(\int x^3\sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^4} + \frac{(3f^2(de-cf))\,\text{Subst}\left(\int x^2\sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^4} + \frac{(3f(de-cf)^2)\,\text{Subst}\left(\int x\sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^4} + \frac{(de-cf)^3\text{Subst}\left(\int \sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^4}$$

$$= -\frac{f^3(c+dx)\cos\left(a+b(c+dx)^3\right)}{3bd^4} + \frac{f^3\text{Subst}\left(\int \cos\left(a+bx^3\right)\,dx,x,c+dx\right)}{3bd^4} + \frac{(f^2(de-cf))\,\text{Subst}\left(\int \sin(a+bx)\,dx,x,(c+dx)^3\right)}{d^4} + \frac{(3if(de-cf)^2)\,\text{Subst}\left(\int e^{-ia-ibx^3}x\,dx,x,c+dx\right)}{2d^4} - \frac{(3if(de-cf)^2)\,\text{Subst}\left(\int e^{ia+ibx^3}x\,dx,x,c+dx\right)}{2d^4} + \frac{(i(de-cf)^3)\,\text{Subst}\left(\int e^{-ia-ibx^3}\,dx,x,c+dx\right)}{2d^4} - \frac{(i(de-cf)^3)\,\text{Subst}\left(\int e^{ia+ibx^3}\,dx,x,c+dx\right)}{2d^4}$$

$$
\begin{aligned}
&= -\frac{f^2(de-cf)\cos\left(a+b(c+dx)^3\right)}{bd^4} - \frac{f^3(c+dx)\cos\left(a+b(c+dx)^3\right)}{3bd^4} \\
&+ \frac{ie^{ia}(de-cf)^3(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^4\sqrt[3]{-ib(c+dx)^3}} \\
&- \frac{ie^{-ia}(de-cf)^3(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^4\sqrt[3]{ib(c+dx)^3}} \\
&+ \frac{ie^{ia}f(de-cf)^2(c+dx)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{2d^4\left(-ib(c+dx)^3\right)^{2/3}} \\
&- \frac{ie^{-ia}f(de-cf)^2(c+dx)^2\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{2d^4\left(ib(c+dx)^3\right)^{2/3}} \\
&+ \frac{f^3\text{Subst}\left(\int e^{-ia-ibx^3}\,dx,x,c+dx\right)}{6bd^4} + \frac{f^3\text{Subst}\left(\int e^{ia+ibx^3}\,dx,x,c+dx\right)}{6bd^4} \\
&= -\frac{f^2(de-cf)\cos\left(a+b(c+dx)^3\right)}{bd^4} - \frac{f^3(c+dx)\cos\left(a+b(c+dx)^3\right)}{3bd^4} \\
&- \frac{e^{ia}f^3(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{18bd^4\sqrt[3]{-ib(c+dx)^3}} + \frac{ie^{ia}(de-cf)^3(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^4\sqrt[3]{-ib(c+dx)^3}} \\
&- \frac{e^{-ia}f^3(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{18bd^4\sqrt[3]{ib(c+dx)^3}} - \frac{ie^{-ia}(de-cf)^3(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^4\sqrt[3]{ib(c+dx)^3}} \\
&+ \frac{ie^{ia}f(de-cf)^2(c+dx)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{2d^4\left(-ib(c+dx)^3\right)^{2/3}} \\
&- \frac{ie^{-ia}f(de-cf)^2(c+dx)^2\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{2d^4\left(ib(c+dx)^3\right)^{2/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 13.94 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.81

$$
\int (e+fx)^3\sin\left(a+b(c+dx)^3\right)\,dx
$$

$$
= \frac{-6f^2(3de-2cf+dfx)\cos\left(a+bc^3\right)\cos\left(bdx(3c^2+3cdx+d^2x^2)\right) + \frac{(c+dx)\left(-\left(\left(f^3+3ib(de-cf)^3\right)\sqrt[3]{ib(c+dx)}\right.\right.}{}}{}
$$

```
[In] Integrate[(e + f*x)^3*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] (-6*f^2*(3*d*e - 2*c*f + d*f*x)*Cos[a + b*c^3]*Cos[b*d*x*(3*c^2 + 3*c*d*x +
 d^2*x^2)] + ((c + d*x)*(-((f^3 + (3*I)*b*(d*e - c*f)^3)*(I*b*(c + d*x)^3)^
(1/3)*Gamma[1/3, I*b*(c + d*x)^3]) - (9*I)*b*f*(d*e - c*f)^2*(c + d*x)*Gamm
a[2/3, I*b*(c + d*x)^3])*(Cos[a] - I*Sin[a]))/(I*b*(c + d*x)^3)^(2/3) + ((c
 + d*x)*(-((f^3 - (3*I)*b*(d*e - c*f)^3)*((-I)*b*(c + d*x)^3)^(1/3)*Gamma[1
```

```
/3, (-I)*b*(c + d*x)^3]) + (9*I)*b*f*(d*e - c*f)^2*(c + d*x)*Gamma[2/3, (-I
)*b*(c + d*x)^3])*(Cos[a] + I*Sin[a]))/((-I)*b*(c + d*x)^3)^(2/3) + 6*f^2*(
3*d*e - 2*c*f + d*f*x)*Sin[a + b*c^3]*Sin[b*d*x*(3*c^2 + 3*c*d*x + d^2*x^2)
])/(18*b*d^4)
```

Maple [F]

$$\int (fx+e)^3 \sin \left(a+b(dx+c)^3\right) dx$$

```
[In] int((f*x+e)^3*sin(a+b*(d*x+c)^3),x)
[Out] int((f*x+e)^3*sin(a+b*(d*x+c)^3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.36

$$\int (e+fx)^3 \sin \left(a+b(c+dx)^3\right) \, dx =$$
$$-\frac{(i\,bd^3)^{\frac{2}{3}} \left(\left(3\,bd^3e^3 - 9\,bcd^2e^2f + 9\,bc^2def^2 - 3\,bc^3f^3 - i\,f^3\right)\cos\left(a\right) - \left(3i\,bd^3e^3 - 9i\,bcd^2e^2f + 9i\,bc^2def^2 \right.\right.}{}$$

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^3),x, algorithm="fricas")
[Out] -1/18*((I*b*d^3)^(2/3)*((3*b*d^3*e^3 - 9*b*c*d^2*e^2*f + 9*b*c^2*d*e*f^2 -
3*b*c^3*f^3 - I*f^3)*cos(a) - (3*I*b*d^3*e^3 - 9*I*b*c*d^2*e^2*f + 9*I*b*c^
2*d*e*f^2 - 3*I*b*c^3*f^3 + f^3)*sin(a))*gamma(1/3, I*b*d^3*x^3 + 3*I*b*c*d
^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(2/3)*((3*b*d^3*e^3 - 9*b*c*
d^2*e^2*f + 9*b*c^2*d*e*f^2 - 3*b*c^3*f^3 + I*f^3)*cos(a) - (-3*I*b*d^3*e^3
 + 9*I*b*c*d^2*e^2*f - 9*I*b*c^2*d*e*f^2 + 3*I*b*c^3*f^3 + f^3)*sin(a))*gam
ma(1/3, -I*b*d^3*x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3) + 9*(I*b*
d^3)^(1/3)*((b*d^3*e^2*f - 2*b*c*d^2*e*f^2 + b*c^2*d*f^3)*cos(a) + (-I*b*d^
3*e^2*f + 2*I*b*c*d^2*e*f^2 - I*b*c^2*d*f^3)*sin(a))*gamma(2/3, I*b*d^3*x^3
 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + 9*(-I*b*d^3)^(1/3)*((b*d^3*
e^2*f - 2*b*c*d^2*e*f^2 + b*c^2*d*f^3)*cos(a) + (I*b*d^3*e^2*f - 2*I*b*c*d^
2*e*f^2 + I*b*c^2*d*f^3)*sin(a))*gamma(2/3, -I*b*d^3*x^3 - 3*I*b*c*d^2*x^2
- 3*I*b*c^2*d*x - I*b*c^3) + 6*(b*d^3*f^3*x + 3*b*d^3*e*f^2 - 2*b*c*d^2*f^3
)*cos(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(b^2*d^6)
```

Sympy [F]

$$\int (e+fx)^3 \sin \left(a+b(c+dx)^3\right)\, dx = \int (e+fx)^3 \sin \left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right) dx$$

```
[In] integrate((f*x+e)**3*sin(a+b*(d*x+c)**3),x)
[Out] Integral((e + f*x)**3*sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d
**3*x**3), x)
```

Maxima [F]

$$\int (e+fx)^3 \sin \left(a+b(c+dx)^3\right)\, dx = \int (fx+e)^3 \sin \left((dx+c)^3 b+a\right)\, dx$$

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^3),x, algorithm="maxima")
[Out] integrate((f*x + e)^3*sin((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int (e+fx)^3 \sin \left(a+b(c+dx)^3\right)\, dx = \int (fx+e)^3 \sin \left((dx+c)^3 b+a\right)\, dx$$

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^3),x, algorithm="giac")
[Out] integrate((f*x + e)^3*sin((d*x + c)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \sin \left(a+b(c+dx)^3\right)\, dx = \int \sin \left(a+b\,(c+d\,x)^3\right)\,(e+f\,x)^3\, dx$$

```
[In] int(sin(a + b*(c + d*x)^3)*(e + f*x)^3,x)
[Out] int(sin(a + b*(c + d*x)^3)*(e + f*x)^3, x)
```

3.172 $\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 280

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right) \, dx = -\frac{f^2 \cos \left(a+b(c+dx)^3\right)}{3bd^3} + \frac{ie^{ia}(de-cf)^2(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^3\sqrt[3]{-ib(c+dx)^3}} - \frac{ie^{-ia}(de-cf)^2(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^3\sqrt[3]{ib(c+dx)^3}} + \frac{ie^{ia}f(de-cf)(c+dx)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{3d^3\left(-ib(c+dx)^3\right)^{2/3}} - \frac{ie^{-ia}f(de-cf)(c+dx)^2\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{3d^3\left(ib(c+dx)^3\right)^{2/3}}$$

```
[Out] -1/3*f^2*cos(a+b*(d*x+c)^3)/b/d^3+1/6*I*exp(I*a)*(-c*f+d*e)^2*(d*x+c)*GAMMA
(1/3,-I*b*(d*x+c)^3)/d^3/(-I*b*(d*x+c)^3)^(1/3)-1/6*I*(-c*f+d*e)^2*(d*x+c)*
GAMMA(1/3,I*b*(d*x+c)^3)/d^3/exp(I*a)/(I*b*(d*x+c)^3)^(1/3)+1/3*I*exp(I*a)*
f*(-c*f+d*e)*(d*x+c)^2*GAMMA(2/3,-I*b*(d*x+c)^3)/d^3/(-I*b*(d*x+c)^3)^(2/3)
-1/3*I*f*(-c*f+d*e)*(d*x+c)^2*GAMMA(2/3,I*b*(d*x+c)^3)/d^3/exp(I*a)/(I*b*(d
*x+c)^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3514, 3436, 2239, 3470, 2250, 3460, 2718}

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right) \, dx = \frac{ie^{ia} f(c+dx)^2 (de-cf)\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{3d^3 \left(-ib(c+dx)^3\right)^{2/3}} - \frac{ie^{-ia} f(c+dx)^2 (de-cf)\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{3d^3 \left(ib(c+dx)^3\right)^{2/3}} + \frac{ie^{ia}(c+dx)(de-cf)^2\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^3 \sqrt[3]{-ib(c+dx)^3}} - \frac{ie^{-ia}(c+dx)(de-cf)^2\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^3 \sqrt[3]{ib(c+dx)^3}} - \frac{f^2 \cos\left(a+b(c+dx)^3\right)}{3bd^3}$$

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] -1/3*(f^2*Cos[a + b*(c + d*x)^3])/(b*d^3) + ((I/6)*E^(I*a)*(d*e - c*f)^2*(c
 + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d^3*((-I)*b*(c + d*x)^3)^(1/3)) -
((I/6)*(d*e - c*f)^2*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(d^3*E^(I*a)*(I
*b*(c + d*x)^3)^(1/3)) + ((I/3)*E^(I*a)*f*(d*e - c*f)*(c + d*x)^2*Gamma[2/3
, (-I)*b*(c + d*x)^3])/(d^3*((-I)*b*(c + d*x)^3)^(2/3)) - ((I/3)*f*(d*e - c
*f)*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(d^3*E^(I*a)*(I*b*(c + d*x)^3)
^(2/3))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(a+bx^3\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(a+bx^3\right)+f^2x^2\sin\left(a+bx^3\right)\right)\,dx,x,c}{d^3}$$

$$= \frac{f^2\text{Subst}\left(\int x^2\sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^3} + \frac{(2f(de-cf))\text{Subst}\left(\int x\sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^3} + \frac{(de-cf)^2\text{Subst}\left(\int \sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^3}$$

$$
\begin{aligned}
&= \frac{f^2 \text{Subst}\left(\int \sin(a+bx)\,dx, x, (c+dx)^3\right)}{3d^3} \\
&\quad + \frac{(if(de-cf))\text{Subst}\left(\int e^{-ia-ibx^3} x\,dx, x, c+dx\right)}{d^3} \\
&\quad - \frac{(if(de-cf))\text{Subst}\left(\int e^{ia+ibx^3} x\,dx, x, c+dx\right)}{d^3} \\
&\quad + \frac{\left(i(de-cf)^2\right)\text{Subst}\left(\int e^{-ia-ibx^3}\,dx, x, c+dx\right)}{2d^3} \\
&\quad - \frac{\left(i(de-cf)^2\right)\text{Subst}\left(\int e^{ia+ibx^3}\,dx, x, c+dx\right)}{2d^3} \\
&= -\frac{f^2 \cos\left(a+b(c+dx)^3\right)}{3bd^3} + \frac{ie^{ia}(de-cf)^2(c+dx)\Gamma\left(\frac{1}{3}, -ib(c+dx)^3\right)}{6d^3\sqrt[3]{-ib(c+dx)^3}} \\
&\quad - \frac{ie^{-ia}(de-cf)^2(c+dx)\Gamma\left(\frac{1}{3}, ib(c+dx)^3\right)}{6d^3\sqrt[3]{ib(c+dx)^3}} \\
&\quad + \frac{ie^{ia}f(de-cf)(c+dx)^2\Gamma\left(\frac{2}{3}, -ib(c+dx)^3\right)}{3d^3\left(-ib(c+dx)^3\right)^{2/3}} \\
&\quad - \frac{ie^{-ia}f(de-cf)(c+dx)^2\Gamma\left(\frac{2}{3}, ib(c+dx)^3\right)}{3d^3\left(ib(c+dx)^3\right)^{2/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.74 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.07

$$
\int (e+fx)^2 \sin\left(a+b(c+dx)^3\right)\,dx
$$

$$
= \frac{-\frac{2f^2\cos(a+bc^3)\cos(bdx(3c^2+3cdx+d^2x^2))}{b} + \frac{(de-cf)(c+dx)\left((de-cf)\sqrt[3]{ib(c+dx)^3}\Gamma\left(\frac{1}{3}, ib(c+dx)^3\right)+2f(c+dx)\Gamma\left(\frac{2}{3}, ib(c+dx)^3\right)\right)(-\cdots}{(ib(c+dx)^3)^{2/3}}}{\cdots}
$$

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] ((-2*f^2*Cos[a + b*c^3]*Cos[b*d*x*(3*c^2 + 3*c*d*x + d^2*x^2)])/b + ((d*e -
 c*f)*(c + d*x)*((d*e - c*f)*(I*b*(c + d*x)^3)^(1/3)*Gamma[1/3, I*b*(c + d*
x)^3] + 2*f*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^3])*((-I)*Cos[a] - Sin[a]))/
(I*b*(c + d*x)^3)^(2/3) + ((d*e - c*f)*(c + d*x)*((d*e - c*f)*((-I)*b*(c +
d*x)^3)^(1/3)*Gamma[1/3, (-I)*b*(c + d*x)^3] + 2*f*(c + d*x)*Gamma[2/3, (-I
)*b*(c + d*x)^3])*(I*Cos[a] - Sin[a]))/((-I)*b*(c + d*x)^3)^(2/3) + (2*f^2*
Sin[a + b*c^3]*Sin[b*d*x*(3*c^2 + 3*c*d*x + d^2*x^2)])/b)/(6*d^3)
```

Maple [F]

$$\int (fx+e)^2 \sin \left(a+b(dx+c)^3\right) dx$$

[In] int((f*x+e)^2*sin(a+b*(d*x+c)^3),x)

[Out] int((f*x+e)^2*sin(a+b*(d*x+c)^3),x)

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.46

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right)\, dx =$$

$$-\frac{2\,d^2f^2\cos\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)+\left(i\,bd^3\right)^{\frac{2}{3}}\left(\left(d^2e^2-2\,cdef+c^2f^2\right)\cos\left(a\right)-\left(i\,d^2e^2-2i\right.\right.}{}$$

[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^3),x, algorithm="fricas")

[Out] -1/6*(2*d^2*f^2*cos(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) + (I*b*d^3)^(2/3)*((d^2*e^2 - 2*c*d*e*f + c^2*f^2)*cos(a) - (I*d^2*e^2 - 2*I*c*d*e*f + I*c^2*f^2)*sin(a))*gamma(1/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(2/3)*((d^2*e^2 - 2*c*d*e*f + c^2*f^2)*cos(a) - (-I*d^2*e^2 + 2*I*c*d*e*f - I*c^2*f^2)*sin(a))*gamma(1/3, -I*b*d^3*x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3) + 2*(I*b*d^3)^(1/3)*((d^2*e*f - c*d*f^2)*cos(a) + (-I*d^2*e*f + I*c*d*f^2)*sin(a))*gamma(2/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + 2*(-I*b*d^3)^(1/3)*((d^2*e*f - c*d*f^2)*cos(a) + (I*d^2*e*f - I*c*d*f^2)*sin(a))*gamma(2/3, -I*b*d^3*x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3))/(b*d^5)

Sympy [F]

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right)\, dx = \int (e+fx)^2 \sin \left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right) dx$$

[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**3),x)

[Out] Integral((e + f*x)**2*sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3), x)

Maxima [F]

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right) \, dx = \int (fx+e)^2 \sin \left((dx+c)^3 b+a\right) \, dx$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^3),x, algorithm="maxima")
[Out] integrate((f*x + e)^2*sin((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right) \, dx = \int (fx+e)^2 \sin \left((dx+c)^3 b+a\right) \, dx$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^3),x, algorithm="giac")
[Out] integrate((f*x + e)^2*sin((d*x + c)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right) \, dx = \int \sin \left(a+b\,(c+d\,x)^3\right)\,(e+f\,x)^2 \, dx$$

```
[In] int(sin(a + b*(c + d*x)^3)*(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^3)*(e + f*x)^2, x)
```

3.173 $\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 235

$$\begin{aligned}\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx =& \frac{ie^{ia}(de-cf)(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^2\sqrt[3]{-ib(c+dx)^3}}\\&-\frac{ie^{-ia}(de-cf)(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^2\sqrt[3]{ib(c+dx)^3}}\\&+\frac{ie^{ia}f(c+dx)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{6d^2\left(-ib(c+dx)^3\right)^{2/3}}\\&-\frac{ie^{-ia}f(c+dx)^2\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{6d^2\left(ib(c+dx)^3\right)^{2/3}}\end{aligned}$$

```
[Out] 1/6*I*exp(I*a)*(-c*f+d*e)*(d*x+c)*GAMMA(1/3,-I*b*(d*x+c)^3)/d^2/(-I*b*(d*x+
c)^3)^(1/3)-1/6*I*(-c*f+d*e)*(d*x+c)*GAMMA(1/3,I*b*(d*x+c)^3)/d^2/exp(I*a)/
(I*b*(d*x+c)^3)^(1/3)+1/6*I*exp(I*a)*f*(d*x+c)^2*GAMMA(2/3,-I*b*(d*x+c)^3)/
d^2/(-I*b*(d*x+c)^3)^(2/3)-1/6*I*f*(d*x+c)^2*GAMMA(2/3,I*b*(d*x+c)^3)/d^2/e
xp(I*a)/(I*b*(d*x+c)^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used

= {3514, 3436, 2239, 3470, 2250}

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx = \frac{ie^{ia}(c+dx)(de-cf)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^2\sqrt[3]{-ib(c+dx)^3}} - \frac{ie^{-ia}(c+dx)(de-cf)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^2\sqrt[3]{ib(c+dx)^3}} + \frac{ie^{ia}f(c+dx)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{6d^2\left(-ib(c+dx)^3\right)^{2/3}} - \frac{ie^{-ia}f(c+dx)^2\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{6d^2\left(ib(c+dx)^3\right)^{2/3}}$$

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] ((I/6)*E^(I*a)*(d*e - c*f)*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d^2*(
(-I)*b*(c + d*x)^3)^(1/3)) - ((I/6)*(d*e - c*f)*(c + d*x)*Gamma[1/3, I*b*(c
 + d*x)^3])/(d^2*E^(I*a)*(I*b*(c + d*x)^3)^(1/3)) + ((I/6)*E^(I*a)*f*(c + d
*x)^2*Gamma[2/3, (-I)*b*(c + d*x)^3])/(d^2*((-I)*b*(c + d*x)^3)^(2/3)) - ((
I/6)*f*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(d^2*E^(I*a)*(I*b*(c + d*x)
^3)^(2/3))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(a+bx^3\right)+fx\sin\left(a+bx^3\right)\right)\,dx,x,c+dx\right)}{d^2}\\
&= \frac{f\text{Subst}\left(\int x\sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\text{Subst}\left(\int \sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^2}\\
&= \frac{(if)\text{Subst}\left(\int e^{-ia-ibx^3}x\,dx,x,c+dx\right)}{2d^2}-\frac{(if)\text{Subst}\left(\int e^{ia+ibx^3}x\,dx,x,c+dx\right)}{2d^2}\\
&\quad+\frac{(i(de-cf))\text{Subst}\left(\int e^{-ia-ibx^3}\,dx,x,c+dx\right)}{2d^2}\\
&\quad-\frac{(i(de-cf))\text{Subst}\left(\int e^{ia+ibx^3}\,dx,x,c+dx\right)}{2d^2}\\
&= \frac{ie^{ia}(de-cf)(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^2\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(de-cf)(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^2\sqrt[3]{ib(c+dx)^3}}\\
&\quad+\frac{ie^{ia}f(c+dx)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{6d^2\left(-ib(c+dx)^3\right)^{2/3}}-\frac{ie^{-ia}f(c+dx)^2\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{6d^2\left(ib(c+dx)^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.63 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.62

$$\begin{aligned}
\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx &= -\frac{ie\cos(a)\left(-\frac{(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{3\sqrt[3]{-ib(c+dx)^3}}+\frac{(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{3\sqrt[3]{ib(c+dx)^3}}\right)}{2d}\\
&\quad+\frac{e\left(-\frac{(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{3\sqrt[3]{-ib(c+dx)^3}}-\frac{(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{3\sqrt[3]{ib(c+dx)^3}}\right)\sin(a)}{2d}\\
&\quad+\frac{f(c+dx)\left(c\sqrt[3]{-ib(c+dx)^3}\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)-(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)\right)(-i\cos(a)+\sin(a))}{6d^2\left(-ib(c+dx)^3\right)^{2/3}}\\
&\quad+\frac{f(c+dx)\left(c\sqrt[3]{ib(c+dx)^3}\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)-(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)\right)(i\cos(a)+\sin(a))}{6d^2\left(ib(c+dx)^3\right)^{2/3}}
\end{aligned}$$

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] ((-1/2*I)*e*Cos[a]*(-1/3*((c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/((-I)*b
*(c + d*x)^3)^(1/3) + ((c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(3*(I*b*(c +
d*x)^3)^(1/3))))/d + (e*(-1/3*((c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/((
-I)*b*(c + d*x)^3)^(1/3) - ((c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(3*(I*b*
(c + d*x)^3)^(1/3)))*Sin[a])/(2*d) + (f*(c + d*x)*(c*((-I)*b*(c + d*x)^3)^(
1/3)*Gamma[1/3, (-I)*b*(c + d*x)^3] - (c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)
^3])*((-I)*Cos[a] + Sin[a]))/(6*d^2*((-I)*b*(c + d*x)^3)^(2/3)) + (f*(c + d
*x)*(c*(I*b*(c + d*x)^3)^(1/3)*Gamma[1/3, I*b*(c + d*x)^3] - (c + d*x)*Gamm
a[2/3, I*b*(c + d*x)^3])*(I*Cos[a] + Sin[a]))/(6*d^2*(I*b*(c + d*x)^3)^(2/3
))
```

Maple [F]

$$\int (fx+e)\sin\left(a+b(dx+c)^3\right)dx$$

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^3),x)
```

```
[Out] int((f*x+e)*sin(a+b*(d*x+c)^3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.12

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx = -\frac{(i\,bd^3)^{\frac{2}{3}}\left((de-cf)\cos(a)-(i\,de-i\,cf)\sin(a)\right)\Gamma\left(\frac{1}{3}, i\,bd^3x^3+3i\,bcd^2x^2+3i\,bc^2dx+i\,bc^3\right)+(-i\,bd^3)^{\frac{2}{3}}}{}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*((I*b*d^3)^(2/3)*((d*e - c*f)*cos(a) - (I*d*e - I*c*f)*sin(a))*gamma(1
/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(
2/3)*((d*e - c*f)*cos(a) - (-I*d*e + I*c*f)*sin(a))*gamma(1/3, -I*b*d^3*x^3
 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3) + (I*b*d^3)^(1/3)*(d*f*cos(a)
 - I*d*f*sin(a))*gamma(2/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x +
 I*b*c^3) + (-I*b*d^3)^(1/3)*(d*f*cos(a) + I*d*f*sin(a))*gamma(2/3, -I*b*d^
3*x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3))/(b*d^4)
```

Sympy [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx=\int (e+fx)\sin\left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right)dx$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**3),x)
[Out] Integral((e + f*x)*sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3
*x**3), x)
```

Maxima [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx=\int (fx+e)\sin\left((dx+c)^3b+a\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^3),x, algorithm="maxima")
[Out] integrate((f*x + e)*sin((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx=\int (fx+e)\sin\left((dx+c)^3b+a\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^3),x, algorithm="giac")
[Out] integrate((f*x + e)*sin((d*x + c)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^3\right)\,(e+f\,x)\,dx$$

```
[In] int(sin(a + b*(c + d*x)^3)*(e + f*x),x)
[Out] int(sin(a + b*(c + d*x)^3)*(e + f*x), x)
```

3.174 $\int \sin\left(a+b(c+dx)^3\right)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 107

$$\int \sin\left(a+b(c+dx)^3\right)\, dx=\frac{ie^{ia}(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d\sqrt[3]{ib(c+dx)^3}}$$

```
[Out] 1/6*I*exp(I*a)*(d*x+c)*GAMMA(1/3,-I*b*(d*x+c)^3)/d/(-I*b*(d*x+c)^3)^(1/3)-1
/6*I*(d*x+c)*GAMMA(1/3,I*b*(d*x+c)^3)/d/exp(I*a)/(I*b*(d*x+c)^3)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3436, 2239}

$$\int \sin\left(a+b(c+dx)^3\right)\, dx=\frac{ie^{ia}(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d\sqrt[3]{ib(c+dx)^3}}$$

```
[In] Int[Sin[a + b*(c + d*x)^3],x]
```

```
[Out] ((I/6)*E^(I*a)*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d*((-I)*b*(c + d*
x)^3)^(1/3)) - ((I/6)*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(d*E^(I*a)*(I*
b*(c + d*x)^3)^(1/3))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}i\int e^{-ia-ib(c+dx)^3}\,dx - \frac{1}{2}i\int e^{ia+ib(c+dx)^3}\,dx\\ &= \frac{ie^{ia}(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d\sqrt[3]{-ib(c+dx)^3}} - \frac{ie^{-ia}(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d\sqrt[3]{ib(c+dx)^3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int \sin\left(a+b(c+dx)^3\right)\,dx = \frac{i(c+dx)\left(-\sqrt[3]{-ib(c+dx)^3}\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)(\cos(a)-i\sin(a))+\sqrt[3]{ib(c+dx)^3}\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)(\cos(a\right.}{6d\sqrt[3]{b^2(c+dx)^6}}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^3],x]
[Out] ((I/6)*(c + d*x)*(-(((-I)*b*(c + d*x)^3)^(1/3)*Gamma[1/3, I*b*(c + d*x)^3]*
(Cos[a] - I*Sin[a])) + (I*b*(c + d*x)^3)^(1/3)*Gamma[1/3, (-I)*b*(c + d*x)^
3]*(Cos[a] + I*Sin[a])))/(d*(b^2*(c + d*x)^6)^(1/3))
```

Maple [F]

$$\int \sin\left(a+b(dx+c)^3\right)dx$$

```
[In] int(sin(a+b*(d*x+c)^3),x)
[Out] int(sin(a+b*(d*x+c)^3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int \sin \left(a+b(c+dx)^3\right)\,dx =$$
$$-\frac{\left(i\,bd^3\right)^{\frac{2}{3}}\left(\cos\left(a\right)-i\,\sin\left(a\right)\right)\Gamma\left(\frac{1}{3},i\,bd^3x^3+3i\,bcd^2x^2+3i\,bc^2dx+i\,bc^3\right)+\left(-i\,bd^3\right)^{\frac{2}{3}}\left(\cos\left(a\right)+i\,\sin\left(a\right)\right)\Gamma}{6\,bd^3}$$

```
[In] integrate(sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*((I*b*d^3)^(2/3)*(cos(a) - I*sin(a))*gamma(1/3, I*b*d^3*x^3 + 3*I*b*c*
d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(2/3)*(cos(a) + I*sin(a))*g
amma(1/3, -I*b*d^3*x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3))/(b*d^3
)
```

Sympy [F]

$$\int \sin \left(a+b(c+dx)^3\right)\,dx = \int \sin \left(a+b(c+dx)^3\right)dx$$

```
[In] integrate(sin(a+b*(d*x+c)**3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**3), x)
```

Maxima [F]

$$\int \sin \left(a+b(c+dx)^3\right)\,dx = \int \sin \left((dx+c)^3b+a\right)\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int \sin \left(a+b(c+dx)^3\right) \, dx = \int \sin \left((dx+c)^3 b+a\right) \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^3),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin \left(a+b(c+dx)^3\right) \, dx = \int \sin \left(a+b\,(c+d\,x)^3\right) \, dx$$

```
[In] int(sin(a + b*(c + d*x)^3),x)
[Out] int(sin(a + b*(c + d*x)^3), x)
```

3.175 $\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx = \text{Int}\left(\frac{\sin\left(a+b(c+dx)^3\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^3)/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx = \int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$$

```
[In] Int[Sin[a + b*(c + d*x)^3]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^3]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+b(c+dx)^3)}{e+fx}\,dx=\int \frac{\sin(a+b(c+dx)^3)}{e+fx}\,dx$$

```
[In] Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x),x]
[Out] Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(dx+c)^3\right)}{fx+e}dx$$

```
[In] int(sin(a+b*(d*x+c)^3)/(f*x+e),x)
[Out] int(sin(a+b*(d*x+c)^3)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{\sin(a+b(c+dx)^3)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^3b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e),x, algorithm="fricas")
[Out] integral(sin(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(f*x + e)
, x)
```

Sympy [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{\sin \left(a+b(c+dx)^3\right)}{e+fx}\,dx=\int \frac{\sin \left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right)}{e+fx}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**3)/(f*x+e),x)
[Out] Integral(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(e
+ f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin \left(a+b(c+dx)^3\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^3b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^3*b + a)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin \left(a+b(c+dx)^3\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^3b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^3*b + a)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 6.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+b(c+dx)^3)}{e+fx}\,dx=\int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^3\right)}{e+f\,x}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^3)/(e + f*x),x)
[Out] int(sin(a + b*(c + d*x)^3)/(e + f*x), x)
```

3.176 $\int \frac{\sin(a+b(c+dx)^3)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx = \text{Int}\left(\frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx$$

```
[In] Int[Sin[a + b*(c + d*x)^3]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^3]/(e + f*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 43.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+b(c+dx)^3)}{(e+fx)^2}\,dx = \int \frac{\sin(a+b(c+dx)^3)}{(e+fx)^2}\,dx$$

```
[In] Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(dx+c)^3\right)}{(fx+e)^2}dx$$

```
[In] int(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x)
[Out] int(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{\sin(a+b(c+dx)^3)}{(e+fx)^2}\,dx = \int \frac{\sin\left((dx+c)^3 b+a\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sin(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(f^2*x^2
+ 2*e*f*x + e^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{\sin(a + b(c + dx)^3)}{(e + fx)^2} \, dx = \int \frac{\sin(a + bc^3 + 3bc^2dx + 3bcd^2x^2 + bd^3x^3)}{(e + fx)^2} \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)**3)/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(e
+ f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a + b(c + dx)^3)}{(e + fx)^2} \, dx = \int \frac{\sin\left((dx + c)^3 b + a\right)}{(fx + e)^2} \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^3*b + a)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 3.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a + b(c + dx)^3)}{(e + fx)^2} \, dx = \int \frac{\sin\left((dx + c)^3 b + a\right)}{(fx + e)^2} \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^3*b + a)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+b(c+dx)^3)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+b\,(c+d\,x)^3\right)}{\left(e+f\,x\right)^2}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^3)/(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^3)/(e + f*x)^2, x)
```

3.177 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^2}\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 371

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = \frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3} - \frac{bf(de-cf)\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3} - \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3} + \frac{2b^{3/2}f^2\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3} + \frac{2b^{3/2}f^2\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{3d^3} + \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d^3} + \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3} + \frac{bf(de-cf)\sin(a)\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{d^3}$$

```
[Out] -b*f*(-c*f+d*e)*Ci(b/(d*x+c)^2)*cos(a)/d^3+2/3*b*f^2*(d*x+c)*cos(a+b/(d*x+c
)^2)/d^3+b*f*(-c*f+d*e)*Si(b/(d*x+c)^2)*sin(a)/d^3+(-c*f+d*e)^2*(d*x+c)*sin
(a+b/(d*x+c)^2)/d^3+f*(-c*f+d*e)*(d*x+c)^2*sin(a+b/(d*x+c)^2)/d^3+1/3*f^2*(
d*x+c)^3*sin(a+b/(d*x+c)^2)/d^3+2/3*b^(3/2)*f^2*cos(a)*FresnelS(b^(1/2)*2^(
1/2)/Pi^(1/2)/(d*x+c))*2^(1/2)*Pi^(1/2)/d^3+2/3*b^(3/2)*f^2*FresnelC(b^(1/2
)*2^(1/2)/Pi^(1/2)/(d*x+c))*sin(a)*2^(1/2)*Pi^(1/2)/d^3-(-c*f+d*e)^2*cos(a)
*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^3+(-
c*f+d*e)^2*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*sin(a)*b^(1/2)*2^(1/2
)*Pi^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3514, 3440, 3468, 3435, 3433, 3432, 3460, 3378, 3384, 3380, 3383, 3490, 3469, 3434}

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)dx=\frac{2\sqrt{2\pi}b^{3/2}f^2\sin(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3}+\frac{2\sqrt{2\pi}b^{3/2}f^2\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3}-\frac{bf\cos(a)(de-cf)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-\frac{\sqrt{2\pi}\sqrt{b}\cos(a)(de-cf)^2\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3}+\frac{\sqrt{2\pi}\sqrt{b}\sin(a)(de-cf)^2\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3}+\frac{bf\sin(a)(de-cf)\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{f(c+dx)^2(de-cf)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{(c+dx)(de-cf)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}+\frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}$$

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^2],x]

[Out] (2*b*f^2*(c + d*x)*Cos[a + b/(c + d*x)^2])/(3*d^3) - (b*f*(d*e - c*f)*Cos[a
]*CosIntegral[b/(c + d*x)^2])/d^3 - (Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*Cos[a
]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d^3 + (2*b^(3/2)*f^2*Sqrt[2*Pi]
*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/(3*d^3) + (2*b^(3/2)*f^2*
Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/(3*d^3) + (Sqrt
[b]*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a
])/d^3 + ((d*e - c*f)^2*(c + d*x)*Sin[a + b/(c + d*x)^2])/d^3 + (f*(d*e - c
*f)*(c + d*x)^2*Sin[a + b/(c + d*x)^2])/d^3 + (f^2*(c + d*x)^3*Sin[a + b/(c
```

```
 + d*x)^2])/(3*d^3) + (b*f*(d*e - c*f)*Sin[a]*SinIntegral[b/(c + d*x)^2])/d
^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(a+\frac{b}{x^2}\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(a+\frac{b}{x^2}\right)+f^2x^2\sin\left(a+\frac{b}{x^2}\right)\right)dx,x,c+\right.}{d^3}$$

$$= \frac{f^2\text{Subst}\left(\int x^2\sin\left(a+\frac{b}{x^2}\right)dx,x,c+dx\right)}{d^3} + \frac{(2f(de-cf))\text{Subst}\left(\int x\sin\left(a+\frac{b}{x^2}\right)dx,x,c+dx\right)}{d^3} + \frac{(de-cf)^2\text{Subst}\left(\int \sin\left(a+\frac{b}{x^2}\right)dx,x,c+dx\right)}{d^3}$$

$$= -\frac{f^2\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^4}dx,x,\frac{1}{c+dx}\right)}{d^3} - \frac{(f(de-cf))\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}dx,x,\frac{1}{(c+dx)^2}\right)}{d^3} - \frac{(de-cf)^2\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^2}dx,x,\frac{1}{c+dx}\right)}{d^3}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3} - \frac{(2bf^2)\text{Subst}\left(\int \frac{\cos(a+bx^2)}{x^2}dx,x,\frac{1}{c+dx}\right)}{3d^3} - \frac{(bf(de-cf))\text{Subst}\left(\int \frac{\cos(a+bx)}{x}dx,x,\frac{1}{(c+dx)^2}\right)}{d^3} - \frac{(2b(de-cf)^2)\text{Subst}\left(\int \cos\left(a+bx^2\right)dx,x,\frac{1}{c+dx}\right)}{d^3}$$

$$= \frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3} + \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3} + \frac{(4b^2f^2)\text{Subst}\left(\int \sin\left(a+bx^2\right)dx,x,\frac{1}{c+dx}\right)}{3d^3} - \frac{(bf(de-cf)\cos(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}dx,x,\frac{1}{(c+dx)^2}\right)}{d^3} - \frac{(2b(de-cf)^2\cos(a))\text{Subst}\left(\int \cos\left(bx^2\right)dx,x,\frac{1}{c+dx}\right)}{d^3} + \frac{(bf(de-cf)\sin(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}dx,x,\frac{1}{(c+dx)^2}\right)}{d^3} + \frac{(2b(de-cf)^2\sin(a))\text{Subst}\left(\int \sin\left(bx^2\right)dx,x,\frac{1}{c+dx}\right)}{d^3}$$

$$= \frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3} - \frac{bf(de-cf)\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3}$$
$$- \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3}$$
$$+ \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d^3}$$
$$+ \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3}$$
$$+ \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3} + \frac{bf(de-cf)\sin(a)\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{d^3}$$
$$+ \frac{(4b^2f^2\cos(a))\,\text{Subst}\left(\int\sin\left(bx^2\right)\,dx, x, \frac{1}{c+dx}\right)}{3d^3}$$
$$+ \frac{(4b^2f^2\sin(a))\,\text{Subst}\left(\int\cos\left(bx^2\right)\,dx, x, \frac{1}{c+dx}\right)}{3d^3}$$

$$= \frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3} - \frac{bf(de-cf)\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3}$$
$$- \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3}$$
$$+ \frac{2b^{3/2}f^2\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3} + \frac{2b^{3/2}f^2\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{3d^3}$$
$$+ \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d^3}$$
$$+ \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3}$$
$$+ \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3} + \frac{bf(de-cf)\sin(a)\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.26

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^2}\right) dx$$
$$= \frac{2bcf^2\cos\left(a+\frac{b}{(c+dx)^2}\right)+2bdf^2x\cos\left(a+\frac{b}{(c+dx)^2}\right)+3bf(-de+cf)\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)+2b^{3/2}f^2}{}$$

[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^2],x]

[Out] (2*b*c*f^2*Cos[a + b/(c + d*x)^2] + 2*b*d*f^2*x*Cos[a + b/(c + d*x)^2] + 3*b*f*(-(d*e) + c*f)*Cos[a]*CosIntegral[b/(c + d*x)^2] + 2*b^(3/2)*f^2*Sqrt[2*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 3*Sqrt[b]*d^2*e^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] - 6*Sqrt[b]*c*d*e*f*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + 3*Sqrt[b]*c^2*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*(-3*(d*e - c*f)^2*Cos[a] + 2*b*f^2*Sin[a]) + 3*c*d^2*e^2*Sin[a + b/(c + d*x)^2] - 3*c^2*d*e*f*Sin[a + b/(c + d*x)^2] + c^3*f^2*Sin[a + b/(c + d*x)^2] + 3*d^3*e^2*x*Sin[a + b/(c + d*x)^2] + 3*d^3*e*f*x^2*Sin[a + b/(c + d*x)^2] + d^3*f^2*x^3*Sin[a + b/(c + d*x)^2] + 3*b*d*e*f*Sin[a]*SinIntegral[b/(c + d*x)^2] - 3*b*c*f^2*Sin[a]*SinIntegral[b/(c + d*x)^2])/(3*d^3)

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{-(cf-de)^2(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+(cf-de)^2\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)\right)+f(cf-de)(d}{}$
default	$-\frac{-(cf-de)^2(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+(cf-de)^2\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)\right)+f(cf-de)(d}{}$
risch	$-\frac{e^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)c^2f^2}{2d^3\sqrt{-ib}}+\frac{e^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)cef}{d^2\sqrt{-ib}}-\frac{e^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)e^2}{2d\sqrt{-ib}}-\frac{e^{ia}b\,\mathrm{Ei}_1\left(-\frac{ib}{(dx+c)^2}\right)cf^2}{2d^3}+\frac{e^{ia}b\,\mathrm{E}}{}$
parts	Expression too large to display

[In] int((f*x+e)^2*sin(a+b/(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] -1/d^3*(-(c*f-d*e)^2*(d*x+c)*sin(a+b/(d*x+c)^2)+(c*f-d*e)^2*b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))+f*(c*f-d*e)*(d*x+c)^2*sin(a+b/(d*x+c)^

```
2)-2*f*(c*f-d*e)*b*(1/2*cos(a)*Ci(b/(d*x+c)^2)-1/2*sin(a)*Si(b/(d*x+c)^2))-
1/3*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^2)+2/3*f^2*b*(-(d*x+c)*cos(a+b/(d*x+c)^2)
-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)
)+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.06

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = -\frac{3\,(bdef-bcf^2)\cos(a)\,\mathrm{Ci}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)-\sqrt{2}(2\,\pi bdf^2\sin(a)-3\,\pi(d^3e^2-2\,cd^2ef+c^2df^2)\cos(a))\sqrt{\frac{\ }{\pi}}$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(b*d*e*f - b*c*f^2)*cos(a)*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2)
) - sqrt(2)*(2*pi*b*d*f^2*sin(a) - 3*pi*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)
*cos(a))*sqrt(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c))
 - sqrt(2)*(2*pi*b*d*f^2*cos(a) + 3*pi*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*
sin(a))*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c))
- 3*(b*d*e*f - b*c*f^2)*sin(a)*sin_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) -
2*(b*d*f^2*x + b*c*f^2)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 +
2*c*d*x + c^2)) - (d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 3*c*d^2*e^2
- 3*c^2*d*e*f + c^3*f^2)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 +
 2*c*d*x + c^2)))/d^3
```

Sympy [F]

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = \int (e+fx)^2 \sin\left(a+\frac{b}{c^2+2cdx+d^2x^2}\right)\,dx$$

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^2}\right) \, dx = \int (fx+e)^2 \sin \left(a+\frac{b}{(dx+c)^2}\right) \, dx$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*b*f^2*x*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
 c^2)) - 3*d^2*integrate(1/3*(2*b^2*d*f^2*x*sin((a*d^2*x^2 + 2*a*c*d*x + a*
c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 - 3*(b*d^3*e*f - b*c*d^2*f
^2)*x^2 - 3*(b*d^3*e^2 - b*c^2*d*f^2)*x)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2
 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^
3*d^2), x) - 3*d^2*integrate(1/3*(2*b^2*d*f^2*x*sin((a*d^2*x^2 + 2*a*c*d*x
+ a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 - 3*(b*d^3*e*f - b*c*d
^2*f^2)*x^2 - 3*(b*d^3*e^2 - b*c^2*d*f^2)*x)*cos((a*d^2*x^2 + 2*a*c*d*x + a
*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x
 + c^3*d^2)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^
2))^2 + (d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2)*sin((a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2), x) + (d^2*f^2*x^3 + 3*d
^2*e*f*x^2 + 3*d^2*e^2*x)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2
+ 2*c*d*x + c^2)))/d^2
```

Giac [F]

$$\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^2}\right) \, dx = \int (fx+e)^2 \sin \left(a+\frac{b}{(dx+c)^2}\right) \, dx$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(a + b/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^2}\right) \, dx = \int \sin \left(a+\frac{b}{(c+d\,x)^2}\right) (e+f\,x)^2 \, dx$$

```
[In] int(sin(a + b/(c + d*x)^2)*(e + f*x)^2,x)
```

```
[Out] int(sin(a + b/(c + d*x)^2)*(e + f*x)^2, x)
```

3.178 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 198

$$\begin{aligned}\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = &-\frac{bf\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2} \\ &-\frac{\sqrt{b}(de-cf)\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2} \\ &+\frac{\sqrt{b}(de-cf)\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d^2} \\ &+\frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^2} \\ &+\frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{2d^2}+\frac{bf\sin(a)\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}\end{aligned}$$

```
[Out] -1/2*b*f*Ci(b/(d*x+c)^2)*cos(a)/d^2+1/2*b*f*Si(b/(d*x+c)^2)*sin(a)/d^2+(-c*
f+d*e)*(d*x+c)*sin(a+b/(d*x+c)^2)/d^2+1/2*f*(d*x+c)^2*sin(a+b/(d*x+c)^2)/d^
2-(-c*f+d*e)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*b^(1/2)*2^(1
/2)*Pi^(1/2)/d^2+(-c*f+d*e)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*sin(
a)*b^(1/2)*2^(1/2)*Pi^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {3514, 3440, 3468, 3435, 3433, 3432, 3460, 3378, 3384, 3380, 3383}

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)dx = -\frac{bf\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2} - \frac{\sqrt{2\pi}\sqrt{b}\cos(a)(de-cf)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2} + \frac{\sqrt{2\pi}\sqrt{b}\sin(a)(de-cf)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2} + \frac{(c+dx)(de-cf)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^2} + \frac{bf\sin(a)\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{2d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{2d^2}$$

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^2],x]
```

```
[Out] -1/2*(b*f*Cos[a]*CosIntegral[b/(c + d*x)^2])/d^2 - (Sqrt[b]*(d*e - c*f)*Sqr
t[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d^2 + (Sqrt[b]*(d*
e - c*f)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/d^2 +
((d*e - c*f)*(c + d*x)*Sin[a + b/(c + d*x)^2])/d^2 + (f*(c + d*x)^2*Sin[a +
 b/(c + d*x)^2])/(2*d^2) + (b*f*Sin[a]*SinIntegral[b/(c + d*x)^2])/(2*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(a+\frac{b}{x^2}\right)+fx\sin\left(a+\frac{b}{x^2}\right)\right)\,dx,x,c+dx\right)}{d^2}\\
&= \frac{f\text{Subst}\left(\int x\sin\left(a+\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\text{Subst}\left(\int \sin\left(a+\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^2}\\
&= -\frac{f\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,\frac{1}{(c+dx)^2}\right)}{2d^2}-\frac{(de-cf)\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^2}\,dx,x,\frac{1}{c+dx}\right)}{d^2}\\
&= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{2d^2}\\
&\quad-\frac{(bf)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx,x,\frac{1}{(c+dx)^2}\right)}{2d^2}\\
&\quad-\frac{(2b(de-cf))\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx,x,\frac{1}{c+dx}\right)}{d^2}\\
&= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{2d^2}\\
&\quad-\frac{(bf\cos(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,\frac{1}{(c+dx)^2}\right)}{2d^2}\\
&\quad-\frac{(2b(de-cf)\cos(a))\text{Subst}\left(\int \cos\left(bx^2\right)\,dx,x,\frac{1}{c+dx}\right)}{d^2}\\
&\quad+\frac{(bf\sin(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx,x,\frac{1}{(c+dx)^2}\right)}{2d^2}\\
&\quad+\frac{(2b(de-cf)\sin(a))\text{Subst}\left(\int \sin\left(bx^2\right)\,dx,x,\frac{1}{c+dx}\right)}{d^2}\\
&= -\frac{bf\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{\sqrt{b}(de-cf)\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2}\\
&\quad+\frac{\sqrt{b}(de-cf)\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d^2}+\frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^2}\\
&\quad+\frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{2d^2}+\frac{bf\sin(a)\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.22

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx$$
$$=\frac{-bf\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)-2\sqrt{b}(de-cf)\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+2\sqrt{b}de\sqrt{2\pi}\,\text{FresnelS}\left(\right.}{}$$

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^2],x]
```

```
[Out] (-(b*f*Cos[a]*CosIntegral[b/(c + d*x)^2]) - 2*Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi
]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 2*Sqrt[b]*d*e*Sqrt[2*Pi
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] - 2*Sqrt[b]*c*f*Sqrt[2*Pi
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + 2*c*d*e*Sin[a + b/(c +
d*x)^2] - c^2*f*Sin[a + b/(c + d*x)^2] + 2*d^2*e*x*Sin[a + b/(c + d*x)^2] +
 d^2*f*x^2*Sin[a + b/(c + d*x)^2] + b*f*Sin[a]*SinIntegral[b/(c + d*x)^2])/
(2*d^2)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{-(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+(cf-de)\sqrt{b}\sqrt{2}\sqrt{\pi}\left(\cos(a)\,\mathrm{C}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)-\sin(a)\,\mathrm{S}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)\right)+\frac{f(dx+c)^2\sin\left(a\right.}{2}}{d^2}$
default	$\frac{-(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+(cf-de)\sqrt{b}\sqrt{2}\sqrt{\pi}\left(\cos(a)\,\mathrm{C}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)-\sin(a)\,\mathrm{S}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)\right)+\frac{f(dx+c)^2\sin\left(a\right.}{2}}{d^2}$
risch	$\frac{e^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)cf}{2d^2\sqrt{-ib}}-\frac{e^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)e}{2d\sqrt{-ib}}+\frac{e^{ia}b\,\mathrm{Ei}_1\left(-\frac{ib}{(dx+c)^2}\right)f}{4d^2}+\frac{e^{-ia}b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)cf}{2d^2\sqrt{ib}}-\frac{e^{-ia}b\sqrt{\pi}\,\mathrm{er}}{2d\sqrt{}}$
parts	$-\frac{\sqrt{\pi}\sqrt{b}\sqrt{2}\cos(a)\,\mathrm{C}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)fx}{d}+\frac{\sqrt{\pi}\sqrt{b}\sqrt{2}\sin(a)\,\mathrm{S}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)fx}{d}-\frac{\sqrt{\pi}\sqrt{b}\sqrt{2}\cos(a)\,\mathrm{C}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)e}{d}+$

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(-(c*f-d*e)*(d*x+c)*sin(a+b/(d*x+c)^2)+(c*f-d*e)*b^(1/2)*2^(1/2)*Pi^(
1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))-sin(a)*FresnelS(b^(
```

```
1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))+1/2*f*(d*x+c)^2*sin(a+b/(d*x+c)^2)-f*b*(1/2
*cos(a)*Ci(b/(d*x+c)^2)-1/2*sin(a)*Si(b/(d*x+c)^2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.18

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx =$$
$$-\frac{2\sqrt{2}\pi(d^2e-cdf)\sqrt{\frac{b}{\pi d^2}}\cos(a)\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-2\sqrt{2}\pi(d^2e-cdf)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{S}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)\sin(a)+bf\cos(a)}{2\,d}$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(2)*pi*(d^2*e - c*d*f)*sqrt(b/(pi*d^2))*cos(a)*fresnel_cos(sqrt
(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) - 2*sqrt(2)*pi*(d^2*e - c*d*f)*sqrt(b/(pi
*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c))*sin(a) + b*f*cos(a
)*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) - b*f*sin(a)*sin_integral(b/(d^
2*x^2 + 2*c*d*x + c^2)) - (d^2*f*x^2 + 2*d^2*e*x + 2*c*d*e - c^2*f)*sin((a*
d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = \int (e+fx)\sin\left(a+\frac{b}{c^2+2cdx+d^2x^2}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = \int (fx+e)\sin\left(a+\frac{b}{(dx+c)^2}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*e*x)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*
d*x + c^2)) + integrate(1/2*(b*d*f*x^2 + 2*b*d*e*x)*cos((a*d^2*x^2 + 2*a*c*
d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*
```

```
d*x + c^3), x) + integrate(1/2*(b*d*f*x^2 + 2*b*d*e*x)*cos((a*d^2*x^2 + 2*a
*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((d^3*x^3 + 3*c*d^2*x^2 + 3*
c^2*d*x + c^3)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
 c^2))^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin((a*d^2*x^2 + 2*a*c
*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2), x)
```

Giac [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx=\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^2}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^2),x, algorithm="giac")
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^2}\right)\,(e+f\,x)\,dx$$

```
[In] int(sin(a + b/(c + d*x)^2)*(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^2)*(e + f*x), x)
```

3.179 $\int \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 105

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = -\frac{\sqrt{b}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{\sqrt{b}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

```
[Out] (d*x+c)*sin(a+b/(d*x+c)^2)/d-cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+
c))*b^(1/2)*2^(1/2)*Pi^(1/2)/d+FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*s
in(a)*b^(1/2)*2^(1/2)*Pi^(1/2)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3440, 3468, 3435, 3433, 3432}

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)dx = -\frac{\sqrt{2\pi}\sqrt{b}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{\sqrt{2\pi}\sqrt{b}\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

```
[In] Int[Sin[a + b/(c + d*x)^2],x]
```

```
[Out] -((Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d) +
 (Sqrt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/d + (
(c + d*x)*Sin[a + b/(c + d*x)^2])/d
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d} - \frac{(2b)\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d} - \frac{(2b\cos(a))\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&\quad + \frac{(2b\sin(a))\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&= -\frac{\sqrt{b}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d}\\
&\quad + \frac{\sqrt{b}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = \frac{-\sqrt{b}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right) + \sqrt{b}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a) + (c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^2],x]
```

```
[Out] (-(Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]) + Sq
rt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + (c + d*x
)*Sin[a + b/(c + d*x)^2])/d
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$-\frac{-(dx+c)\sin\left(a+\frac{b}{(dx+c)^{2}}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)\right)}{d}$	80
default	$-\frac{-(dx+c)\sin\left(a+\frac{b}{(dx+c)^{2}}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)\right)}{d}$	80
risch	$-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)\mathrm{e}^{-ia}}{2d\sqrt{ib}}-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)\mathrm{e}^{ia}}{2d\sqrt{-ib}}-\frac{(-dx-c)\sin\left(\frac{a\,d^{2}x^{2}+2acdx+a\,c^{2}+b}{(dx+c)^{2}}\right)}{d}$	115

```
[In] int(sin(a+b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-(d*x+c)*sin(a+b/(d*x+c)^2)+b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC
(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)
/(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int \sin\left(a+\frac{b}{(c+dx)^{2}}\right)\,dx = -\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d^{2}}}\cos\left(a\right)\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^{2}}}}{dx+c}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d^{2}}}\,\mathrm{S}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^{2}}}}{dx+c}\right)\sin\left(a\right)-(dx+c)\sin\left(\frac{ad^{2}x^{2}+2\,acdx+ac^{2}+b}{d^{2}x^{2}+2\,cdx+c^{2}}\right)}{d}$$

```
[In] integrate(sin(a+b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*pi*d*sqrt(b/(pi*d^2))*cos(a)*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2
))/(d*x + c)) - sqrt(2)*pi*d*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/
(pi*d^2))/(d*x + c))*sin(a) - (d*x + c)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2
+ b)/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]

$$\int \sin \left(a+\frac{b}{(c+dx)^2}\right) dx = \int \sin \left(a+\frac{b}{\left(c+dx\right)^{2}}\right) dx$$

[In] integrate(sin(a+b/(d*x+c)**2),x)

[Out] Integral(sin(a + b/(c + d*x)**2), x)

Maxima [F]

$$\int \sin \left(a+\frac{b}{(c+dx)^2}\right) dx = \int \sin \left(a+\frac{b}{\left(dx+c\right)^{2}}\right) dx$$

[In] integrate(sin(a+b/(d*x+c)^2),x, algorithm="maxima")

[Out] b*d*integrate(x*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x) + b*d*integrate(x*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2), x) + x*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))

Giac [F]

$$\int \sin \left(a+\frac{b}{(c+dx)^2}\right) dx = \int \sin \left(a+\frac{b}{\left(dx+c\right)^{2}}\right) dx$$

[In] integrate(sin(a+b/(d*x+c)^2),x, algorithm="giac")

[Out] integrate(sin(a + b/(d*x + c)^2), x)

Mupad [F(-1)]

Timed out.

$$\int \sin \left(a+\frac{b}{(c+dx)^2}\right) dx = \int \sin \left(a+\frac{b}{\left(c+d\,x\right)^{2}}\right) dx$$

[In] int(sin(a + b/(c + d*x)^2),x)

[Out] int(sin(a + b/(c + d*x)^2), x)

3.180 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \operatorname{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^2)/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

```
[In] Int[Sin[a + b/(c + d*x)^2]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[a + b/(c + d*x)^2]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

```
[In] Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x),x]
[Out] Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{fx+e}dx$$

```
[In] int(sin(a+b/(d*x+c)^2)/(f*x+e),x)
[Out] int(sin(a+b/(d*x+c)^2)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
[Out] integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 18.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{c^2+2cdx+d^2x^2}\right)}{e+fx}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**2)/(f*x+e),x)
[Out] Integral(sin(a + b/(c**2 + 2*c*d*x + d**2*x**2))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin(a + b/(d*x + c)^2)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^2)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 6.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^2}\right)}{e+f\,x}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^2)/(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^2)/(e + f*x), x)
```

3.181 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

```
[In] Int[Sin[a + b/(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b/(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 27.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

```
[In] Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}dx$$

```
[In] int(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x)
[Out] int(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2} dx = \text{Timed out}$$

```
[In] integrate(sin(a+b/(d*x+c)**2)/(f*x+e)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2} dx = \int \frac{\sin \left(a+\frac{b}{(dx+c)^2}\right)}{(fx+e)^2} dx$$

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin(a + b/(d*x + c)^2)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2} dx = \int \frac{\sin \left(a+\frac{b}{(dx+c)^2}\right)}{(fx+e)^2} dx$$

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^2)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 9.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^2}\right)}{\left(e+f\,x\right)^2}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^2)/(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^2)/(e + f*x)^2, x)
```

3.182 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 330

$$\begin{aligned}\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx &= -\frac{bf^2\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^3}\right)}{3d^3} \\ &\quad - \frac{ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3} \\ &\quad + \frac{ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3d^3} \\ &\quad - \frac{ie^{ia}(de-cf)^2\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^3} \\ &\quad + \frac{ie^{-ia}(de-cf)^2\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^3} \\ &\quad + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^3}\right)}{3d^3} + \frac{bf^2\sin(a)\text{Si}\left(\frac{b}{(c+dx)^3}\right)}{3d^3}\end{aligned}$$

```
[Out] -1/3*b*f^2*Ci(b/(d*x+c)^3)*cos(a)/d^3-1/3*I*exp(I*a)*f*(-c*f+d*e)*(-I*b/(d*
x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,-I*b/(d*x+c)^3)/d^3+1/3*I*f*(-c*f+d*e)*(
I*b/(d*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,I*b/(d*x+c)^3)/d^3/exp(I*a)-1/6*I
*exp(I*a)*(-c*f+d*e)^2*(-I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,-I*b/(d*x+
c)^3)/d^3+1/6*I*(-c*f+d*e)^2*(I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,I*b/(
```

```
d*x+c)^3)/d^3/exp(I*a)+1/3*b*f^2*Si(b/(d*x+c)^3)*sin(a)/d^3+1/3*f^2*(d*x+c)
^3*sin(a+b/(d*x+c)^3)/d^3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3514, 3446, 2239, 3504, 2250, 3460, 3378, 3384, 3380, 3383}

$$\begin{aligned}
&\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^3}\right)dx\\
&=-\frac{bf^2\cos(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^3}\right)}{3d^3}\\
&-\frac{ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(de-cf)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3}\\
&+\frac{ie^{-ia}f(c+dx)^2\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(de-cf)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3d^3}\\
&-\frac{ie^{ia}(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}(de-cf)^2\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^3}\\
&+\frac{ie^{-ia}(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}(de-cf)^2\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^3}\\
&+\frac{bf^2\sin(a)\text{Si}\left(\frac{b}{(c+dx)^3}\right)}{3d^3}+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^3}\right)}{3d^3}
\end{aligned}$$

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^3],x]
```

```
[Out] -1/3*(b*f^2*Cos[a]*CosIntegral[b/(c + d*x)^3])/d^3 - ((I/3)*E^(I*a)*f*(d*e
- c*f)*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gamma[-2/3, ((-I)*b)/(c + d
*x)^3])/d^3 + ((I/3)*f*(d*e - c*f)*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Ga
mma[-2/3, (I*b)/(c + d*x)^3])/(d^3*E^(I*a)) - ((I/6)*E^(I*a)*(d*e - c*f)^2*
(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*b)/(c + d*x)^3])/d
^3 + ((I/6)*(d*e - c*f)^2*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, (
I*b)/(c + d*x)^3])/(d^3*E^(I*a)) + (f^2*(c + d*x)^3*Sin[a + b/(c + d*x)^3])
/(3*d^3) + (b*f^2*Sin[a]*SinIntegral[b/(c + d*x)^3])/(3*d^3)
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
```

```
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(a+\frac{b}{x^3}\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(a+\frac{b}{x^3}\right)+f^2x^2\sin\left(a+\frac{b}{x^3}\right)\right)dx,x,c+\right.}{d^3}$$

$$= \frac{f^2\text{Subst}\left(\int x^2\sin\left(a+\frac{b}{x^3}\right)\,dx,x,c+dx\right)}{d^3} + \frac{(2f(de-cf))\text{Subst}\left(\int x\sin\left(a+\frac{b}{x^3}\right)\,dx,x,c+dx\right)}{d^3} + \frac{(de-cf)^2\text{Subst}\left(\int \sin\left(a+\frac{b}{x^3}\right)\,dx,x,c+dx\right)}{d^3}$$

$$= -\frac{f^2\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,\frac{1}{(c+dx)^3}\right)}{3d^3} + \frac{(if(de-cf))\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}x\,dx,x,c+dx\right)}{d^3} - \frac{(if(de-cf))\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}}x\,dx,x,c+dx\right)}{d^3} + \frac{\left(i(de-cf)^2\right)\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}\,dx,x,c+dx\right)}{2d^3} - \frac{\left(i(de-cf)^2\right)\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}}\,dx,x,c+dx\right)}{2d^3}$$

$$= -\frac{ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3} + \frac{ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3d^3} - \frac{ie^{ia}(de-cf)^2\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^3} + \frac{ie^{-ia}(de-cf)^2\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^3}\right)}{3d^3} - \frac{(bf^2)\operatorname{Subst}\left(\int\frac{\cos(a+bx)}{x}\,dx,x,\frac{1}{(c+dx)^3}\right)}{3d^3}$$

$$= -\frac{ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3} + \frac{ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3d^3} - \frac{ie^{ia}(de-cf)^2\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^3} + \frac{ie^{-ia}(de-cf)^2\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^3}\right)}{3d^3} - \frac{(bf^2\cos(a))\operatorname{Subst}\left(\int\frac{\cos(bx)}{x}\,dx,x,\frac{1}{(c+dx)^3}\right)}{3d^3} + \frac{(bf^2\sin(a))\operatorname{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,\frac{1}{(c+dx)^3}\right)}{3d^3}$$

$$= -\frac{bf^2\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^3}\right)}{3d^3} - \frac{ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3} + \frac{ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3d^3} - \frac{ie^{ia}(de-cf)^2\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^3} + \frac{ie^{-ia}(de-cf)^2\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^3}\right)}{3d^3} + \frac{bf^2\sin(a)\mathrm{Si}\left(\frac{b}{(c+dx)^3}\right)}{3d^3}$$

Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.88

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx$$

$$= \frac{\frac{3bdef\cos(a)\left(\frac{\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{\sqrt[3]{-\frac{ib}{(c+dx)^3}}}+\frac{\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{\sqrt[3]{\frac{ib}{(c+dx)^3}}}\right)}{c+dx} - \frac{3bcf^2\cos(a)\left(\frac{\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{\sqrt[3]{-\frac{ib}{(c+dx)^3}}}+\frac{\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{\sqrt[3]{\frac{ib}{(c+dx)^3}}}\right)}{c+dx} - 3i(de-cf)^2\sqrt[3]{\frac{}{(c}}}{}$$

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^3],x]
```

```
[Out] ((3*b*d*e*f*Cos[a]*(Gamma[1/3, ((-I)*b)/(c + d*x)^3]/(((-I)*b)/(c + d*x)^3)
^(1/3) + Gamma[1/3, (I*b)/(c + d*x)^3]/((I*b)/(c + d*x)^3)^(1/3)))/(c + d*x
) - (3*b*c*f^2*Cos[a]*(Gamma[1/3, ((-I)*b)/(c + d*x)^3]/(((-I)*b)/(c + d*x)
^3)^(1/3) + Gamma[1/3, (I*b)/(c + d*x)^3]/((I*b)/(c + d*x)^3)^(1/3)))/(c +
d*x) - (3*I)*(d*e - c*f)^2*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[2/3, (
I*b)/(c + d*x)^3]*(Cos[a] - I*Sin[a]) + (3*I)*(d*e - c*f)^2*(((-I)*b)/(c +
d*x)^3)^(1/3)*(c + d*x)*Gamma[2/3, ((-I)*b)/(c + d*x)^3]*(Cos[a] + I*Sin[a]
) + 2*(c + d*x)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x
^2))*Cos[b/(c + d*x)^3]*Sin[a] + ((3*I)*b*d*e*f*(Gamma[1/3, ((-I)*b)/(c + d
*x)^3]/(((-I)*b)/(c + d*x)^3)^(1/3) - Gamma[1/3, (I*b)/(c + d*x)^3]/((I*b)/
```

```
(c + d*x)^3)^(1/3))*Sin[a])/(c + d*x) - ((3*I)*b*c*f^2*(Gamma[1/3, ((-I)*b)
/(c + d*x)^3]/(((-I)*b)/(c + d*x)^3)^(1/3) - Gamma[1/3, (I*b)/(c + d*x)^3]/
((I*b)/(c + d*x)^3)^(1/3))*Sin[a])/(c + d*x) + 2*(c + d*x)*(c^2*f^2 - c*d*f
*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Cos[a]*Sin[b/(c + d*x)^3] -
 2*b*f^2*(Cos[a]*CosIntegral[b/(c + d*x)^3] - Sin[a]*SinIntegral[b/(c + d*x
)^3]))/(6*d^3)
```

Maple [F]

$$\int (fx+e)^2 \sin\left(a+\frac{b}{(dx+c)^3}\right) dx$$

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^3),x)
```

```
[Out] int((f*x+e)^2*sin(a+b/(d*x+c)^3),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(264) = 528.

Time = 0.11 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.75

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^3}\right) dx = -\frac{2\,bf^2\cos(a)\,\mathrm{Ci}\left(\frac{b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)-2\,bf^2\sin(a)\,\mathrm{Si}\left(\frac{b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+3\left(\left(i\,d^3ef-i\,cd^2f^2\right)\cos\right.}{}$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*f^2*cos(a)*cos_integral(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^
3)) - 2*b*f^2*sin(a)*sin_integral(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^
3)) + 3*((I*d^3*e*f - I*c*d^2*f^2)*cos(a) + (d^3*e*f - c*d^2*f^2)*sin(a))*(
I*b/d^3)^(2/3)*gamma(1/3, I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) +
3*((-I*d^3*e*f + I*c*d^2*f^2)*cos(a) + (d^3*e*f - c*d^2*f^2)*sin(a))*(-I*b/
d^3)^(2/3)*gamma(1/3, -I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + 3*(
(I*d^3*e^2 - 2*I*c*d^2*e*f + I*c^2*d*f^2)*cos(a) + (d^3*e^2 - 2*c*d^2*e*f +
 c^2*d*f^2)*sin(a))*(I*b/d^3)^(1/3)*gamma(2/3, I*b/(d^3*x^3 + 3*c*d^2*x^2 +
 3*c^2*d*x + c^3)) + 3*((-I*d^3*e^2 + 2*I*c*d^2*e*f - I*c^2*d*f^2)*cos(a) +
 (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sin(a))*(-I*b/d^3)^(1/3)*gamma(2/3, -I
*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - 2*(d^3*f^2*x^3 + 3*d^3*e*f*
x^2 + 3*d^3*e^2*x + 3*c*d^2*e^2 - 3*c^2*d*e*f + c^3*f^2)*sin((a*d^3*x^3 + 3
*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
+ c^3)))/d^3
```

Sympy [F]

$$\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^3}\right) dx = \int (e+fx)^2 \sin \left(a+\frac{b}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right) dx$$

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**3),x)
[Out] Integral((e + f*x)**2*sin(a + b/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3)), x)
```

Maxima [F]

$$\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^3}\right) dx = \int (fx+e)^2 \sin \left(a+\frac{b}{(dx+c)^3}\right) dx$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^3),x, algorithm="maxima")
[Out] 1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^
2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate(1
/2*(b*d*f^2*x^3 + 3*b*d*e*f*x^2 + 3*b*d*e^2*x)*cos((a*d^3*x^3 + 3*a*c*d^2*x
^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d
^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x) + integrate(1/2
*(b*d*f^2*x^3 + 3*b*d*e*f*x^2 + 3*b*d*e^2*x)*cos((a*d^3*x^3 + 3*a*c*d^2*x^2
 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/((d^
4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*cos((a*d^3*x^3 + 3*a
*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
c^3))^2 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*sin((a*
d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 +
 3*c^2*d*x + c^3))^2), x)
```

Giac [F]

$$\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^3}\right) dx = \int (fx+e)^2 \sin \left(a+\frac{b}{(dx+c)^3}\right) dx$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^3),x, algorithm="giac")
[Out] integrate((f*x + e)^2*sin(a + b/(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^3}\right)\,{\left(e+f\,x\right)}^2\,dx$$

```
[In] int(sin(a + b/(c + d*x)^3)*(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^3)*(e + f*x)^2, x)
```

3.183 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 235

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=-\frac{ie^{ia}f\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2}+\frac{ie^{-ia}f\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2}-\frac{ie^{ia}(de-cf)\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2}+\frac{ie^{-ia}(de-cf)\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2}$$

```
[Out] -1/6*I*exp(I*a)*f*(-I*b/(d*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,-I*b/(d*x+c)^
3)/d^2+1/6*I*f*(I*b/(d*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,I*b/(d*x+c)^3)/d^
2/exp(I*a)-1/6*I*exp(I*a)*(-c*f+d*e)*(-I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-
1/3,-I*b/(d*x+c)^3)/d^2+1/6*I*(-c*f+d*e)*(I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMM
A(-1/3,I*b/(d*x+c)^3)/d^2/exp(I*a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3514, 3446, 2239, 3504, 2250}

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=-\frac{ie^{ia}(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}(de-cf)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2}+\frac{ie^{-ia}(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}(de-cf)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2}-\frac{ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2}+\frac{ie^{-ia}f(c+dx)^2\left(\frac{ib}{(c+dx)^3}\right)^{2/3}\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2}$$

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^3],x]
```

```
[Out] ((-1/6*I)*E^(I*a)*f*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gamma[-2/3, ((
-I)*b)/(c + d*x)^3])/d^2 + ((I/6)*f*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*G
amma[-2/3, (I*b)/(c + d*x)^3])/(d^2*E^(I*a)) - ((I/6)*E^(I*a)*(d*e - c*f)*(
((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*b)/(c + d*x)^3])/d^
2 + ((I/6)*(d*e - c*f)*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, (I*b
)/(c + d*x)^3])/(d^2*E^(I*a))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(a+\frac{b}{x^3}\right)+fx\sin\left(a+\frac{b}{x^3}\right)\right)\,dx,x,c+dx\right)}{d^2}\\
&= \frac{f\text{Subst}\left(\int x\sin\left(a+\frac{b}{x^3}\right)\,dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\text{Subst}\left(\int \sin\left(a+\frac{b}{x^3}\right)\,dx,x,c+dx\right)}{d^2}\\
&= \frac{(if)\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}x\,dx,x,c+dx\right)}{2d^2}-\frac{(if)\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}}x\,dx,x,c+dx\right)}{2d^2}\\
&\quad+\frac{(i(de-cf))\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}\,dx,x,c+dx\right)}{2d^2}-\frac{(i(de-cf))\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}}\,dx,x,c+dx\right)}{2d^2}\\
&= -\frac{ie^{ia}f\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2}\\
&\quad+\frac{ie^{-ia}f\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2}\\
&\quad-\frac{ie^{ia}(de-cf)\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2}\\
&\quad+\frac{ie^{-ia}(de-cf)\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 705 vs. 2(235) = 470.

Time = 1.36 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.00

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)dx$$
$$=\frac{3bf\cos(a)\left(\frac{\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{3\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)}+\frac{\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{3\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)}\right)}{4d^2}$$
$$+\frac{3be\cos(a)\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}+\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}\right)}{2d}$$
$$-\frac{3bcf\cos(a)\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}+\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}\right)}{2d^2}$$
$$+\frac{e(c+dx)\cos\left(\frac{b}{(c+dx)^3}\right)\sin(a)}{d}+\frac{f(-c+dx)(c+dx)\cos\left(\frac{b}{(c+dx)^3}\right)\sin(a)}{2d^2}$$
$$+\frac{3ibf\left(\frac{\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{3\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)}-\frac{\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{3\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)}\right)\sin(a)}{4d^2}$$
$$+\frac{3ibe\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}-\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}\right)\sin(a)}{2d}$$
$$-\frac{3ibcf\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}-\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}\right)\sin(a)}{2d^2}$$
$$+\frac{e(c+dx)\cos(a)\sin\left(\frac{b}{(c+dx)^3}\right)}{d}+\frac{f(-c+dx)(c+dx)\cos(a)\sin\left(\frac{b}{(c+dx)^3}\right)}{2d^2}$$

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^3],x]
```

```
[Out] (3*b*f*Cos[a]*(Gamma[1/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(
1/3)*(c + d*x)) + Gamma[1/3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(1/3
```

```
)*(c + d*x))))/(4*d^2) + (3*b*e*Cos[a]*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(3
*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2) + Gamma[2/3, (I*b)/(c + d*x)^3]/
(3*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2)))/(2*d) - (3*b*c*f*Cos[a]*(Gamma[
2/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2) + G
amma[2/3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2)))/(2
*d^2) + (e*(c + d*x)*Cos[b/(c + d*x)^3]*Sin[a])/d + (f*(-c + d*x)*(c + d*x)
*Cos[b/(c + d*x)^3]*Sin[a])/(2*d^2) + (((3*I)/4)*b*f*(Gamma[1/3, ((-I)*b)/(
c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)) - Gamma[1/3, (I*b)/(
c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)))*Sin[a])/d^2 + (((3*I)/
2)*b*e*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(2/3)*(c
 + d*x)^2) - Gamma[2/3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(2/3)*(c
+ d*x)^2))*Sin[a])/d - (((3*I)/2)*b*c*f*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(
3*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2) - Gamma[2/3, (I*b)/(c + d*x)^3]
/(3*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2))*Sin[a])/d^2 + (e*(c + d*x)*Cos[
a]*Sin[b/(c + d*x)^3])/d + (f*(-c + d*x)*(c + d*x)*Cos[a]*Sin[b/(c + d*x)^3
])/(2*d^2)
```

Maple [F]

$$\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^3}\right)dx$$

[In] int((f*x+e)*sin(a+b/(d*x+c)^3),x)

[Out] int((f*x+e)*sin(a+b/(d*x+c)^3),x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(176) = 352.

Time = 0.10 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.56

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)dx$$
$$= \frac{\left(-i\,d^2f\cos(a)-d^2f\sin(a)\right)\left(\frac{ib}{d^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+\left(i\,d^2f\cos(a)-d^2f\sin(a)\right)\left(-\frac{ib}{d^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},-\frac{}{d}\right.}{}$$

[In] integrate((f*x+e)*sin(a+b/(d*x+c)^3),x, algorithm="fricas")

```
[Out] 1/4*((-I*d^2*f*cos(a) - d^2*f*sin(a))*(I*b/d^3)^(2/3)*gamma(1/3, I*b/(d^3*x
^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + (I*d^2*f*cos(a) - d^2*f*sin(a))*(-I*
b/d^3)^(2/3)*gamma(1/3, -I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - 2
*((I*d^2*e - I*c*d*f)*cos(a) + (d^2*e - c*d*f)*sin(a))*(I*b/d^3)^(1/3)*gamm
a(2/3, I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - 2*((-I*d^2*e + I*c*
```

```
d*f)*cos(a) + (d^2*e - c*d*f)*sin(a))*(-I*b/d^3)^(1/3)*gamma(2/3, -I*b/(d^3
*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + 2*(d^2*f*x^2 + 2*d^2*e*x + 2*c*d*e
 - c^2*f)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^
3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int (e+fx)\sin\left(a+\frac{b}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**3),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
)), x)
```

Maxima [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^3}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*e*x)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 +
b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate(3/4*(b*d*f*x^2 +
2*b*d*e*x)*cos((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x
^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2
 + 4*c^3*d*x + c^4), x) + integrate(3/4*(b*d*f*x^2 + 2*b*d*e*x)*cos((a*d^3*
x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c
^2*d*x + c^3))/((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*c
os((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2
*x^2 + 3*c^2*d*x + c^3))^2 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3
*d*x + c^4)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*
x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))^2), x)
```

Giac [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx = \int (fx+e)\sin\left(a+\frac{b}{(dx+c)^3}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^3),x, algorithm="giac")
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx = \int \sin\left(a+\frac{b}{(c+d\,x)^3}\right)\,(e+f\,x)\,dx$$

```
[In] int(sin(a + b/(c + d*x)^3)*(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^3)*(e + f*x), x)
```

3.184 $\int \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 107

$$\int \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx = -\frac{ie^{ia}\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d} + \frac{ie^{-ia}\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d}$$

```
[Out] -1/6*I*exp(I*a)*(-I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,-I*b/(d*x+c)^3)/d
+1/6*I*(I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,I*b/(d*x+c)^3)/d/exp(I*a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3446, 2239}

$$\int \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx = \frac{ie^{-ia}(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d} - \frac{ie^{ia}(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d}$$

```
[In] Int[Sin[a + b/(c + d*x)^3],x]
```

```
[Out] ((-1/6*I)*E^(I*a)*(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*
b)/(c + d*x)^3])/d + ((I/6)*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3,
 (I*b)/(c + d*x)^3])/(d*E^(I*a))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}i\int e^{-ia-\frac{ib}{(c+dx)^3}}\,dx - \frac{1}{2}i\int e^{ia+\frac{ib}{(c+dx)^3}}\,dx$$

$$= -\frac{ie^{ia}\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d} + \frac{ie^{-ia}\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.90

$$\int \sin\left(a+\frac{b}{(c+dx)^3}\right)dx$$

$$= \frac{b\cos(a)\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}}+\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{\left(\frac{ib}{(c+dx)^3}\right)^{2/3}}\right)+2(c+dx)^3\cos\left(\frac{b}{(c+dx)^3}\right)\sin(a)+ib\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}}-\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{\left(\frac{ib}{(c+dx)^3}\right)^{2/3}}\right)}{2d(c+dx)^2}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^3],x]
```

```
[Out] (b*Cos[a]*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(((-I)*b)/(c + d*x)^3)^(2/3) +
Gamma[2/3, (I*b)/(c + d*x)^3]/((I*b)/(c + d*x)^3)^(2/3)) + 2*(c + d*x)^3*Co
s[b/(c + d*x)^3]*Sin[a] + I*b*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(((-I)*b)/(
c + d*x)^3)^(2/3) - Gamma[2/3, (I*b)/(c + d*x)^3]/((I*b)/(c + d*x)^3)^(2/3)
)*Sin[a] + 2*(c + d*x)^3*Cos[a]*Sin[b/(c + d*x)^3])/(2*d*(c + d*x)^2)
```

Maple [F]

$$\int \sin\left(a + \frac{b}{(dx+c)^3}\right) dx$$

```
[In] int(sin(a+b/(d*x+c)^3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^3),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(77) = 154.

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.73

$$\int \sin\left(a + \frac{b}{(c+dx)^3}\right) dx = \frac{(-i d \cos(a) - d \sin(a))\left(\frac{ib}{d^3}\right)^{\frac{1}{3}} \Gamma\left(\frac{2}{3}, \frac{ib}{d^3x^3+3cd^2x^2+3c^2dx+c^3}\right) + (i d \cos(a) - d \sin(a))\left(-\frac{ib}{d^3}\right)^{\frac{1}{3}} \Gamma\left(\frac{2}{3}, -\frac{ib}{d^3x^3+3cd^2x^2+3c^2dx+c^3}\right) + \cdots}{2d}$$

```
[In] integrate(sin(a+b/(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*d*cos(a) - d*sin(a))*(I*b/d^3)^(1/3)*gamma(2/3, I*b/(d^3*x^3 + 3*c
*d^2*x^2 + 3*c^2*d*x + c^3)) + (I*d*cos(a) - d*sin(a))*(-I*b/d^3)^(1/3)*gam
ma(2/3, -I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + 2*(d*x + c)*sin((
a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2
 + 3*c^2*d*x + c^3)))/d
```

Sympy [F]

$$\int \sin\left(a + \frac{b}{(c+dx)^3}\right) dx = \int \sin\left(a + \frac{b}{(c+dx)^3}\right) dx$$

```
[In] integrate(sin(a+b/(d*x+c)**3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**3), x)
```

Maxima [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int \sin\left(a+\frac{b}{(dx+c)^3}\right)\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 3*b*d*integrate(1/2*x*cos((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3
+ b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*
c^2*d^2*x^2 + 4*c^3*d*x + c^4), x) + 3*b*d*integrate(1/2*x*cos((a*d^3*x^3 +
 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*
x + c^3))/((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*cos((a
*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2
+ 3*c^2*d*x + c^3))^2 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x
+ c^4)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3))^2), x) + x*sin((a*d^3*x^3 + 3*a*c*d^2*x^2
+ 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))
```

Giac [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int \sin\left(a+\frac{b}{(dx+c)^3}\right)\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^3}\right)\,dx$$

```
[In] int(sin(a + b/(c + d*x)^3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^3), x)
```

3.185 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx = \text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^3)/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$$

```
[In] Int[Sin[a + b/(c + d*x)^3]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[a + b/(c + d*x)^3]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$$

```
[In] Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x),x]
[Out] Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{fx+e}dx$$

```
[In] int(sin(a+b/(d*x+c)^3)/(f*x+e),x)
[Out] int(sin(a+b/(d*x+c)^3)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.70

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e),x, algorithm="fricas")
[Out] integral(sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3
 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx = \text{Timed out}$$

```
[In] integrate(sin(a+b/(d*x+c)**3)/(f*x+e),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin(a + b/(d*x + c)^3)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 5.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^3)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 7.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^3}\right)}{e+f\,x}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^3)/(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^3)/(e + f*x), x)
```

3.186 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx = \text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx$$

```
[In] Int[Sin[a + b/(c + d*x)^3]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b/(c + d*x)^3]/(e + f*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 21.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx$$

```
[In] Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{(fx+e)^2}dx$$

```
[In] int(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x)
[Out] int(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 4.25

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3
 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(a+b/(d*x+c)**3)/(f*x+e)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin(a + b/(d*x + c)^3)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 7.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^3)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 14.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^3}\right)}{{(e+f\,x)}^2}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^3)/(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^3)/(e + f*x)^2, x)
```

3.187 $\int (e+fx)^2 \sin\left(a+b\sqrt{c+dx}\right)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 410

$$\begin{aligned}\int (e+fx)^2 \sin\left(a+b\sqrt{c+dx}\right)\, dx = &-\frac{240f^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^5d^3} \\ &+\frac{24f(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3} \\ &-\frac{2(de-cf)^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} \\ &+\frac{40f^2(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3} \\ &-\frac{4f(de-cf)(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} \\ &-\frac{2f^2(c+dx)^{5/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} \\ &+\frac{240f^2\sin\left(a+b\sqrt{c+dx}\right)}{b^6d^3} \\ &-\frac{24f(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3} \\ &+\frac{2(de-cf)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\ &-\frac{120f^2(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3} \\ &+\frac{12f(de-cf)(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\ &+\frac{10f^2(c+dx)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}\end{aligned}$$

```
[Out] 40*f^2*(d*x+c)^(3/2)*cos(a+b*(d*x+c)^(1/2))/b^3/d^3-4*f*(-c*f+d*e)*(d*x+c)^
(3/2)*cos(a+b*(d*x+c)^(1/2))/b/d^3-2*f^2*(d*x+c)^(5/2)*cos(a+b*(d*x+c)^(1/2
))/b/d^3+240*f^2*sin(a+b*(d*x+c)^(1/2))/b^6/d^3-24*f*(-c*f+d*e)*sin(a+b*(d*
x+c)^(1/2))/b^4/d^3+2*(-c*f+d*e)^2*sin(a+b*(d*x+c)^(1/2))/b^2/d^3-120*f^2*(
d*x+c)*sin(a+b*(d*x+c)^(1/2))/b^4/d^3+12*f*(-c*f+d*e)*(d*x+c)*sin(a+b*(d*x+
c)^(1/2))/b^2/d^3+10*f^2*(d*x+c)^2*sin(a+b*(d*x+c)^(1/2))/b^2/d^3-240*f^2*c
os(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^5/d^3+24*f*(-c*f+d*e)*cos(a+b*(d*x+c)
^(1/2))*(d*x+c)^(1/2)/b^3/d^3-2*(-c*f+d*e)^2*cos(a+b*(d*x+c)^(1/2))*(d*x+c)
^(1/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3512, 3377, 2717}

$$\begin{aligned}\int (e+fx)^2 \sin\left(a+b\sqrt{c+dx}\right)\,dx = {} & \frac{240f^2\sin\left(a+b\sqrt{c+dx}\right)}{b^6d^3} \\ & -\frac{240f^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^5d^3} \\ & -\frac{24f(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3} \\ & -\frac{120f^2(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3} \\ & +\frac{24f\sqrt{c+dx}(de-cf)\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3} \\ & +\frac{40f^2(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3} \\ & +\frac{12f(c+dx)(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\ & +\frac{2(de-cf)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\ & +\frac{10f^2(c+dx)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\ & -\frac{4f(c+dx)^{3/2}(de-cf)\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} \\ & -\frac{2\sqrt{c+dx}(de-cf)^2\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} \\ & -\frac{2f^2(c+dx)^{5/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}\end{aligned}$$

```
[In] Int[(e + f*x)^2*Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-240*f^2*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^5*d^3) + (24*f*(d*e -
c*f)*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^3*d^3) - (2*(d*e - c*f)^2*S
qrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d^3) + (40*f^2*(c + d*x)^(3/2)*Co
s[a + b*Sqrt[c + d*x]])/(b^3*d^3) - (4*f*(d*e - c*f)*(c + d*x)^(3/2)*Cos[a
+ b*Sqrt[c + d*x]])/(b*d^3) - (2*f^2*(c + d*x)^(5/2)*Cos[a + b*Sqrt[c + d*x
]])/(b*d^3) + (240*f^2*Sin[a + b*Sqrt[c + d*x]])/(b^6*d^3) - (24*f*(d*e - c
*f)*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^3) + (2*(d*e - c*f)^2*Sin[a + b*Sqrt[c
 + d*x]])/(b^2*d^3) - (120*f^2*(c + d*x)*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^3
) + (12*f*(d*e - c*f)*(c + d*x)*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^3) + (10*f
^2*(c + d*x)^2*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^3)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \left(\frac{(de-cf)^2 x\sin(a+bx)}{d^2}+\frac{2f(de-cf)x^3\sin(a+bx)}{d^2}+\frac{f^2x^5\sin(a+bx)}{d^2}\right)\,dx, x, \sqrt{c+dx}\right)}{d}$$

$$= \frac{(2f^2)\,\text{Subst}\left(\int x^5\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3} + \frac{(4f(de-cf))\text{Subst}\left(\int x^3\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3} + \frac{(2(de-cf)^2)\,\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= -\frac{2(de-cf)^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{4f(de-cf)(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{2f^2(c+dx)^{5/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{(10f^2)\,\text{Subst}\left(\int x^4\cos(a+bx)\,dx,x,\sqrt{c+dx}\right)}{bd^3} + \frac{(12f(de-cf))\text{Subst}\left(\int x^2\cos(a+bx)\,dx,x,\sqrt{c+dx}\right)}{bd^3} + \frac{(2(de-cf)^2)\,\text{Subst}\left(\int\cos(a+bx)\,dx,x,\sqrt{c+dx}\right)}{bd^3}$$

$$= -\frac{2(de-cf)^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{4f(de-cf)(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{2f^2(c+dx)^{5/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(de-cf)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{12f(de-cf)(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{10f^2(c+dx)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{(40f^2)\,\text{Subst}\left(\int x^3\sin(a+bx)\,dx,x,\sqrt{c+dx}\right)}{b^2d^3} - \frac{(24f(de-cf))\text{Subst}\left(\int x\sin(a+bx)\,dx,x,\sqrt{c+dx}\right)}{b^2d^3}$$

$$= \frac{24f(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - \frac{2(de-cf)^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{40f^2(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - \frac{4f(de-cf)(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{2f^2(c+dx)^{5/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(de-cf)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{12f(de-cf)(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{10f^2(c+dx)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{(120f^2)\,\text{Subst}\left(\int x^2\cos(a+bx)\,dx,x,\sqrt{c+dx}\right)}{b^3d^3} - \frac{(24f(de-cf))\text{Subst}\left(\int\cos(a+bx)\,dx,x,\sqrt{c+dx}\right)}{b^3d^3}$$

$$= \frac{24f(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3}$$
$$-\frac{2(de-cf)^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}+\frac{40f^2(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3}$$
$$-\frac{4f(de-cf)(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}$$
$$-\frac{2f^2(c+dx)^{5/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}-\frac{24f(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3}$$
$$+\frac{2(de-cf)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{120f^2(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3}$$
$$+\frac{12f(de-cf)(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{10f^2(c+dx)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}$$
$$+\frac{(240f^2)\,\text{Subst}\left(\int x\sin(a+bx)\,dx,x,\sqrt{c+dx}\right)}{b^4d^3}$$

$$= -\frac{240f^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^5d^3}+\frac{24f(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3}$$
$$-\frac{2(de-cf)^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}+\frac{40f^2(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3}$$
$$-\frac{4f(de-cf)(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}$$
$$-\frac{2f^2(c+dx)^{5/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}-\frac{24f(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3}$$
$$+\frac{2(de-cf)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{120f^2(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3}$$
$$+\frac{12f(de-cf)(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{10f^2(c+dx)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}$$
$$+\frac{(240f^2)\,\text{Subst}\left(\int\cos(a+bx)\,dx,x,\sqrt{c+dx}\right)}{b^5d^3}$$

$$= -\frac{240f^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^5d^3}+\frac{24f(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3}$$
$$-\frac{2(de-cf)^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}+\frac{40f^2(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3}$$
$$-\frac{4f(de-cf)(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}-\frac{2f^2(c+dx)^{5/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}$$
$$+\frac{240f^2\sin\left(a+b\sqrt{c+dx}\right)}{b^6d^3}-\frac{24f(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3}$$
$$+\frac{2(de-cf)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{120f^2(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3}$$
$$+\frac{12f(de-cf)(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{10f^2(c+dx)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.34

$$\int (e+fx)^2\sin\left(a+b\sqrt{c+dx}\right)\,dx$$
$$=\frac{-2b\sqrt{c+dx}(120f^2+b^4d^2(e+fx)^2-4b^2f(3de+2cf+5dfx))\cos\left(a+b\sqrt{c+dx}\right)+2(120f^2-12b^2f(4c}{b^6d^3}$$

```
[In] Integrate[(e + f*x)^2*Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*b*Sqrt[c + d*x]*(120*f^2 + b^4*d^2*(e + f*x)^2 - 4*b^2*f*(3*d*e + 2*c*f
 + 5*d*f*x))*Cos[a + b*Sqrt[c + d*x]] + 2*(120*f^2 - 12*b^2*f*(4*c*f + d*(e
 + 5*f*x)) + b^4*d*(e + f*x)*(4*c*f + d*(e + 5*f*x)))*Sin[a + b*Sqrt[c + d*
x]])/(b^6*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. 2(374) = 748.

Time = 0.42 (sec) , antiderivative size = 1161, normalized size of antiderivative = 2.83

method	result	size
parts	Expression too large to display	1161
derivativedivides	Expression too large to display	1246
default	Expression too large to display	1246

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/b*cos(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)*f^2*x^2-4/d/b*cos(a+b*(d*x+c)^(
1/2))*(d*x+c)^(1/2)*e*f*x-2/d/b*cos(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)*e^2+2/
d/b^2*sin(a+b*(d*x+c)^(1/2))*f^2*x^2+4/d/b^2*sin(a+b*(d*x+c)^(1/2))*e*f*x+2
/d/b^2*sin(a+b*(d*x+c)^(1/2))*e^2-8/d^3/b^4*f*(-c*f*(sin(a+b*(d*x+c)^(1/2))
-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))-c*f*a*cos(a+b*(d*x+c)^(1/2))+d
*e*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+d*e*
a*cos(a+b*(d*x+c)^(1/2))+c*f*((a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-
2*sin(a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))-d*e*
((a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-2*sin(a+b*(d*x+c)^(1/2))+2*(a
+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+a^2*c*f*sin(a+b*(d*x+c)^(1/2))-a^
2*d*e*sin(a+b*(d*x+c)^(1/2))-2*c*f*a*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(
1/2))*sin(a+b*(d*x+c)^(1/2))))+3/b^2*a^2*f*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x
+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))-3/b^2*a*f*(-(a+b*(d*x+c)^(1/2))^2*cos(a+
b*(d*x+c)^(1/2))+2*cos(a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*
x+c)^(1/2)))-6/b^2*a^2*f*((a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-2*si
n(a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+4/b^2*a^
```

```
3*f*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))+4/b
^2*a*f*((a+b*(d*x+c)^(1/2))^3*sin(a+b*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^
2*cos(a+b*(d*x+c)^(1/2))-6*cos(a+b*(d*x+c)^(1/2))-6*(a+b*(d*x+c)^(1/2))*sin
(a+b*(d*x+c)^(1/2)))+2*d*e*a*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*si
n(a+b*(d*x+c)^(1/2)))+1/b^2*f*(-(a+b*(d*x+c)^(1/2))^3*cos(a+b*(d*x+c)^(1/2)
)+3*(a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-6*sin(a+b*(d*x+c)^(1/2))+6
*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))-1/b^2*f*((a+b*(d*x+c)^(1/2))^4
*sin(a+b*(d*x+c)^(1/2))+4*(a+b*(d*x+c)^(1/2))^3*cos(a+b*(d*x+c)^(1/2))-12*(
a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))+24*sin(a+b*(d*x+c)^(1/2))-24*(a
+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))-1/b^2*a^4*f*sin(a+b*(d*x+c)^(1/2)
)+1/b^2*a^3*f*cos(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.48

$$\int (e+fx)^2 \sin\left(a+b\sqrt{c+dx}\right)\,dx = -\frac{2\left(\left(b^5d^2f^2x^2+b^5d^2e^2-12\,b^3def-8\,(b^3c-15\,b)f^2+2\,(b^5d^2ef-10\,b^3df^2)x\right)\sqrt{dx+c}\cos\left(\sqrt{dx+c}b+\right.\right.}{}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*((b^5*d^2*f^2*x^2 + b^5*d^2*e^2 - 12*b^3*d*e*f - 8*(b^3*c - 15*b)*f^2 +
2*(b^5*d^2*e*f - 10*b^3*d*f^2)*x)*sqrt(d*x + c)*cos(sqrt(d*x + c)*b + a) -
(5*b^4*d^2*f^2*x^2 + b^4*d^2*e^2 + 4*(b^4*c - 3*b^2)*d*e*f - 24*(2*b^2*c -
5)*f^2 + 2*(3*b^4*d^2*e*f + 2*(b^4*c - 15*b^2)*d*f^2)*x)*sin(sqrt(d*x + c)*
b + a))/(b^6*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.29

$$\int (e+fx)^2 \sin\left(a+b\sqrt{c+dx}\right)\,dx = \begin{cases} \left(e^2x+efx^2+\frac{f^2x^3}{3}\right)\sin(a) \\ \left(e^2x+efx^2+\frac{f^2x^3}{3}\right)\sin\left(a+b\sqrt{c}\right) \\ -\frac{2e^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{4efx\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2f^2x^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{8cef\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{8cf^2x\sin}{} \end{cases}$$

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise(((e**2*x + e*f*x**2 + f**2*x**3/3)*sin(a), Eq(b, 0) & (Eq(b, 0) |
 Eq(d, 0))), ((e**2*x + e*f*x**2 + f**2*x**3/3)*sin(a + b*sqrt(c)), Eq(d, 0
)), (-2*e**2*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d) - 4*e*f*x*sqrt(c
+ d*x)*cos(a + b*sqrt(c + d*x))/(b*d) - 2*f**2*x**2*sqrt(c + d*x)*cos(a + b
*sqrt(c + d*x))/(b*d) + 8*c*e*f*sin(a + b*sqrt(c + d*x))/(b**2*d**2) + 8*c*
f**2*x*sin(a + b*sqrt(c + d*x))/(b**2*d**2) + 2*e**2*sin(a + b*sqrt(c + d*x
))/(b**2*d) + 12*e*f*x*sin(a + b*sqrt(c + d*x))/(b**2*d) + 10*f**2*x**2*sin
(a + b*sqrt(c + d*x))/(b**2*d) + 16*c*f**2*sqrt(c + d*x)*cos(a + b*sqrt(c +
 d*x))/(b**3*d**3) + 24*e*f*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b**3*d*
*2) + 40*f**2*x*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b**3*d**2) - 96*c*f
**2*sin(a + b*sqrt(c + d*x))/(b**4*d**3) - 24*e*f*sin(a + b*sqrt(c + d*x))/
(b**4*d**2) - 120*f**2*x*sin(a + b*sqrt(c + d*x))/(b**4*d**2) - 240*f**2*sq
rt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b**5*d**3) + 240*f**2*sin(a + b*sqrt(
c + d*x))/(b**6*d**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(374) = 748$.

Time = 0.25 (sec) , antiderivative size = 1101, normalized size of antiderivative = 2.69

$$\int (e+fx)^2 \sin\left(a+b\sqrt{c+dx}\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*(a*e^2*cos(sqrt(d*x + c)*b + a) - 2*a*c*e*f*cos(sqrt(d*x + c)*b + a)/d +
a*c^2*f^2*cos(sqrt(d*x + c)*b + a)/d^2 - ((sqrt(d*x + c)*b + a)*cos(sqrt(d*
x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*e^2 + 2*((sqrt(d*x + c)*b + a)*co
s(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*c*e*f/d - ((sqrt(d*x + c
)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*c^2*f^2/d^2 +
 2*a^3*e*f*cos(sqrt(d*x + c)*b + a)/(b^2*d) - 2*a^3*c*f^2*cos(sqrt(d*x + c)
*b + a)/(b^2*d^2) - 6*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin
(sqrt(d*x + c)*b + a))*a^2*e*f/(b^2*d) + 6*((sqrt(d*x + c)*b + a)*cos(sqrt(
d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*a^2*c*f^2/(b^2*d^2) + a^5*f^2*c
os(sqrt(d*x + c)*b + a)/(b^4*d^2) + 6*(((sqrt(d*x + c)*b + a)^2 - 2)*cos(sq
rt(d*x + c)*b + a) - 2*(sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a))*a*e*
f/(b^2*d) - 5*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*
x + c)*b + a))*a^4*f^2/(b^4*d^2) - 6*(((sqrt(d*x + c)*b + a)^2 - 2)*cos(sqr
t(d*x + c)*b + a) - 2*(sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a))*a*c*f
^2/(b^2*d^2) - 2*(((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*cos(s
qrt(d*x + c)*b + a) - 3*((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b +
 a))*e*f/(b^2*d) + 10*(((sqrt(d*x + c)*b + a)^2 - 2)*cos(sqrt(d*x + c)*b +
a) - 2*(sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a))*a^3*f^2/(b^4*d^2) +
2*(((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*cos(sqrt(d*x + c)*b
+ a) - 3*((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b + a))*c*f^2/(b^2
```

```
*d^2) - 10*(((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*cos(sqrt(d*
x + c)*b + a) - 3*((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b + a))*a
^2*f^2/(b^4*d^2) + 5*(((sqrt(d*x + c)*b + a)^4 - 12*(sqrt(d*x + c)*b + a)^2
 + 24)*cos(sqrt(d*x + c)*b + a) - 4*((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x +
 c)*b - 6*a)*sin(sqrt(d*x + c)*b + a))*a*f^2/(b^4*d^2) - (((sqrt(d*x + c)*b
 + a)^5 - 20*(sqrt(d*x + c)*b + a)^3 + 120*sqrt(d*x + c)*b + 120*a)*cos(sqr
t(d*x + c)*b + a) - 5*((sqrt(d*x + c)*b + a)^4 - 12*(sqrt(d*x + c)*b + a)^2
 + 24)*sin(sqrt(d*x + c)*b + a))*f^2/(b^4*d^2))/(b^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.71

$$\int (e+fx)^2 \sin\left(a+b\sqrt{c+dx}\right)\, dx = -\frac{2\left(\frac{\left(\sqrt{dx+c}b\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+a\right)\right)e^2}{b}+\frac{f^2\left(\frac{\left(\left(\sqrt{dx+c}b+a\right)b^4c^2-ab^4c^2-2\left(\sqrt{dx+c}b+a\right)^3b^2c+6\left(\sqrt{dx+c}b+a\right)^2ab^2c-6\left(\sqrt{dx+c}b+a\right)a\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*((sqrt(d*x + c)*b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*e
^2/b + f^2*(((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b
 + a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)
*a^2*b^2*c + 2*a^3*b^2*c + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x + c)*b + a
)^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^2*a^3 + 5
*(sqrt(d*x + c)*b + a)*a^4 - a^5 + 12*(sqrt(d*x + c)*b + a)*b^2*c - 12*a*b^
2*c - 20*(sqrt(d*x + c)*b + a)^3 + 60*(sqrt(d*x + c)*b + a)^2*a - 60*(sqrt(
d*x + c)*b + a)*a^2 + 20*a^3 + 120*sqrt(d*x + c)*b)*cos(sqrt(d*x + c)*b + a
)/(b^4*d^2) - (b^4*c^2 - 6*(sqrt(d*x + c)*b + a)^2*b^2*c + 12*(sqrt(d*x + c
)*b + a)*a*b^2*c - 6*a^2*b^2*c + 5*(sqrt(d*x + c)*b + a)^4 - 20*(sqrt(d*x +
 c)*b + a)^3*a + 30*(sqrt(d*x + c)*b + a)^2*a^2 - 20*(sqrt(d*x + c)*b + a)*
a^3 + 5*a^4 + 12*b^2*c - 60*(sqrt(d*x + c)*b + a)^2 + 120*(sqrt(d*x + c)*b
+ a)*a - 60*a^2 + 120)*sin(sqrt(d*x + c)*b + a)/(b^4*d^2))/b - 2*e*f*(((sqr
t(d*x + c)*b + a)*b^2*c - a*b^2*c - (sqrt(d*x + c)*b + a)^3 + 3*(sqrt(d*x +
 c)*b + a)^2*a - 3*(sqrt(d*x + c)*b + a)*a^2 + a^3 + 6*sqrt(d*x + c)*b)*cos
(sqrt(d*x + c)*b + a)/b^2 - (b^2*c - 3*(sqrt(d*x + c)*b + a)^2 + 6*(sqrt(d*
x + c)*b + a)*a - 3*a^2 + 6)*sin(sqrt(d*x + c)*b + a)/b^2)/(b*d))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+b\sqrt{c+dx}\right)\,dx=\int \sin\left(a+b\,\sqrt{c+d\,x}\right)\,(e+f\,x)^2\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/2))*(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(1/2))*(e + f*x)^2, x)
```

3.188 $\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 185

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx = \frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{2f(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{12f\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2} + \frac{2(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{6f(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}$$

```
[Out] -2*f*(d*x+c)^(3/2)*cos(a+b*(d*x+c)^(1/2))/b/d^2-12*f*sin(a+b*(d*x+c)^(1/2))
/b^4/d^2+2*(-c*f+d*e)*sin(a+b*(d*x+c)^(1/2))/b^2/d^2+6*f*(d*x+c)*sin(a+b*(d
*x+c)^(1/2))/b^2/d^2+12*f*cos(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^3/d^2-2*(-
c*f+d*e)*cos(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b/d^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3512, 3377, 2717}

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx = -\frac{12f\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2} + \frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2} + \frac{2(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{6f(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{2\sqrt{c+dx}(de-cf)\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{2f(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2}$$

```
[In] Int[(e + f*x)*Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (12*f*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*(d*e - c*f)*Sq
rt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d^2) - (2*f*(c + d*x)^(3/2)*Cos[a
+ b*Sqrt[c + d*x]])/(b*d^2) - (12*f*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (
2*(d*e - c*f)*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^2) + (6*f*(c + d*x)*Sin[a +
b*Sqrt[c + d*x]])/(b^2*d^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \left(\frac{(de-cf)x\sin(a+bx)}{d} + \frac{fx^3\sin(a+bx)}{d}\right) dx, x, \sqrt{c+dx}\right)}{d}$$

$$= \frac{(2f)\text{Subst}\left(\int x^3\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^2} + \frac{(2(de-cf))\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= -\frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{2f(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{(6f)\text{Subst}\left(\int x^2\cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd^2} + \frac{(2(de-cf))\text{Subst}\left(\int \cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd^2}$$

$$= -\frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{2f(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{6f(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{(12f)\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^2d^2}$$

$$= \frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{2f(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{6f(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{(12f)\text{Subst}\left(\int \cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^3d^2}$$

$$= \frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{2f(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{12f\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2} + \frac{2(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{6f(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx$$
$$=\frac{-2b\sqrt{c+dx}(-6f+b^2d(e+fx))\cos\left(a+b\sqrt{c+dx}\right)+2(-6f+b^2(2cf+d(e+3fx)))\sin\left(a+b\sqrt{c+dx}\right.}{b^4d^2}$$

[In] Integrate[(e + f*x)*Sin[a + b*Sqrt[c + d*x]],x]

[Out] (-2*b*Sqrt[c + d*x]*(-6*f + b^2*d*(e + f*x))*Cos[a + b*Sqrt[c + d*x]] + 2*(-6*f + b^2*(2*c*f + d*(e + 3*f*x)))*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(167) = 334.

Time = 0.37 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.88

method	result
parts	$-\frac{2\sqrt{dx+c}\,\cos\left(a+b\sqrt{dx+c}\right)fx}{db}-\frac{2\sqrt{dx+c}\,\cos\left(a+b\sqrt{dx+c}\right)e}{db}+\frac{2\sin\left(a+b\sqrt{dx+c}\right)fx}{d\,b^2}+\frac{2\sin\left(a+b\sqrt{dx+c}\right)e}{d\,b^2}-\frac{2f(}{}$
derivativedivides	$\frac{-2cfa\cos\left(a+b\sqrt{dx+c}\right)+2dea\cos\left(a+b\sqrt{dx+c}\right)-2cf\left(\sin\left(a+b\sqrt{dx+c}\right)-\left(a+b\sqrt{dx+c}\right)\cos\left(a+b\sqrt{dx+c}\right)\right)+2de\left(\sin\left(a+b\sqrt{}\right.\right.}{}$
default	$\frac{-2cfa\cos\left(a+b\sqrt{dx+c}\right)+2dea\cos\left(a+b\sqrt{dx+c}\right)-2cf\left(\sin\left(a+b\sqrt{dx+c}\right)-\left(a+b\sqrt{dx+c}\right)\cos\left(a+b\sqrt{dx+c}\right)\right)+2de\left(\sin\left(a+b\sqrt{}\right.\right.}{}$

[In] int((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)

[Out] -2/d/b*(d*x+c)^(1/2)*cos(a+b*(d*x+c)^(1/2))*f*x-2/d/b*(d*x+c)^(1/2)*cos(a+b*(d*x+c)^(1/2))*e+2/d/b^2*sin(a+b*(d*x+c)^(1/2))*f*x+2/d/b^2*sin(a+b*(d*x+c)^(1/2))*e-2/d/b^2*f*(2*a/d/b^2*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2))-a*sin(a+b*(d*x+c)^(1/2)))-2/d/b^2*((a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-2*sin(a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2))-a*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2))))+2/d/b^2*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2))+a*cos(a+b*(d*x+c)^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.46

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx = -\frac{2\left((b^3dfx+b^3de-6\,bf)\sqrt{dx+c}\cos\left(\sqrt{dx+c}b+a\right)-(3\,b^2dfx+b^2de+2\,(b^2c-3)f)\sin\left(\sqrt{dx+c}b+\right.\right.}{b^4d^2}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*((b^3*d*f*x + b^3*d*e - 6*b*f)*sqrt(d*x + c)*cos(sqrt(d*x + c)*b + a) -
(3*b^2*d*f*x + b^2*d*e + 2*(b^2*c - 3)*f)*sin(sqrt(d*x + c)*b + a))/(b^4*d^
2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.19

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx = \begin{cases}\left(ex+\frac{fx^2}{2}\right)\sin(a) \\ \left(ex+\frac{fx^2}{2}\right)\sin\left(a+b\sqrt{c}\right) \\ -\frac{2e\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2fx\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{4cf\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{2e\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{6fx\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}+\end{cases}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise(((e*x + f*x**2/2)*sin(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), ((e*
x + f*x**2/2)*sin(a + b*sqrt(c)), Eq(d, 0)), (-2*e*sqrt(c + d*x)*cos(a + b*
sqrt(c + d*x))/(b*d) - 2*f*x*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d) +
 4*c*f*sin(a + b*sqrt(c + d*x))/(b**2*d**2) + 2*e*sin(a + b*sqrt(c + d*x))/
(b**2*d) + 6*f*x*sin(a + b*sqrt(c + d*x))/(b**2*d) + 12*f*sqrt(c + d*x)*cos
(a + b*sqrt(c + d*x))/(b**3*d**2) - 12*f*sin(a + b*sqrt(c + d*x))/(b**4*d**
2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(167) = 334.

Time = 0.20 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.88

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx$$
$$= \frac{2\left(ae\cos\left(\sqrt{dx+c}b+a\right) - \frac{acf\cos\left(\sqrt{dx+c}b+a\right)}{d} - \left(\left(\sqrt{dx+c}b+a\right)\cos\left(\sqrt{dx+c}b+a\right) - \sin\left(\sqrt{dx+c}b+a\right)\right)\right.}{}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*(a*e*cos(sqrt(d*x + c)*b + a) - a*c*f*cos(sqrt(d*x + c)*b + a)/d - ((sqrt
(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*e + (
(sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*
c*f/d + a^3*f*cos(sqrt(d*x + c)*b + a)/(b^2*d) - 3*((sqrt(d*x + c)*b + a)*c
os(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*a^2*f/(b^2*d) + 3*(((sq
rt(d*x + c)*b + a)^2 - 2)*cos(sqrt(d*x + c)*b + a) - 2*(sqrt(d*x + c)*b + a
)*sin(sqrt(d*x + c)*b + a))*a*f/(b^2*d) - (((sqrt(d*x + c)*b + a)^3 - 6*sqr
t(d*x + c)*b - 6*a)*cos(sqrt(d*x + c)*b + a) - 3*((sqrt(d*x + c)*b + a)^2 -
 2)*sin(sqrt(d*x + c)*b + a))*f/(b^2*d))/(b^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.18

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx =$$
$$-\frac{2\left(\frac{\left(\sqrt{dx+c}b\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+a\right)\right)e}{b} - \frac{f\left(\frac{\left(\left(\sqrt{dx+c}b+a\right)b^2c-ab^2c-\left(\sqrt{dx+c}b+a\right)^3+3\left(\sqrt{dx+c}b+a\right)^2a-3\left(\sqrt{dx+c}b+a\right)a^2+a^3+6\sqrt{dx+c}\cdots\right.}{b^2}\right.}{bd}\right.}{bd}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*((sqrt(d*x + c)*b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*e
/b - f*(((sqrt(d*x + c)*b + a)*b^2*c - a*b^2*c - (sqrt(d*x + c)*b + a)^3 +
3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(d*x + c)*b + a)*a^2 + a^3 + 6*sqrt(d*
x + c)*b)*cos(sqrt(d*x + c)*b + a)/b^2 - (b^2*c - 3*(sqrt(d*x + c)*b + a)^2
 + 6*(sqrt(d*x + c)*b + a)*a - 3*a^2 + 6)*sin(sqrt(d*x + c)*b + a)/b^2)/(b*
d))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx=\int \sin\left(a+b\sqrt{c+d\,x}\right)\,(e+f\,x)\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/2))*(e + f*x),x)
[Out] int(sin(a + b*(c + d*x)^(1/2))*(e + f*x), x)
```

3.189 $\int \sin\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \sin\left(a+b\sqrt{c+dx}\right)\,dx=-\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] 2*sin(a+b*(d*x+c)^(1/2))/b^2/d-2*cos(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3442, 3377, 2717}

$$\int \sin\left(a+b\sqrt{c+dx}\right)\,dx=\frac{2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}-\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}$$

```
[In] Int[Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d) + (2*Sin[a + b*Sqrt[c + d
*x]])/(b^2*d)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d} \\
&= -\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd} + \frac{2\text{Subst}\left(\int \cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd} \\
&= -\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd} + \frac{2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \sin\left(a+b\sqrt{c+dx}\right)\,dx = \frac{-2b\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)+2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[In] Integrate[Sin[a + b*Sqrt[c + d*x]],x]
[Out] (-2*b*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]] + 2*Sin[a + b*Sqrt[c + d*x]])/
(b^2*d)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{2\sin\left(a+b\sqrt{dx+c}\right)-2\left(a+b\sqrt{dx+c}\right)\cos\left(a+b\sqrt{dx+c}\right)+2a\cos\left(a+b\sqrt{dx+c}\right)}{b^2d}$	61
default	$\frac{2\sin\left(a+b\sqrt{dx+c}\right)-2\left(a+b\sqrt{dx+c}\right)\cos\left(a+b\sqrt{dx+c}\right)+2a\cos\left(a+b\sqrt{dx+c}\right)}{b^2d}$	61

```
[In] int(sin(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/d/b^2*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2))+
a*cos(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \sin\left(a+b\sqrt{c+dx}\right)\,dx=-\frac{2\left(\sqrt{dx+c}b\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
[Out] -2*(sqrt(d*x + c)*b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))/(b
^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \sin\left(a+b\sqrt{c+dx}\right)\,dx=\begin{cases} x\sin\left(a\right) & \text{for}\ b=0\wedge\left(b=0\vee d=0\right)\\ x\sin\left(a+b\sqrt{c}\right) & \text{for}\ d=0\\ -\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d} & \text{otherwise}\end{cases}$$

```
[In] integrate(sin(a+b*(d*x+c)**(1/2)),x)
[Out] Piecewise((x*sin(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sin(a + b*sqrt(c
)), Eq(d, 0)), (-2*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d) + 2*sin(a +
 b*sqrt(c + d*x))/(b**2*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \sin\left(a+b\sqrt{c+dx}\right)\,dx=-\frac{2\left(\left(\sqrt{dx+c}b+a\right)\cos\left(\sqrt{dx+c}b+a\right)-a\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
[Out] -2*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - a*cos(sqrt(d*x + c)*b
+ a) - sin(sqrt(d*x + c)*b + a))/(b^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \sin \left(a+b\sqrt{c+dx}\right)\,dx=-\frac{2\left(\sqrt{dx+c}b\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
[Out] -2*(sqrt(d*x + c)*b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))/(b
^2*d)
```

Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \sin \left(a+b\sqrt{c+dx}\right)\,dx=\frac{2\left(\sin\left(a+b\sqrt{c+d\,x}\right)-b\,\cos\left(a+b\sqrt{c+d\,x}\right)\,\sqrt{c+d\,x}\right)}{b^2\,d}$$

```
[In] int(sin(a + b*(c + d*x)^(1/2)),x)
[Out] (2*(sin(a + b*(c + d*x)^(1/2)) - b*cos(a + b*(c + d*x)^(1/2))*(c + d*x)^(1/
2)))/(b^2*d)
```

3.190 $\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 238

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx = \frac{\text{CosIntegral}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)\sin\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}{f} + \frac{\text{CosIntegral}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)\sin\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}{f} - \frac{\cos\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Si}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{f} + \frac{\cos\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Si}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{f}$$

```
[Out] cos(a+b*(c*f-d*e)^(1/2)/f^(1/2))*Si(-b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x+c)^(1
/2))/f+cos(a-b*(c*f-d*e)^(1/2)/f^(1/2))*Si(b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x
+c)^(1/2))/f+Ci(b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x+c)^(1/2))*sin(a-b*(c*f-d*e
)^(1/2)/f^(1/2))/f+Ci(b*(c*f-d*e)^(1/2)/f^(1/2)-b*(d*x+c)^(1/2))*sin(a+b*(c
*f-d*e)^(1/2)/f^(1/2))/f
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3512, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx = \frac{\sin\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{\sqrt{cf-de}b}{\sqrt{f}}+\sqrt{c+dx}b\right)}{f} + \frac{\sin\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{f} - \frac{\cos\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{Si}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{f} + \frac{\cos\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{Si}\left(\frac{\sqrt{cf-de}b}{\sqrt{f}}+\sqrt{c+dx}b\right)}{f}$$

```
[In] Int[Sin[a + b*Sqrt[c + d*x]]/(e + f*x),x]
```

```
[Out] (CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]]*Sin[a - (b*S
qrt[-(d*e) + c*f])/Sqrt[f]])/f + (CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f
] - b*Sqrt[c + d*x]]*Sin[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]])/f - (Cos[a +
(b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f]
- b*Sqrt[c + d*x]])/f + (Cos[a - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegra
l[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]])/f
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \left(-\frac{d\sin(a+bx)}{2\sqrt{f}\left(\sqrt{-de+cf}-\sqrt{f}x\right)} + \frac{d\sin(a+bx)}{2\sqrt{f}\left(\sqrt{-de+cf}+\sqrt{f}x\right)}\right)dx, x, \sqrt{c+dx}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{-de+cf}-\sqrt{f}x}dx, x, \sqrt{c+dx}\right)}{\sqrt{f}} + \frac{\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{-de+cf}+\sqrt{f}x}dx, x, \sqrt{c+dx}\right)}{\sqrt{f}}$$

$$= \frac{\cos\left(a - \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+bx\right)}{\sqrt{-de+cf}+\sqrt{f}x}dx, x, \sqrt{c+dx}\right)}{\sqrt{f}}$$
$$+ \frac{\cos\left(a + \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-bx\right)}{\sqrt{-de+cf}-\sqrt{f}x}dx, x, \sqrt{c+dx}\right)}{\sqrt{f}}$$
$$+ \frac{\sin\left(a - \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+bx\right)}{\sqrt{-de+cf}+\sqrt{f}x}dx, x, \sqrt{c+dx}\right)}{\sqrt{f}}$$
$$- \frac{\sin\left(a + \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-bx\right)}{\sqrt{-de+cf}-\sqrt{f}x}dx, x, \sqrt{c+dx}\right)}{\sqrt{f}}$$

$$= \frac{\text{CosIntegral}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}} + b\sqrt{c+dx}\right)\sin\left(a - \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}{f}$$
$$+ \frac{\text{CosIntegral}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}} - b\sqrt{c+dx}\right)\sin\left(a + \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}{f}$$
$$- \frac{\cos\left(a + \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Si}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}} - b\sqrt{c+dx}\right)}{f}$$
$$+ \frac{\cos\left(a - \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Si}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}} + b\sqrt{c+dx}\right)}{f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx$$
$$= \frac{ie^{-i\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}\left(\text{ExpIntegralEi}\left(-ib\left(-\frac{\sqrt{-de+cf}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)-e^{2i\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}\text{ExpIntegralEi}\left(ib\left(-\frac{\sqrt{-}\right.\right.\right.}{}$$

```
[In] Integrate[Sin[a + b*Sqrt[c + d*x]]/(e + f*x),x]
```

```
[Out] ((I/2)*(ExpIntegralEi[(-I)*b*(-(Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x]
)] - E^((2*I)*(a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]))*ExpIntegralEi[I*b*(-(Sq
rt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x])] + E^(((2*I)*b*Sqrt[-(d*e) + c*f
])/Sqrt[f])*ExpIntegralEi[(-I)*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c + d*x
])] - E^((2*I)*a)*ExpIntegralEi[I*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c +
d*x])]))/(E^(I*(a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]))*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. 2(197) = 394.

Time = 0.25 (sec) , antiderivative size = 793, normalized size of antiderivative = 3.33

method	result
derivativedivides	$-\frac{b^2\left(fa+\sqrt{b^2cf^2-b^2def}\right)\left(-\text{Si}\left(-b\sqrt{dx+c}-a+\frac{fa+\sqrt{b^2cf^2-b^2def}}{f}\right)\cos\left(\frac{fa+\sqrt{b^2cf^2-b^2def}}{f}\right)+\text{Ci}\left(b\sqrt{dx+c}+a-\frac{fa+\sqrt{b^2cf}}{f}\right.\right.}{f^2\left(-\frac{fa+\sqrt{b^2cf^2-b^2def}}{f}+a\right)}$
default	$-\frac{b^2\left(fa+\sqrt{b^2cf^2-b^2def}\right)\left(-\text{Si}\left(-b\sqrt{dx+c}-a+\frac{fa+\sqrt{b^2cf^2-b^2def}}{f}\right)\cos\left(\frac{fa+\sqrt{b^2cf^2-b^2def}}{f}\right)+\text{Ci}\left(b\sqrt{dx+c}+a-\frac{fa+\sqrt{b^2cf}}{f}\right.\right.}{f^2\left(-\frac{fa+\sqrt{b^2cf^2-b^2def}}{f}+a\right)}$

```
[In] int(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(-1/2*b^2*(f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f^2/(-(f*a+(b^2*c*f^2-b^2
*d*e*f)^(1/2))/f+a)*(-Si(-b*(d*x+c)^(1/2)-a+(f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2
))/f)*cos((f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a-(f*a+(b
^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/2
*b^2*(-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f^2/((-f*a+(b^2*c*f^2-b^2*d*e*f)^(1
/2))/f+a)*(-Si(-b*(d*x+c)^(1/2)-a-(-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos
((-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)-Ci(b*(d*x+c)^(1/2)+a+(-f*a+(b^2*c*f^
```

```
2-b^2*d*e*f)^(1/2))/f)*sin((-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))-b^2*a*(-1
/2/f/(-(f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(-b*(d*x+c)^(1/2)-a+(f*a+
(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci
(b*(d*x+c)^(1/2)+a-(f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((f*a+(b^2*c*f^2
-b^2*d*e*f)^(1/2))/f))-1/2/f/((-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(
-b*(d*x+c)^(1/2)-a-(-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-f*a+(b^2*c*f
^2-b^2*d*e*f)^(1/2))/f)-Ci(b*(d*x+c)^(1/2)+a+(-f*a+(b^2*c*f^2-b^2*d*e*f)^(1
/2))/f)*sin((-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.05

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx = \frac{-i\,\mathrm{Ei}\left(i\sqrt{dx+c}b-\sqrt{\frac{b^2de-b^2cf}{f}}\right)e^{\left(ia+\sqrt{\frac{b^2de-b^2cf}{f}}\right)}-i\,\mathrm{Ei}\left(i\sqrt{dx+c}b+\sqrt{\frac{b^2de-b^2cf}{f}}\right)e^{\left(ia-\sqrt{\frac{b^2de-b^2cf}{f}}\right)}+i\,\mathrm{Ei}}{2f}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*Ei(I*sqrt(d*x + c)*b - sqrt((b^2*d*e - b^2*c*f)/f))*e^(I*a + sqrt((
b^2*d*e - b^2*c*f)/f)) - I*Ei(I*sqrt(d*x + c)*b + sqrt((b^2*d*e - b^2*c*f)/
f))*e^(I*a - sqrt((b^2*d*e - b^2*c*f)/f)) + I*Ei(-I*sqrt(d*x + c)*b - sqrt(
(b^2*d*e - b^2*c*f)/f))*e^(-I*a + sqrt((b^2*d*e - b^2*c*f)/f)) + I*Ei(-I*sq
rt(d*x + c)*b + sqrt((b^2*d*e - b^2*c*f)/f))*e^(-I*a - sqrt((b^2*d*e - b^2*
c*f)/f)))/f
```

Sympy [F]

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(1/2))/(f*x+e),x)
```

```
[Out] Integral(sin(a + b*sqrt(c + d*x))/(e + f*x), x)
```

Maxima [F]

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx = \int \frac{\sin\left(\sqrt{dx+c}b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e), x)
```

Giac [F]

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx = \int \frac{\sin\left(\sqrt{dx+c}b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x, algorithm="giac")
[Out] integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+b\sqrt{c+d\,x}\right)}{e+f\,x}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/2))/(e + f*x),x)
[Out] int(sin(a + b*(c + d*x)^(1/2))/(e + f*x), x)
```

3.191 $\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 339

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx = \frac{bd\cos\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\operatorname{CosIntegral}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}} - \frac{bd\cos\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\operatorname{CosIntegral}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}} - \frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} + \frac{bd\sin\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\operatorname{Si}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}} + \frac{bd\sin\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\operatorname{Si}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}}$$

```
[Out] -sin(a+b*(d*x+c)^(1/2))/f/(f*x+e)-1/2*b*d*Ci(b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d
*x+c)^(1/2))*cos(a-b*(c*f-d*e)^(1/2)/f^(1/2))/f^(3/2)/(c*f-d*e)^(1/2)+1/2*b
*d*Ci(b*(c*f-d*e)^(1/2)/f^(1/2)-b*(d*x+c)^(1/2))*cos(a+b*(c*f-d*e)^(1/2)/f^
(1/2))/f^(3/2)/(c*f-d*e)^(1/2)+1/2*b*d*Si(b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x+
c)^(1/2))*sin(a-b*(c*f-d*e)^(1/2)/f^(1/2))/f^(3/2)/(c*f-d*e)^(1/2)-1/2*b*d*
Si(-b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x+c)^(1/2))*sin(a+b*(c*f-d*e)^(1/2)/f^(1
/2))/f^(3/2)/(c*f-d*e)^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3512, 3422, 3415, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx = \frac{bd\cos\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{cf-de}} - \frac{bd\cos\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{\sqrt{cf-de}b}{\sqrt{f}}+\sqrt{c+dx}b\right)}{2f^{3/2}\sqrt{cf-de}} + \frac{bd\sin\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{Si}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{cf-de}} + \frac{bd\sin\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{Si}\left(\frac{\sqrt{cf-de}b}{\sqrt{f}}+\sqrt{c+dx}b\right)}{2f^{3/2}\sqrt{cf-de}} - \frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)}$$

```
[In] Int[Sin[a + b*Sqrt[c + d*x]]/(e + f*x)^2,x]
```

```
[Out] (b*d*Cos[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*CosIntegral[(b*Sqrt[-(d*e) + c
*f])/Sqrt[f] - b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d*e) + c*f]) - (b*d*Cos[
a - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt
[f] + b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d*e) + c*f]) - Sin[a + b*Sqrt[c +
 d*x]]/(f*(e + f*x)) + (b*d*Sin[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinInte
gral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] - b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d
*e) + c*f]) + (b*d*Sin[a - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegral[(b*S
qrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d*e) + c*f
])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x\sin(a+bx)}{\left(e-\frac{cf}{d}+\frac{fx^2}{d}\right)^2}\,dx, x, \sqrt{c+dx}\right)}{d}$$

$$= -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} + \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{e-\frac{cf}{d}+\frac{fx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{f}$$

$$= -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} + \frac{b\text{Subst}\left(\int \left(\frac{\sqrt{-de+cf}\cos(a+bx)}{2\left(e-\frac{cf}{d}\right)\left(\sqrt{-de+cf}-\sqrt{f}x\right)} + \frac{\sqrt{-de+cf}\cos(a+bx)}{2\left(e-\frac{cf}{d}\right)\left(\sqrt{-de+cf}+\sqrt{f}x\right)}\right)dx, x, \sqrt{c+dx}\right)}{f}$$

$$= -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{-de+cf}-\sqrt{f}x}\,dx, x, \sqrt{c+dx}\right)}{2f\sqrt{-de+cf}} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{-de+cf}+\sqrt{f}x}\,dx, x, \sqrt{c+dx}\right)}{2f\sqrt{-de+cf}}$$

$$= -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} - \frac{\left(bd\cos\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+bx\right)}{\sqrt{-de+cf}+\sqrt{f}x}dx,x,\sqrt{c+dx}\right)}{2f\sqrt{-de+cf}}$$
$$-\frac{\left(bd\cos\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-bx\right)}{\sqrt{-de+cf}-\sqrt{f}x}dx,x,\sqrt{c+dx}\right)}{2f\sqrt{-de+cf}}$$
$$+\frac{\left(bd\sin\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+bx\right)}{\sqrt{-de+cf}+\sqrt{f}x}dx,x,\sqrt{c+dx}\right)}{2f\sqrt{-de+cf}}$$
$$-\frac{\left(bd\sin\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-bx\right)}{\sqrt{-de+cf}-\sqrt{f}x}dx,x,\sqrt{c+dx}\right)}{2f\sqrt{-de+cf}}$$
$$=\frac{bd\cos\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}}$$
$$-\frac{bd\cos\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}}$$
$$-\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)}+\frac{bd\sin\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Si}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}}$$
$$+\frac{bd\sin\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Si}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.17

$$\int\frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}dx$$
$$=\frac{ide^{-ia}\left(-\frac{2e^{-ib\sqrt{c+dx}}\sqrt{f}}{de+dfx}-\frac{ibe^{-\frac{ib\sqrt{-de+cf}}{\sqrt{f}}}\text{ExpIntegralEi}\left(-ib\left(-\frac{\sqrt{-de+cf}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)}{\sqrt{-de+cf}}+\frac{ibe^{\frac{ib\sqrt{-de+cf}}{\sqrt{f}}}\text{ExpIntegralEi}\left(-ib\left(\frac{\sqrt{-de+cf}}{\sqrt{f}}\right.\right.}{\sqrt{-de+cf}}\right.}{}$$

```
[In] Integrate[Sin[a + b*Sqrt[c + d*x]]/(e + f*x)^2,x]
```

```
[Out] ((I/4)*d*((-2*Sqrt[f])/(E^(I*b*Sqrt[c + d*x])*(d*e + d*f*x)) - (I*b*ExpInte
gralEi[(-I)*b*(-(Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x])])/(E^((I*b*Sq
rt[-(d*e) + c*f])/Sqrt[f])*Sqrt[-(d*e) + c*f]) + (I*b*E^((I*b*Sqrt[-(d*e) +
 c*f])/Sqrt[f])*ExpIntegralEi[(-I)*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c +
```

```
d*x])])/Sqrt[-(d*e) + c*f] + E^((2*I)*a)*((2*E^(I*b*Sqrt[c + d*x])*Sqrt[f]
)/(d*e + d*f*x) - (I*b*E^((I*b*Sqrt[-(d*e) + c*f])/Sqrt[f])*ExpIntegralEi[I
*b*(-(Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x])])/Sqrt[-(d*e) + c*f] + (
I*b*ExpIntegralEi[I*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c + d*x])])/(E^((I
*b*Sqrt[-(d*e) + c*f])/Sqrt[f])*Sqrt[-(d*e) + c*f]))))/(E^(I*a)*f^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1830 vs. $2(273) = 546$.

Time = 0.30 (sec) , antiderivative size = 1831, normalized size of antiderivative = 5.40

method	result	size
derivativedivides	Expression too large to display	1831
default	Expression too large to display	1831

```
[In] int(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d/b^2*(sin(a+b*(d*x+c)^(1/2))*(-1/2*a*b^2/(c*f-d*e)*(a+b*(d*x+c)^(1/2))+1
/2*b^2*(-b^2*c*f+b^2*d*e+a^2*f)/(c*f-d*e)/f)/(-c*f*b^2+d*e*b^2+a^2*f-2*a*f*
(a+b*(d*x+c)^(1/2))+f*(a+b*(d*x+c)^(1/2))^2)+1/4*a*b^2/(c*f-d*e)/f/(-(f*a+(
b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(-b*(d*x+c)^(1/2)-a+(f*a+(b^2*c*f^2-b^
2*d*e*f)^(1/2))/f)*cos((f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1
/2)+a-(f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((f*a+(b^2*c*f^2-b^2*d*e*f)^(
1/2))/f))+1/4*a*b^2/(c*f-d*e)/f/((-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-
Si(-b*(d*x+c)^(1/2)-a-(-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-f*a+(b^2*
c*f^2-b^2*d*e*f)^(1/2))/f)-Ci(b*(d*x+c)^(1/2)+a+(-f*a+(b^2*c*f^2-b^2*d*e*f)
^(1/2))/f)*sin((-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/4*b^2*(-c*f*b^2+d*e
*b^2+a^2*f-a*(f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2)))/(c*f-d*e)/f^2/(-(f*a+(b^2*c
*f^2-b^2*d*e*f)^(1/2))/f+a)*(Si(-b*(d*x+c)^(1/2)-a+(f*a+(b^2*c*f^2-b^2*d*e*
f)^(1/2))/f)*sin((f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a-
(f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/
f))+1/4*b^2*(-c*f*b^2+d*e*b^2+a^2*f+a*(-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2)))/(
c*f-d*e)/f^2/((-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(-b*(d*x+c)^(1/2)
-a-(-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((-f*a+(b^2*c*f^2-b^2*d*e*f)^(1
/2))/f)+Ci(b*(d*x+c)^(1/2)+a+(-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-f*
a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))-a*b^4*(sin(a+b*(d*x+c)^(1/2))*(-1/2/b^2/
(c*f-d*e)*(a+b*(d*x+c)^(1/2))+1/2*a/b^2/(c*f-d*e))/(-c*f*b^2+d*e*b^2+a^2*f-
2*a*f*(a+b*(d*x+c)^(1/2))+f*(a+b*(d*x+c)^(1/2))^2)+1/4/b^2/(c*f-d*e)/f/(-(f
*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(-b*(d*x+c)^(1/2)-a+(f*a+(b^2*c*f^
2-b^2*d*e*f)^(1/2))/f)*cos((f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c
)^(1/2)+a-(f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((f*a+(b^2*c*f^2-b^2*d*e*
f)^(1/2))/f))+1/4/b^2/(c*f-d*e)/f/((-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*
(-Si(-b*(d*x+c)^(1/2)-a-(-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-f*a+(b^
2*c*f^2-b^2*d*e*f)^(1/2))/f)-Ci(b*(d*x+c)^(1/2)+a+(-f*a+(b^2*c*f^2-b^2*d*e*
```

```
f)^(1/2))/f)*sin((-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/4/b^2/f/(c*f-d*e)
*(Si(-b*(d*x+c)^(1/2)-a+(f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((f*a+(b^2*
c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a-(f*a+(b^2*c*f^2-b^2*d*e*f)^
(1/2))/f)*cos((f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/4/b^2/f/(c*f-d*e)*(-S
i(-b*(d*x+c)^(1/2)-a-(-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((-f*a+(b^2*c
*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a+(-f*a+(b^2*c*f^2-b^2*d*e*f)^
(1/2))/f)*cos((-f*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.23

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx =$$

$$-\frac{(-i\,dfx-i\,de)\sqrt{\frac{b^2de-b^2cf}{f}}\text{Ei}\left(i\sqrt{dx+c}b-\sqrt{\frac{b^2de-b^2cf}{f}}\right)e^{\left(ia+\sqrt{\frac{b^2de-b^2cf}{f}}\right)}+(i\,dfx+i\,de)\sqrt{\frac{b^2de-b^2cf}{f}}\text{Ei}\left(\right.}{}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((-I*d*f*x - I*d*e)*sqrt((b^2*d*e - b^2*c*f)/f)*Ei(I*sqrt(d*x + c)*b -
 sqrt((b^2*d*e - b^2*c*f)/f))*e^(I*a + sqrt((b^2*d*e - b^2*c*f)/f)) + (I*d*
f*x + I*d*e)*sqrt((b^2*d*e - b^2*c*f)/f)*Ei(I*sqrt(d*x + c)*b + sqrt((b^2*d
*e - b^2*c*f)/f))*e^(I*a - sqrt((b^2*d*e - b^2*c*f)/f)) + (I*d*f*x + I*d*e)
*sqrt((b^2*d*e - b^2*c*f)/f)*Ei(-I*sqrt(d*x + c)*b - sqrt((b^2*d*e - b^2*c*
f)/f))*e^(-I*a + sqrt((b^2*d*e - b^2*c*f)/f)) + (-I*d*f*x - I*d*e)*sqrt((b^
2*d*e - b^2*c*f)/f)*Ei(-I*sqrt(d*x + c)*b + sqrt((b^2*d*e - b^2*c*f)/f))*e^
(-I*a - sqrt((b^2*d*e - b^2*c*f)/f)) + 4*(d*e - c*f)*sin(sqrt(d*x + c)*b +
a))/(d*e^2*f - c*e*f^2 + (d*e*f^2 - c*f^3)*x)
```

Sympy [F]

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{\left(e+fx\right)^2}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(1/2))/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*sqrt(c + d*x))/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\sqrt{dx+c}b+a\right)}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e)^2, x)
```

Giac [F]

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\sqrt{dx+c}b+a\right)}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+b\,\sqrt{c+d\,x}\right)}{\left(e+f\,x\right)^2}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/2))/(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(1/2))/(e + f*x)^2, x)
```

3.192 $\int (e+fx)^2 \sin\left(a+b(c+dx)^{3/2}\right)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 382

$$\begin{aligned}\int (e+fx)^2 \sin\left(a+b(c+dx)^{3/2}\right)\, dx &= -\frac{4f(de-cf)\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3} \\ &- \frac{2f^2(c+dx)^{3/2}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3} \\ &- \frac{2e^{ia}f(de-cf)\sqrt{c+dx}\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{-ib(c+dx)^{3/2}}} \\ &- \frac{2e^{-ia}f(de-cf)\sqrt{c+dx}\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{ib(c+dx)^{3/2}}} \\ &+ \frac{ie^{ia}(de-cf)^2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^3\left(-ib(c+dx)^{3/2}\right)^{2/3}} \\ &- \frac{ie^{-ia}(de-cf)^2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^3\left(ib(c+dx)^{3/2}\right)^{2/3}} + \frac{2f^2\sin\left(a+b(c+dx)^{3/2}\right)}{3b^2d^3}\end{aligned}$$

```
[Out] -2/3*f^2*(d*x+c)^(3/2)*cos(a+b*(d*x+c)^(3/2))/b/d^3+1/3*I*exp(I*a)*(-c*f+d*
e)^2*(d*x+c)*GAMMA(2/3,-I*b*(d*x+c)^(3/2))/d^3/(-I*b*(d*x+c)^(3/2))^(2/3)-1
/3*I*(-c*f+d*e)^2*(d*x+c)*GAMMA(2/3,I*b*(d*x+c)^(3/2))/d^3/exp(I*a)/(I*b*(d
*x+c)^(3/2))^(2/3)+2/3*f^2*sin(a+b*(d*x+c)^(3/2))/b^2/d^3-4/3*f*(-c*f+d*e)*
cos(a+b*(d*x+c)^(3/2))*(d*x+c)^(1/2)/b/d^3-2/9*exp(I*a)*f*(-c*f+d*e)*GAMMA(
1/3,-I*b*(d*x+c)^(3/2))*(d*x+c)^(1/2)/b/d^3/(-I*b*(d*x+c)^(3/2))^(1/3)-2/9*
f*(-c*f+d*e)*GAMMA(1/3,I*b*(d*x+c)^(3/2))*(d*x+c)^(1/2)/b/d^3/exp(I*a)/(I*b
*(d*x+c)^(3/2))^(1/3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {3514, 3470, 2250, 3466, 3437, 2239, 3460, 3377, 2717}

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^{3/2}\right)\,dx = \frac{2f^2\sin\left(a+b(c+dx)^{3/2}\right)}{3b^2d^3} - \frac{4f\sqrt{c+dx}(de-cf)\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3} - \frac{2e^{ia}f\sqrt{c+dx}(de-cf)\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{-ib(c+dx)^{3/2}}} - \frac{2e^{-ia}f\sqrt{c+dx}(de-cf)\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{ib(c+dx)^{3/2}}} + \frac{ie^{ia}(c+dx)(de-cf)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^3\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{ie^{-ia}(c+dx)(de-cf)^2\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^3\left(ib(c+dx)^{3/2}\right)^{2/3}} - \frac{2f^2(c+dx)^{3/2}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3}$$

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] (-4*f*(d*e - c*f)*Sqrt[c + d*x]*Cos[a + b*(c + d*x)^(3/2)])/(3*b*d^3) - (2*
f^2*(c + d*x)^(3/2)*Cos[a + b*(c + d*x)^(3/2)])/(3*b*d^3) - (2*E^(I*a)*f*(d
*e - c*f)*Sqrt[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(9*b*d^3*((-I)*
b*(c + d*x)^(3/2))^(1/3)) - (2*f*(d*e - c*f)*Sqrt[c + d*x]*Gamma[1/3, I*b*(
c + d*x)^(3/2)])/(9*b*d^3*E^(I*a)*(I*b*(c + d*x)^(3/2))^(1/3)) + ((I/3)*E^(
I*a)*(d*e - c*f)^2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(d^3*((-I)
*b*(c + d*x)^(3/2))^(2/3)) - ((I/3)*(d*e - c*f)^2*(c + d*x)*Gamma[2/3, I*b*
(c + d*x)^(3/2)])/(d^3*E^(I*a)*(I*b*(c + d*x)^(3/2))^(2/3)) + (2*f^2*Sin[a
+ b*(c + d*x)^(3/2)])/(3*b^2*d^3)
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{2\text{Subst}\left(\int \left((de-cf)^2 x \sin\left(a+bx^3\right) - 2f(-de+cf)x^3 \sin\left(a+bx^3\right) + f^2 x^5 \sin\left(a+bx^3\right)\right) dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{(2f^2)\,\text{Subst}\left(\int x^5 \sin\left(a+bx^3\right) dx, x, \sqrt{c+dx}\right)}{d^3} + \frac{(4f(de-cf))\text{Subst}\left(\int x^3 \sin\left(a+bx^3\right) dx, x, \sqrt{c+dx}\right)}{d^3} + \frac{(2(de-cf)^2)\,\text{Subst}\left(\int x \sin\left(a+bx^3\right) dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= -\frac{4f(de-cf)\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3} + \frac{(2f^2)\,\text{Subst}\left(\int x \sin(a+bx)\,dx, x, (c+dx)^{3/2}\right)}{3d^3} + \frac{(4f(de-cf))\text{Subst}\left(\int \cos\left(a+bx^3\right) dx, x, \sqrt{c+dx}\right)}{3bd^3} + \frac{(i(de-cf)^2)\,\text{Subst}\left(\int e^{-ia-ibx^3} x\,dx, x, \sqrt{c+dx}\right)}{d^3} - \frac{(i(de-cf)^2)\,\text{Subst}\left(\int e^{ia+ibx^3} x\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= -\frac{4f(de-cf)\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3} - \frac{2f^2(c+dx)^{3/2}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3} + \frac{ie^{ia}(de-cf)^2(c+dx)\Gamma\left(\frac{2}{3}, -ib(c+dx)^{3/2}\right)}{3d^3\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{ie^{-ia}(de-cf)^2(c+dx)\Gamma\left(\frac{2}{3}, ib(c+dx)^{3/2}\right)}{3d^3\left(ib(c+dx)^{3/2}\right)^{2/3}} + \frac{(2f^2)\,\text{Subst}\left(\int \cos(a+bx)\,dx, x, (c+dx)^{3/2}\right)}{3bd^3} + \frac{(2f(de-cf))\text{Subst}\left(\int e^{-ia-ibx^3} dx, x, \sqrt{c+dx}\right)}{3bd^3} + \frac{(2f(de-cf))\text{Subst}\left(\int e^{ia+ibx^3} dx, x, \sqrt{c+dx}\right)}{3bd^3}$$

$$= -\frac{4f(de-cf)\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3} - \frac{2f^2(c+dx)^{3/2}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3} - \frac{2e^{ia}f(de-cf)\sqrt{c+dx}\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{-ib(c+dx)^{3/2}}} - \frac{2e^{-ia}f(de-cf)\sqrt{c+dx}\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{ib(c+dx)^{3/2}}} + \frac{ie^{ia}(de-cf)^2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^3\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{ie^{-ia}(de-cf)^2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^3\left(ib(c+dx)^{3/2}\right)^{2/3}} + \frac{2f^2\sin\left(a+b(c+dx)^{3/2}\right)}{3b^2d^3}$$

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.10

$$\int (e+fx)^2\sin\left(a+b(c+dx)^{3/2}\right)\,dx = -\frac{i\left((\cos(a)+i\sin(a))\left(\frac{f^2\cos\left(b(c+dx)^{3/2}\right)}{b^2} - \frac{(de-cf)^2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{2f(de-cf)(c+dx)^2\Gamma\left(\frac{4}{3},-ib(c+dx)^{3/2}\right)}{\left(-ib(c+dx)^{3/2}\right)^{4/3}} + if^2\cdots\right.\right.}{}$$

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] ((-1/3*I)*((Cos[a] + I*Sin[a])*((f^2*Cos[b*(c + d*x)^(3/2)])/b^2 - ((d*e -
c*f)^2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/((-I)*b*(c + d*x)^(3/2
))^(2/3) - (2*f*(d*e - c*f)*(c + d*x)^2*Gamma[4/3, (-I)*b*(c + d*x)^(3/2)])
/((-I)*b*(c + d*x)^(3/2))^(4/3) + (I*f^2*Sin[b*(c + d*x)^(3/2)])/b^2 + (f^2
*(c + d*x)^(3/2)*((-I)*Cos[b*(c + d*x)^(3/2)] + Sin[b*(c + d*x)^(3/2)]))/b)
 - (Cos[a] - I*Sin[a])*((f^2*Cos[b*(c + d*x)^(3/2)])/b^2 - ((d*e - c*f)^2*(
c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(I*b*(c + d*x)^(3/2))^(2/3) - (2*
f*(d*e - c*f)*(c + d*x)^2*Gamma[4/3, I*b*(c + d*x)^(3/2)])/(I*b*(c + d*x)^(
3/2))^(4/3) - (I*f^2*Sin[b*(c + d*x)^(3/2)])/b^2 + (f^2*(c + d*x)^(3/2)*(I*
Cos[b*(c + d*x)^(3/2)] + Sin[b*(c + d*x)^(3/2)]))/b)))/d^3
```

Maple [F]

$$\int (fx+e)^2 \sin\left(a+b(dx+c)^{\frac{3}{2}}\right) dx$$

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x)
```

```
[Out] int((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.96

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^{3/2}\right) dx = \frac{6 f^2 \sin\left((bdx+bc)\sqrt{dx+c}+a\right) - 2\left(\left(-i\,def+i\,cf^2\right)\cos(a) - \left(def-cf^2\right)\sin(a)\right)(i\,b)^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right.}{}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/9*(6*f^2*sin((b*d*x + b*c)*sqrt(d*x + c) + a) - 2*((-I*d*e*f + I*c*f^2)*c
os(a) - (d*e*f - c*f^2)*sin(a))*(I*b)^(2/3)*gamma(1/3, (I*b*d*x + I*b*c)*sq
rt(d*x + c)) - 2*((I*d*e*f - I*c*f^2)*cos(a) - (d*e*f - c*f^2)*sin(a))*(-I*
b)^(2/3)*gamma(1/3, (-I*b*d*x - I*b*c)*sqrt(d*x + c)) - 6*(b*d*f^2*x + 2*b*
d*e*f - b*c*f^2)*sqrt(d*x + c)*cos((b*d*x + b*c)*sqrt(d*x + c) + a) - 3*((b
*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cos(a) + (-I*b*d^2*e^2 + 2*I*b*c*d*e*f
- I*b*c^2*f^2)*sin(a))*(I*b)^(1/3)*gamma(2/3, (I*b*d*x + I*b*c)*sqrt(d*x +
c)) - 3*((b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cos(a) + (I*b*d^2*e^2 - 2*I*
b*c*d*e*f + I*b*c^2*f^2)*sin(a))*(-I*b)^(1/3)*gamma(2/3, (-I*b*d*x - I*b*c)
*sqrt(d*x + c)))/(b^2*d^3)
```

Sympy [F]

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^{3/2}\right) dx = \int (e+fx)^2 \sin\left(a+bc\sqrt{c+dx}+bdx\sqrt{c+dx}\right) dx$$

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**(3/2)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 694 vs. 2(290) = 580.

Time = 0.52 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.82

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^{3/2}\right) \, dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] -1/18*(3*((d*x + c)^(3/2)*b)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(
3/2)*b) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt
(3) - 1)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(
d*x + c)^(3/2)*b))*sin(a))*e^2/(sqrt(d*x + c)*b) - 6*((d*x + c)^(3/2)*b)^(1
/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) - I)*gamma(2
/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt(3) - 1)*gamma(2/3, I*(d*x + c)
^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(a))*c*e*
f/(sqrt(d*x + c)*b*d) + 3*((d*x + c)^(3/2)*b)^(1/3)*(((sqrt(3) + I)*gamma(2
/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*
cos(a) - ((I*sqrt(3) - 1)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (-I*sqrt(3) - 1
)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(a))*c^2*f^2/(sqrt(d*x + c)*b*d^2) +
 2*(12*((d*x + c)^(3/2)*b)^(1/3)*sqrt(d*x + c)*cos((d*x + c)^(3/2)*b + a) +
 sqrt(d*x + c)*(((sqrt(3) - I)*gamma(1/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) +
 I)*gamma(1/3, -I*(d*x + c)^(3/2)*b))*cos(a) + ((-I*sqrt(3) - 1)*gamma(1/3,
 I*(d*x + c)^(3/2)*b) + (I*sqrt(3) - 1)*gamma(1/3, -I*(d*x + c)^(3/2)*b))*s
in(a)))*e*f/(((d*x + c)^(3/2)*b)^(1/3)*b*d) - 2*(12*((d*x + c)^(3/2)*b)^(1/
3)*sqrt(d*x + c)*cos((d*x + c)^(3/2)*b + a) + sqrt(d*x + c)*(((sqrt(3) - I)
*gamma(1/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) + I)*gamma(1/3, -I*(d*x + c)^(3
/2)*b))*cos(a) + ((-I*sqrt(3) - 1)*gamma(1/3, I*(d*x + c)^(3/2)*b) + (I*sqr
t(3) - 1)*gamma(1/3, -I*(d*x + c)^(3/2)*b))*sin(a)))*c*f^2/(((d*x + c)^(3/2
)*b)^(1/3)*b*d^2) + 12*((d*x + c)^(3/2)*b*cos((d*x + c)^(3/2)*b + a) - sin(
(d*x + c)^(3/2)*b + a))*f^2/(b^2*d^2))/d
```

Giac [F]

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^{3/2}\right) \, dx = \int (fx+e)^2 \sin \left((dx+c)^{\frac{3}{2}}b+a\right) \, dx$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin((d*x + c)^(3/2)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+b(c+dx)^{3/2}\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^{3/2}\right)\,(e+f\,x)^2\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(3/2))*(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(3/2))*(e + f*x)^2, x)
```

3.193 $\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 291

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx=-\frac{2f\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2}-\frac{e^{ia}f\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^2\sqrt[3]{-ib(c+dx)^{3/2}}}-\frac{e^{-ia}f\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{9bd^2\sqrt[3]{ib(c+dx)^{3/2}}}+\frac{ie^{ia}(de-cf)(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^2\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie^{-ia}(de-cf)(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^2\left(ib(c+dx)^{3/2}\right)^{2/3}}$$

```
[Out] 1/3*I*exp(I*a)*(-c*f+d*e)*(d*x+c)*GAMMA(2/3,-I*b*(d*x+c)^(3/2))/d^2/(-I*b*(
d*x+c)^(3/2))^(2/3)-1/3*I*(-c*f+d*e)*(d*x+c)*GAMMA(2/3,I*b*(d*x+c)^(3/2))/d
^2/exp(I*a)/(I*b*(d*x+c)^(3/2))^(2/3)-2/3*f*cos(a+b*(d*x+c)^(3/2))*(d*x+c)^
(1/2)/b/d^2-1/9*exp(I*a)*f*GAMMA(1/3,-I*b*(d*x+c)^(3/2))*(d*x+c)^(1/2)/b/d^
2/(-I*b*(d*x+c)^(3/2))^(1/3)-1/9*f*GAMMA(1/3,I*b*(d*x+c)^(3/2))*(d*x+c)^(1/
2)/b/d^2/exp(I*a)/(I*b*(d*x+c)^(3/2))^(1/3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3514, 3470, 2250, 3466, 3437, 2239}

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx = \frac{ie^{ia}(c+dx)(de-cf)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^2\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{ie^{-ia}(c+dx)(de-cf)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^2\left(ib(c+dx)^{3/2}\right)^{2/3}} - \frac{2f\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2} - \frac{e^{ia}f\sqrt{c+dx}\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^2\sqrt[3]{-ib(c+dx)^{3/2}}} - \frac{e^{-ia}f\sqrt{c+dx}\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{9bd^2\sqrt[3]{ib(c+dx)^{3/2}}}$$

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] (-2*f*Sqrt[c + d*x]*Cos[a + b*(c + d*x)^(3/2)])/(3*b*d^2) - (E^(I*a)*f*Sqrt
[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(9*b*d^2*((-I)*b*(c + d*x)^(3
/2))^(1/3)) - (f*Sqrt[c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(9*b*d^2*E^
(I*a)*(I*b*(c + d*x)^(3/2))^(1/3)) + ((I/3)*E^(I*a)*(d*e - c*f)*(c + d*x)*G
amma[2/3, (-I)*b*(c + d*x)^(3/2)])/(d^2*((-I)*b*(c + d*x)^(3/2))^(2/3)) - (
(I/3)*(d*e - c*f)*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(d^2*E^(I*a)*(
I*b*(c + d*x)^(3/2))^(2/3))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
```

```
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \left((de-cf)x\sin\left(a+bx^3\right)+fx^3\sin\left(a+bx^3\right)\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= \frac{(2f)\text{Subst}\left(\int x^3\sin\left(a+bx^3\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&\quad+\frac{(2(de-cf))\text{Subst}\left(\int x\sin\left(a+bx^3\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= -\frac{2f\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2}+\frac{(2f)\text{Subst}\left(\int \cos\left(a+bx^3\right)\,dx, x, \sqrt{c+dx}\right)}{3bd^2}\\
&\quad+\frac{(i(de-cf))\text{Subst}\left(\int e^{-ia-ibx^3}x\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&\quad-\frac{(i(de-cf))\text{Subst}\left(\int e^{ia+ibx^3}x\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= -\frac{2f\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2}+\frac{ie^{ia}(de-cf)(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^2\left(-ib(c+dx)^{3/2}\right)^{2/3}}\\
&\quad-\frac{ie^{-ia}(de-cf)(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^2\left(ib(c+dx)^{3/2}\right)^{2/3}}\\
&\quad+\frac{f\text{Subst}\left(\int e^{-ia-ibx^3}\,dx, x, \sqrt{c+dx}\right)}{3bd^2}+\frac{f\text{Subst}\left(\int e^{ia+ibx^3}\,dx, x, \sqrt{c+dx}\right)}{3bd^2}
\end{aligned}$$

$$= -\frac{2f\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2} - \frac{e^{ia}f\sqrt{c+dx}\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^2\sqrt[3]{-ib(c+dx)^{3/2}}} - \frac{e^{-ia}f\sqrt{c+dx}\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{9bd^2\sqrt[3]{ib(c+dx)^{3/2}}} + \frac{ie^{ia}(de-cf)(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^2\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{ie^{-ia}(de-cf)(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^2\left(ib(c+dx)^{3/2}\right)^{2/3}}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 705 vs. 2(291) = 582.

Time = 1.79 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.42

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx = -\frac{2f\sqrt{c+dx}\cos(a)\cos\left(b(c+dx)^{3/2}\right)}{3bd^2} + \frac{f\cos(a)\left(-\frac{2\sqrt{c+dx}\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{3\sqrt[3]{-ib(c+dx)^{3/2}}} - \frac{2\sqrt{c+dx}\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{3\sqrt[3]{ib(c+dx)^{3/2}}}\right)}{6bd^2} - \frac{ie\cos(a)\left(-\frac{2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3\left(-ib(c+dx)^{3/2}\right)^{2/3}} + \frac{2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)}{2d} + \frac{icf\cos(a)\left(-\frac{2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3\left(-ib(c+dx)^{3/2}\right)^{2/3}} + \frac{2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)}{2d^2} + \frac{if\left(-\frac{2\sqrt{c+dx}\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{3\sqrt[3]{-ib(c+dx)^{3/2}}} + \frac{2\sqrt{c+dx}\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{3\sqrt[3]{ib(c+dx)^{3/2}}}\right)\sin(a)}{6bd^2} + \frac{e\left(-\frac{2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)\sin(a)}{2d} - \frac{cf\left(-\frac{2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)\sin(a)}{2d^2} + \frac{2f\sqrt{c+dx}\sin(a)\sin\left(b(c+dx)^{3/2}\right)}{3bd^2}$$

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] (-2*f*Sqrt[c + d*x]*Cos[a]*Cos[b*(c + d*x)^(3/2)])/(3*b*d^2) + (f*Cos[a]*((
-2*Sqrt[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(
```

```
3/2))^(1/3)) - (2*Sqrt[c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c
 + d*x)^(3/2))^(1/3))))/(6*b*d^2) - ((I/2)*e*Cos[a]*((-2*(c + d*x)*Gamma[2/
3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2))^(2/3)) + (2*(c + d*
x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c + d*x)^(3/2))^(2/3))))/d + (
(I/2)*c*f*Cos[a]*((-2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I
)*b*(c + d*x)^(3/2))^(2/3)) + (2*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])
/(3*(I*b*(c + d*x)^(3/2))^(2/3))))/d^2 + ((I/6)*f*((-2*Sqrt[c + d*x]*Gamma[
1/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2))^(1/3)) + (2*Sqrt[
c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c + d*x)^(3/2))^(1/3)))*
Sin[a])/(b*d^2) + (e*((-2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(3*
((-I)*b*(c + d*x)^(3/2))^(2/3)) - (2*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/
2)])/(3*(I*b*(c + d*x)^(3/2))^(2/3)))*Sin[a])/(2*d) - (c*f*((-2*(c + d*x)*G
amma[2/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2))^(2/3)) - (2*
(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c + d*x)^(3/2))^(2/3)))
*Sin[a])/(2*d^2) + (2*f*Sqrt[c + d*x]*Sin[a]*Sin[b*(c + d*x)^(3/2)])/(3*b*d
^2)
```

Maple [F]

$$\int (fx+e)\sin\left(a+b(dx+c)^{\frac{3}{2}}\right)dx$$

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x)
```

```
[Out] int((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.77

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx =$$

$$-\frac{6\sqrt{dx+c}bf\cos\left((bdx+bc)\sqrt{dx+c}+a\right)-(if\cos(a)+f\sin(a))(ib)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},(ibdx+ibc)\sqrt{dx+c}\right)-(-\cdots}{\cdots}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/9*(6*sqrt(d*x + c)*b*f*cos((b*d*x + b*c)*sqrt(d*x + c) + a) - (I*f*cos(a
) + f*sin(a))*(I*b)^(2/3)*gamma(1/3, (I*b*d*x + I*b*c)*sqrt(d*x + c)) - (-I
*f*cos(a) + f*sin(a))*(-I*b)^(2/3)*gamma(1/3, (-I*b*d*x - I*b*c)*sqrt(d*x +
 c)) + 3*((b*d*e - b*c*f)*cos(a) + (-I*b*d*e + I*b*c*f)*sin(a))*(I*b)^(1/3)
*gamma(2/3, (I*b*d*x + I*b*c)*sqrt(d*x + c)) + 3*((b*d*e - b*c*f)*cos(a) +
(I*b*d*e - I*b*c*f)*sin(a))*(-I*b)^(1/3)*gamma(2/3, (-I*b*d*x - I*b*c)*sqrt
(d*x + c)))/(b^2*d^2)
```

Sympy [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx=\int (e+fx)\sin\left(a+bc\sqrt{c+dx}+bdx\sqrt{c+dx}\right)dx$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**(3/2)),x)
[Out] Integral((e + f*x)*sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.29

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx=$$

$$-\frac{\frac{3\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},i\,(dx+c)^{\frac{3}{2}}b\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-i\,(dx+c)^{\frac{3}{2}}b\right)\right)\cos(a)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},i\,(dx+c)^{\frac{3}{2}}b\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-i\,(dx+c)^{\frac{3}{2}}b\right.\right.\right.}{\sqrt{dx+c}b}}{}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x, algorithm="maxima")
[Out] -1/18*(3*((d*x + c)^(3/2)*b)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(
3/2)*b) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt
(3) - 1)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(
d*x + c)^(3/2)*b))*sin(a))*e/(sqrt(d*x + c)*b) - 3*((d*x + c)^(3/2)*b)^(1/3
)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) - I)*gamma(2/3
, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt(3) - 1)*gamma(2/3, I*(d*x + c)^(
3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(a))*c*f/(s
qrt(d*x + c)*b*d) + (12*((d*x + c)^(3/2)*b)^(1/3)*sqrt(d*x + c)*cos((d*x +
c)^(3/2)*b + a) + sqrt(d*x + c)*(((sqrt(3) - I)*gamma(1/3, I*(d*x + c)^(3/2
)*b) + (sqrt(3) + I)*gamma(1/3, -I*(d*x + c)^(3/2)*b))*cos(a) + ((-I*sqrt(3
) - 1)*gamma(1/3, I*(d*x + c)^(3/2)*b) + (I*sqrt(3) - 1)*gamma(1/3, -I*(d*x
 + c)^(3/2)*b))*sin(a)))*f/(((d*x + c)^(3/2)*b)^(1/3)*b*d))/d
```

Giac [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx=\int (fx+e)\sin\left((dx+c)^{\frac{3}{2}}b+a\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x, algorithm="giac")
[Out] integrate((f*x + e)*sin((d*x + c)^(3/2)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^{3/2}\right)\,(e+f\,x)\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(3/2))*(e + f*x),x)
[Out] int(sin(a + b*(c + d*x)^(3/2))*(e + f*x), x)
```

3.194 $\int \sin \left(a+b(c+dx)^{3/2}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \sin \left(a+b(c+dx)^{3/2}\right) \, dx = \frac{ie^{ia}(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{ie^{-ia}(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d\left(ib(c+dx)^{3/2}\right)^{2/3}}$$

```
[Out] 1/3*I*exp(I*a)*(d*x+c)*GAMMA(2/3,-I*b*(d*x+c)^(3/2))/d/(-I*b*(d*x+c)^(3/2))
^(2/3)-1/3*I*(d*x+c)*GAMMA(2/3,I*b*(d*x+c)^(3/2))/d/exp(I*a)/(I*b*(d*x+c)^(
3/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3444, 3470, 2250}

$$\int \sin \left(a+b(c+dx)^{3/2}\right) \, dx = \frac{ie^{ia}(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{ie^{-ia}(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d\left(ib(c+dx)^{3/2}\right)^{2/3}}$$

```
[In] Int[Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] ((I/3)*E^(I*a)*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(d*((-I)*b*(c
+ d*x)^(3/2))^(2/3)) - ((I/3)*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(d
*E^(I*a)*(I*b*(c + d*x)^(3/2))^(2/3))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3444

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Dist[k/f, Subst[Int[x^(k - 1)*(a + b
*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int x\sin\left(a+bx^3\right)\,dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{i\text{Subst}\left(\int e^{-ia-ibx^3}x\,dx,x,\sqrt{c+dx}\right)}{d}-\frac{i\text{Subst}\left(\int e^{ia+ibx^3}x\,dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{ie^{ia}(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie^{-ia}(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d\left(ib(c+dx)^{3/2}\right)^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx=\frac{i(c+dx)\left(-\left(-ib(c+dx)^{3/2}\right)^{2/3}\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)(\cos(a)-i\sin(a))+\left(ib(c+dx)^{3/2}\right)^{2/3}\Gamma}{3d\left(b^2(c+dx)^3\right)^{2/3}}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] ((I/3)*(c + d*x)*(-(((-I)*b*(c + d*x)^(3/2))^(2/3)*Gamma[2/3, I*b*(c + d*x)
^(3/2)]*(Cos[a] - I*Sin[a])) + (I*b*(c + d*x)^(3/2))^(2/3)*Gamma[2/3, (-I)*
b*(c + d*x)^(3/2)]*(Cos[a] + I*Sin[a])))/(d*(b^2*(c + d*x)^3)^(2/3))
```

Maple [F]

$$\int \sin\left(a+b(dx+c)^{\frac{3}{2}}\right)dx$$

```
[In] int(sin(a+b*(d*x+c)^(3/2)),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(3/2)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx = -\frac{(i\,b)^{\frac{1}{3}}\left(\cos\left(a\right)-i\,\sin\left(a\right)\right)\Gamma\left(\frac{2}{3},(i\,bdx+i\,bc)\sqrt{dx+c}\right)+(-i\,b)^{\frac{1}{3}}\left(\cos\left(a\right)+i\,\sin\left(a\right)\right)\Gamma\left(\frac{2}{3},(-i\,bdx-i\,bc)\sqrt{dx+c}\right)}{3\,bd}$$

```
[In] integrate(sin(a+b*(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/3*((I*b)^(1/3)*(cos(a) - I*sin(a))*gamma(2/3, (I*b*d*x + I*b*c)*sqrt(d*x
 + c)) + (-I*b)^(1/3)*(cos(a) + I*sin(a))*gamma(2/3, (-I*b*d*x - I*b*c)*sqr
t(d*x + c)))/(b*d)
```

Sympy [F]

$$\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx = \int \sin\left(a+b(c+dx)^{\frac{3}{2}}\right)dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(3/2)),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx = -\frac{\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},i\,(dx+c)^{\frac{3}{2}}b\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-i\,(dx+c)^{\frac{3}{2}}b\right)\right)\cos\left(a\right)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},\right.\right.\right.}{6\sqrt{dx+c}bd}$$

```
[In] integrate(sin(a+b*(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] -1/6*((d*x + c)^(3/2)*b)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(3/2)
*b) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt(3)
- 1)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(d*x
+ c)^(3/2)*b))*sin(a))/(sqrt(d*x + c)*b*d)
```

Giac [F]

$$\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx=\int \sin\left((dx+c)^{\frac{3}{2}}b+a\right)\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(3/2)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx=\int \sin\left(a+b\,{\left(c+d\,x\right)}^{3/2}\right)\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(3/2)),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(3/2)), x)
```

3.195 $\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx} dx = \text{Int}\left(\frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}, x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx} dx = \int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx} dx$$

```
[In] Int[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x),x]
[Out] Defer[Int][Sin[a + b*(c + d*x)^(3/2)]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx} dx$$

Mathematica [N/A]

Not integrable

Time = 13.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x),x]
[Out] Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{3}{2}}\right)}{fx+e}dx$$

```
[In] int(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x)
[Out] int(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^{\frac{3}{2}}b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x, algorithm="fricas")
[Out] integral(sin((b*d*x + b*c)*sqrt(d*x + c) + a)/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 6.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin \left(a+bc\sqrt{c+dx}+bdx\sqrt{c+dx}\right)}{e+fx}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(3/2))/(f*x+e),x)
[Out] Integral(sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^{\frac{3}{2}}b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^{\frac{3}{2}}b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 6.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b\,(c+d\,x)^{3/2}\right)}{e+f\,x}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(3/2))/(e + f*x),x)
[Out] int(sin(a + b*(c + d*x)^(3/2))/(e + f*x), x)
```

3.196 $\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} dx = \operatorname{Int}\left(\frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}, x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} dx = \int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} dx$$

```
[In] Int[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 17.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{3}{2}}\right)}{\left(fx+e\right)^{2}}dx$$

```
[In] int(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x)
[Out] int(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left((dx+c)^{\frac{3}{2}}b+a\right)}{\left(fx+e\right)^{2}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sin((b*d*x + b*c)*sqrt(d*x + c) + a)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [N/A]

Not integrable

Time = 41.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} \, dx=\int \frac{\sin \left(a+bc\sqrt{c+dx}+bdx\sqrt{c+dx}\right)}{\left(e+fx\right)^{2}} \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(3/2))/(f*x+e)**2,x)
[Out] Integral(sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x))/(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} \, dx=\int \frac{\sin \left({\left(dx+c\right)}^{\frac{3}{2}}b+a\right)}{{\left(fx+e\right)}^{2}} \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} \, dx=\int \frac{\sin \left({\left(dx+c\right)}^{\frac{3}{2}}b+a\right)}{{\left(fx+e\right)}^{2}} \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(a+b\,{\left(c+d\,x\right)}^{3/2}\right)}{{\left(e+f\,x\right)}^2}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(3/2))/(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(3/2))/(e + f*x)^2, x)
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3.197 $\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 611

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx = \frac{b^5 f^2 \sqrt{c+dx} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^3 f(de-cf)\sqrt{c+dx} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b(de-cf)^2\sqrt{c+dx} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} - \frac{b^3 f^2 (c+dx)^{3/2} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{180d^3} + \frac{bf(de-cf)(c+dx)^{3/2} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} + \frac{bf^2(c+dx)^{5/2} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{15d^3} + \frac{b^6 f^2 \operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{360d^3} - \frac{b^4 f(de-cf) \operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{6d^3} + \frac{b^2(de-cf)^2 \operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d^3} + \frac{b^4 f^2 (c+dx) \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^2 f(de-cf)(c+dx) \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{(de-cf)^2(c+dx) \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} - \frac{b^2 f^2 (c+dx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{60d^3} + \frac{f(de-cf)(c+dx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{f^2(c+dx)^3 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} + \frac{b^6 f^2 \cos(a)\operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^4 f(de-cf) \cos(a)\operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b^2(de-cf)^2 \cos(a)\operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^3}$$

```
[Out] -1/180*b^3*f^2*(d*x+c)^(3/2)*cos(a+b/(d*x+c)^(1/2))/d^3+1/3*b*f*(-c*f+d*e)*
(d*x+c)^(3/2)*cos(a+b/(d*x+c)^(1/2))/d^3+1/15*b*f^2*(d*x+c)^(5/2)*cos(a+b/(
d*x+c)^(1/2))/d^3+1/360*b^6*f^2*cos(a)*Si(b/(d*x+c)^(1/2))/d^3-1/6*b^4*f*(-
c*f+d*e)*cos(a)*Si(b/(d*x+c)^(1/2))/d^3+b^2*(-c*f+d*e)^2*cos(a)*Si(b/(d*x+c
)^(1/2))/d^3+1/360*b^6*f^2*Ci(b/(d*x+c)^(1/2))*sin(a)/d^3-1/6*b^4*f*(-c*f+d
*e)*Ci(b/(d*x+c)^(1/2))*sin(a)/d^3+b^2*(-c*f+d*e)^2*Ci(b/(d*x+c)^(1/2))*sin
(a)/d^3+1/360*b^4*f^2*(d*x+c)*sin(a+b/(d*x+c)^(1/2))/d^3-1/6*b^2*f*(-c*f+d*
e)*(d*x+c)*sin(a+b/(d*x+c)^(1/2))/d^3+(-c*f+d*e)^2*(d*x+c)*sin(a+b/(d*x+c)^
(1/2))/d^3-1/60*b^2*f^2*(d*x+c)^2*sin(a+b/(d*x+c)^(1/2))/d^3+f*(-c*f+d*e)*(
d*x+c)^2*sin(a+b/(d*x+c)^(1/2))/d^3+1/3*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^(1/2)
)/d^3+1/360*b^5*f^2*cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/d^3-1/6*b^3*f*(-c*
f+d*e)*cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/d^3+b*(-c*f+d*e)^2*cos(a+b/(d*x
+c)^(1/2))*(d*x+c)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used

$= \{3512, 3378, 3384, 3380, 3383\}$

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx = \frac{b^6 f^2 \sin(a) \operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{360d^3} + \frac{b^6 f^2 \cos(a) \operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{360d^3} + \frac{b^5 f^2 \sqrt{c+dx} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^4 f \sin(a)(de-cf) \operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{6d^3} - \frac{b^4 f \cos(a)(de-cf) \operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b^4 f^2 (c+dx) \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^3 f \sqrt{c+dx}(de-cf) \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} - \frac{b^3 f^2 (c+dx)^{3/2} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{180d^3} + \frac{b^2 \sin(a)(de-cf)^2 \operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{b^2 \cos(a)(de-cf)^2 \operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^3} - \frac{b^2 f (c+dx)(de-cf) \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} - \frac{b^2 f^2 (c+dx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{60d^3} + \frac{f (c+dx)^2 (de-cf) \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{(c+dx)(de-cf)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{b f (c+dx)^{3/2} (de-cf) \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} + \frac{b \sqrt{c+dx}(de-cf)^2 \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{f^2 (c+dx)^3 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} + \frac{b f^2 (c+dx)^{5/2} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{15d^3}$$

[In] Int[(e + f*x)^2*Sin[a + b/Sqrt[c + d*x]],x]

[Out] (b^5*f^2*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(360*d^3) - (b^3*f*(d*e - c*f)*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(6*d^3) + (b*(d*e - c*f)^2*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d^3 - (b^3*f^2*(c + d*x)^(3/2)*Cos[a + b/Sqrt[c + d*x]])/(180*d^3) + (b*f*(d*e - c*f)*(c + d*x)^(3/2)*Cos[a + b/Sqrt[c + d*x]])/(3*d^3) + (b*f^2*(c + d*x)^(5/2)*Cos[a + b/Sqrt[c + d*x]])/(15*d^3) + (b^6*f^2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(360*d^3) - (b^4*f*(d*e - c*f)*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(6*d^3) + (b^2*(d*e - c*f)^2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/d^3 + (b^4*f^2*(c + d*x)*Sin[a + b/Sqrt[c + d*x]])/(360*d^3) - (b^2*f*(d*e - c*f)*(c + d*x)*Sin[a + b/Sqrt[c + d*x]])/(6*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin[a + b/Sqrt[c + d*x]])/d^3 - (b^2*f^2*(c + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/(60*d^3) + (f*(d*e - c*f)*(c + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/d^3 + (f^2*(c + d*x)^3*Sin[a + b/Sqrt[c + d*x]])/(3*d^3) + (b^6*f^2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(360*d^3) - (b^4*f*(d*e - c*f)*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(6*d^3) + (b^2*(d*e - c*f)^2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/d^3

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
```

```
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int\left(\frac{f^2\sin(a+bx)}{d^2x^7}+\frac{2f(de-cf)\sin(a+bx)}{d^2x^5}+\frac{(de-cf)^2\sin(a+bx)}{d^2x^3}\right)dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= -\frac{(2f^2)\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^7}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^3} - \frac{(4f(de-cf))\text{Subst}\left(\int\frac{\sin(a+bx)}{x^5}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^3} - \frac{(2(de-cf)^2)\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^3}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^3}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} - \frac{(bf^2)\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^6}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{3d^3} - \frac{(bf(de-cf))\text{Subst}\left(\int\frac{\cos(a+bx)}{x^4}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^3} - \frac{(b(de-cf)^2)\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^2}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^3}$$

$$= \frac{b(de-cf)^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{bf(de-cf)(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} + \frac{bf^2(c+dx)^{5/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{15d^3} + \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} + \frac{(b^2f^2)\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^5}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{15d^3} + \frac{(b^2f(de-cf))\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^3}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{3d^3} + \frac{(b^2(de-cf)^2)\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^3}$$

$$= \frac{b(de-cf)^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3}+\frac{bf(de-cf)(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3}$$
$$+\frac{bf^2(c+dx)^{5/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{15d^3}-\frac{b^2f(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3}$$
$$+\frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3}-\frac{b^2f^2(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{60d^3}$$
$$+\frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3}+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3}$$
$$+\frac{(b^3f^2)\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^4}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{60d^3}$$
$$+\frac{(b^3f(de-cf))\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^2}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{6d^3}$$
$$+\frac{(b^2(de-cf)^2\cos(a))\,\text{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^3}$$
$$+\frac{(b^2(de-cf)^2\sin(a))\,\text{Subst}\left(\int\frac{\cos(bx)}{x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^3}$$

$$= -\frac{b^3f(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3}+\frac{b(de-cf)^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3}$$
$$-\frac{b^3f^2(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{180d^3}+\frac{bf(de-cf)(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3}$$
$$+\frac{bf^2(c+dx)^{5/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{15d^3}+\frac{b^2(de-cf)^2\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d^3}$$
$$-\frac{b^2f(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3}$$
$$+\frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3}-\frac{b^2f^2(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{60d^3}$$
$$+\frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3}+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3}$$
$$+\frac{b^2(de-cf)^2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^3}-\frac{(b^4f^2)\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^3}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{180d^3}$$
$$-\frac{(b^4f(de-cf))\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{6d^3}$$

$$
\begin{aligned}
&= -\frac{b^3 f(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b(de-cf)^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} \\
&- \frac{b^3 f^2 (c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{180d^3} + \frac{bf(de-cf)(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} \\
&+ \frac{bf^2(c+dx)^{5/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{15d^3} + \frac{b^2(de-cf)^2\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d^3} \\
&+ \frac{b^4 f^2 (c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^2 f(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} \\
&+ \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} - \frac{b^2 f^2 (c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{60d^3} \\
&+ \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} \\
&+ \frac{b^2(de-cf)^2\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^3} - \frac{(b^5 f^2)\,\mathrm{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{360d^3} \\
&- \frac{(b^4 f(de-cf)\cos(a))\,\mathrm{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{6d^3} \\
&- \frac{(b^4 f(de-cf)\sin(a))\,\mathrm{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{6d^3}
\end{aligned}
$$

$$= \frac{b^5 f^2 \sqrt{c+dx} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360 d^3} - \frac{b^3 f(de-cf)\sqrt{c+dx} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b(de-cf)^2 \sqrt{c+dx} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} - \frac{b^3 f^2 (c+dx)^{3/2} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{180 d^3} + \frac{bf(de-cf)(c+dx)^{3/2} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} + \frac{bf^2 (c+dx)^{5/2} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{15 d^3} - \frac{b^4 f(de-cf) \operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right) \sin(a)}{6d^3} + \frac{b^2 (de-cf)^2 \operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right) \sin(a)}{d^3} + \frac{b^4 f^2 (c+dx) \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360 d^3} - \frac{b^2 f(de-cf)(c+dx) \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{(de-cf)^2 (c+dx) \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} - \frac{b^2 f^2 (c+dx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{60 d^3} + \frac{f(de-cf)(c+dx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{f^2 (c+dx)^3 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} - \frac{b^4 f(de-cf) \cos(a) \operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b^2 (de-cf)^2 \cos(a) \operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{(b^6 f^2) \operatorname{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{360 d^3}$$

$$
\begin{aligned}
&= \frac{b^5 f^2 \sqrt{c+dx} \cos \left(a+\frac{b}{\sqrt{c+dx}}\right)}{360 d^3} - \frac{b^3 f (de-cf) \sqrt{c+dx} \cos \left(a+\frac{b}{\sqrt{c+dx}}\right)}{6 d^3} \\
&+ \frac{b (de-cf)^2 \sqrt{c+dx} \cos \left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} - \frac{b^3 f^2 (c+dx)^{3/2} \cos \left(a+\frac{b}{\sqrt{c+dx}}\right)}{180 d^3} \\
&+ \frac{b f (de-cf) (c+dx)^{3/2} \cos \left(a+\frac{b}{\sqrt{c+dx}}\right)}{3 d^3} + \frac{b f^2 (c+dx)^{5/2} \cos \left(a+\frac{b}{\sqrt{c+dx}}\right)}{15 d^3} \\
&- \frac{b^4 f (de-cf) \operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right) \sin(a)}{6 d^3} \\
&+ \frac{b^2 (de-cf)^2 \operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right) \sin(a)}{d^3} + \frac{b^4 f^2 (c+dx) \sin \left(a+\frac{b}{\sqrt{c+dx}}\right)}{360 d^3} \\
&- \frac{b^2 f (de-cf) (c+dx) \sin \left(a+\frac{b}{\sqrt{c+dx}}\right)}{6 d^3} + \frac{(de-cf)^2 (c+dx) \sin \left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} \\
&- \frac{b^2 f^2 (c+dx)^2 \sin \left(a+\frac{b}{\sqrt{c+dx}}\right)}{60 d^3} + \frac{f (de-cf) (c+dx)^2 \sin \left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} \\
&+ \frac{f^2 (c+dx)^3 \sin \left(a+\frac{b}{\sqrt{c+dx}}\right)}{3 d^3} - \frac{b^4 f (de-cf) \cos(a) \operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{6 d^3} \\
&+ \frac{b^2 (de-cf)^2 \cos(a) \operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{\left(b^6 f^2 \cos(a)\right) \operatorname{Subst}\left(\int \frac{\sin(bx)}{x} \, dx, x, \frac{1}{\sqrt{c+dx}}\right)}{360 d^3} \\
&+ \frac{\left(b^6 f^2 \sin(a)\right) \operatorname{Subst}\left(\int \frac{\cos(bx)}{x} \, dx, x, \frac{1}{\sqrt{c+dx}}\right)}{360 d^3}
\end{aligned}
$$

$$= \frac{b^5 f^2 \sqrt{c+dx} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^3 f(de-cf)\sqrt{c+dx} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b(de-cf)^2\sqrt{c+dx} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} - \frac{b^3 f^2 (c+dx)^{3/2} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{180d^3} + \frac{bf(de-cf)(c+dx)^{3/2} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} + \frac{bf^2(c+dx)^{5/2} \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{15d^3} + \frac{b^6 f^2 \,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{360d^3} - \frac{b^4 f(de-cf)\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{6d^3} + \frac{b^2(de-cf)^2\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d^3} + \frac{b^4 f^2(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^2 f(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} - \frac{b^2 f^2(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{60d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} + \frac{b^6 f^2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^4 f(de-cf)\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b^2(de-cf)^2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 557, normalized size of antiderivative = 0.91

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$

$$= \frac{ie^{-ia}\left(e^{-\frac{ib}{\sqrt{c+dx}}}\sqrt{c+dx}\left(-ib^5f^2+b^4f^2\sqrt{c+dx}+2ib^3f(30de-29cf+dfx)-6b^2f\sqrt{c+dx}(10de-9cf+\right.\right.}{}$$

```
[In] Integrate[(e + f*x)^2*Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] ((I/720)*((Sqrt[c + d*x]*((-I)*b^5*f^2 + b^4*f^2*Sqrt[c + d*x] + (2*I)*b^3*
f*(30*d*e - 29*c*f + d*f*x) - 6*b^2*f*Sqrt[c + d*x]*(10*d*e - 9*c*f + d*f*x
) + 120*Sqrt[c + d*x]*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x +
 f^2*x^2)) - (24*I)*b*(11*c^2*f^2 - c*d*f*(25*e + 3*f*x) + d^2*(15*e^2 + 5*
e*f*x + f^2*x^2))))/E^((I*b)/Sqrt[c + d*x]) - E^(I*(2*a + b/Sqrt[c + d*x]))
*Sqrt[c + d*x]*(I*b^5*f^2 + b^4*f^2*Sqrt[c + d*x] - (2*I)*b^3*f*(30*d*e - 2
9*c*f + d*f*x) - 6*b^2*f*Sqrt[c + d*x]*(10*d*e - 9*c*f + d*f*x) + 120*Sqrt[
```

```
c + d*x]*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2)) +
(24*I)*b*(11*c^2*f^2 - c*d*f*(25*e + 3*f*x) + d^2*(15*e^2 + 5*e*f*x + f^2*x
^2))) + b^2*(360*d^2*e^2 - 60*(b^2 + 12*c)*d*e*f + (b^4 + 60*b^2*c + 360*c^
2)*f^2)*ExpIntegralEi[((-I)*b)/Sqrt[c + d*x]] - b^2*E^((2*I)*a)*(360*d^2*e^
2 - 60*(b^2 + 12*c)*d*e*f + (b^4 + 60*b^2*c + 360*c^2)*f^2)*ExpIntegralEi[(
I*b)/Sqrt[c + d*x]]))/(d^3*E^(I*a))
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.14

method	result
derivativedivides	$-\frac{2b^2\left(-2cdef\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\frac{\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)}{2}\right)-2b^2cf^2\left(\cdots\right.\right.}{}$
default	$-\frac{2b^2\left(-2cdef\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\frac{\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)}{2}\right)-2b^2cf^2\left(\cdots\right.\right.}{}$
parts	Expression too large to display

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d^3*b^2*(-2*c*d*e*f*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b
/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*
x+c)^(1/2))*sin(a))-2*b^2*c*f^2*(-1/4*sin(a+b/(d*x+c)^(1/2))/b^4*(d*x+c)^2-
1/12*cos(a+b/(d*x+c)^(1/2))/b^3*(d*x+c)^(3/2)+1/24*sin(a+b/(d*x+c)^(1/2))/b
^2*(d*x+c)+1/24*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)+1/24*Si(b/(d*x+c)^(1
/2))*cos(a)+1/24*Ci(b/(d*x+c)^(1/2))*sin(a))+d^2*e^2*(-1/2*sin(a+b/(d*x+c)^
(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x
+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^(1/2))*sin(a))+b^4*f^2*(-1/6*sin(a+b/(d*
x+c)^(1/2))/b^6*(d*x+c)^3-1/30*cos(a+b/(d*x+c)^(1/2))/b^5*(d*x+c)^(5/2)+1/1
20*sin(a+b/(d*x+c)^(1/2))/b^4*(d*x+c)^2+1/360*cos(a+b/(d*x+c)^(1/2))/b^3*(d
*x+c)^(3/2)-1/720*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/720*cos(a+b/(d*x+c)^
(1/2))/b*(d*x+c)^(1/2)-1/720*Si(b/(d*x+c)^(1/2))*cos(a)-1/720*Ci(b/(d*x+c)^
(1/2))*sin(a))+c^2*f^2*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b
/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*
x+c)^(1/2))*sin(a))+2*b^2*d*e*f*(-1/4*sin(a+b/(d*x+c)^(1/2))/b^4*(d*x+c)^2-
1/12*cos(a+b/(d*x+c)^(1/2))/b^3*(d*x+c)^(3/2)+1/24*sin(a+b/(d*x+c)^(1/2))/b
^2*(d*x+c)+1/24*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)+1/24*Si(b/(d*x+c)^(1
/2))*cos(a)+1/24*Ci(b/(d*x+c)^(1/2))*sin(a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.64

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \frac{(360\,b^2d^2e^2 - 60\,(b^4+12\,b^2c)def + (b^6+60\,b^4c+360\,b^2c^2)f^2)\operatorname{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a) + (360\,b^2d^2e^2 - 60\,(b^4+\cdots}{\cdots}$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x, algorithm="fricas")
[Out] 1/360*((360*b^2*d^2*e^2 - 60*(b^4 + 12*b^2*c)*d*e*f + (b^6 + 60*b^4*c + 360
*b^2*c^2)*f^2)*cos_integral(b/sqrt(d*x + c))*sin(a) + (360*b^2*d^2*e^2 - 60
*(b^4 + 12*b^2*c)*d*e*f + (b^6 + 60*b^4*c + 360*b^2*c^2)*f^2)*cos(a)*sin_in
tegral(b/sqrt(d*x + c)) + (24*b*d^2*f^2*x^2 + 360*b*d^2*e^2 - 60*(b^3 + 10*
b*c)*d*e*f + (b^5 + 58*b^3*c + 264*b*c^2)*f^2 + 2*(60*b*d^2*e*f - (b^3 + 36
*b*c)*d*f^2)*x)*sqrt(d*x + c)*cos((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c)
) + (120*d^3*f^2*x^3 + 360*c*d^2*e^2 - 60*(b^2*c + 6*c^2)*d*e*f + (b^4*c +
54*b^2*c^2 + 120*c^3)*f^2 - 6*(b^2*d^2*f^2 - 60*d^3*e*f)*x^2 - (60*b^2*d^2*
e*f - 360*d^3*e^2 - (b^4 + 48*b^2*c)*d*f^2)*x)*sin((a*d*x + a*c + sqrt(d*x
+ c)*b)/(d*x + c)))/d^3
```

Sympy [F]

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**(1/2)),x)
[Out] Integral((e + f*x)**2*sin(a + b/sqrt(c + d*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.44

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/720*(360*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) +
 (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x
+ c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x
 + c)*a + b)/sqrt(d*x + c)))*e^2 - 720*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-
I*b/sqrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)
))*sin(a))*b^2 + 2*sqrt(d*x + c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))
 + 2*(d*x + c)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*c*e*f/d + 360*(((-
I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d
*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x + c)*b*cos((sqr
t(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x + c)*a + b)/sq
rt(d*x + c)))*c^2*f^2/d^2 + 60*(((I*Ei(I*b/sqrt(d*x + c)) - I*Ei(-I*b/sqrt(
d*x + c)))*cos(a) - (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a)
)*b^4 - 2*(sqrt(d*x + c)*b^3 - 2*(d*x + c)^(3/2)*b)*cos((sqrt(d*x + c)*a +
b)/sqrt(d*x + c)) - 2*((d*x + c)*b^2 - 6*(d*x + c)^2)*sin((sqrt(d*x + c)*a
+ b)/sqrt(d*x + c)))*e*f/d - 60*(((I*Ei(I*b/sqrt(d*x + c)) - I*Ei(-I*b/sqrt
(d*x + c)))*cos(a) - (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a
))*b^4 - 2*(sqrt(d*x + c)*b^3 - 2*(d*x + c)^(3/2)*b)*cos((sqrt(d*x + c)*a +
 b)/sqrt(d*x + c)) - 2*((d*x + c)*b^2 - 6*(d*x + c)^2)*sin((sqrt(d*x + c)*a
 + b)/sqrt(d*x + c)))*c*f^2/d^2 + (((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/s
qrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*si
n(a))*b^6 + 2*(sqrt(d*x + c)*b^5 - 2*(d*x + c)^(3/2)*b^3 + 24*(d*x + c)^(5/
2)*b)*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*((d*x + c)*b^4 - 6*(d*x
+ c)^2*b^2 + 120*(d*x + c)^3)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*f^2
/d^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6606 vs. 2(537) = 1074.

Time = 0.64 (sec) , antiderivative size = 6606, normalized size of antiderivative = 10.81

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/360*(360*(a^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*
sin(a) - a^2*b^3*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c
)) - 2*(sqrt(d*x + c)*a + b)*a*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/
sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 2*(sqrt(d*x + c)*a + b)*a*b^3*cos(a)*
sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + (sqrt
(d*x + c)*a + b)^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c
))*sin(a)/(d*x + c) - (sqrt(d*x + c)*a + b)^2*b^3*cos(a)*sin_integral(a - (
sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - a*b^3*cos((sqrt(d*x + c)*a
+ b)/sqrt(d*x + c)) + (sqrt(d*x + c)*a + b)*b^3*cos((sqrt(d*x + c)*a + b)/s
qrt(d*x + c))/sqrt(d*x + c) + b^3*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))
```

```
*e^2/((a^2 - 2*(sqrt(d*x + c)*a + b)*a/sqrt(d*x + c) + (sqrt(d*x + c)*a + b
)^2/(d*x + c))*b) + (a^6*b^7*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d
*x + c))*sin(a) - a^6*b^7*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqr
t(d*x + c)) - 6*(sqrt(d*x + c)*a + b)*a^5*b^7*cos_integral(-a + (sqrt(d*x +
 c)*a + b)/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 6*(sqrt(d*x + c)*a + b)*a^
5*b^7*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x
 + c) + 15*(sqrt(d*x + c)*a + b)^2*a^4*b^7*cos_integral(-a + (sqrt(d*x + c)
*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c) + 60*a^6*b^5*c*cos_integral(-a + (s
qrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a) - 15*(sqrt(d*x + c)*a + b)^2*a^4*
b^7*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)
- 60*a^6*b^5*c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))
 - 20*(sqrt(d*x + c)*a + b)^3*a^3*b^7*cos_integral(-a + (sqrt(d*x + c)*a +
b)/sqrt(d*x + c))*sin(a)/(d*x + c)^(3/2) - 360*(sqrt(d*x + c)*a + b)*a^5*b^
5*c*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/sqrt(d*x
+ c) + 20*(sqrt(d*x + c)*a + b)^3*a^3*b^7*cos(a)*sin_integral(a - (sqrt(d*x
 + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) + 360*(sqrt(d*x + c)*a + b)*a^5
*b^5*c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*
x + c) - a^5*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 15*(sqrt(d*x +
c)*a + b)^4*a^2*b^7*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*
sin(a)/(d*x + c)^2 + 900*(sqrt(d*x + c)*a + b)^2*a^4*b^5*c*cos_integral(-a
+ (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c) + 360*a^6*b^3*c^2*c
os_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a) - 15*(sqrt(d*x
 + c)*a + b)^4*a^2*b^7*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d
*x + c))/(d*x + c)^2 - 900*(sqrt(d*x + c)*a + b)^2*a^4*b^5*c*cos(a)*sin_int
egral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - 360*a^6*b^3*c^2*
cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 5*(sqrt(d*x
+ c)*a + b)*a^4*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c)
- 6*(sqrt(d*x + c)*a + b)^5*a*b^7*cos_integral(-a + (sqrt(d*x + c)*a + b)/s
qrt(d*x + c))*sin(a)/(d*x + c)^(5/2) - 1200*(sqrt(d*x + c)*a + b)^3*a^3*b^5
*c*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^
(3/2) - 2160*(sqrt(d*x + c)*a + b)*a^5*b^3*c^2*cos_integral(-a + (sqrt(d*x
+ c)*a + b)/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 6*(sqrt(d*x + c)*a + b)^5
*a*b^7*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x +
c)^(5/2) + 1200*(sqrt(d*x + c)*a + b)^3*a^3*b^5*c*cos(a)*sin_integral(a - (
sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) + 2160*(sqrt(d*x + c)*a
 + b)*a^5*b^3*c^2*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x +
c))/sqrt(d*x + c) - 10*(sqrt(d*x + c)*a + b)^2*a^3*b^7*cos((sqrt(d*x + c)*a
 + b)/sqrt(d*x + c))/(d*x + c) - 60*a^5*b^5*c*cos((sqrt(d*x + c)*a + b)/sqr
t(d*x + c)) + (sqrt(d*x + c)*a + b)^6*b^7*cos_integral(-a + (sqrt(d*x + c)*
a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^3 + 900*(sqrt(d*x + c)*a + b)^4*a^2*
b^5*c*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x +
c)^2 + 5400*(sqrt(d*x + c)*a + b)^2*a^4*b^3*c^2*cos_integral(-a + (sqrt(d*x
 + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c) + a^4*b^7*sin((sqrt(d*x + c)*a
 + b)/sqrt(d*x + c)) - (sqrt(d*x + c)*a + b)^6*b^7*cos(a)*sin_integral(a -
(sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^3 - 900*(sqrt(d*x + c)*a + b
```

)^4*a^2*b^5*c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 - 5400*(sqrt(d*x + c)*a + b)^2*a^4*b^3*c^2*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) + 10*(sqrt(d*x + c)*a + b)^3*a^2*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) + 300*(sqrt(d*x + c)*a + b)*a^4*b^5*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 360*(sqrt(d*x + c)*a + b)^5*a*b^5*c*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^(5/2) - 7200*(sqrt(d*x + c)*a + b)^3*a^3*b^3*c^2*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^(3/2) - 4*(sqrt(d*x + c)*a + b)*a^3*b^7*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + 360*(sqrt(d*x + c)*a + b)^5*a*b^5*c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(5/2) + 7200*(sqrt(d*x + c)*a + b)^3*a^3*b^3*c^2*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) - 5*(sqrt(d*x + c)*a + b)^4*a*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 + 2*a^3*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 600*(sqrt(d*x + c)*a + b)^2*a^3*b^5*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - 360*a^5*b^3*c^2*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 60*(sqrt(d*x + c)*a + b)^6*b^5*c*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^3 + 5400*(sqrt(d*x + c)*a + b)^4*a^2*b^3*c^2*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^2 + 6*(sqrt(d*x + c)*a + b)^2*a^2*b^7*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) + 60*a^4*b^5*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 60*(sqrt(d*x + c)*a + b)^6*b^5*c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^3 - 5400*(sqrt(d*x + c)*a + b)^4*a^2*b^3*c^2*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 + (sqrt(d*x + c)*a + b)^5*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(5/2) - 6*(sqrt(d*x + c)*a + b)*a^2*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + 600*(sqrt(d*x + c)*a + b)^3*a^2*b^5*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) + 1800*(sqrt(d*x + c)*a + b)*a^4*b^3*c^2*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 2160*(sqrt(d*x + c)*a + b)^5*a*b^3*c^2*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^(5/2) - 4*(sqrt(d*x + c)*a + b)^3*a*b^7*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) - 240*(sqrt(d*x + c)*a + b)*a^3*b^5*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + 2160*(sqrt(d*x + c)*a + b)^5*a*b^3*c^2*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(5/2) + 6*(sqrt(d*x + c)*a + b)^2*a*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - 300*(sqrt(d*x + c)*a + b)^4*a*b^5*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 + 120*a^3*b^5*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 3600*(sqrt(d*x + c)*a + b)^2*a^3*b^3*c^2*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) + 360*(sqrt(d*x + c)*a + b)^6*b^3*c^2*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^3 + (sqrt(d*x + c)*a + b)^4*b^7*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 - 6*a^2*b^7*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 360*(sqrt(d*x + c)*a + b)^2*a^2*b^5*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) + 360*a^4*b^3*c^2*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 360*(sqrt(d*x + c)*a + b)^

6*b^3*c^2*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^3 - 2*(sqrt(d*x + c)*a + b)^3*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) + 60*(sqrt(d*x + c)*a + b)^5*b^5*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(5/2) - 360*(sqrt(d*x + c)*a + b)*a^2*b^5*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + 3600*(sqrt(d*x + c)*a + b)^3*a^2*b^3*c^2*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) + 12*(sqrt(d*x + c)*a + b)*a*b^7*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 240*(sqrt(d*x + c)*a + b)^3*a*b^5*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) - 1440*(sqrt(d*x + c)*a + b)*a^3*b^3*c^2*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 24*a*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 360*(sqrt(d*x + c)*a + b)^2*a*b^5*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - 1800*(sqrt(d*x + c)*a + b)^4*a*b^3*c^2*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 - 6*(sqrt(d*x + c)*a + b)^2*b^7*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) + 60*(sqrt(d*x + c)*a + b)^4*b^5*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 - 360*a^2*b^5*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2160*(sqrt(d*x + c)*a + b)^2*a^2*b^3*c^2*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) + 24*(sqrt(d*x + c)*a + b)*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 120*(sqrt(d*x + c)*a + b)^3*b^5*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) + 360*(sqrt(d*x + c)*a + b)^5*b^3*c^2*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(5/2) + 720*(sqrt(d*x + c)*a + b)*a*b^5*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 1440*(sqrt(d*x + c)*a + b)^3*a*b^3*c^2*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) + 120*b^7*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 360*(sqrt(d*x + c)*a + b)^2*b^5*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) + 360*(sqrt(d*x + c)*a + b)^4*b^3*c^2*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2)*f^2/((a^6*d^2 - 6*(sqrt(d*x + c)*a + b)*a^5*d^2/sqrt(d*x + c) + 15*(sqrt(d*x + c)*a + b)^2*a^4*d^2/(d*x + c) - 20*(sqrt(d*x + c)*a + b)^3*a^3*d^2/(d*x + c)^(3/2) + 15*(sqrt(d*x + c)*a + b)^4*a^2*d^2/(d*x + c)^2 - 6*(sqrt(d*x + c)*a + b)^5*a*d^2/(d*x + c)^(5/2) + (sqrt(d*x + c)*a + b)^6*d^2/(d*x + c)^3)*b) - 60*(a^4*b^5*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a) - a^4*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 4*(sqrt(d*x + c)*a + b)*a^3*b^5*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 4*(sqrt(d*x + c)*a + b)*a^3*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + 6*(sqrt(d*x + c)*a + b)^2*a^2*b^5*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c) + 12*a^4*b^3*c*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a) - 6*(sqrt(d*x + c)*a + b)^2*a^2*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - 12*a^4*b^3*c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 4*(sqrt(d*x + c)*a + b)^3*a*b^5*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^(3/2) - 48*(sqrt(d*x + c)*a + b)*a^3*b^3*c*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 4*(sqrt(d*x + c)*a + b)^3*a*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) + 48*(sqrt(d

```
*x + c)*a + b)*a^3*b^3*c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt
(d*x + c))/sqrt(d*x + c) - a^3*b^5*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))
 + (sqrt(d*x + c)*a + b)^4*b^5*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt
(d*x + c))*sin(a)/(d*x + c)^2 + 72*(sqrt(d*x + c)*a + b)^2*a^2*b^3*c*cos_in
tegral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c) - (sqrt(d
*x + c)*a + b)^4*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x
 + c))/(d*x + c)^2 - 72*(sqrt(d*x + c)*a + b)^2*a^2*b^3*c*cos(a)*sin_integr
al(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) + 3*(sqrt(d*x + c)*a
+ b)*a^2*b^5*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 48*(s
qrt(d*x + c)*a + b)^3*a*b^3*c*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(
d*x + c))*sin(a)/(d*x + c)^(3/2) + 48*(sqrt(d*x + c)*a + b)^3*a*b^3*c*cos(a
)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) - 3
*(sqrt(d*x + c)*a + b)^2*a*b^5*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*
x + c) - 12*a^3*b^3*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 12*(sqrt(d
*x + c)*a + b)^4*b^3*c*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c
))*sin(a)/(d*x + c)^2 + a^2*b^5*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) -
12*(sqrt(d*x + c)*a + b)^4*b^3*c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a +
 b)/sqrt(d*x + c))/(d*x + c)^2 + (sqrt(d*x + c)*a + b)^3*b^5*cos((sqrt(d*x
+ c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) + 36*(sqrt(d*x + c)*a + b)*a^2*b
^3*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 2*(sqrt(d*x +
 c)*a + b)*a*b^5*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + 2
*a*b^5*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 36*(sqrt(d*x + c)*a + b)^
2*a*b^3*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) + (sqrt(d*x +
c)*a + b)^2*b^5*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) + 12*a^2
*b^3*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 2*(sqrt(d*x + c)*a + b)*b
^5*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + 12*(sqrt(d*x +
c)*a + b)^3*b^3*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2)
- 24*(sqrt(d*x + c)*a + b)*a*b^3*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))
/sqrt(d*x + c) - 6*b^5*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 12*(sqrt(
d*x + c)*a + b)^2*b^3*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c))
*e*f/((a^4 - 4*(sqrt(d*x + c)*a + b)*a^3/sqrt(d*x + c) + 6*(sqrt(d*x + c)*a
 + b)^2*a^2/(d*x + c) - 4*(sqrt(d*x + c)*a + b)^3*a/(d*x + c)^(3/2) + (sqrt
(d*x + c)*a + b)^4/(d*x + c)^2)*b*d))/d
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx = \int \sin\left(a+\frac{b}{\sqrt{c+d\,x}}\right) (e+f\,x)^2 \,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/2))*(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^(1/2))*(e + f*x)^2, x)
```

3.198 $\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 301

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = -\frac{b^3f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2} + \frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{bf(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^2} - \frac{b^4f\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{12d^2} + \frac{b^2(de-cf)\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d^2} - \frac{b^2f(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2d^2} - \frac{b^4f\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{12d^2} + \frac{b^2(de-cf)\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2}$$

```
[Out] 1/6*b*f*(d*x+c)^(3/2)*cos(a+b/(d*x+c)^(1/2))/d^2-1/12*b^4*f*cos(a)*Si(b/(d*
x+c)^(1/2))/d^2+b^2*(-c*f+d*e)*cos(a)*Si(b/(d*x+c)^(1/2))/d^2-1/12*b^4*f*Ci
```

```
(b/(d*x+c)^(1/2))*sin(a)/d^2+b^2*(-c*f+d*e)*Ci(b/(d*x+c)^(1/2))*sin(a)/d^2-
1/12*b^2*f*(d*x+c)*sin(a+b/(d*x+c)^(1/2))/d^2+(-c*f+d*e)*(d*x+c)*sin(a+b/(d
*x+c)^(1/2))/d^2+1/2*f*(d*x+c)^2*sin(a+b/(d*x+c)^(1/2))/d^2-1/12*b^3*f*cos(
a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/d^2+b*(-c*f+d*e)*cos(a+b/(d*x+c)^(1/2))*(d
*x+c)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3512, 3378, 3384, 3380, 3383}

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx = -\frac{b^4 f\sin(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{12d^2} - \frac{b^4 f\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{12d^2} - \frac{b^3 f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2} + \frac{b^2\sin(a)(de-cf)\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{b^2\cos(a)(de-cf)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2} - \frac{b^2 f(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2} + \frac{(c+dx)(de-cf)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{b\sqrt{c+dx}(de-cf)\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2d^2} + \frac{bf(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^2}$$

```
[In] Int[(e + f*x)*Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] -1/12*(b^3*f*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d^2 + (b*(d*e - c*f)*S
qrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d^2 + (b*f*(c + d*x)^(3/2)*Cos[a + b
/Sqrt[c + d*x]])/(6*d^2) - (b^4*f*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(12*
d^2) + (b^2*(d*e - c*f)*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/d^2 - (b^2*f*(
c + d*x)*Sin[a + b/Sqrt[c + d*x]])/(12*d^2) + ((d*e - c*f)*(c + d*x)*Sin[a
+ b/Sqrt[c + d*x]])/d^2 + (f*(c + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/(2*d^2)
```

```
- (b^4*f*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(12*d^2) + (b^2*(d*e - c*f)*C
os[a]*SinIntegral[b/Sqrt[c + d*x]])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \left(\frac{f\sin(a+bx)}{dx^5} + \frac{(de-cf)\sin(a+bx)}{dx^3}\right)\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= -\frac{(2f)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^5}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d^2} - \frac{(2(de-cf))\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d^2}$$

$$=\frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2d^2}-\frac{(bf)\text{Subst}\left(\int\frac{\cos(a+bx)}{x^4}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{2d^2}-\frac{(b(de-cf))\text{Subst}\left(\int\frac{\cos(a+bx)}{x^2}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^2}$$

$$=\frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{bf(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^2}+\frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2d^2}+\frac{(b^2f)\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^3}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{6d^2}+\frac{(b^2(de-cf))\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^2}$$

$$=\frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{bf(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^2}-\frac{b^2f(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2d^2}+\frac{(b^3f)\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^2}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{12d^2}+\frac{(b^2(de-cf)\cos(a))\,\text{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^2}+\frac{(b^2(de-cf)\sin(a))\,\text{Subst}\left(\int\frac{\cos(bx)}{x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^2}$$

$$=-\frac{b^3f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{bf(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^2}+\frac{b^2(de-cf)\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d^2}-\frac{b^2f(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2d^2}+\frac{b^2(de-cf)\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2}-\frac{(b^4f)\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{12d^2}$$

$$=-\frac{b^3f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{bf(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^2}+\frac{b^2(de-cf)\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d^2}-\frac{b^2f(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2d^2}+\frac{b^2(de-cf)\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2}-\frac{(b^4f\cos(a))\,\text{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{12d^2}-\frac{(b^4f\sin(a))\,\text{Subst}\left(\int\frac{\cos(bx)}{x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{12d^2}$$

$$=-\frac{b^3f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{bf(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^2}-\frac{b^4f\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{12d^2}+\frac{b^2(de-cf)\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d^2}-\frac{b^2f(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2d^2}-\frac{b^4f\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{b^2(de-cf)\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.22

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \frac{e\sqrt{c+dx}\cos\left(\frac{b}{\sqrt{c+dx}}\right)\left(b\cos(a)+\sqrt{c+dx}\sin(a)\right)}{d}$$

$$+\frac{f\sqrt{c+dx}\cos\left(\frac{b}{\sqrt{c+dx}}\right)\left(-b^3\cos(a)-12bc\cos(a)+2b(c+dx)\cos(a)-b^2\sqrt{c+dx}\sin(a)-12c\sqrt{c+dx}\ldots\right.}{12d^2}$$

$$+\frac{e\sqrt{c+dx}\left(\sqrt{c+dx}\cos(a)-b\sin(a)\right)\sin\left(\frac{b}{\sqrt{c+dx}}\right)}{d}$$

$$+\frac{f\sqrt{c+dx}\left(-b^2\sqrt{c+dx}\cos(a)-12c\sqrt{c+dx}\cos(a)+6(c+dx)^{3/2}\cos(a)+b^3\sin(a)+12bc\sin(a)-2b\ldots\right.}{12d^2}$$

$$+\frac{b^2e\left(\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)+\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)\right)}{d}$$

$$-\frac{b^2(b^2+12c)\,f\left(\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)+\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)\right)}{12d^2}$$

```
[In] Integrate[(e + f*x)*Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] (e*Sqrt[c + d*x]*Cos[b/Sqrt[c + d*x]]*(b*Cos[a] + Sqrt[c + d*x]*Sin[a]))/d
+ (f*Sqrt[c + d*x]*Cos[b/Sqrt[c + d*x]]*(-(b^3*Cos[a]) - 12*b*c*Cos[a] + 2*
b*(c + d*x)*Cos[a] - b^2*Sqrt[c + d*x]*Sin[a] - 12*c*Sqrt[c + d*x]*Sin[a] +
 6*(c + d*x)^(3/2)*Sin[a]))/(12*d^2) + (e*Sqrt[c + d*x]*(Sqrt[c + d*x]*Cos[
a] - b*Sin[a])*Sin[b/Sqrt[c + d*x]])/d + (f*Sqrt[c + d*x]*(-(b^2*Sqrt[c + d
*x]*Cos[a]) - 12*c*Sqrt[c + d*x]*Cos[a] + 6*(c + d*x)^(3/2)*Cos[a] + b^3*Si
n[a] + 12*b*c*Sin[a] - 2*b*(c + d*x)*Sin[a])*Sin[b/Sqrt[c + d*x]])/(12*d^2)
 + (b^2*e*(CosIntegral[b/Sqrt[c + d*x]]*Sin[a] + Cos[a]*SinIntegral[b/Sqrt[
c + d*x]]))/d - (b^2*(b^2 + 12*c)*f*(CosIntegral[b/Sqrt[c + d*x]]*Sin[a] +
Cos[a]*SinIntegral[b/Sqrt[c + d*x]]))/(12*d^2)
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{2b^2\left(-cf\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\frac{\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)}{2}\right)+de\left(-\frac{\sin\left(a+\frac{b}{\sqrt{}}\right.}{}\right.\right.}{}$
default	$-\frac{2b^2\left(-cf\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\frac{\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)}{2}\right)+de\left(-\frac{\sin\left(a+\frac{b}{\sqrt{}}\right.}{}\right.\right.}{}$
parts	$\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)x^2f+\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)xe+\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)cfx}{d}+\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)ce}{d}+\frac{b\cos\left(a+\frac{}{\sqrt{d}}\right.}{}$

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d^2*b^2*(-c*f*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x+
c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^(
1/2))*sin(a))+d*e*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x
+c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^
(1/2))*sin(a))+f*b^2*(-1/4*sin(a+b/(d*x+c)^(1/2))/b^4*(d*x+c)^2-1/12*cos(a+
b/(d*x+c)^(1/2))/b^3*(d*x+c)^(3/2)+1/24*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)+
1/24*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)+1/24*Si(b/(d*x+c)^(1/2))*cos(a)
+1/24*Ci(b/(d*x+c)^(1/2))*sin(a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.67

$$\int(e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx$$
$$=\frac{(12\,b^2de-(b^4+12\,b^2c)f)\,\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)+(12\,b^2de-(b^4+12\,b^2c)f)\cos(a)\,\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)+(2\,bdfx+12}{}$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/12*((12*b^2*d*e - (b^4 + 12*b^2*c)*f)*cos_integral(b/sqrt(d*x + c))*sin(a
) + (12*b^2*d*e - (b^4 + 12*b^2*c)*f)*cos(a)*sin_integral(b/sqrt(d*x + c))
+ (2*b*d*f*x + 12*b*d*e - (b^3 + 10*b*c)*f)*sqrt(d*x + c)*cos((a*d*x + a*c
+ sqrt(d*x + c)*b)/(d*x + c)) + (6*d^2*f*x^2 + 12*c*d*e - (b^2*c + 6*c^2)*f
 - (b^2*d*f - 12*d^2*e)*x)*sin((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c)))/
d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**(1/2)),x)
[Out] Integral((e + f*x)*sin(a + b/sqrt(c + d*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.35

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$
$$= \frac{12\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{i\,b}{\sqrt{dx+c}}\right)+i\,\mathrm{Ei}\left(-\frac{i\,b}{\sqrt{dx+c}}\right)\right)\cos(a)+\left(\mathrm{Ei}\left(\frac{i\,b}{\sqrt{dx+c}}\right)+\mathrm{Ei}\left(-\frac{i\,b}{\sqrt{dx+c}}\right)\right)\sin(a)\right)b^2+2\sqrt{dx+c}b\,\mathrm{co}}{}$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x, algorithm="maxima")
[Out] 1/24*(12*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) + (
Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x +
c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x +
 c)*a + b)/sqrt(d*x + c)))*e - 12*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/s
qrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*si
n(a))*b^2 + 2*sqrt(d*x + c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*
(d*x + c)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*c*f/d + (((I*Ei(I*b/sqr
t(d*x + c)) - I*Ei(-I*b/sqrt(d*x + c)))*cos(a) - (Ei(I*b/sqrt(d*x + c)) + E
i(-I*b/sqrt(d*x + c)))*sin(a))*b^4 - 2*(sqrt(d*x + c)*b^3 - 2*(d*x + c)^(3/
2)*b)*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 2*((d*x + c)*b^2 - 6*(d*x
+ c)^2)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*f/d)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2158 vs. $2(263) = 526$.

Time = 0.43 (sec) , antiderivative size = 2158, normalized size of antiderivative = 7.17

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x, algorithm="giac")
```

[Out] 1/12*(12*(a^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*si
n(a) - a^2*b^3*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))
 - 2*(sqrt(d*x + c)*a + b)*a*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sq
rt(d*x + c))*sin(a)/sqrt(d*x + c) + 2*(sqrt(d*x + c)*a + b)*a*b^3*cos(a)*si
n_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + (sqrt(d
*x + c)*a + b)^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))
*sin(a)/(d*x + c) - (sqrt(d*x + c)*a + b)^2*b^3*cos(a)*sin_integral(a - (sq
rt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - a*b^3*cos((sqrt(d*x + c)*a +
b)/sqrt(d*x + c)) + (sqrt(d*x + c)*a + b)*b^3*cos((sqrt(d*x + c)*a + b)/sqr
t(d*x + c))/sqrt(d*x + c) + b^3*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*e
/((a^2 - 2*(sqrt(d*x + c)*a + b)*a/sqrt(d*x + c) + (sqrt(d*x + c)*a + b)^2/
(d*x + c))*b) - (a^4*b^5*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x +
 c))*sin(a) - a^4*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*
x + c)) - 4*(sqrt(d*x + c)*a + b)*a^3*b^5*cos_integral(-a + (sqrt(d*x + c)*
a + b)/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 4*(sqrt(d*x + c)*a + b)*a^3*b^
5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c
) + 6*(sqrt(d*x + c)*a + b)^2*a^2*b^5*cos_integral(-a + (sqrt(d*x + c)*a +
b)/sqrt(d*x + c))*sin(a)/(d*x + c) + 12*a^4*b^3*c*cos_integral(-a + (sqrt(d
*x + c)*a + b)/sqrt(d*x + c))*sin(a) - 6*(sqrt(d*x + c)*a + b)^2*a^2*b^5*co
s(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - 12*a
^4*b^3*c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 4*(
sqrt(d*x + c)*a + b)^3*a*b^5*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d
*x + c))*sin(a)/(d*x + c)^(3/2) - 48*(sqrt(d*x + c)*a + b)*a^3*b^3*c*cos_in
tegral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 4*(
sqrt(d*x + c)*a + b)^3*a*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/
sqrt(d*x + c))/(d*x + c)^(3/2) + 48*(sqrt(d*x + c)*a + b)*a^3*b^3*c*cos(a)*
sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - a^3*b
^5*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + (sqrt(d*x + c)*a + b)^4*b^5*c
os_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^2 +
72*(sqrt(d*x + c)*a + b)^2*a^2*b^3*c*cos_integral(-a + (sqrt(d*x + c)*a + b
)/sqrt(d*x + c))*sin(a)/(d*x + c) - (sqrt(d*x + c)*a + b)^4*b^5*cos(a)*sin_
integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 - 72*(sqrt(d*
x + c)*a + b)^2*a^2*b^3*c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqr
t(d*x + c))/(d*x + c) + 3*(sqrt(d*x + c)*a + b)*a^2*b^5*cos((sqrt(d*x + c)*
a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 48*(sqrt(d*x + c)*a + b)^3*a*b^3*c*co
s_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^(3/2)
 + 48*(sqrt(d*x + c)*a + b)^3*a*b^3*c*cos(a)*sin_integral(a - (sqrt(d*x + c
)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) - 3*(sqrt(d*x + c)*a + b)^2*a*b^5*c
os((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - 12*a^3*b^3*c*cos((sqrt(
d*x + c)*a + b)/sqrt(d*x + c)) + 12*(sqrt(d*x + c)*a + b)^4*b^3*c*cos_integ
ral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^2 + a^2*b^5*
sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 12*(sqrt(d*x + c)*a + b)^4*b^3*c
*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 +
 (sqrt(d*x + c)*a + b)^3*b^5*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x
+ c)^(3/2) + 36*(sqrt(d*x + c)*a + b)*a^2*b^3*c*cos((sqrt(d*x + c)*a + b)/s

```
qrt(d*x + c))/sqrt(d*x + c) - 2*(sqrt(d*x + c)*a + b)*a*b^5*sin((sqrt(d*x +
 c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + 2*a*b^5*cos((sqrt(d*x + c)*a + b)
/sqrt(d*x + c)) - 36*(sqrt(d*x + c)*a + b)^2*a*b^3*c*cos((sqrt(d*x + c)*a +
 b)/sqrt(d*x + c))/(d*x + c) + (sqrt(d*x + c)*a + b)^2*b^5*sin((sqrt(d*x +
c)*a + b)/sqrt(d*x + c))/(d*x + c) + 12*a^2*b^3*c*sin((sqrt(d*x + c)*a + b)
/sqrt(d*x + c)) - 2*(sqrt(d*x + c)*a + b)*b^5*cos((sqrt(d*x + c)*a + b)/sqr
t(d*x + c))/sqrt(d*x + c) + 12*(sqrt(d*x + c)*a + b)^3*b^3*c*cos((sqrt(d*x
+ c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) - 24*(sqrt(d*x + c)*a + b)*a*b^3
*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 6*b^5*sin((sqrt
(d*x + c)*a + b)/sqrt(d*x + c)) + 12*(sqrt(d*x + c)*a + b)^2*b^3*c*sin((sqr
t(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c))*f/((a^4 - 4*(sqrt(d*x + c)*a +
b)*a^3/sqrt(d*x + c) + 6*(sqrt(d*x + c)*a + b)^2*a^2/(d*x + c) - 4*(sqrt(d*
x + c)*a + b)^3*a/(d*x + c)^(3/2) + (sqrt(d*x + c)*a + b)^4/(d*x + c)^2)*b*
d))/d
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx=\int \sin\left(a+\frac{b}{\sqrt{c+d\,x}}\right)\,(e+f\,x)\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/2))*(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^(1/2))*(e + f*x), x)
```

3.199 $\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 94

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx = \frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{b^2 \operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{b^2\cos(a)\operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d}$$

```
[Out] b^2*cos(a)*Si(b/(d*x+c)^(1/2))/d+b^2*Ci(b/(d*x+c)^(1/2))*sin(a)/d+(d*x+c)*s
in(a+b/(d*x+c)^(1/2))/d+b*cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3442, 3378, 3384, 3380, 3383}

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx = \frac{b^2\sin(a)\operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{b^2\cos(a)\operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d}$$

```
[In] Int[Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] (b*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d + (b^2*CosIntegral[b/Sqrt[c +
d*x]]*Sin[a])/d + ((c + d*x)*Sin[a + b/Sqrt[c + d*x]])/d + (b^2*Cos[a]*SinI
ntegral[b/Sqrt[c + d*x]])/d
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} - \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= \frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{b^2\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= \frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{(b^2\cos(a))\operatorname{Subst}\left(\int\frac{\sin(bx)}{x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d} + \frac{(b^2\sin(a))\operatorname{Subst}\left(\int\frac{\cos(bx)}{x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= \frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{b^2\operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{b^2\cos(a)\operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$

$$= \frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right) + b^2\operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a) + c\sin\left(a+\frac{b}{\sqrt{c+dx}}\right) + dx\sin\left(a+\frac{b}{\sqrt{c+dx}}\right) + b^2}{d}$$

```
[In] Integrate[Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] (b*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]] + b^2*CosIntegral[b/Sqrt[c + d*x]
]*Sin[a] + c*Sin[a + b/Sqrt[c + d*x]] + d*x*Sin[a + b/Sqrt[c + d*x]] + b^2*
Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/d
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{2b^2\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\operatorname{Si}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\frac{\operatorname{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)}{2}\right)}{d}$	84
default	$-\frac{2b^2\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\operatorname{Si}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\frac{\operatorname{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)}{2}\right)}{d}$	84

```
[In] int(sin(a+b/(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

[Out] -2/d*b^2*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x+c)^(1/2)
)/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^(1/2))*si
n(a))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$
$$=\frac{b^2\,\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)+b^2\cos(a)\,\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)+\sqrt{dx+c}b\cos\left(\frac{adx+ac+\sqrt{dx+c}b}{dx+c}\right)+(dx+c)\sin\left(\frac{adx+ac+\sqrt{dx+}}{dx+c}\right.}{d}$$

[In] integrate(sin(a+b/(d*x+c)^(1/2)),x, algorithm="fricas")

[Out] (b^2*cos_integral(b/sqrt(d*x + c))*sin(a) + b^2*cos(a)*sin_integral(b/sqrt(
d*x + c)) + sqrt(d*x + c)*b*cos((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c))
+ (d*x + c)*sin((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c)))/d

Sympy [F]

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx=\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$

[In] integrate(sin(a+b/(d*x+c)**(1/2)),x)

[Out] Integral(sin(a + b/sqrt(c + d*x)), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.32

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$
$$=\frac{\left(\left(-i\,\mathrm{Ei}\left(\frac{ib}{\sqrt{dx+c}}\right)+i\,\mathrm{Ei}\left(-\frac{ib}{\sqrt{dx+c}}\right)\right)\cos(a)+\left(\mathrm{Ei}\left(\frac{ib}{\sqrt{dx+c}}\right)+\mathrm{Ei}\left(-\frac{ib}{\sqrt{dx+c}}\right)\right)\sin(a)\right)b^2+2\sqrt{dx+c}b\cos\left(\right.}{2d}$$

[In] integrate(sin(a+b/(d*x+c)^(1/2)),x, algorithm="maxima")

[Out] 1/2*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) + (Ei(I*
b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x + c)*b*
cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x + c)*a
 + b)/sqrt(d*x + c)))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(84) = 168$.

Time = 0.35 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.39

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$
$$= \frac{a^2b^3\,\mathrm{Ci}\left(-a+\frac{\sqrt{dx+c}a+b}{\sqrt{dx+c}}\right)\sin(a) - a^2b^3\cos(a)\,\mathrm{Si}\left(a-\frac{\sqrt{dx+c}a+b}{\sqrt{dx+c}}\right) - \frac{2\left(\sqrt{dx+c}a+b\right)ab^3\,\mathrm{Ci}\left(-a+\frac{\sqrt{dx+c}a+b}{\sqrt{dx+c}}\right)\sin(a)}{\sqrt{dx+c}} + \frac{2(}{}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] (a^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a) - a^
2*b^3*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 2*(sqr
t(d*x + c)*a + b)*a*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x +
c))*sin(a)/sqrt(d*x + c) + 2*(sqrt(d*x + c)*a + b)*a*b^3*cos(a)*sin_integra
l(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + (sqrt(d*x + c)*a
 + b)^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(
d*x + c) - (sqrt(d*x + c)*a + b)^2*b^3*cos(a)*sin_integral(a - (sqrt(d*x +
c)*a + b)/sqrt(d*x + c))/(d*x + c) - a*b^3*cos((sqrt(d*x + c)*a + b)/sqrt(d
*x + c)) + (sqrt(d*x + c)*a + b)*b^3*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c
))/sqrt(d*x + c) + b^3*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))/((a^2 - 2*
(sqrt(d*x + c)*a + b)*a/sqrt(d*x + c) + (sqrt(d*x + c)*a + b)^2/(d*x + c))*
b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \int \sin\left(a+\frac{b}{\sqrt{c+d\,x}}\right)\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/2)),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/2)), x)
```

3.200 $\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 276

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx = -\frac{2\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{f} + \frac{\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)\sin\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)}{f} + \frac{\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)\sin\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)}{f} - \frac{2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{f} - \frac{\cos\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\text{Si}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)}{f} + \frac{\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\text{Si}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)}{f}$$

```
[Out] -cos(a+b*f^(1/2)/(c*f-d*e)^(1/2))*Si(b*f^(1/2)/(c*f-d*e)^(1/2)-b/(d*x+c)^(1
/2))/f+cos(a-b*f^(1/2)/(c*f-d*e)^(1/2))*Si(b*f^(1/2)/(c*f-d*e)^(1/2)+b/(d*x
+c)^(1/2))/f-2*cos(a)*Si(b/(d*x+c)^(1/2))/f-2*Ci(b/(d*x+c)^(1/2))*sin(a)/f+
Ci(b*f^(1/2)/(c*f-d*e)^(1/2)+b/(d*x+c)^(1/2))*sin(a-b*f^(1/2)/(c*f-d*e)^(1/
2))/f+Ci(b*f^(1/2)/(c*f-d*e)^(1/2)-b/(d*x+c)^(1/2))*sin(a+b*f^(1/2)/(c*f-d*
e)^(1/2))/f
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3512, 3384, 3380, 3383, 3426}

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx = \frac{\sin\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{\sqrt{f}b}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{f} + \frac{\sin\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{f} - \frac{2\sin(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{f} - \frac{\cos\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{Si}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{f} + \frac{\cos\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{Si}\left(\frac{\sqrt{f}b}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{f} - \frac{2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{f}$$

```
[In] Int[Sin[a + b/Sqrt[c + d*x]]/(e + f*x),x]
```

```
[Out] (-2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/f + (CosIntegral[(b*Sqrt[f])/Sqrt[
-(d*e) + c*f] + b/Sqrt[c + d*x]]*Sin[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*f]])/f
 + (CosIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] - b/Sqrt[c + d*x]]*Sin[a + (
b*Sqrt[f])/Sqrt[-(d*e) + c*f]])/f - (2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])
/f - (Cos[a + (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[
-(d*e) + c*f] - b/Sqrt[c + d*x]])/f + (Cos[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*
f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] + b/Sqrt[c + d*x]])/f
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \left(\frac{d\sin(a+bx)}{fx}+\frac{d(-de+cf)x\sin(a+bx)}{f(f+(de-cf)x^2)}\right)\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d}\\
&= -\frac{2\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{f}+\frac{(2(de-cf))\text{Subst}\left(\int \frac{x\sin(a+bx)}{f+(de-cf)x^2}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{f}\\
&= \frac{(2(de-cf))\text{Subst}\left(\int \left(-\frac{\sqrt{-de+cf}\sin(a+bx)}{2(de-cf)\left(\sqrt{f}-\sqrt{-de+cf}x\right)}+\frac{\sqrt{-de+cf}\sin(a+bx)}{2(de-cf)\left(\sqrt{f}+\sqrt{-de+cf}x\right)}\right)\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{f}\\
&\quad -\frac{(2\cos(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{f}-\frac{(2\sin(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{f}\\
&= -\frac{2\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{f}-\frac{2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{f}\\
&\quad -\frac{\sqrt{-de+cf}\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{f}-\sqrt{-de+cf}x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{f}\\
&\quad +\frac{\sqrt{-de+cf}\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{f}+\sqrt{-de+cf}x}\,dx,x,\frac{1}{\sqrt{c+dx}}\right)}{f}
\end{aligned}$$

$$= -\frac{2\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{f} - \frac{2\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{f}$$
$$+ \frac{\left(\sqrt{-de+cf}\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\right)\mathrm{Subst}\left(\int\frac{\sin\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+bx\right)}{\sqrt{f}+\sqrt{-de+cf}x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{f}$$
$$+ \frac{\left(\sqrt{-de+cf}\cos\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\right)\mathrm{Subst}\left(\int\frac{\sin\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-bx\right)}{\sqrt{f}-\sqrt{-de+cf}x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{f}$$
$$+ \frac{\left(\sqrt{-de+cf}\sin\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\right)\mathrm{Subst}\left(\int\frac{\cos\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+bx\right)}{\sqrt{f}+\sqrt{-de+cf}x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{f}$$
$$- \frac{\left(\sqrt{-de+cf}\sin\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\right)\mathrm{Subst}\left(\int\frac{\cos\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-bx\right)}{\sqrt{f}-\sqrt{-de+cf}x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{f}$$
$$= -\frac{2\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{f} + \frac{\mathrm{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)\sin\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)}{f}$$
$$+ \frac{\mathrm{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)\sin\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)}{f}$$
$$- \frac{2\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{f} - \frac{\cos\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\mathrm{Si}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)}{f}$$
$$+ \frac{\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\mathrm{Si}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)}{f}$$

Mathematica [F]

$$\int\frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx = \int\frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$$

```
[In] Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x),x]
[Out] Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x), x]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.60

method	result
derivativedivides	$-2b^2\left(-\frac{-\mathrm{Si}\left(-\frac{b}{\sqrt{dx+c}}-a+\frac{acf-ade+\sqrt{b^2cf^2-b^2def}}{cf-de}\right)\cos\left(\frac{acf-ade+\sqrt{b^2cf^2-b^2def}}{cf-de}\right)+\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}+a-\frac{acf-ade+}{c}\right.}{2fb^2}\right.$
default	$-2b^2\left(-\frac{-\mathrm{Si}\left(-\frac{b}{\sqrt{dx+c}}-a+\frac{acf-ade+\sqrt{b^2cf^2-b^2def}}{cf-de}\right)\cos\left(\frac{acf-ade+\sqrt{b^2cf^2-b^2def}}{cf-de}\right)+\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}+a-\frac{acf-ade+}{c}\right.}{2fb^2}\right.$

```
[In] int(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^2*(-1/2/f/b^2*(-Si(-b/(d*x+c)^(1/2)-a+(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*
f)^(1/2))/(c*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e
))+Ci(b/(d*x+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)
)*sin((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))-1/2/f/b^2*(Si(b
/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos(
(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))-Ci(b/(d*x+c)^(1/2)+a+
(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^
2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))+1/f/b^2*(Ci(b/(d*x+c)^(1/2))*sin(a)+S
i(b/(d*x+c)^(1/2))*cos(a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.12

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$$
$$=\frac{-i\,\mathrm{Ei}\left(-\frac{\sqrt{\frac{b^2f}{de-cf}}(dx+c)-i\sqrt{dx+c}\,b}{dx+c}\right)e^{\left(ia+\sqrt{\frac{b^2f}{de-cf}}\right)}-i\,\mathrm{Ei}\left(\frac{\sqrt{\frac{b^2f}{de-cf}}(dx+c)+i\sqrt{dx+c}\,b}{dx+c}\right)e^{\left(ia-\sqrt{\frac{b^2f}{de-cf}}\right)}+i\,\mathrm{Ei}\left(-\frac{\sqrt{\frac{b^2}{de-}}}{}\right.}{}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*Ei(-(sqrt(b^2*f/(d*e - c*f))*(d*x + c) - I*sqrt(d*x + c)*b)/(d*x +
c))*e^(I*a + sqrt(b^2*f/(d*e - c*f))) - I*Ei((sqrt(b^2*f/(d*e - c*f))*(d*x
+ c) + I*sqrt(d*x + c)*b)/(d*x + c))*e^(I*a - sqrt(b^2*f/(d*e - c*f))) + I*
Ei(-(sqrt(b^2*f/(d*e - c*f))*(d*x + c) + I*sqrt(d*x + c)*b)/(d*x + c))*e^(-
I*a + sqrt(b^2*f/(d*e - c*f))) + I*Ei((sqrt(b^2*f/(d*e - c*f))*(d*x + c) -
```

```
I*sqrt(d*x + c)*b)/(d*x + c))*e^(-I*a - sqrt(b^2*f/(d*e - c*f))) - 4*cos_in
tegral(b/sqrt(d*x + c))*sin(a) - 4*cos(a)*sin_integral(b/sqrt(d*x + c)))/f
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(1/2))/(f*x+e),x)
[Out] Integral(sin(a + b/sqrt(c + d*x))/(e + f*x), x)
```

Maxima [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin(a + b/sqrt(d*x + c))/(f*x + e), x)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x, algorithm="giac")
[Out] integrate(sin(a + b/sqrt(d*x + c))/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt{c+d\,x}}\right)}{e+f\,x}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/2))/(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^(1/2))/(e + f*x), x)
```

3.201 $\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 350

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx =& -\frac{bd\cos\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\operatorname{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}} \\ &+\frac{bd\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\operatorname{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}} \\ &+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} \\ &-\frac{bd\sin\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\operatorname{Si}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}} \\ &-\frac{bd\sin\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\operatorname{Si}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}}\end{aligned}$$

```
[Out] (d*x+c)*sin(a+b/(d*x+c)^(1/2))/(-c*f+d*e)/(f*x+e)+1/2*b*d*Ci(b*f^(1/2)/(c*f
-d*e)^(1/2)+b/(d*x+c)^(1/2))*cos(a-b*f^(1/2)/(c*f-d*e)^(1/2))/(c*f-d*e)^(3/
2)/f^(1/2)-1/2*b*d*Ci(b*f^(1/2)/(c*f-d*e)^(1/2)-b/(d*x+c)^(1/2))*cos(a+b*f^
(1/2)/(c*f-d*e)^(1/2))/(c*f-d*e)^(3/2)/f^(1/2)-1/2*b*d*Si(b*f^(1/2)/(c*f-d*
e)^(1/2)+b/(d*x+c)^(1/2))*sin(a-b*f^(1/2)/(c*f-d*e)^(1/2))/(c*f-d*e)^(3/2)/
f^(1/2)-1/2*b*d*Si(b*f^(1/2)/(c*f-d*e)^(1/2)-b/(d*x+c)^(1/2))*sin(a+b*f^(1/
2)/(c*f-d*e)^(1/2))/(c*f-d*e)^(3/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3512, 3422, 3415, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx = -\frac{bd\cos\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(cf-de)^{3/2}} + \frac{bd\cos\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{\sqrt{f}b}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(cf-de)^{3/2}} - \frac{bd\sin\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{Si}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(cf-de)^{3/2}} - \frac{bd\sin\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{Si}\left(\frac{\sqrt{f}b}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(cf-de)^{3/2}} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)(de-cf)}$$

```
[In] Int[Sin[a + b/Sqrt[c + d*x]]/(e + f*x)^2,x]
```

```
[Out] -1/2*(b*d*Cos[a + (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*CosIntegral[(b*Sqrt[f])/S
qrt[-(d*e) + c*f] - b/Sqrt[c + d*x]])/(Sqrt[f]*(-(d*e) + c*f)^(3/2)) + (b*d
*Cos[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*CosIntegral[(b*Sqrt[f])/Sqrt[-(d*e
) + c*f] + b/Sqrt[c + d*x]])/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2)) + ((c + d*x)*
Sin[a + b/Sqrt[c + d*x]])/((d*e - c*f)*(e + f*x)) - (b*d*Sin[a + (b*Sqrt[f]
)/Sqrt[-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] - b/Sqrt[c
 + d*x]])/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2)) - (b*d*Sin[a - (b*Sqrt[f])/Sqrt[
-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] + b/Sqrt[c + d*x]
])/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{x\sin(a+bx)}{\left(\frac{f}{d}+\left(e-\frac{cf}{d}\right)x^2\right)^2}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{\frac{f}{d}+\left(e-\frac{cf}{d}\right)x^2}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{de-cf}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{b\text{Subst}\left(\int \left(\frac{d\cos(a+bx)}{2\sqrt{f}\left(\sqrt{f}-\sqrt{-de+cf}x\right)} + \frac{d\cos(a+bx)}{2\sqrt{f}\left(\sqrt{f}+\sqrt{-de+cf}x\right)}\right)dx, x, \frac{1}{\sqrt{c+dx}}\right)}{de-cf}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{f}-\sqrt{-de+cfx}}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{2\sqrt{f}(de-cf)} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{f}+\sqrt{-de+cfx}}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{2\sqrt{f}(de-cf)}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{\left(bd\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+bx\right)}{\sqrt{f}+\sqrt{-de+cf}x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{2\sqrt{f}(de-cf)} - \frac{\left(bd\cos\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-bx\right)}{\sqrt{f}-\sqrt{-de+cf}x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{2\sqrt{f}(de-cf)} + \frac{\left(bd\sin\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+bx\right)}{\sqrt{f}+\sqrt{-de+cf}x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{2\sqrt{f}(de-cf)} - \frac{\left(bd\sin\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-bx\right)}{\sqrt{f}-\sqrt{-de+cf}x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{2\sqrt{f}(de-cf)}$$

$$= -\frac{bd\cos\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}} + \frac{bd\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{bd\sin\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\text{Si}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}} - \frac{bd\sin\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\text{Si}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}}$$

Mathematica [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx$$

```
[In] Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x)^2, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2733 vs. $2(284) = 568$.

Time = 0.96 (sec) , antiderivative size = 2734, normalized size of antiderivative = 7.81

method	result	size
derivativedivides	Expression too large to display	2734
default	Expression too large to display	2734

```
[In] int(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*d*b^2*(sin(a+b/(d*x+c)^(1/2))*(-1/2*a/f/b^2*(a+b/(d*x+c)^(1/2))+1/2*(a^2
*c*f-a^2*d*e-b^2*f)/f/b^2/(c*f-d*e))/(a^2*c*f-a^2*d*e-2*a*c*f*(a+b/(d*x+c)^
(1/2))+2*a*d*e*(a+b/(d*x+c)^(1/2))+c*f*(a+b/(d*x+c)^(1/2))^2-d*e*(a+b/(d*x+
c)^(1/2))^2-b^2*f)+1/4*a/f/b^2/(a*c*f-a*d*e-c*f*(a*c*f-a*d*e+(b^2*c*f^2-b^2
*d*e*f)^(1/2))/(c*f-d*e)+d*e*(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f
-d*e))*(-Si(-b/(d*x+c)^(1/2)-a+(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c
*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))+Ci(b/(d*x
+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((a*c*f
-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))+1/4*a/f/b^2/(a*c*f-a*d*e+c*
f*(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)-d*e*(-a*c*f+a*d*e+(b
^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*(Si(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(
b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e
*f)^(1/2))/(c*f-d*e))-Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e
*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d
*e)))+1/4*(a^2*c*f-a^2*d*e-a*c*f*(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/
(c*f-d*e)+a*d*e*(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)-b^2*f)/
b^2/(c*f-d*e)/f/(a*c*f-a*d*e-c*f*(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/
(c*f-d*e)+d*e*(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*(Si(-b/(
d*x+c)^(1/2)-a+(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((a*
c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))+Ci(b/(d*x+c)^(1/2)+a-(a*c
*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^
2-b^2*d*e*f)^(1/2))/(c*f-d*e)))+1/4*(a^2*c*f-a^2*d*e+a*c*f*(-a*c*f+a*d*e+(b
^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)-a*d*e*(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e
*f)^(1/2))/(c*f-d*e)-b^2*f)/b^2/(c*f-d*e)/f/(a*c*f-a*d*e+c*f*(-a*c*f+a*d*e+
(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)-d*e*(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e
*f)^(1/2))/(c*f-d*e))*(Si(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*
e*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-
d*e))+Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-
d*e))*cos((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))-a*(sin(a+b
/(d*x+c)^(1/2))*(-1/2/f/b^2*(a+b/(d*x+c)^(1/2))+1/2*a/f/b^2)/(a^2*c*f-a^2*d
*e-2*a*c*f*(a+b/(d*x+c)^(1/2))+2*a*d*e*(a+b/(d*x+c)^(1/2))+c*f*(a+b/(d*x+c)
^(1/2))^2-d*e*(a+b/(d*x+c)^(1/2))^2-b^2*f)+1/4/f/b^2/(a*c*f-a*d*e-c*f*(a*c*
f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)+d*e*(a*c*f-a*d*e+(b^2*c*f^2-
b^2*d*e*f)^(1/2))/(c*f-d*e))*(-Si(-b/(d*x+c)^(1/2)-a+(a*c*f-a*d*e+(b^2*c*f^
```

```
2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2)
)/(c*f-d*e))+Ci(b/(d*x+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))
/(c*f-d*e))*sin((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))+1/4/f
/b^2/(a*c*f-a*d*e+c*f*(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)-
d*e*(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*(Si(b/(d*x+c)^(1/
2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((-a*c*f+a*d*
e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))-Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*
e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*
d*e*f)^(1/2))/(c*f-d*e)))+1/4/b^2/(c*f-d*e)/f*(Si(-b/(d*x+c)^(1/2)-a+(a*c*f
-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((a*c*f-a*d*e+(b^2*c*f^2-
b^2*d*e*f)^(1/2))/(c*f-d*e))+Ci(b/(d*x+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b
^2*d*e*f)^(1/2))/(c*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(
c*f-d*e)))+1/4/b^2/(c*f-d*e)/f*(Si(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f
^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/
2))/(c*f-d*e))+Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/
2))/(c*f-d*e))*cos((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.28

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx = -\frac{(i\,dfx+i\,de)\sqrt{\frac{b^2f}{de-cf}}\operatorname{Ei}\left(-\frac{\sqrt{\frac{b^2f}{de-cf}}(dx+c)-i\sqrt{dx+c}b}{dx+c}\right)e^{\left(i\,a+\sqrt{\frac{b^2f}{de-cf}}\right)}+(-i\,dfx-i\,de)\sqrt{\frac{b^2f}{de-cf}}\operatorname{Ei}\left(\frac{\sqrt{\frac{b^2f}{de-cf}}(dx+c}{dx+}\right.}{}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((I*d*f*x + I*d*e)*sqrt(b^2*f/(d*e - c*f))*Ei(-(sqrt(b^2*f/(d*e - c*f)
)*(d*x + c) - I*sqrt(d*x + c)*b)/(d*x + c))*e^(I*a + sqrt(b^2*f/(d*e - c*f)
)) + (-I*d*f*x - I*d*e)*sqrt(b^2*f/(d*e - c*f))*Ei((sqrt(b^2*f/(d*e - c*f))
*(d*x + c) + I*sqrt(d*x + c)*b)/(d*x + c))*e^(I*a - sqrt(b^2*f/(d*e - c*f))
) + (-I*d*f*x - I*d*e)*sqrt(b^2*f/(d*e - c*f))*Ei(-(sqrt(b^2*f/(d*e - c*f))
*(d*x + c) + I*sqrt(d*x + c)*b)/(d*x + c))*e^(-I*a + sqrt(b^2*f/(d*e - c*f)
)) + (I*d*f*x + I*d*e)*sqrt(b^2*f/(d*e - c*f))*Ei((sqrt(b^2*f/(d*e - c*f))*
(d*x + c) - I*sqrt(d*x + c)*b)/(d*x + c))*e^(-I*a - sqrt(b^2*f/(d*e - c*f))
) - 4*(d*f*x + c*f)*sin((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c)))/(d*e^2*
f - c*e*f^2 + (d*e*f^2 - c*f^3)*x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{\left(e+fx\right)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(1/2))/(f*x+e)**2,x)
[Out] Integral(sin(a + b/sqrt(c + d*x))/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin(a + b/sqrt(d*x + c))/(f*x + e)^2, x)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin(a + b/sqrt(d*x + c))/(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt{c+d\,x}}\right)}{\left(e+f\,x\right)^2}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/2))/(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^(1/2))/(e + f*x)^2, x)
```

3.202 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right) dx$

Optimal result

Integrand size = 22, antiderivative size = 390

$$\begin{aligned}\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right) dx &= \frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} \\ &- \frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} \\ &+ \frac{2ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} \\ &- \frac{ie^{ia}(de-cf)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} \\ &+ \frac{ie^{-ia}(de-cf)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} \\ &+ \frac{b^2f^2\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{3/2}}\right)\sin(a)}{3d^3} \\ &+ \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{b^2f^2\cos(a)\text{Si}\left(\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}\end{aligned}$$

```
[Out] 1/3*b*f^2*(d*x+c)^(3/2)*cos(a+b/(d*x+c)^(3/2))/d^3-2/3*I*exp(I*a)*f*(-c*f+d
*e)*(-I*b/(d*x+c)^(3/2))^(4/3)*(d*x+c)^2*GAMMA(-4/3,-I*b/(d*x+c)^(3/2))/d^3
+2/3*I*f*(-c*f+d*e)*(I*b/(d*x+c)^(3/2))^(4/3)*(d*x+c)^2*GAMMA(-4/3,I*b/(d*x
+c)^(3/2))/d^3/exp(I*a)-1/3*I*exp(I*a)*(-c*f+d*e)^2*(-I*b/(d*x+c)^(3/2))^(2
/3)*(d*x+c)*GAMMA(-2/3,-I*b/(d*x+c)^(3/2))/d^3+1/3*I*(-c*f+d*e)^2*(I*b/(d*x
+c)^(3/2))^(2/3)*(d*x+c)*GAMMA(-2/3,I*b/(d*x+c)^(3/2))/d^3/exp(I*a)+1/3*b^2
```

```
*f^2*cos(a)*Si(b/(d*x+c)^(3/2))/d^3+1/3*b^2*f^2*Ci(b/(d*x+c)^(3/2))*sin(a)/
d^3+1/3*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^(3/2))/d^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3514, 3504, 2250, 3460, 3378, 3384, 3380, 3383}

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right) dx = \frac{b^2 f^2 \sin(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{b^2 f^2 \cos(a)\text{Si}\left(\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} - \frac{2ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(de-cf)\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{2ie^{-ia}f(c+dx)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(de-cf)\Gamma\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} - \frac{ie^{ia}(c+dx)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{ie^{-ia}(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)^2\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}$$

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] (b*f^2*(c + d*x)^(3/2)*Cos[a + b/(c + d*x)^(3/2)])/(3*d^3) - (((2*I)/3)*E^(
I*a)*f*(d*e - c*f)*(((-I)*b)/(c + d*x)^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3,
 ((-I)*b)/(c + d*x)^(3/2)])/d^3 + (((2*I)/3)*f*(d*e - c*f)*((I*b)/(c + d*x)
^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3, (I*b)/(c + d*x)^(3/2)])/(d^3*E^(I*a))
 - ((I/3)*E^(I*a)*(d*e - c*f)^2*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*
Gamma[-2/3, ((-I)*b)/(c + d*x)^(3/2)])/d^3 + ((I/3)*(d*e - c*f)^2*((I*b)/(c
 + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/(d^3*E^(
I*a)) + (b^2*f^2*CosIntegral[b/(c + d*x)^(3/2)]*Sin[a])/(3*d^3) + (f^2*(c +
 d*x)^3*Sin[a + b/(c + d*x)^(3/2)])/(3*d^3) + (b^2*f^2*Cos[a]*SinIntegral[b
/(c + d*x)^(3/2)])/(3*d^3)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
```

```
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
```

```
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{2\text{Subst}\left(\int \left((de-cf)^2 x \sin\left(a+\frac{b}{x^3}\right) - 2f(-de+cf)x^3 \sin\left(a+\frac{b}{x^3}\right) + f^2 x^5 \sin\left(a+\frac{b}{x^3}\right)\right) dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{(2f^2)\,\text{Subst}\left(\int x^5 \sin\left(a+\frac{b}{x^3}\right) dx, x, \sqrt{c+dx}\right)}{d^3} + \frac{(4f(de-cf))\text{Subst}\left(\int x^3 \sin\left(a+\frac{b}{x^3}\right) dx, x, \sqrt{c+dx}\right)}{d^3} + \frac{(2(de-cf)^2)\,\text{Subst}\left(\int x \sin\left(a+\frac{b}{x^3}\right) dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= -\frac{(2f^2)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3} dx, x, \frac{1}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{(2if(de-cf))\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}} x^3 dx, x, \sqrt{c+dx}\right)}{d^3} - \frac{(2if(de-cf))\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}} x^3 dx, x, \sqrt{c+dx}\right)}{d^3} + \frac{(i(de-cf)^2)\,\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}} x\, dx, x, \sqrt{c+dx}\right)}{d^3} - \frac{(i(de-cf)^2)\,\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}} x\, dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= -\frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{2ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} - \frac{ie^{ia}(de-cf)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{ie^{-ia}(de-cf)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} - \frac{(bf^2)\,\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2} dx, x, \frac{1}{(c+dx)^{3/2}}\right)}{3d^3}$$

$$=\frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}-\frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{2ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}-\frac{ie^{ia}(de-cf)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{ie^{-ia}(de-cf)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{(b^2f^2)\operatorname{Subst}\left(\int\frac{\sin(a+bx)}{x}\,dx,x,\frac{1}{(c+dx)^{3/2}}\right)}{3d^3}$$

$$=\frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}-\frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{2ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}-\frac{ie^{ia}(de-cf)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{ie^{-ia}(de-cf)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{(b^2f^2\cos(a))\operatorname{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,\frac{1}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{(b^2f^2\sin(a))\operatorname{Subst}\left(\int\frac{\cos(bx)}{x}\,dx,x,\frac{1}{(c+dx)^{3/2}}\right)}{3d^3}$$

$$= \frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} - \frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{2ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} - \frac{ie^{ia}(de-cf)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{ie^{-ia}(de-cf)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{b^2f^2\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{3/2}}\right)\sin(a)}{3d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{b^2f^2\cos(a)\text{Si}\left(\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}$$

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.19

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx = \frac{i\left((\cos(a)-i\sin(a))\left(b^2f^2\,\text{ExpIntegralEi}\left(-\frac{ib}{(c+dx)^{3/2}}\right)+4f(de-cf)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c\right.\right.}{}$$

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] ((I/6)*((Cos[a] - I*Sin[a])*(b^2*f^2*ExpIntegralEi[((-I)*b)/(c + d*x)^(3/2)
] + 4*f*(d*e - c*f)*((I*b)/(c + d*x)^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3, (
I*b)/(c + d*x)^(3/2)] + 2*(d*e - c*f)^2*((I*b)/(c + d*x)^(3/2))^(2/3)*(c +
d*x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)] - I*b*f^2*(c + d*x)^(3/2)*(Cos[b/(c
 + d*x)^(3/2)] - I*Sin[b/(c + d*x)^(3/2)]) + f^2*(c + d*x)^3*(Cos[b/(c + d*
x)^(3/2)] - I*Sin[b/(c + d*x)^(3/2)])) - (Cos[a] + I*Sin[a])*(b^2*f^2*ExpIn
tegralEi[(I*b)/(c + d*x)^(3/2)] + 4*f*(d*e - c*f)*(((-I)*b)/(c + d*x)^(3/2)
)^(4/3)*(c + d*x)^2*Gamma[-4/3, ((-I)*b)/(c + d*x)^(3/2)] + 2*(d*e - c*f)^2
*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((-I)*b)/(c + d*x)^
(3/2)] + I*b*f^2*(c + d*x)^(3/2)*(Cos[b/(c + d*x)^(3/2)] + I*Sin[b/(c + d*x
)^(3/2)]) + f^2*(c + d*x)^3*(Cos[b/(c + d*x)^(3/2)] + I*Sin[b/(c + d*x)^(3/
2)]))))/d^3
```

Maple [F]

$$\int (fx+e)^2 \sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right) dx$$

[In] int((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x)

[Out] int((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x)

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.50

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right) dx = \frac{2\,b^2 f^2 \operatorname{Ci}\left(\frac{\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)\sin(a) + 2\,b^2 f^2 \cos(a) \operatorname{Si}\left(\frac{\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right) - 3\,((i\,d^2e^2 - 2i\,cdef +$$

[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x, algorithm="fricas")

[Out] 1/6*(2*b^2*f^2*cos_integral(sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2))*sin(a) + 2*b^2*f^2*cos(a)*sin_integral(sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*((I*d^2*e^2 - 2*I*c*d*e*f + I*c^2*f^2)*cos(a) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*sin(a))*(I*b)^(2/3)*gamma(1/3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*((-I*d^2*e^2 + 2*I*c*d*e*f - I*c^2*f^2)*cos(a) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*sin(a))*(-I*b)^(2/3)*gamma(1/3, -I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(b*d*f^2*x + 9*b*d*e*f - 8*b*c*f^2)*sqrt(d*x + c)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2*c*d*x + c^2)) - 9*((b*d*e*f - b*c*f^2)*cos(a) + (-I*b*d*e*f + I*b*c*f^2)*sin(a))*(I*b)^(1/3)*gamma(2/3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) - 9*((b*d*e*f - b*c*f^2)*cos(a) + (I*b*d*e*f - I*b*c*f^2)*sin(a))*(-I*b)^(1/3)*gamma(2/3, -I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 3*c*d^2*e^2 - 3*c^2*d*e*f + c^3*f^2)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2*c*d*x + c^2)))/d^3

Sympy [F]

$$\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) dx = \int (e+fx)^2 \sin \left(a+\frac{b}{c\sqrt{c+dx}+dx\sqrt{c+dx}}\right) dx$$

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**(3/2)),x)
[Out] Integral((e + f*x)**2*sin(a + b/(c*sqrt(c + d*x) + d*x*sqrt(c + d*x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 993 vs. 2(296) = 592.

Time = 0.59 (sec) , antiderivative size = 993, normalized size of antiderivative = 2.55

$$\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x, algorithm="maxima")
[Out] 1/12*(3*(4*(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a
 + b)/(d*x + c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) +
(sqrt(3) + I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*
gamma(1/3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)
^(3/2)))*sin(a))*b)*e^2/(sqrt(d*x + c)*(b/(d*x + c)^(3/2))^(1/3)) - 6*(4*(d
*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x +
c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (sqrt(3) + I)
*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*gamma(1/3, I*
b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(
a))*b)*c*e*f/(sqrt(d*x + c)*d*(b/(d*x + c)^(3/2))^(1/3)) + 3*(4*(d*x + c)^(
3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2))
 + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (sqrt(3) + I)*gamma(1/
3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*gamma(1/3, I*b/(d*x +
c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(a))*b)*c^
2*f^2/(sqrt(d*x + c)*d^2*(b/(d*x + c)^(3/2))^(1/3)) + 2*(2*(d*x + c)^3*sin(
((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 2*(d*x + c)^(3/2)*b*cos(((d*x +
c)^(3/2)*a + b)/(d*x + c)^(3/2)) + ((-I*Ei(I*b/(d*x + c)^(3/2)) + I*Ei(-I*b
/(d*x + c)^(3/2)))*cos(a) + (Ei(I*b/(d*x + c)^(3/2)) + Ei(-I*b/(d*x + c)^(3
/2)))*sin(a))*b^2)*f^2/d^2 + 3*(4*(d*x + c)^3*(b/(d*x + c)^(3/2))^(2/3)*sin
(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 12*(d*x + c)^(3/2)*b*(b/(d*x +
c)^(3/2))^(2/3)*cos(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) - 3*(((sqrt(3)
 + I)*gamma(2/3, I*b/(d*x + c)^(3/2)) + (sqrt(3) - I)*gamma(2/3, -I*b/(d*x
+ c)^(3/2)))*cos(a) + ((-I*sqrt(3) + 1)*gamma(2/3, I*b/(d*x + c)^(3/2)) + (
I*sqrt(3) + 1)*gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(a))*b^2)*e*f/((d*x + c
```

```
)*d*(b/(d*x + c)^(3/2))^(2/3)) - 3*(4*(d*x + c)^3*(b/(d*x + c)^(3/2))^(2/3)
*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 12*(d*x + c)^(3/2)*b*(b/(d*
x + c)^(3/2))^(2/3)*cos(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) - 3*(((sqr
t(3) + I)*gamma(2/3, I*b/(d*x + c)^(3/2)) + (sqrt(3) - I)*gamma(2/3, -I*b/(
d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) + 1)*gamma(2/3, I*b/(d*x + c)^(3/2))
 + (I*sqrt(3) + 1)*gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(a))*b^2)*c*f^2/((d
*x + c)*d^2*(b/(d*x + c)^(3/2))^(2/3)))/d
```

Giac [F]

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\int (fx+e)^2\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)\,dx$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*sin(a + b/(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{3/2}}\right)\,(e+f\,x)^2\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(3/2))*(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^(3/2))*(e + f*x)^2, x)
```

3.203 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 251

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right) dx = -\frac{ie^{ia}f\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2} + \frac{ie^{-ia}f\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2} - \frac{ie^{ia}(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2} + \frac{ie^{-ia}(de-cf)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}$$

```
[Out] -1/3*I*exp(I*a)*f*(-I*b/(d*x+c)^(3/2))^(4/3)*(d*x+c)^2*GAMMA(-4/3,-I*b/(d*x
+c)^(3/2))/d^2+1/3*I*f*(I*b/(d*x+c)^(3/2))^(4/3)*(d*x+c)^2*GAMMA(-4/3,I*b/(
d*x+c)^(3/2))/d^2/exp(I*a)-1/3*I*exp(I*a)*(-c*f+d*e)*(-I*b/(d*x+c)^(3/2))^(
2/3)*(d*x+c)*GAMMA(-2/3,-I*b/(d*x+c)^(3/2))/d^2+1/3*I*(-c*f+d*e)*(I*b/(d*x+
c)^(3/2))^(2/3)*(d*x+c)*GAMMA(-2/3,I*b/(d*x+c)^(3/2))/d^2/exp(I*a)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3514, 3504, 2250}

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx = -\frac{ie^{ia}(c+dx)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2} + \frac{ie^{-ia}(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2} - \frac{ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2} + \frac{ie^{-ia}f(c+dx)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}\Gamma\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}$$

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] ((-1/3*I)*E^(I*a)*f*(((-I)*b)/(c + d*x)^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3
, ((-I)*b)/(c + d*x)^(3/2)])/d^2 + ((I/3)*f*((I*b)/(c + d*x)^(3/2))^(4/3)*(
c + d*x)^2*Gamma[-4/3, (I*b)/(c + d*x)^(3/2)])/(d^2*E^(I*a)) - ((I/3)*E^(I*
a)*(d*e - c*f)*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((-I)
*b)/(c + d*x)^(3/2)])/d^2 + ((I/3)*(d*e - c*f)*((I*b)/(c + d*x)^(3/2))^(2/3
)*(c + d*x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/(d^2*E^(I*a))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
```

```
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \left((de-cf)x\sin\left(a+\frac{b}{x^3}\right)+fx^3\sin\left(a+\frac{b}{x^3}\right)\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= \frac{(2f)\text{Subst}\left(\int x^3\sin\left(a+\frac{b}{x^3}\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&\quad+\frac{(2(de-cf))\text{Subst}\left(\int x\sin\left(a+\frac{b}{x^3}\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= \frac{(if)\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}x^3\,dx,x,\sqrt{c+dx}\right)}{d^2}-\frac{(if)\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}}x^3\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&\quad+\frac{(i(de-cf))\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}x\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&\quad-\frac{(i(de-cf))\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}}x\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= -\frac{ie^{ia}f\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}\\
&\quad+\frac{ie^{-ia}f\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}\\
&\quad-\frac{ie^{ia}(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}\\
&\quad+\frac{ie^{-ia}(de-cf)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 835 vs. 2(251) = 502.

Time = 1.92 (sec) , antiderivative size = 835, normalized size of antiderivative = 3.33

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx = \frac{3be\cos(a)\left(\frac{2\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3\sqrt[3]{-\frac{ib}{(c+dx)^{3/2}}}\sqrt{c+dx}}+\frac{2\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3\sqrt[3]{\frac{ib}{(c+dx)^{3/2}}}\sqrt{c+dx}}\right)}{4d}$$

$$-\frac{3bcf\cos(a)\left(\frac{2\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3\sqrt[3]{-\frac{ib}{(c+dx)^{3/2}}}\sqrt{c+dx}}+\frac{2\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3\sqrt[3]{\frac{ib}{(c+dx)^{3/2}}}\sqrt{c+dx}}\right)}{4d^2}$$

$$+\frac{9ib^2f\cos(a)\left(\frac{2\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)}-\frac{2\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)}\right)}{8d^2}$$

$$+\frac{e(c+dx)\cos\left(\frac{b}{(c+dx)^{3/2}}\right)\sin(a)}{d}$$

$$+\frac{3ibe\left(\frac{2\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3\sqrt[3]{-\frac{ib}{(c+dx)^{3/2}}}\sqrt{c+dx}}-\frac{2\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3\sqrt[3]{\frac{ib}{(c+dx)^{3/2}}}\sqrt{c+dx}}\right)\sin(a)}{4d}$$

$$-\frac{3ibcf\left(\frac{2\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3\sqrt[3]{-\frac{ib}{(c+dx)^{3/2}}}\sqrt{c+dx}}-\frac{2\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3\sqrt[3]{\frac{ib}{(c+dx)^{3/2}}}\sqrt{c+dx}}\right)\sin(a)}{4d^2}$$

$$-\frac{9b^2f\left(\frac{2\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)}+\frac{2\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)}\right)\sin(a)}{8d^2}$$

$$+\frac{f\sqrt{c+dx}\cos\left(\frac{b}{(c+dx)^{3/2}}\right)\left(3b\cos(a)-2c\sqrt{c+dx}\sin(a)+(c+dx)^{3/2}\sin(a)\right)}{2d^2}$$

$$+\frac{e(c+dx)\cos(a)\sin\left(\frac{b}{(c+dx)^{3/2}}\right)}{d}$$

$$+\frac{f\sqrt{c+dx}\left(-2c\sqrt{c+dx}\cos(a)+(c+dx)^{3/2}\cos(a)-3b\sin(a)\right)\sin\left(\frac{b}{(c+dx)^{3/2}}\right)}{2d^2}$$

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] (3*b*e*Cos[a]*((2*Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d
*x)^(3/2))^(1/3)*Sqrt[c + d*x]) + (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*
((I*b)/(c + d*x)^(3/2))^(1/3)*Sqrt[c + d*x])))/(4*d) - (3*b*c*f*Cos[a]*((2*
Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(1/3)*S
qrt[c + d*x]) + (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(
3/2))^(1/3)*Sqrt[c + d*x])))/(4*d^2) + (((9*I)/8)*b^2*f*Cos[a]*((2*Gamma[2/
3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)
) - (2*Gamma[2/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(3/2))^(2/3)*
(c + d*x))))/d^2 + (e*(c + d*x)*Cos[b/(c + d*x)^(3/2)]*Sin[a])/d + (((3*I)/
4)*b*e*((2*Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/
2))^(1/3)*Sqrt[c + d*x]) - (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/
(c + d*x)^(3/2))^(1/3)*Sqrt[c + d*x]))*Sin[a])/d - (((3*I)/4)*b*c*f*((2*Gam
ma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(1/3)*Sqrt
[c + d*x]) - (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(3/2
))^(1/3)*Sqrt[c + d*x]))*Sin[a])/d^2 - (9*b^2*f*((2*Gamma[2/3, ((-I)*b)/(c
+ d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)) + (2*Gamma[2/
3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)))*Sin
[a])/(8*d^2) + (f*Sqrt[c + d*x]*Cos[b/(c + d*x)^(3/2)]*(3*b*Cos[a] - 2*c*Sq
rt[c + d*x]*Sin[a] + (c + d*x)^(3/2)*Sin[a]))/(2*d^2) + (e*(c + d*x)*Cos[a]
*Sin[b/(c + d*x)^(3/2)])/d + (f*Sqrt[c + d*x]*(-2*c*Sqrt[c + d*x]*Cos[a] +
(c + d*x)^(3/2)*Cos[a] - 3*b*Sin[a])*Sin[b/(c + d*x)^(3/2)])/(2*d^2)
```

Maple [F]

$$\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)dx$$

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x)
```

```
[Out] int((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(176) = 352.

Time = 0.12 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.45

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx = \frac{6\sqrt{dx+c}\,bf\cos\left(\frac{ad^2x^2+2\,acdx+ac^2+\sqrt{dx+c}\,b}{d^2x^2+2\,cdx+c^2}\right)-2\left((i\,de-i\,cf)\cos(a)+(de-cf)\sin(a)\right)(}{}$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(6*sqrt(d*x + c)*b*f*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)
*b)/(d^2*x^2 + 2*c*d*x + c^2)) - 2*((I*d*e - I*c*f)*cos(a) + (d*e - c*f)*si
n(a))*(I*b)^(2/3)*gamma(1/3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) -
 2*((-I*d*e + I*c*f)*cos(a) + (d*e - c*f)*sin(a))*(-I*b)^(2/3)*gamma(1/3, -
I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(b*f*cos(a) - I*b*f*sin(a)
)*(I*b)^(1/3)*gamma(2/3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(
b*f*cos(a) + I*b*f*sin(a))*(-I*b)^(1/3)*gamma(2/3, -I*sqrt(d*x + c)*b/(d^2*
x^2 + 2*c*d*x + c^2)) + 2*(d^2*f*x^2 + 2*d^2*e*x + 2*c*d*e - c^2*f)*sin((a*
d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2*c*d*x + c^2)))/
d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\int (e+fx)\sin\left(a+\frac{b}{c\sqrt{c+dx}+dx\sqrt{c+dx}}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**(3/2)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/(c*sqrt(c + d*x) + d*x*sqrt(c + d*x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(176)=352$.

Time = 0.35 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.00

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\frac{\frac{2\left(4\,(dx+c)^{\frac{3}{2}}\left(\frac{b}{(dx+c)^{\frac{3}{2}}}\right)^{\frac{1}{3}}\sin\left(\frac{(dx+c)^{\frac{3}{2}}a+b}{(dx+c)^{\frac{3}{2}}}\right)+\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},\frac{i\,b}{(dx+c)^{\frac{3}{2}}}\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-\frac{i\,b}{(dx+c)^{\frac{3}{2}}}\right)\right)\cos(a)+\left(\left(\cdots\right.\right.\right.\right.}{\sqrt{dx+c}\left(\frac{b}{(dx+c)^{\frac{3}{2}}}\right)^{\frac{1}{3}}}}{\cdots}$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(4*(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a
+ b)/(d*x + c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (
sqrt(3) + I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*g
amma(1/3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)^
(3/2)))*sin(a))*b)*e/(sqrt(d*x + c)*(b/(d*x + c)^(3/2))^(1/3)) - 2*(4*(d*x
+ c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^
(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (sqrt(3) + I)*ga
```

```
mma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*gamma(1/3, I*b/(
d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(a))
*b)*c*f/(sqrt(d*x + c)*d*(b/(d*x + c)^(3/2))^(1/3)) + (4*(d*x + c)^3*(b/(d*
x + c)^(3/2))^(2/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 12*(d*x
+ c)^(3/2)*b*(b/(d*x + c)^(3/2))^(2/3)*cos(((d*x + c)^(3/2)*a + b)/(d*x + c
)^(3/2)) - 3*(((sqrt(3) + I)*gamma(2/3, I*b/(d*x + c)^(3/2)) + (sqrt(3) - I
)*gamma(2/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) + 1)*gamma(2/3, I
*b/(d*x + c)^(3/2)) + (I*sqrt(3) + 1)*gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin
(a))*b^2)*f/((d*x + c)*d*(b/(d*x + c)^(3/2))^(2/3)))/d
```

Giac [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x, algorithm="giac")
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{3/2}}\right)\,(e+f\,x)\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(3/2))*(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^(3/2))*(e + f*x), x)
```

3.204 $\int \sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) \, dx = -\frac{ie^{ia}\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d} + \frac{ie^{-ia}\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d}$$

```
[Out] -1/3*I*exp(I*a)*(-I*b/(d*x+c)^(3/2))^(2/3)*(d*x+c)*GAMMA(-2/3,-I*b/(d*x+c)^
(3/2))/d+1/3*I*(I*b/(d*x+c)^(3/2))^(2/3)*(d*x+c)*GAMMA(-2/3,I*b/(d*x+c)^(3/
2))/d/exp(I*a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3444, 3504, 2250}

$$\int \sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) \, dx = \frac{ie^{-ia}(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d} - \frac{ie^{ia}(c+dx)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d}$$

```
[In] Int[Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] ((-1/3*I)*E^(I*a)*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((
-I)*b)/(c + d*x)^(3/2)])/d + ((I/3)*((I*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)
*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/(d*E^(I*a))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3444

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Dist[k/f, Subst[Int[x^(k - 1)*(a + b
*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\text{Subst}\left(\int x\sin\left(a+\frac{b}{x^3}\right)\,dx,x,\sqrt{c+dx}\right)}{d}\\ &= \frac{i\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}x\,dx,x,\sqrt{c+dx}\right)}{d}-\frac{i\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}}x\,dx,x,\sqrt{c+dx}\right)}{d}\\ &= -\frac{ie^{ia}\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d}\\ &\quad+\frac{ie^{-ia}\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.44

$$\int\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\frac{b\sqrt[3]{-\frac{ib}{(c+dx)^{3/2}}}\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^{3/2}}\right)(\cos(a)-i\sin(a))+b\sqrt[3]{\frac{ib}{(c+dx)^{3/2}}}\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^{3/2}}\right)(c}{2d\sqrt[3]{\frac{b^2}{(c+dx)^3}}\sqrt{c+dx}}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] (b*(((-I)*b)/(c + d*x)^(3/2))^(1/3)*Gamma[1/3, (I*b)/(c + d*x)^(3/2)]*(Cos[
a] - I*Sin[a]) + b*((I*b)/(c + d*x)^(3/2))^(1/3)*Gamma[1/3, ((-I)*b)/(c + d
*x)^(3/2)]*(Cos[a] + I*Sin[a]) + 2*(b^2/(c + d*x)^3)^(1/3)*(c + d*x)^(3/2)*
Sin[a + b/(c + d*x)^(3/2)])/(2*d*(b^2/(c + d*x)^3)^(1/3)*Sqrt[c + d*x])
```

Maple [F]

$$\int \sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)dx$$

```
[In] int(sin(a+b/(d*x+c)^(3/2)),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(3/2)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.32

$$\int \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx = \frac{(i\,b)^{\frac{2}{3}}\,(-i\,\cos{(a)}-\sin{(a)})\Gamma\left(\frac{1}{3},\frac{i\,\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)+(-i\,b)^{\frac{2}{3}}\,(i\,\cos{(a)}-\sin{(a)})\Gamma\left(\frac{1}{3},-\frac{i\,\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)+\ldots}{2\,d}$$

```
[In] integrate(sin(a+b/(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/2*((I*b)^(2/3)*(-I*cos(a) - sin(a))*gamma(1/3, I*sqrt(d*x + c)*b/(d^2*x^2
 + 2*c*d*x + c^2)) + (-I*b)^(2/3)*(I*cos(a) - sin(a))*gamma(1/3, -I*sqrt(d*
x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(d*x + c)*sin((a*d^2*x^2 + 2*a*c*d*
x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx = \int \sin\left(a+\frac{b}{(c+dx)^{\frac{3}{2}}}\right)dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(3/2)),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.31

$$\int \sin \left(a + \frac{b}{(c+dx)^{3/2}}\right) dx = \frac{4\,(dx+c)^{\frac{3}{2}}\left(\frac{b}{(dx+c)^{\frac{3}{2}}}\right)^{\frac{1}{3}}\sin\left(\frac{(dx+c)^{\frac{3}{2}}a+b}{(dx+c)^{\frac{3}{2}}}\right)+\left(\left((\sqrt{3}-i)\Gamma\left(\frac{1}{3},\frac{ib}{(dx+c)^{\frac{3}{2}}}\right)+(\sqrt{3}+i)\Gamma\left(\frac{1}{3}\right.\right.\right.}{4\sqrt{dx+}}$$

```
[In] integrate(sin(a+b/(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b
)/(d*x + c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (sqr
t(3) + I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*gamm
a(1/3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)^(3/
2)))*sin(a))*b)/(sqrt(d*x + c)*d*(b/(d*x + c)^(3/2))^(1/3))
```

Giac [F]

$$\int \sin \left(a + \frac{b}{(c+dx)^{3/2}}\right) dx = \int \sin \left(a + \frac{b}{(dx+c)^{\frac{3}{2}}}\right) dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin \left(a + \frac{b}{(c+dx)^{3/2}}\right) dx = \int \sin \left(a + \frac{b}{(c+d\,x)^{3/2}}\right) dx$$

```
[In] int(sin(a + b/(c + d*x)^(3/2)),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(3/2)), x)
```

3.205 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx} dx = \text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}, x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx} dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx} dx$$

```
[In] Int[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x),x]
[Out] Defer[Int][Sin[a + b/(c + d*x)^(3/2)]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx} dx$$

Mathematica [N/A]

Not integrable

Time = 11.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x),x]
[Out] Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{fx+e}dx$$

```
[In] int(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x)
[Out] int(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x, algorithm="fricas")
[Out] integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2
*c*d*x + c^2))/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 28.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{c\sqrt{c+dx}+dx\sqrt{c+dx}}\right)}{e+fx}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(3/2))/(f*x+e),x)
[Out] Integral(sin(a + b/(c*sqrt(c + d*x) + d*x*sqrt(c + d*x)))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 5.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{3/2}}\right)}{e+f\,x} \,dx$$

```
[In] int(sin(a + b/(c + d*x)^(3/2))/(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^(3/2))/(e + f*x), x)
```

3.206 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2} dx = \text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}, x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2} dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2} dx$$

```
[In] Int[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 15.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{(fx+e)^2}dx$$

```
[In] int(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x)
[Out] int(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.18

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2
*c*d*x + c^2))/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx=\text{Timed out}$$

```
[In] integrate(sin(a+b/(d*x+c)**(3/2))/(f*x+e)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 5.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{3/2}}\right)}{{\left(e+f\,x\right)}^2}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(3/2))/(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^(3/2))/(e + f*x)^2, x)
```

3.207 $\int (e+fx)^2 \sin \left(a+b\sqrt[3]{c+dx}\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 633

$$\int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{120960f^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3} + \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{60480f^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} - \frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{120f(de-cf)(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{5040f^2(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{6f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{168f^2(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{3f^2(c+dx)^{8/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{720f(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{120960f^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} + \frac{6(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{360f(de-cf)(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{20160f^2(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{30f(de-cf)(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{1008f^2(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{24f^2(c+dx)^{7/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$

```
[Out] -120960*f^2*cos(a+b*(d*x+c)^(1/3))/b^9/d^3+6*(-c*f+d*e)^2*cos(a+b*(d*x+c)^(
1/3))/b^3/d^3-720*f*(-c*f+d*e)*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^5/d^3
+60480*f^2*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b^7/d^3-3*(-c*f+d*e)^2*(d*x
+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d^3+120*f*(-c*f+d*e)*(d*x+c)*cos(a+b*(d*
x+c)^(1/3))/b^3/d^3-5040*f^2*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(1/3))/b^5/d^3-6
*f*(-c*f+d*e)*(d*x+c)^(5/3)*cos(a+b*(d*x+c)^(1/3))/b/d^3+168*f^2*(d*x+c)^2*
cos(a+b*(d*x+c)^(1/3))/b^3/d^3-3*f^2*(d*x+c)^(8/3)*cos(a+b*(d*x+c)^(1/3))/b
/d^3+720*f*(-c*f+d*e)*sin(a+b*(d*x+c)^(1/3))/b^6/d^3-120960*f^2*(d*x+c)^(1/
3)*sin(a+b*(d*x+c)^(1/3))/b^8/d^3+6*(-c*f+d*e)^2*(d*x+c)^(1/3)*sin(a+b*(d*x
+c)^(1/3))/b^2/d^3-360*f*(-c*f+d*e)*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(1/3))/b^
4/d^3+20160*f^2*(d*x+c)*sin(a+b*(d*x+c)^(1/3))/b^6/d^3+30*f*(-c*f+d*e)*(d*x
+c)^(4/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d^3-1008*f^2*(d*x+c)^(5/3)*sin(a+b*(d*
x+c)^(1/3))/b^4/d^3+24*f^2*(d*x+c)^(7/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d^3
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used

$= \{3512, 3377, 2718, 2717\}$

$$\int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{120960f^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3} - \frac{120960f^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} + \frac{60480f^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} + \frac{720f(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{20160f^2(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{720f\sqrt[3]{c+dx}(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{5040f^2(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{360f(c+dx)^{2/3}(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{1008f^2(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{120f(c+dx)(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{168f^2(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{30f(c+dx)^{4/3}(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{6\sqrt[3]{c+dx}(de-cf)^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{24f^2(c+dx)^{7/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{6f(c+dx)^{5/3}(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{3(c+dx)^{2/3}(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{3f^2(c+dx)^{8/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-120960*f^2*Cos[a + b*(c + d*x)^(1/3)])/(b^9*d^3) + (6*(d*e - c*f)^2*Cos[a
 + b*(c + d*x)^(1/3)])/(b^3*d^3) - (720*f*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a
 + b*(c + d*x)^(1/3)])/(b^5*d^3) + (60480*f^2*(c + d*x)^(2/3)*Cos[a + b*(c
+ d*x)^(1/3)])/(b^7*d^3) - (3*(d*e - c*f)^2*(c + d*x)^(2/3)*Cos[a + b*(c +
d*x)^(1/3)])/(b*d^3) + (120*f*(d*e - c*f)*(c + d*x)*Cos[a + b*(c + d*x)^(1/
3)])/(b^3*d^3) - (5040*f^2*(c + d*x)^(4/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5
*d^3) - (6*f*(d*e - c*f)*(c + d*x)^(5/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^3
) + (168*f^2*(c + d*x)^2*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d^3) - (3*f^2*(c
+ d*x)^(8/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^3) + (720*f*(d*e - c*f)*Sin[a
 + b*(c + d*x)^(1/3)])/(b^6*d^3) - (120960*f^2*(c + d*x)^(1/3)*Sin[a + b*(c
 + d*x)^(1/3)])/(b^8*d^3) + (6*(d*e - c*f)^2*(c + d*x)^(1/3)*Sin[a + b*(c +
 d*x)^(1/3)])/(b^2*d^3) - (360*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a + b*(c +
 d*x)^(1/3)])/(b^4*d^3) + (20160*f^2*(c + d*x)*Sin[a + b*(c + d*x)^(1/3)])/
(b^6*d^3) + (30*f*(d*e - c*f)*(c + d*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)])/(
b^2*d^3) - (1008*f^2*(c + d*x)^(5/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^4*d^3)
+ (24*f^2*(c + d*x)^(7/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d^3)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{3\text{Subst}\left(\int \left(\frac{(de-cf)^2x^2\sin(a+bx)}{d^2}+\frac{2f(de-cf)x^5\sin(a+bx)}{d^2}+\frac{f^2x^8\sin(a+bx)}{d^2}\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{(3f^2)\,\text{Subst}\left(\int x^8\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3} + \frac{(6f(de-cf))\text{Subst}\left(\int x^5\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3} + \frac{(3(de-cf)^2)\,\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}$$

$$= -\frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{3f^2(c+dx)^{8/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{(24f^2)\,\text{Subst}\left(\int x^7\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd^3} + \frac{(30f(de-cf))\text{Subst}\left(\int x^4\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd^3} + \frac{(6(de-cf)^2)\,\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd^3}$$

$$= -\frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{3f^2(c+dx)^{8/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{6(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{30f(de-cf)(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{24f^2(c+dx)^{7/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{(168f^2)\,\text{Subst}\left(\int x^6\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{(120f(de-cf))\text{Subst}\left(\int x^3\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{(6(de-cf)^2)\,\text{Subst}\left(\int \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d^3}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$+ \frac{120f(de-cf)(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}$$
$$- \frac{6f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$+ \frac{168f^2(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{3f^2(c+dx)^{8/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$+ \frac{6(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$+ \frac{30f(de-cf)(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$+ \frac{24f^2(c+dx)^{7/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$- \frac{(1008f^2)\,\mathrm{Subst}\left(\int x^5\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^3d^3}$$
$$- \frac{(360f(de-cf))\mathrm{Subst}\left(\int x^2\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^3d^3}$$

$$
\begin{aligned}
&= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+ \frac{120f(de-cf)(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&- \frac{6f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+ \frac{168f^2(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{3f^2(c+dx)^{8/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+ \frac{6(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&- \frac{360f(de-cf)(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} \\
&+ \frac{30f(de-cf)(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&- \frac{1008f^2(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{24f^2(c+dx)^{7/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&+ \frac{(5040f^2)\,\text{Subst}\left(\int x^4\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^4d^3} \\
&+ \frac{(720f(de-cf))\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^4d^3}
\end{aligned}
$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}$$
$$- \frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$+ \frac{120f(de-cf)(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}$$
$$- \frac{5040f^2(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}$$
$$- \frac{6f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$+ \frac{168f^2(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{3f^2(c+dx)^{8/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$+ \frac{6(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$- \frac{360f(de-cf)(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3}$$
$$+ \frac{30f(de-cf)(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$- \frac{1008f^2(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{24f^2(c+dx)^{7/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$+ \frac{(20160f^2)\,\text{Subst}\left(\int x^3\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^5d^3}$$
$$+ \frac{(720f(de-cf))\text{Subst}\left(\int \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^5d^3}$$

$$
\begin{aligned}
&= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} \\
&\quad - \frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&\quad + \frac{120f(de-cf)(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&\quad - \frac{5040f^2(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} \\
&\quad - \frac{6f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&\quad + \frac{168f^2(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{3f^2(c+dx)^{8/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&\quad + \frac{720f(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{6(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&\quad - \frac{360f(de-cf)(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} \\
&\quad + \frac{20160f^2(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} \\
&\quad + \frac{30f(de-cf)(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&\quad - \frac{1008f^2(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{24f^2(c+dx)^{7/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&\quad - \frac{(60480f^2)\,\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^6d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} \\
&+ \frac{60480f^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} \\
&- \frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+ \frac{120f(de-cf)(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&- \frac{5040f^2(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} \\
&- \frac{6f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+ \frac{168f^2(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{3f^2(c+dx)^{8/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+ \frac{720f(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{6(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&- \frac{360f(de-cf)(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} \\
&+ \frac{20160f^2(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} \\
&+ \frac{30f(de-cf)(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&- \frac{1008f^2(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{24f^2(c+dx)^{7/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&- \frac{(120960f^2)\,\mathrm{Subst}\left(\int x\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^7d^3}
\end{aligned}
$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}$$
$$+ \frac{60480f^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} - \frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$+ \frac{120f(de-cf)(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{5040f^2(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}$$
$$- \frac{6f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{168f^2(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}$$
$$- \frac{3f^2(c+dx)^{8/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{720f(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3}$$
$$- \frac{120960f^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} + \frac{6(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$- \frac{360f(de-cf)(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{20160f^2(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3}$$
$$+ \frac{30f(de-cf)(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{1008f^2(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3}$$
$$+ \frac{24f^2(c+dx)^{7/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{(120960f^2)\,\mathrm{Subst}\left(\int\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^8d^3}$$

$$= -\frac{120960 f^2 \cos\left(a+b\sqrt[3]{c+dx}\right)}{b^9 d^3} + \frac{6(de-cf)^2 \cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3 d^3}$$
$$- \frac{720 f(de-cf)\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5 d^3}$$
$$+ \frac{60480 f^2 (c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7 d^3}$$
$$- \frac{3(de-cf)^2 (c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b d^3}$$
$$+ \frac{120 f(de-cf)(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3 d^3}$$
$$- \frac{5040 f^2 (c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5 d^3}$$
$$- \frac{6 f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b d^3}$$
$$+ \frac{168 f^2 (c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3 d^3} - \frac{3 f^2 (c+dx)^{8/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b d^3}$$
$$+ \frac{720 f(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6 d^3} - \frac{120960 f^2 \sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^8 d^3}$$
$$+ \frac{6(de-cf)^2 \sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2 d^3}$$
$$- \frac{360 f(de-cf)(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4 d^3}$$
$$+ \frac{20160 f^2 (c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6 d^3}$$
$$+ \frac{30 f(de-cf)(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2 d^3}$$
$$- \frac{1008 f^2 (c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4 d^3} + \frac{24 f^2 (c+dx)^{7/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2 d^3}$$

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.40

$$\int (e+fx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\frac{-3\Big(40320f^2-20160b^2f^2(c+dx)^{2/3}+b^8d^2(c+dx)^{2/3}(e+fx)^2+240b^4f\sqrt[3]{c+dx}(6cf+d(e+7fx))-}{}$$

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*(40320*f^2 - 20160*b^2*f^2*(c + d*x)^(2/3) + b^8*d^2*(c + d*x)^(2/3)*(e
 + f*x)^2 + 240*b^4*f*(c + d*x)^(1/3)*(6*c*f + d*(e + 7*f*x)) - 2*b^6*(9*c^
2*f^2 + 18*c*d*f*(e + 2*f*x) + d^2*(e^2 + 20*e*f*x + 28*f^2*x^2)))*Cos[a +
b*(c + d*x)^(1/3)] + 6*b*(-20160*f^2*(c + d*x)^(1/3) - 12*b^4*f*(c + d*x)^(
2/3)*(5*d*e + 9*c*f + 14*d*f*x) + b^6*d*(c + d*x)^(1/3)*(e + f*x)*(3*c*f +
d*(e + 4*f*x)) + 120*b^2*f*(27*c*f + d*(e + 28*f*x)))*Sin[a + b*(c + d*x)^(
1/3)])/(b^9*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2703 vs. 2(573) = 1146.

Time = 0.46 (sec) , antiderivative size = 2704, normalized size of antiderivative = 4.27

method	result	size
derivativedivides	Expression too large to display	2704
default	Expression too large to display	2704
parts	Expression too large to display	3867

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^3/b^3*(2*a^2*c*d*e*f*cos(a+b*(d*x+c)^(1/3))+2/b^3*a^5*d*e*f*cos(a+b*(d*
x+c)^(1/3))+10/b^3*a^4*d*e*f*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*co
s(a+b*(d*x+c)^(1/3)))-20/b^3*a^3*d*e*f*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x
+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(
1/3)))+20/b^3*a^2*d*e*f*(-(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))+3*(a
+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-6*sin(a+b*(d*x+c)^(1/3))+6*(a+b*
(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-10/b^3*a*d*e*f*(-(a+b*(d*x+c)^(1/3))
^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))+12
*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-24*cos(a+b*(d*x+c)^(1/3))-24*
(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+d^2*e^2*(-(a+b*(d*x+c)^(1/3))^2
*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(
a+b*(d*x+c)^(1/3)))+1/b^6*f^2*(-(a+b*(d*x+c)^(1/3))^8*cos(a+b*(d*x+c)^(1/3)
)+8*(a+b*(d*x+c)^(1/3))^7*sin(a+b*(d*x+c)^(1/3))+56*(a+b*(d*x+c)^(1/3))^6*c
```

os(a+b*(d*x+c)^(1/3))-336*(a+b*(d*x+c)^(1/3))^5*sin(a+b*(d*x+c)^(1/3))-1680
(a+b(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+6720*(a+b*(d*x+c)^(1/3))^3*si
n(a+b*(d*x+c)^(1/3))+20160*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-403
20*cos(a+b*(d*x+c)^(1/3))-40320*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))
+c^2*f^2*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(
1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+20/b^3*a^3*c*f^2*(-(a+b
*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d
*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-20/b^3*a^2*c*f^2*(-(a+b*(d*x+c)^(1/3))
^3*cos(a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-6*
sin(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+70/b^6
*a^4*f^2*(-(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3
))^3*sin(a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))
-24*cos(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-5
6/b^6*a^3*f^2*(-(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)
^(1/3))^4*sin(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(
1/3))-60*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+120*sin(a+b*(d*x+c)^(
1/3))-120*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+28/b^6*a^2*f^2*(-(a+b
*(d*x+c)^(1/3))^6*cos(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))^5*sin(a+b*(d
x+c)^(1/3))+30(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))-120*(a+b*(d*x+
c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))-360*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c
)^(1/3))+720*cos(a+b*(d*x+c)^(1/3))+720*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)
^(1/3)))-8/b^6*a*f^2*(-(a+b*(d*x+c)^(1/3))^7*cos(a+b*(d*x+c)^(1/3))+7*(a+b*
(d*x+c)^(1/3))^6*sin(a+b*(d*x+c)^(1/3))+42*(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d
x+c)^(1/3))-210(a+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))-840*(a+b*(d*x
+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))+2520*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x
+c)^(1/3))-5040*sin(a+b*(d*x+c)^(1/3))+5040*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*
x+c)^(1/3)))-2*a*c^2*f^2*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+
b*(d*x+c)^(1/3)))-2*a*d^2*e^2*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*c
os(a+b*(d*x+c)^(1/3)))-a^2*d^2*e^2*cos(a+b*(d*x+c)^(1/3))-1/b^6*a^8*f^2*cos
(a+b*(d*x+c)^(1/3))-2/b^3*a^5*c*f^2*cos(a+b*(d*x+c)^(1/3))+10/b^3*a*c*f^2*(
-(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*sin(a
+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-24*cos(a+
b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+2/b^3*d*e*f
(-(a+b(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*sin
(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))-60*(a+b
*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+120*sin(a+b*(d*x+c)^(1/3))-120*(a+
b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-2*c*d*e*f*(-(a+b*(d*x+c)^(1/3))^2*
cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a
+b*(d*x+c)^(1/3)))-10/b^3*a^4*c*f^2*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1
/3))*cos(a+b*(d*x+c)^(1/3)))-a^2*c^2*f^2*cos(a+b*(d*x+c)^(1/3))-2/b^3*c*f^2
(-(a+b(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*sin
(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))-60*(a+b
*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+120*sin(a+b*(d*x+c)^(1/3))-120*(a+
b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-8/b^6*a^7*f^2*(sin(a+b*(d*x+c)^(1/
3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+28/b^6*a^6*f^2*(-(a+b*(d*x+

```
c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^
(1/3))*sin(a+b*(d*x+c)^(1/3)))-56/b^6*a^5*f^2*(-(a+b*(d*x+c)^(1/3))^3*cos(a
+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-6*sin(a+b*
(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+4*a*c*d*e*f*(s
in(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.53

$$\int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$= \frac{3\left(\left(56\,b^6d^2f^2x^2+2\,b^6d^2e^2+36\,b^6cdef+18\,(b^6c^2-2240)f^2+8\,(5\,b^6d^2ef+9\,b^6cdf^2)x-(b^8d^2f^2x^2+2\,b\right.\right.}{}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((56*b^6*d^2*f^2*x^2 + 2*b^6*d^2*e^2 + 36*b^6*c*d*e*f + 18*(b^6*c^2 - 224
0)*f^2 + 8*(5*b^6*d^2*e*f + 9*b^6*c*d*f^2)*x - (b^8*d^2*f^2*x^2 + 2*b^8*d^2
*e*f*x + b^8*d^2*e^2 - 20160*b^2*f^2)*(d*x + c)^(2/3) - 240*(7*b^4*d*f^2*x
+ b^4*d*e*f + 6*b^4*c*f^2)*(d*x + c)^(1/3))*cos((d*x + c)^(1/3)*b + a) + 2*
(3360*b^3*d*f^2*x + 120*b^3*d*e*f + 3240*b^3*c*f^2 - 12*(14*b^5*d*f^2*x + 5
*b^5*d*e*f + 9*b^5*c*f^2)*(d*x + c)^(2/3) + (4*b^7*d^2*f^2*x^2 + b^7*d^2*e^
2 + 3*b^7*c*d*e*f - 20160*b*f^2 + (5*b^7*d^2*e*f + 3*b^7*c*d*f^2)*x)*(d*x +
 c)^(1/3))*sin((d*x + c)^(1/3)*b + a))/(b^9*d^3)
```

Sympy [F]

$$\int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)dx$$

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*(c + d*x)**(1/3)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2151 vs. 2(573) = 1146.

Time = 0.31 (sec) , antiderivative size = 2151, normalized size of antiderivative = 3.40

$$\int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right) dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*(a^2*e^2*cos((d*x + c)^(1/3)*b + a) - 2*a^2*c*e*f*cos((d*x + c)^(1/3)*b
+ a)/d + a^2*c^2*f^2*cos((d*x + c)^(1/3)*b + a)/d^2 - 2*(((d*x + c)^(1/3)*b
 + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a*e^2 + 4*((
(d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b +
 a))*a*c*e*f/d - 2*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - si
n((d*x + c)^(1/3)*b + a))*a*c^2*f^2/d^2 - 2*a^5*e*f*cos((d*x + c)^(1/3)*b +
 a)/(b^3*d) + 2*a^5*c*f^2*cos((d*x + c)^(1/3)*b + a)/(b^3*d^2) + ((((d*x +
c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b +
a)*sin((d*x + c)^(1/3)*b + a))*e^2 + 10*(((d*x + c)^(1/3)*b + a)*cos((d*x +
 c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^4*e*f/(b^3*d) - 2*((((d*x
+ c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b
+ a)*sin((d*x + c)^(1/3)*b + a))*c*e*f/d - 10*(((d*x + c)^(1/3)*b + a)*cos(
(d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^4*c*f^2/(b^3*d^2) +
((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^
(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*c^2*f^2/d^2 + a^8*f^2*cos((d*x + c
)^(1/3)*b + a)/(b^6*d^2) - 20*((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c
)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*a^3*
e*f/(b^3*d) - 8*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((
d*x + c)^(1/3)*b + a))*a^7*f^2/(b^6*d^2) + 20*((((d*x + c)^(1/3)*b + a)^2 -
 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1
/3)*b + a))*a^3*c*f^2/(b^3*d^2) + 20*((((d*x + c)^(1/3)*b + a)^3 - 6*(d*x +
 c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) - 3*(((d*x + c)^(1/3)*b + a)^
2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^2*e*f/(b^3*d) + 28*((((d*x + c)^(1/3)*
b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d
*x + c)^(1/3)*b + a))*a^6*f^2/(b^6*d^2) - 20*((((d*x + c)^(1/3)*b + a)^3 -
6*(d*x + c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) - 3*(((d*x + c)^(1/3)
*b + a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^2*c*f^2/(b^3*d^2) - 10*((((d*x
 + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/
3)*b + a) - 4*(((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((
d*x + c)^(1/3)*b + a))*a*e*f/(b^3*d) - 56*((((d*x + c)^(1/3)*b + a)^3 - 6*(
d*x + c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) - 3*(((d*x + c)^(1/3)*b
+ a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^5*f^2/(b^6*d^2) + 10*((((d*x + c)
^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/3)*b
+ a) - 4*(((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((d*x +
 c)^(1/3)*b + a))*a*c*f^2/(b^3*d^2) + 2*((((d*x + c)^(1/3)*b + a)^5 - 20*((
```

```
d*x + c)^(1/3)*b + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3
)*b + a) - 5*(((d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24
)*sin((d*x + c)^(1/3)*b + a))*e*f/(b^3*d) + 70*((((d*x + c)^(1/3)*b + a)^4
- 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/3)*b + a) - 4*(((d*x
+ c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((d*x + c)^(1/3)*b + a)
)*a^4*f^2/(b^6*d^2) - 2*((((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b
 + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b + a) - 5*(((
d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*sin((d*x + c)^
(1/3)*b + a))*c*f^2/(b^3*d^2) - 56*((((d*x + c)^(1/3)*b + a)^5 - 20*((d*x +
 c)^(1/3)*b + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b +
 a) - 5*(((d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*sin
((d*x + c)^(1/3)*b + a))*a^3*f^2/(b^6*d^2) + 28*((((d*x + c)^(1/3)*b + a)^6
 - 30*((d*x + c)^(1/3)*b + a)^4 + 360*((d*x + c)^(1/3)*b + a)^2 - 720)*cos(
(d*x + c)^(1/3)*b + a) - 6*(((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)
*b + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*sin((d*x + c)^(1/3)*b + a))*a^2*
f^2/(b^6*d^2) - 8*((((d*x + c)^(1/3)*b + a)^7 - 42*((d*x + c)^(1/3)*b + a)^
5 + 840*((d*x + c)^(1/3)*b + a)^3 - 5040*(d*x + c)^(1/3)*b - 5040*a)*cos((d
*x + c)^(1/3)*b + a) - 7*(((d*x + c)^(1/3)*b + a)^6 - 30*((d*x + c)^(1/3)*b
 + a)^4 + 360*((d*x + c)^(1/3)*b + a)^2 - 720)*sin((d*x + c)^(1/3)*b + a))*
a*f^2/(b^6*d^2) + ((((d*x + c)^(1/3)*b + a)^8 - 56*((d*x + c)^(1/3)*b + a)^
6 + 1680*((d*x + c)^(1/3)*b + a)^4 - 20160*((d*x + c)^(1/3)*b + a)^2 + 4032
0)*cos((d*x + c)^(1/3)*b + a) - 8*(((d*x + c)^(1/3)*b + a)^7 - 42*((d*x + c
)^(1/3)*b + a)^5 + 840*((d*x + c)^(1/3)*b + a)^3 - 5040*(d*x + c)^(1/3)*b -
 5040*a)*sin((d*x + c)^(1/3)*b + a))*f^2/(b^6*d^2))/(b^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1558 vs. $2(573) = 1146$.

Time = 0.34 (sec) , antiderivative size = 1558, normalized size of antiderivative = 2.46

$$\int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3*(e^2*(2*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a)/b - (((d*x + c)^(1/3)*
b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*cos((d*x + c)^(1/3)*b + a
)/b^2) - f^2*((((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a
)*a*b^6*c^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x +
c)^(1/3)*b + a)^4*a*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d
*x + c)^(1/3)*b + a)^2*a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2
*a^5*b^3*c + ((d*x + c)^(1/3)*b + a)^8 - 8*((d*x + c)^(1/3)*b + a)^7*a + 28
*((d*x + c)^(1/3)*b + a)^6*a^2 - 56*((d*x + c)^(1/3)*b + a)^5*a^3 + 70*((d*
x + c)^(1/3)*b + a)^4*a^4 - 56*((d*x + c)^(1/3)*b + a)^3*a^5 + 28*((d*x + c
)^(1/3)*b + a)^2*a^6 - 8*((d*x + c)^(1/3)*b + a)*a^7 + a^8 - 2*b^6*c^2 + 40
```

```
*((d*x + c)^(1/3)*b + a)^3*b^3*c - 120*((d*x + c)^(1/3)*b + a)^2*a*b^3*c +
120*((d*x + c)^(1/3)*b + a)*a^2*b^3*c - 40*a^3*b^3*c - 56*((d*x + c)^(1/3)*
b + a)^6 + 336*((d*x + c)^(1/3)*b + a)^5*a - 840*((d*x + c)^(1/3)*b + a)^4*
a^2 + 1120*((d*x + c)^(1/3)*b + a)^3*a^3 - 840*((d*x + c)^(1/3)*b + a)^2*a^
4 + 336*((d*x + c)^(1/3)*b + a)*a^5 - 56*a^6 - 240*((d*x + c)^(1/3)*b + a)*
b^3*c + 240*a*b^3*c + 1680*((d*x + c)^(1/3)*b + a)^4 - 6720*((d*x + c)^(1/3
)*b + a)^3*a + 10080*((d*x + c)^(1/3)*b + a)^2*a^2 - 6720*((d*x + c)^(1/3)*
b + a)*a^3 + 1680*a^4 - 20160*((d*x + c)^(1/3)*b + a)^2 + 40320*((d*x + c)^
(1/3)*b + a)*a - 20160*a^2 + 40320)*cos((d*x + c)^(1/3)*b + a)/(b^8*d^2) -
2*(((d*x + c)^(1/3)*b + a)*b^6*c^2 - a*b^6*c^2 - 5*((d*x + c)^(1/3)*b + a)^
4*b^3*c + 20*((d*x + c)^(1/3)*b + a)^3*a*b^3*c - 30*((d*x + c)^(1/3)*b + a)
^2*a^2*b^3*c + 20*((d*x + c)^(1/3)*b + a)*a^3*b^3*c - 5*a^4*b^3*c + 4*((d*x
 + c)^(1/3)*b + a)^7 - 28*((d*x + c)^(1/3)*b + a)^6*a + 84*((d*x + c)^(1/3)
*b + a)^5*a^2 - 140*((d*x + c)^(1/3)*b + a)^4*a^3 + 140*((d*x + c)^(1/3)*b
+ a)^3*a^4 - 84*((d*x + c)^(1/3)*b + a)^2*a^5 + 28*((d*x + c)^(1/3)*b + a)*
a^6 - 4*a^7 + 60*((d*x + c)^(1/3)*b + a)^2*b^3*c - 120*((d*x + c)^(1/3)*b +
 a)*a*b^3*c + 60*a^2*b^3*c - 168*((d*x + c)^(1/3)*b + a)^5 + 840*((d*x + c)
^(1/3)*b + a)^4*a - 1680*((d*x + c)^(1/3)*b + a)^3*a^2 + 1680*((d*x + c)^(1
/3)*b + a)^2*a^3 - 840*((d*x + c)^(1/3)*b + a)*a^4 + 168*a^5 - 120*b^3*c +
3360*((d*x + c)^(1/3)*b + a)^3 - 10080*((d*x + c)^(1/3)*b + a)^2*a + 10080*
((d*x + c)^(1/3)*b + a)*a^2 - 3360*a^3 - 20160*(d*x + c)^(1/3)*b)*sin((d*x
+ c)^(1/3)*b + a)/(b^8*d^2)) + 2*e*f*((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*
((d*x + c)^(1/3)*b + a)*a*b^3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5
*((d*x + c)^(1/3)*b + a)^4*a - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x
+ c)^(1/3)*b + a)^2*a^3 - 5*((d*x + c)^(1/3)*b + a)*a^4 + a^5 - 2*b^3*c + 2
0*((d*x + c)^(1/3)*b + a)^3 - 60*((d*x + c)^(1/3)*b + a)^2*a + 60*((d*x + c
)^(1/3)*b + a)*a^2 - 20*a^3 - 120*(d*x + c)^(1/3)*b)*cos((d*x + c)^(1/3)*b
+ a)/b^5 - (2*((d*x + c)^(1/3)*b + a)*b^3*c - 2*a*b^3*c - 5*((d*x + c)^(1/3
)*b + a)^4 + 20*((d*x + c)^(1/3)*b + a)^3*a - 30*((d*x + c)^(1/3)*b + a)^2*
a^2 + 20*((d*x + c)^(1/3)*b + a)*a^3 - 5*a^4 + 60*((d*x + c)^(1/3)*b + a)^2
 - 120*((d*x + c)^(1/3)*b + a)*a + 60*a^2 - 120)*sin((d*x + c)^(1/3)*b + a)
/b^5)/d)/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \sin \left(a+b\sqrt[3]{c+dx}\right) \, dx = \int \sin \left(a+b\,(c+d\,x)^{1/3}\right)\,(e+f\,x)^2 \, dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/3))*(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(1/3))*(e + f*x)^2, x)
```

3.208 $\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 288

$$\begin{aligned}\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx &= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} \\ &- \frac{360f\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} \\ &- \frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} \\ &+ \frac{60f(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} \\ &- \frac{3f(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} \\ &+ \frac{360f\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2} \\ &+ \frac{6(de-cf)\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} \\ &- \frac{180f(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} \\ &+ \frac{15f(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}\end{aligned}$$

```
[Out] 6*(-c*f+d*e)*cos(a+b*(d*x+c)^(1/3))/b^3/d^2-360*f*(d*x+c)^(1/3)*cos(a+b*(d*
x+c)^(1/3))/b^5/d^2-3*(-c*f+d*e)*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d^2
+60*f*(d*x+c)*cos(a+b*(d*x+c)^(1/3))/b^3/d^2-3*f*(d*x+c)^(5/3)*cos(a+b*(d*x
```

```
+c)^(1/3))/b/d^2+360*f*sin(a+b*(d*x+c)^(1/3))/b^6/d^2+6*(-c*f+d*e)*(d*x+c)^
(1/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d^2-180*f*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(1
/3))/b^4/d^2+15*f*(d*x+c)^(4/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3512, 3377, 2718, 2717}

$$
\begin{aligned}
\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = {} & \frac{360f\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2} \\
& -\frac{360f\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} \\
& -\frac{180f(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} \\
& +\frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} \\
& +\frac{60f(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} \\
& +\frac{6\sqrt[3]{c+dx}(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} \\
& +\frac{15f(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} \\
& -\frac{3(c+dx)^{2/3}(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} \\
& -\frac{3f(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}
\end{aligned}
$$

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (6*(d*e - c*f)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d^2) - (360*f*(c + d*x)^(1/
3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5*d^2) - (3*(d*e - c*f)*(c + d*x)^(2/3)*C
os[a + b*(c + d*x)^(1/3)])/(b*d^2) + (60*f*(c + d*x)*Cos[a + b*(c + d*x)^(1
/3)])/(b^3*d^2) - (3*f*(c + d*x)^(5/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^2)
+ (360*f*Sin[a + b*(c + d*x)^(1/3)])/(b^6*d^2) + (6*(d*e - c*f)*(c + d*x)^(
1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d^2) - (180*f*(c + d*x)^(2/3)*Sin[a +
 b*(c + d*x)^(1/3)])/(b^4*d^2) + (15*f*(c + d*x)^(4/3)*Sin[a + b*(c + d*x)^
(1/3)])/(b^2*d^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3\text{Subst}\left(\int \left(\frac{(de-cf)x^2\sin(a+bx)}{d}+\frac{fx^5\sin(a+bx)}{d}\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d} \\
&= \frac{(3f)\text{Subst}\left(\int x^5\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^2} \\
&\quad + \frac{(3(de-cf))\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^2} \\
&= -\frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} - \frac{3f(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} \\
&\quad + \frac{(15f)\text{Subst}\left(\int x^4\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd^2} \\
&\quad + \frac{(6(de-cf))\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd^2}
\end{aligned}$$

$$= -\frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} - \frac{3f(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{6(de-cf)\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} + \frac{15f(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{(60f)\text{Subst}\left(\int x^3\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{(6(de-cf))\text{Subst}\left(\int \sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^2d^2}$$

$$= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{60f(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3f(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{6(de-cf)\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} + \frac{15f(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{(180f)\text{Subst}\left(\int x^2\cos(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^3d^2}$$

$$= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{60f(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3f(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{6(de-cf)\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{180f(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} + \frac{15f(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} + \frac{(360f)\text{Subst}\left(\int x\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^4d^2}$$

$$= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{360f\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{60f(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3f(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{6(de-cf)\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{180f(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} + \frac{15f(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} + \frac{(360f)\text{Subst}\left(\int \cos(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^5d^2}$$

$$= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{360f\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{60f(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3f(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{360f\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2} + \frac{6(de-cf)\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{180f(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} + \frac{15f(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.51

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{-3b\left(120f\sqrt[3]{c+dx}+b^4d(c+dx)^{2/3}(e+fx)-2b^2(9cf+d(e+10fx))\right)\cos\left(a+b\sqrt[3]{c+dx}\right)+3\left(2b^4de\cdots\right.}{b^6d^2}$$

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*b*(120*f*(c + d*x)^(1/3) + b^4*d*(c + d*x)^(2/3)*(e + f*x) - 2*b^2*(9*c
*f + d*(e + 10*f*x)))*Cos[a + b*(c + d*x)^(1/3)] + 3*(2*b^4*d*e*(c + d*x)^(
1/3) + f*(120 - 60*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(1/3)*(3*c + 5*d*x))
)*Sin[a + b*(c + d*x)^(1/3)])/(b^6*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. 2(258) = 516.

Time = 0.53 (sec) , antiderivative size = 801, normalized size of antiderivative = 2.78

method	result	size
derivativedivides	Expression too large to display	801
default	Expression too large to display	801
parts	Expression too large to display	1288

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^2/b^3*(a^2*c*f*cos(a+b*(d*x+c)^(1/3))-a^2*d*e*cos(a+b*(d*x+c)^(1/3))+2*
a*c*f*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-2
```

```
*a*d*e*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-
c*f*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))
+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+d*e*(-(a+b*(d*x+c)^(1/3))^2*
cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a
+b*(d*x+c)^(1/3)))+1/b^3*a^5*f*cos(a+b*(d*x+c)^(1/3))+5/b^3*a^4*f*(sin(a+b*
(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-10/b^3*a^3*f*(-(
a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b
*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+10/b^3*a^2*f*(-(a+b*(d*x+c)^(1/3))^
3*cos(a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-6*s
in(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-5/b^3*a
*f*(-(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*s
in(a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-24*co
s(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+1/b^3*f
*(-(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*sin
(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))-60*(a+b
*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+120*sin(a+b*(d*x+c)^(1/3))-120*(a+
b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.49

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\frac{3\left(\left(20\,b^3dfx+2\,b^3de+18\,b^3cf-120\,(dx+c)^{\frac{1}{3}}bf-(b^5dfx+b^5de)(dx+c)^{\frac{2}{3}}\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-\left(6\right.\right.}{b^6d^2}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((20*b^3*d*f*x + 2*b^3*d*e + 18*b^3*c*f - 120*(d*x + c)^(1/3)*b*f - (b^5*
d*f*x + b^5*d*e)*(d*x + c)^(2/3))*cos((d*x + c)^(1/3)*b + a) - (60*(d*x + c
)^(2/3)*b^2*f - (5*b^4*d*f*x + 2*b^4*d*e + 3*b^4*c*f)*(d*x + c)^(1/3) - 120
*f)*sin((d*x + c)^(1/3)*b + a))/(b^6*d^2)
```

Sympy [F]

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b*(c + d*x)**(1/3)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(258) = 516$.

Time = 0.23 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.36

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*(a^2*e*cos((d*x + c)^(1/3)*b + a) - a^2*c*f*cos((d*x + c)^(1/3)*b + a)/d
 - 2*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1
/3)*b + a))*a*e + 2*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - s
in((d*x + c)^(1/3)*b + a))*a*c*f/d - a^5*f*cos((d*x + c)^(1/3)*b + a)/(b^3*
d) + ((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x
+ c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*e + 5*(((d*x + c)^(1/3)*b + a
)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^4*f/(b^3*d) -
((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^
(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*c*f/d - 10*((((d*x + c)^(1/3)*b +
a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x +
 c)^(1/3)*b + a))*a^3*f/(b^3*d) + 10*((((d*x + c)^(1/3)*b + a)^3 - 6*(d*x +
 c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) - 3*(((d*x + c)^(1/3)*b + a)^
2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^2*f/(b^3*d) - 5*((((d*x + c)^(1/3)*b +
 a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/3)*b + a) - 4*(
((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((d*x + c)^(1/3)*
b + a))*a*f/(b^3*d) + ((((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b +
 a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b + a) - 5*(((d*
x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*sin((d*x + c)^(1
/3)*b + a))*f/(b^3*d))/(b^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.57

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

$$= \frac{3\left(e\left(\frac{2(dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{b} - \frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2 - 2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^2-2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{b^2}\right) + \frac{f\left(\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2 b^3c-\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3*(e*(2*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a)/b - (((d*x + c)^(1/3)*b
+ a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*cos((d*x + c)^(1/3)*b + a)/
b^2) + f*((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b + a)*a*b^
3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*b + a)^4*a
 - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^2*a^3 - 5*
((d*x + c)^(1/3)*b + a)*a^4 + a^5 - 2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^3
- 60*((d*x + c)^(1/3)*b + a)^2*a + 60*((d*x + c)^(1/3)*b + a)*a^2 - 20*a^3
- 120*(d*x + c)^(1/3)*b)*cos((d*x + c)^(1/3)*b + a)/b^5 - (2*((d*x + c)^(1/
3)*b + a)*b^3*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a)^4 + 20*((d*x + c)^(
1/3)*b + a)^3*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 20*((d*x + c)^(1/3)*b
+ a)*a^3 - 5*a^4 + 60*((d*x + c)^(1/3)*b + a)^2 - 120*((d*x + c)^(1/3)*b +
a)*a + 60*a^2 - 120)*sin((d*x + c)^(1/3)*b + a)/b^5)/d)/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^{1/3}\right)\,(e+f\,x)\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/3))*(e + f*x),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/3))*(e + f*x), x)
```

3.209 $\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}$$

```
[Out] 6*cos(a+b*(d*x+c)^(1/3))/b^3/d-3*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d+6
*(d*x+c)^(1/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3442, 3377, 2718}

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (6*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (3*(c + d*x)^(2/3)*Cos[a + b*(c +
d*x)^(1/3)])/(b*d) + (6*(c + d*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d} \\
&= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{6\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd} \\
&= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} \\
&\quad - \frac{6\text{Subst}\left(\int \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d} \\
&= \frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{\left(6-3b^2(c+dx)^{2/3}\right)\cos\left(a+b\sqrt[3]{c+dx}\right)+6b\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)],x]
[Out] ((6 - 3*b^2*(c + d*x)^(2/3))*Cos[a + b*(c + d*x)^(1/3)] + 6*b*(c + d*x)^(1/
3)*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{-3a^2\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)-\left(a+b(dx+c)^{\frac{1}{3}}\right)\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)-3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\cos\left(a+b(d\right.}{d\,b^3}$
default	$\frac{-3a^2\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)-\left(a+b(dx+c)^{\frac{1}{3}}\right)\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)-3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\cos\left(a+b(d\right.}{d\,b^3}$

```
[In] int(sin(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d/b^3*(-a^2*cos(a+b*(d*x+c)^(1/3))-2*a*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+
c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/
3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\left(2\,(dx+c)^{\frac{1}{3}}b\sin\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^2-2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^3d}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*(2*(d*x + c)^(1/3)*b*sin((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^2 -
2)*cos((d*x + c)^(1/3)*b + a))/(b^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \begin{cases} x\sin(a) & \text{for } b=0\wedge(b=0\vee d= \\ x\sin\left(a+b\sqrt[3]{c}\right) & \text{for } d=0 \\ -\frac{3(c+dx)^{\frac{2}{3}}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Piecewise((x*sin(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sin(a + b*c**(1/
3)), Eq(d, 0)), (-3*(c + d*x)**(2/3)*cos(a + b*(c + d*x)**(1/3))/(b*d) + 6*
(c + d*x)**(1/3)*sin(a + b*(c + d*x)**(1/3))/(b**2*d) + 6*cos(a + b*(c + d*
x)**(1/3))/(b**3*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.41

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{3\left(a^2\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-2\left(\left((dx+c)^{\frac{1}{3}}b+a\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)a+\left(\right.}{b^3d}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*(a^2*cos((d*x + c)^(1/3)*b + a) - 2*(((d*x + c)^(1/3)*b + a)*cos((d*x +
c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a + (((d*x + c)^(1/3)*b + a)^
2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x + c)
^(1/3)*b + a))/(b^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\left(\frac{2\,(dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{b}-\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^2-2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{b^2}\right)}{bd}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3*(2*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a)/b - (((d*x + c)^(1/3)*b + a
)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*cos((d*x + c)^(1/3)*b + a)/b^2
)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

$$=\frac{3\left(2\,\cos\left(a+b\,(c+d\,x)^{1/3}\right)+2\,b\,\sin\left(a+b\,(c+d\,x)^{1/3}\right)\,(c+d\,x)^{1/3}-b^2\,\cos\left(a+b\,(c+d\,x)^{1/3}\right)\,(c+\right.}{b^3\,d}$$

```
[In] int(sin(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] (3*(2*cos(a + b*(c + d*x)^(1/3)) + 2*b*sin(a + b*(c + d*x)^(1/3))*(c + d*x)
^(1/3) - b^2*cos(a + b*(c + d*x)^(1/3))*(c + d*x)^(2/3)))/(b^3*d)
```

3.210 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 396

$$\begin{aligned}\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx &= \frac{\text{CosIntegral}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)}{f}\\ &+\frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)}{f}\\ &+\frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)}{f}\\ &-\frac{\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{f}\\ &+\frac{\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{f}\\ &+\frac{\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{f}\end{aligned}$$

```
[Out] cos(a+(-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))*Si(-(-1)^(1/3)*b*(-c*f+d*e)^(1
/3)/f^(1/3)+b*(d*x+c)^(1/3))/f+cos(a-b*(-c*f+d*e)^(1/3)/f^(1/3))*Si(b*(-c*f
```

```
+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))/f+cos(a-(-1)^(2/3)*b*(-c*f+d*e)^(1/3)/
f^(1/3))*Si((-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))/f+Ci(b*(
-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))*sin(a-b*(-c*f+d*e)^(1/3)/f^(1/3))/
f+Ci((-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)-b*(d*x+c)^(1/3))*sin(a+(-1)^(1/3
)*b*(-c*f+d*e)^(1/3)/f^(1/3))/f+Ci((-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)+b*
(d*x+c)^(1/3))*sin(a-(-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))/f
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3512, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$$
$$= \frac{\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{de-cf}b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}b\right)}{f}$$
$$+\frac{\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{f}$$
$$+\frac{\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{de-cf}b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}b\right)}{f}$$
$$-\frac{\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{f}$$
$$+\frac{\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{de-cf}b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}b\right)}{f}$$
$$+\frac{\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{de-cf}b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}b\right)}{f}$$

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x),x]
```

```
[Out] (CosIntegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)]*Sin[a - (b*
(d*e - c*f)^(1/3))/f^(1/3)])/f + (CosIntegral[((-1)^(1/3)*b*(d*e - c*f)^(1/
3))/f^(1/3) - b*(c + d*x)^(1/3)]*Sin[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f
^(1/3)])/f + (CosIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c +
 d*x)^(1/3)]*Sin[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3)])/f - (Cos[a
+ ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*SinIntegral[((-1)^(1/3)*b*(d*e
```

```
- c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/f + (Cos[a - (b*(d*e - c*f)^(1/
3))/f^(1/3)]*SinIntegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)]
)/f + (Cos[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*SinIntegral[((-1)^
(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/f
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

integral

$$= \frac{3\text{Subst}\left(\int \left(\frac{(de-cf)\sin(a+bx)}{3f^{2/3}\left(e-\frac{cf}{d}\right)\left(\sqrt[3]{de-cf}+\sqrt[3]{f}x\right)}+\frac{(de-cf)\sin(a+bx)}{3f^{2/3}\left(e-\frac{cf}{d}\right)\left(-\sqrt[3]{-1}\sqrt[3]{de-cf}+\sqrt[3]{f}x\right)}+\frac{(de-cf)\sin(a+bx)}{3f^{2/3}\left(e-\frac{cf}{d}\right)\left((-1)^{2/3}\sqrt[3]{\right.}\right.}\right.\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt[3]{de-cf}+\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{f^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sin(a+bx)}{-\sqrt[3]{-1}\sqrt[3]{de-cf}+\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{f^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sin(a+bx)}{(-1)^{2/3}\sqrt[3]{de-cf}+\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{f^{2/3}}$$

$$= \frac{\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+bx\right)}{\sqrt[3]{de-cf}+\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{f^{2/3}} - \frac{\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-bx\right)}{-\sqrt[3]{-1}\sqrt[3]{de-cf}+\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{f^{2/3}} + \frac{\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+bx\right)}{(-1)^{2/3}\sqrt[3]{de-cf}+\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{f^{2/3}} + \frac{\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+bx\right)}{\sqrt[3]{de-cf}+\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{f^{2/3}} + \frac{\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-bx\right)}{-\sqrt[3]{-1}\sqrt[3]{de-cf}+\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{f^{2/3}} + \frac{\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+bx\right)}{(-1)^{2/3}\sqrt[3]{de-cf}+\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{f^{2/3}}$$

$$= \frac{\text{CosIntegral}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)}{f}$$
$$+\frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)}{f}$$
$$+\frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)}{f}$$
$$-\frac{\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{f}$$
$$+\frac{\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{f}$$
$$+\frac{\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 25.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.30

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$$
$$= \frac{i\left(\text{RootSum}\left[de-cf+f\#1^3\&,e^{-ia-ib\#1}\,\text{ExpIntegralEi}\left(-ib\left(\sqrt[3]{c+dx}-\#1\right)\right)\&\right]-\text{RootSum}\left[de-cf\right.\right.}{2f}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x),x]
```

```
[Out] ((I/2)*(RootSum[d*e - c*f + f*#1^3 & , E^((-I)*a - I*b*#1)*ExpIntegralEi[(-
I)*b*((c + d*x)^(1/3) - #1)] & ] - RootSum[d*e - c*f + f*#1^3 & , E^(I*a +
I*b*#1)*ExpIntegralEi[I*b*((c + d*x)^(1/3) - #1)] & ]))/f
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{b^3 a^2\left(\sum_{\_R1=\mathrm{RootOf}\left(-b^3cf+b^3de+f\_Z^3-3fa\_Z^2+3a^2f\_Z-a^3f\right)}\frac{-\mathrm{Si}\left(-b(dx+c)^{\frac{1}{3}}+\_R1-a\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(b(dx+c)^{\frac{1}{3}}\right.}{\_R1^2-2\_R1a+a^2}\right.}{f}$
default	$\frac{b^3 a^2\left(\sum_{\_R1=\mathrm{RootOf}\left(-b^3cf+b^3de+f\_Z^3-3fa\_Z^2+3a^2f\_Z-a^3f\right)}\frac{-\mathrm{Si}\left(-b(dx+c)^{\frac{1}{3}}+\_R1-a\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(b(dx+c)^{\frac{1}{3}}\right.}{\_R1^2-2\_R1a+a^2}\right.}{f}$

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/3*b^3*a^2/f*sum(1/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)
*cos(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_
Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))-2/3*b^3*a/f*sum(_R1/(_R1^2-2*_R1*a+a^2)
*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*sin(_R1)),
_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))+1/3*b^3/f*
sum(_R1^2/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d
*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+
3*_Z*a^2*f-a^3*f)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.13

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$$

$$= \frac{i\,\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b+\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(\frac{i\,b^3de-i\,b^3cf}{f}\right)^{\frac{1}{3}}\right)e^{\left(\frac{1}{2}\left(i\sqrt{3}+1\right)\left(\frac{i\,b^3de-i\,b^3cf}{f}\right)^{\frac{1}{3}}-i\,a\right)}+i\,\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b\right.}{}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(I*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((I*b^3*d*e - I*b^3*c
*f)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3) - I*
a) + I*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((I*b^3*d*e - I*b^3*c*
f)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3) - I*
```

```
a) - I*Ei(I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((-I*b^3*d*e + I*b^3*c
*f)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3) + I
*a) - I*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((-I*b^3*d*e + I*b^3*c
*f)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3) +
I*a) - I*Ei(I*(d*x + c)^(1/3)*b + ((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3))*e^(I*
a - ((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3)) + I*Ei(-I*(d*x + c)^(1/3)*b + ((I*b
^3*d*e - I*b^3*c*f)/f)^(1/3))*e^(-I*a - ((I*b^3*d*e - I*b^3*c*f)/f)^(1/3)))
/f
```

Sympy [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(f*x+e),x)
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e + f*x), x)
```

Maxima [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e), x)
```

Giac [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{e+f\,x}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(e + f*x),x)
[Out] int(sin(a + b*(c + d*x)^(1/3))/(e + f*x), x)
```

3.211 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 555

$$\begin{aligned}\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx &= -\frac{\sqrt[3]{-1}bd\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}\\
&+\frac{bd\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}\\
&+\frac{(-1)^{2/3}bd\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}\\
&-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)}\\
&-\frac{\sqrt[3]{-1}bd\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}\\
&-\frac{bd\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}\\
&-\frac{(-1)^{2/3}bd\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}\end{aligned}$$

```
[Out] 1/3*b*d*Ci(b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))*cos(a-b*(-c*f+d*e)^(
1/3)/f^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)-1/3*(-1)^(1/3)*b*d*Ci((-1)^(1/3)*b*(
-c*f+d*e)^(1/3)/f^(1/3)-b*(d*x+c)^(1/3))*cos(a+(-1)^(1/3)*b*(-c*f+d*e)^(1/3
)/f^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)+1/3*(-1)^(2/3)*b*d*Ci((-1)^(2/3)*b*(-c*
f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))*cos(a-(-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f
^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)-1/3*b*d*Si(b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d
*x+c)^(1/3))*sin(a-b*(-c*f+d*e)^(1/3)/f^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)+1/3
*(-1)^(1/3)*b*d*Si(-(-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))*
sin(a+(-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)-1/3*(
-1)^(2/3)*b*d*Si((-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))*sin
(a-(-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)-sin(a+b*
(d*x+c)^(1/3))/f/(f*x+e)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3512, 3422, 3415, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx$$

$$= -\frac{\sqrt[3]{-1}bd\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}$$
$$+\frac{bd\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{de-cf}b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}b\right)}{3f^{4/3}(de-cf)^{2/3}}$$
$$+\frac{(-1)^{2/3}bd\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{de-cf}b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}b\right)}{3f^{4/3}(de-cf)^{2/3}}$$
$$-\frac{\sqrt[3]{-1}bd\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}$$
$$-\frac{bd\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{de-cf}b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}b\right)}{3f^{4/3}(de-cf)^{2/3}}$$
$$-\frac{(-1)^{2/3}bd\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{de-cf}b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}b\right)}{3f^{4/3}(de-cf)^{2/3}}$$
$$-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)}$$

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

```
[Out] -1/3*((-1)^(1/3)*b*d*Cos[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*CosI
ntegral[((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/(f^(
4/3)*(d*e - c*f)^(2/3)) + (b*d*Cos[a - (b*(d*e - c*f)^(1/3))/f^(1/3)]*CosIn
tegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/(3*f^(4/3)*(d*e
- c*f)^(2/3)) + ((-1)^(2/3)*b*d*Cos[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^
(1/3)]*CosIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(
1/3)])/(3*f^(4/3)*(d*e - c*f)^(2/3)) - Sin[a + b*(c + d*x)^(1/3)]/(f*(e + f
*x)) - ((-1)^(1/3)*b*d*Sin[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*Si
nIntegral[((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/(3
*f^(4/3)*(d*e - c*f)^(2/3)) - (b*d*Sin[a - (b*(d*e - c*f)^(1/3))/f^(1/3)]*S
inIntegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/(3*f^(4/3)*(
d*e - c*f)^(2/3)) - ((-1)^(2/3)*b*d*Sin[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3)
)/f^(1/3)]*SinIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*
x)^(1/3)])/(3*f^(4/3)*(d*e - c*f)^(2/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
```

```
ntegerQ[n] || GtQ[e, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{3\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{\left(e-\frac{cf}{d}+\frac{fx^3}{d}\right)^2}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} + \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{e-\frac{cf}{d}+\frac{fx^3}{d}}\,dx, x, \sqrt[3]{c+dx}\right)}{f}$$

$$= -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} + \frac{b\text{Subst}\left(\int\left(-\frac{\sqrt[3]{de-cf}\cos(a+bx)}{3\left(e-\frac{cf}{d}\right)\left(-\sqrt[3]{de-cf}-\sqrt[3]{f}x\right)} - \frac{\sqrt[3]{de-cf}\cos(a+bx)}{3\left(e-\frac{cf}{d}\right)\left(-\sqrt[3]{de-cf}+\sqrt[3]{-1}\sqrt[3]{f}x\right)} - \frac{\sqrt[3]{de}}{3\left(e-\frac{cf}{d}\right)\left(-\sqrt[3]{}\right.}\right.\right.}{f}$$

$$= -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{-\sqrt[3]{de-cf}-\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{3f(de-cf)^{2/3}} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{-\sqrt[3]{de-cf}+\sqrt[3]{-1}\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{3f(de-cf)^{2/3}} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{-\sqrt[3]{de-cf}-(-1)^{2/3}\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{3f(de-cf)^{2/3}}$$

$$= -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)}$$

$$-\frac{\left(bd\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+bx\right)}{-\sqrt[3]{de-cf}-\sqrt[3]{f}x}dx,x,\sqrt[3]{c+dx}\right)}{3f(de-cf)^{2/3}}$$

$$-\frac{\left(bd\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-bx\right)}{-\sqrt[3]{de-cf}-(-1)^{2/3}\sqrt[3]{f}x}dx,x,\sqrt[3]{c+dx}\right)}{3f(de-cf)^{2/3}}$$

$$-\frac{\left(bd\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+bx\right)}{-\sqrt[3]{de-cf}+\sqrt[3]{-1}\sqrt[3]{f}x}dx,x,\sqrt[3]{c+dx}\right)}{3f(de-cf)^{2/3}}$$

$$+\frac{\left(bd\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+bx\right)}{-\sqrt[3]{de-cf}-\sqrt[3]{f}x}dx,x,\sqrt[3]{c+dx}\right)}{3f(de-cf)^{2/3}}$$

$$-\frac{\left(bd\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-bx\right)}{-\sqrt[3]{de-cf}-(-1)^{2/3}\sqrt[3]{f}x}dx,x,\sqrt[3]{c+dx}\right)}{3f(de-cf)^{2/3}}$$

$$+\frac{\left(bd\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+bx\right)}{-\sqrt[3]{de-cf}+\sqrt[3]{-1}\sqrt[3]{f}x}dx,x,\sqrt[3]{c+dx}\right)}{3f(de-cf)^{2/3}}$$

$$
= -\frac{\sqrt[3]{-1}bd\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}} + \frac{bd\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}} + \frac{(-1)^{2/3}bd\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}} - \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} - \frac{\sqrt[3]{-1}bd\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}} - \frac{bd\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}} - \frac{(-1)^{2/3}bd\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.87 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.32

$$
\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx = \frac{\frac{3ie^{-i\left(a+b\sqrt[3]{c+dx}\right)}\left(-1+e^{2i\left(a+b\sqrt[3]{c+dx}\right)}\right)f}{e+fx} + bd\text{RootSum}\left[de-cf+f\#1^3\&, \frac{e^{-ia-ib\#1}\,\text{ExpIntegralEi}\left(-ib\left(\sqrt[3]{c+dx}-\#\right.\right.}{\#1^2}\right.}{6f^2}
$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

```
[Out] (((3*I)*(-1 + E^((2*I)*(a + b*(c + d*x)^(1/3))))*f)/(E^(I*(a + b*(c + d*x)^
(1/3)))*(e + f*x)) + b*d*RootSum[d*e - c*f + f*#1^3 & , (E^((-I)*a - I*b*#1
)*ExpIntegralEi[(-I)*b*((c + d*x)^(1/3) - #1)])/#1^2 & ] + b*d*RootSum[d*e
- c*f + f*#1^3 & , (E^(I*a + I*b*#1)*ExpIntegralEi[I*b*((c + d*x)^(1/3) - #
1)])/#1^2 & ])/(6*f^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 1176, normalized size of antiderivative = 2.12

method	result	size
derivativedivides	Expression too large to display	1176
default	Expression too large to display	1176

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*d/b^3*(b^6*a^2*(sin(a+b*(d*x+c)^(1/3))*(1/3/b^3/(c*f-d*e)*(a+b*(d*x+c)^(1
/3))-1/3*a/b^3/(c*f-d*e))/(b^3*c*f-b^3*d*e+a^3*f-3*a^2*f*(a+b*(d*x+c)^(1/3)
)+3*a*f*(a+b*(d*x+c)^(1/3))^2-f*(a+b*(d*x+c)^(1/3))^3)-2/9/b^3/f*sum(1/(c*f
-d*e)/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d*x+c
)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z
*a^2*f-a^3*f))-1/9/b^3/f*sum(1/(-_RR1+a)/(c*f-d*e)*(Si(-b*(d*x+c)^(1/3)+_RR
1-a)*sin(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf(-b^3*c*f+b
^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f)))+sin(a+b*(d*x+c)^(1/3))*(-2/3*a
*b^3/(c*f-d*e)*(a+b*(d*x+c)^(1/3))^2+2/3*a^2*b^3/(c*f-d*e)*(a+b*(d*x+c)^(1/
3)))/(b^3*c*f-b^3*d*e+a^3*f-3*a^2*f*(a+b*(d*x+c)^(1/3))+3*a*f*(a+b*(d*x+c)^
(1/3))^2-f*(a+b*(d*x+c)^(1/3))^3)+2/9*a*b^3/f*sum((_R1+a)/(c*f-d*e)/(_R1^2-
2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a
)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f)
)+2/9*a*b^3/f*sum(_RR1/(-_RR1+a)/(c*f-d*e)*(Si(-b*(d*x+c)^(1/3)+_RR1-a)*sin
(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf(-b^3*c*f+b^3*d*e+_
Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))+sin(a+b*(d*x+c)^(1/3))*(2/3*a*b^3/(c*f-
d*e)*(a+b*(d*x+c)^(1/3))^2-a^2*b^3/(c*f-d*e)*(a+b*(d*x+c)^(1/3))+1/3*b^3*(b
^3*c*f-b^3*d*e+a^3*f)/f/(c*f-d*e))/(b^3*c*f-b^3*d*e+a^3*f-3*a^2*f*(a+b*(d*x
+c)^(1/3))+3*a*f*(a+b*(d*x+c)^(1/3))^2-f*(a+b*(d*x+c)^(1/3))^3)-2/9*a*b^3/f
*sum(_R1/(c*f-d*e)/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1
)+Ci(b*(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*
_Z^2*a*f+3*_Z*a^2*f-a^3*f))+1/9*b^3/f^2*sum((b^3*c*f-b^3*d*e+2*_RR1^2*a*f-3
*_RR1*a^2*f+a^3*f)/(c*f-d*e)/(_RR1^2-2*_RR1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_RR
1-a)*sin(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf(-b^3*c*f+b
^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.32

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx =$$

$$-\frac{\left(i\,dfx+i\,de-\sqrt{3}(dfx+de)\right)\left(\frac{i\,b^3de-i\,b^3cf}{f}\right)^{\frac{1}{3}}\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b+\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(\frac{i\,b^3de-i\,b^3cf}{f}\right)^{\frac{1}{3}}\right)e^{\left(\frac{1}{2}\left(i\right.\right.}}{}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/12*((I*d*f*x + I*d*e - sqrt(3)*(d*f*x + d*e))*((I*b^3*d*e - I*b^3*c*f)/f
)^(1/3)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((I*b^3*d*e - I*b^3*
c*f)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3) - I
*a) + (I*d*f*x + I*d*e + sqrt(3)*(d*f*x + d*e))*((I*b^3*d*e - I*b^3*c*f)/f)
^(1/3)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((I*b^3*d*e - I*b^3*c*
f)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3) - I*
a) + (-I*d*f*x - I*d*e + sqrt(3)*(d*f*x + d*e))*((-I*b^3*d*e + I*b^3*c*f)/f
)^(1/3)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((-I*b^3*d*e + I*b^3*
c*f)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3) +
I*a) + (-I*d*f*x - I*d*e - sqrt(3)*(d*f*x + d*e))*((-I*b^3*d*e + I*b^3*c*f)
/f)^(1/3)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((-I*b^3*d*e + I*b^3
*c*f)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3)
+ I*a) - 2*(-I*d*f*x - I*d*e)*((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3)*Ei(I*(d*x
+ c)^(1/3)*b + ((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3))*e^(I*a - ((-I*b^3*d*e +
I*b^3*c*f)/f)^(1/3)) - 2*(I*d*f*x + I*d*e)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3
)*Ei(-I*(d*x + c)^(1/3)*b + ((I*b^3*d*e - I*b^3*c*f)/f)^(1/3))*e^(-I*a - ((
I*b^3*d*e - I*b^3*c*f)/f)^(1/3)) + 12*(d*e - c*f)*sin((d*x + c)^(1/3)*b + a
))/(d*e^2*f - c*e*f^2 + (d*e*f^2 - c*f^3)*x)
```

Sympy [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{\sin\left(a + b\sqrt[3]{c + dx}\right)}{(e + fx)^2}\,dx = \int \frac{\sin\left((dx + c)^{\frac{1}{3}}b + a\right)}{\left(fx + e\right)^2}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e)^2, x)
```

Giac [F]

$$\int \frac{\sin\left(a + b\sqrt[3]{c + dx}\right)}{(e + fx)^2}\,dx = \int \frac{\sin\left((dx + c)^{\frac{1}{3}}b + a\right)}{\left(fx + e\right)^2}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a + b\sqrt[3]{c + dx}\right)}{(e + fx)^2}\,dx = \int \frac{\sin\left(a + b\,(c + d\,x)^{1/3}\right)}{\left(e + f\,x\right)^2}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(1/3))/(e + f*x)^2, x)
```

3.212 $\int (e+fx)^2 \sin\left(a+b(c+dx)^{2/3}\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 513

$$\begin{aligned}\int (e+fx)^2 \sin\left(a+b(c+dx)^{2/3}\right) \, dx = {} & \frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3} \\
& - \frac{3(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} \\
& + \frac{105f^2(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d^3} \\
& - \frac{3f(de-cf)(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{bd^3} \\
& - \frac{3f^2(c+dx)^{7/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} \\
& + \frac{3(de-cf)^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3} \\
& + \frac{315f^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{16b^{9/2}d^3} \\
& + \frac{315f^2\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{16b^{9/2}d^3} \\
& - \frac{3(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d^3} \\
& - \frac{315f^2\sqrt[3]{c+dx}\sin\left(a+b(c+dx)^{2/3}\right)}{16b^4d^3} \\
& + \frac{6f(de-cf)(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^3} \\
& + \frac{21f^2(c+dx)^{5/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d^3}\end{aligned}$$

```
[Out] 6*f*(-c*f+d*e)*cos(a+b*(d*x+c)^(2/3))/b^3/d^3-3/2*(-c*f+d*e)^2*(d*x+c)^(1/3
)*cos(a+b*(d*x+c)^(2/3))/b/d^3+105/8*f^2*(d*x+c)*cos(a+b*(d*x+c)^(2/3))/b^3
/d^3-3*f*(-c*f+d*e)*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(2/3))/b/d^3-3/2*f^2*(d*x
+c)^(7/3)*cos(a+b*(d*x+c)^(2/3))/b/d^3-315/16*f^2*(d*x+c)^(1/3)*sin(a+b*(d*
x+c)^(2/3))/b^4/d^3+6*f*(-c*f+d*e)*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(2/3))/b^2
/d^3+21/4*f^2*(d*x+c)^(5/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d^3+3/4*(-c*f+d*e)^2
*cos(a)*FresnelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b
^(3/2)/d^3+315/32*f^2*cos(a)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2
))*2^(1/2)*Pi^(1/2)/b^(9/2)/d^3+315/32*f^2*FresnelC((d*x+c)^(1/3)*b^(1/2)*2
^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(9/2)/d^3-3/4*(-c*f+d*e)^2*Fresn
elS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(3/2)
/d^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules

used = {3514, 3466, 3435, 3433, 3432, 3460, 3377, 2718, 3467, 3434}

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^{2/3}\right) dx = \frac{3\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)^2 \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3} - \frac{3\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)^2 \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3} + \frac{315\sqrt{\frac{\pi}{2}}f^2\sin(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{16b^{9/2}d^3} + \frac{315\sqrt{\frac{\pi}{2}}f^2\cos(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{16b^{9/2}d^3} - \frac{315f^2\sqrt[3]{c+dx}\sin\left(a+b(c+dx)^{2/3}\right)}{16b^4d^3} + \frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3} + \frac{105f^2(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d^3} + \frac{6f(c+dx)^{2/3}(de-cf)\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^3} + \frac{21f^2(c+dx)^{5/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d^3} - \frac{3f(c+dx)^{4/3}(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{bd^3} - \frac{3\sqrt[3]{c+dx}(de-cf)^2\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} - \frac{3f^2(c+dx)^{7/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3}$$

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (6*f*(d*e - c*f)*Cos[a + b*(c + d*x)^(2/3)])/(b^3*d^3) - (3*(d*e - c*f)^2*(
c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d^3) + (105*f^2*(c + d*x)*C
os[a + b*(c + d*x)^(2/3)])/(8*b^3*d^3) - (3*f*(d*e - c*f)*(c + d*x)^(4/3)*C
os[a + b*(c + d*x)^(2/3)])/(b*d^3) - (3*f^2*(c + d*x)^(7/3)*Cos[a + b*(c +
d*x)^(2/3)])/(2*b*d^3) + (3*(d*e - c*f)^2*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b
]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(2*b^(3/2)*d^3) + (315*f^2*Sqrt[Pi/2]*Cos[a]
*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(16*b^(9/2)*d^3) + (315*f^2*
Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(16*b^(9/2)
*d^3) - (3*(d*e - c*f)^2*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(
1/3)]*Sin[a])/(2*b^(3/2)*d^3) - (315*f^2*(c + d*x)^(1/3)*Sin[a + b*(c + d*x
)^(2/3)])/(16*b^4*d^3) + (6*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a + b*(c + d*
x)^(2/3)])/(b^2*d^3) + (21*f^2*(c + d*x)^(5/3)*Sin[a + b*(c + d*x)^(2/3)])/
(4*b^2*d^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
```

```
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{3\text{Subst}\left(\int \left((de-cf)^2x^2\sin\left(a+bx^2\right)-2f(-de+cf)x^5\sin\left(a+bx^2\right)+f^2x^8\sin\left(a+bx^2\right)\right)\,dx,x,\sqrt[3]{c+d}\right.}{d^3}$$

$$= \frac{(3f^2)\,\text{Subst}\left(\int x^8\sin\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^3} + \frac{(6f(de-cf))\text{Subst}\left(\int x^5\sin\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^3} + \frac{(3(de-cf)^2)\,\text{Subst}\left(\int x^2\sin\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^3}$$

$$= -\frac{3(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} - \frac{3f^2(c+dx)^{7/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} + \frac{(21f^2)\,\text{Subst}\left(\int x^6\cos\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{2bd^3} + \frac{(3f(de-cf))\text{Subst}\left(\int x^2\sin(a+bx)\,dx,x,(c+dx)^{2/3}\right)}{d^3} + \frac{(3(de-cf)^2)\,\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{2bd^3}$$

$$= -\frac{3(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} - \frac{3f(de-cf)(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{bd^3} - \frac{3f^2(c+dx)^{7/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} + \frac{21f^2(c+dx)^{5/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d^3} - \frac{(105f^2)\,\text{Subst}\left(\int x^4\sin\left(a+bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{4b^2d^3} + \frac{(6f(de-cf))\text{Subst}\left(\int x\cos(a+bx)\,dx, x, (c+dx)^{2/3}\right)}{bd^3} + \frac{(3(de-cf)^2\cos(a))\,\text{Subst}\left(\int\cos\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{2bd^3} - \frac{(3(de-cf)^2\sin(a))\,\text{Subst}\left(\int\sin\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{2bd^3}$$

$$= -\frac{3(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} + \frac{105f^2(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d^3} - \frac{3f(de-cf)(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{bd^3} - \frac{3f^2(c+dx)^{7/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} + \frac{3(de-cf)^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3} - \frac{3(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d^3} + \frac{6f(de-cf)(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^3} + \frac{21f^2(c+dx)^{5/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d^3} - \frac{(315f^2)\,\text{Subst}\left(\int x^2\cos\left(a+bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{8b^3d^3} - \frac{(6f(de-cf))\text{Subst}\left(\int\sin(a+bx)\,dx, x, (c+dx)^{2/3}\right)}{b^2d^3}$$

$$
\begin{aligned}
&= \frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3} - \frac{3(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} \\
&+ \frac{105f^2(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d^3} \\
&- \frac{3f(de-cf)(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{bd^3} \\
&- \frac{3f^2(c+dx)^{7/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} \\
&+ \frac{3(de-cf)^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3} \\
&- \frac{3(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d^3} \\
&- \frac{315f^2\sqrt[3]{c+dx}\sin\left(a+b(c+dx)^{2/3}\right)}{16b^4d^3} \\
&+ \frac{6f(de-cf)(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^3} \\
&+ \frac{21f^2(c+dx)^{5/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d^3} \\
&+ \frac{(315f^2)\,\text{Subst}\left(\int\sin\left(a+bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{16b^4d^3}
\end{aligned}
$$

$$= \frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3} - \frac{3(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3}$$
$$+ \frac{105f^2(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d^3}$$
$$- \frac{3f(de-cf)(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{bd^3}$$
$$- \frac{3f^2(c+dx)^{7/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3}$$
$$+ \frac{3(de-cf)^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3}$$
$$- \frac{3(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d^3}$$
$$- \frac{315f^2\sqrt[3]{c+dx}\sin\left(a+b(c+dx)^{2/3}\right)}{16b^4d^3}$$
$$+ \frac{6f(de-cf)(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^3}$$
$$+ \frac{21f^2(c+dx)^{5/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d^3}$$
$$+ \frac{(315f^2\cos(a))\,\text{Subst}\left(\int\sin\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{16b^4d^3}$$
$$+ \frac{(315f^2\sin(a))\,\text{Subst}\left(\int\cos\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{16b^4d^3}$$

$$
\begin{aligned}
&= \frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3} - \frac{3(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} \\
&\quad + \frac{105f^2(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d^3} \\
&\quad - \frac{3f(de-cf)(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{bd^3} \\
&\quad - \frac{3f^2(c+dx)^{7/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} \\
&\quad + \frac{3(de-cf)^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3} \\
&\quad + \frac{315f^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{16b^{9/2}d^3} \\
&\quad + \frac{315f^2\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{16b^{9/2}d^3} \\
&\quad - \frac{3(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d^3} \\
&\quad - \frac{315f^2\sqrt[3]{c+dx}\sin\left(a+b(c+dx)^{2/3}\right)}{16b^4d^3} \\
&\quad + \frac{6f(de-cf)(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^3} \\
&\quad + \frac{21f^2(c+dx)^{5/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.84

$$
\int (e+fx)^2\sin\left(a+b(c+dx)^{2/3}\right)\,dx = -\frac{3i\left((\cos(a)+i\sin(a))\left((1+i)\left(-105if^2+8b^3(de-cf)^2\right)\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{(1+i)\sqrt{b}\sqrt[3]{c+dx}}{\sqrt{2}}\right)+2\sqrt{b}\left(-105f^2\sqrt[3]{c-\right.\right.\right.}{}
$$

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (((-3*I)/64)*((Cos[a] + I*Sin[a])*((1 + I)*((-105*I)*f^2 + 8*b^3*(d*e - c*f
)^2)*Sqrt[Pi/2]*Erfi[((1 + I)*Sqrt[b]*(c + d*x)^(1/3))/Sqrt[2]] + 2*Sqrt[b]
*(-105*f^2*(c + d*x)^(1/3) - (8*I)*b^3*d^2*(c + d*x)^(1/3)*(e + f*x)^2 + 4*
b^2*f*(c + d*x)^(2/3)*(8*d*e - c*f + 7*d*f*x) + (2*I)*b*f*(16*d*e + 19*c*f
+ 35*d*f*x))*(Cos[b*(c + d*x)^(2/3)] + I*Sin[b*(c + d*x)^(2/3)])) + (2*Sqrt
```

```
[b]*(105*f^2*(c + d*x)^(1/3) - (8*I)*b^3*d^2*(c + d*x)^(1/3)*(e + f*x)^2 +
4*b^2*f*(c + d*x)^(2/3)*(-8*d*e + c*f - 7*d*f*x) + (2*I)*b*f*(16*d*e + 19*c
*f + 35*d*f*x)) + (1 + I)*((105*I)*f^2 + 8*b^3*(d*e - c*f)^2)*Sqrt[Pi/2]*Er
f[((1 + I)*Sqrt[b]*(c + d*x)^(1/3))/Sqrt[2]]*(Cos[b*(c + d*x)^(2/3)] + I*Si
n[b*(c + d*x)^(2/3)]))*(Cos[a + b*(c + d*x)^(2/3)] - I*Sin[a + b*(c + d*x)^
(2/3)])))/(b^(9/2)*d^3)
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-\frac{3f^{2}(dx+c)^{\frac{7}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{21f^{2}\left(\frac{(dx+c)^{\frac{5}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}-\frac{5\left(-\frac{(dx+c)\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{3(dx+c)^{\frac{1}{3}}\sin\left(a+b(dx+\right.}{4b}\right.}{}\right)}{2b}}{}$
default	$\frac{-\frac{3f^{2}(dx+c)^{\frac{7}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{21f^{2}\left(\frac{(dx+c)^{\frac{5}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}-\frac{5\left(-\frac{(dx+c)\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{3(dx+c)^{\frac{1}{3}}\sin\left(a+b(dx+\right.}{4b}\right.}{}\right)}{2b}}{}$
parts	Expression too large to display

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^3*(-1/2*f^2/b*(d*x+c)^(7/3)*cos(a+b*(d*x+c)^(2/3))+7/2*f^2/b*(1/2/b*(d*
x+c)^(5/3)*sin(a+b*(d*x+c)^(2/3))-5/2/b*(-1/2/b*(d*x+c)*cos(a+b*(d*x+c)^(2/
3))+3/2/b*(1/2/b*(d*x+c)^(1/3)*sin(a+b*(d*x+c)^(2/3))-1/4/b^(3/2)*2^(1/2)*P
i^(1/2)*(cos(a)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*Fre
snelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))))))+(c*f-d*e)*f/b*(d*x+c)^(4/
3)*cos(a+b*(d*x+c)^(2/3))-4*(c*f-d*e)*f/b*(1/2/b*(d*x+c)^(2/3)*sin(a+b*(d*x
+c)^(2/3))+1/2/b^2*cos(a+b*(d*x+c)^(2/3)))-1/2*(c*f-d*e)^2/b*(d*x+c)^(1/3)*
cos(a+b*(d*x+c)^(2/3))+1/4*(c*f-d*e)^2/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*Fre
```

```
snelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS((d*x+c)^(1/3)
*b^(1/2)*2^(1/2)/Pi^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.60

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^{2/3}\right)\,dx = \frac{3\left(\sqrt{2}(105\,\pi f^2\sin(a)+8\,\pi(b^3d^2e^2-2\,b^3cdef+b^3c^2f^2)\cos(a))\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)+\right.}{}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 3/32*(sqrt(2)*(105*pi*f^2*sin(a) + 8*pi*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*
c^2*f^2)*cos(a))*sqrt(b/pi)*fresnel_cos(sqrt(2)*(d*x + c)^(1/3)*sqrt(b/pi))
 + sqrt(2)*(105*pi*f^2*cos(a) - 8*pi*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2
*f^2)*sin(a))*sqrt(b/pi)*fresnel_sin(sqrt(2)*(d*x + c)^(1/3)*sqrt(b/pi)) +
4*(35*b^2*d*f^2*x + 16*b^2*d*e*f + 19*b^2*c*f^2 - 4*(b^4*d^2*f^2*x^2 + 2*b^
4*d^2*e*f*x + b^4*d^2*e^2)*(d*x + c)^(1/3))*cos((d*x + c)^(2/3)*b + a) - 2*
(105*(d*x + c)^(1/3)*b*f^2 - 4*(7*b^3*d*f^2*x + 8*b^3*d*e*f - b^3*c*f^2)*(d
*x + c)^(2/3))*sin((d*x + c)^(2/3)*b + a))/(b^5*d^3)
```

Sympy [F]

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^{2/3}\right)\,dx = \int (e+fx)^2 \sin\left(a+b(c+dx)^{\frac{2}{3}}\right)dx$$

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.09

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^{2/3}\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/128*(8*(sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c
)^(1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)
*sqrt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))*e
^2/b^3 - 16*(sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x +
 c)^(1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/
3)*sqrt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))
*c*e*f/(b^3*d) + 8*(sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf
((d*x + c)^(1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x +
 c)^(1/3)*sqrt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*
b + a))*c^2*f^2/(b^3*d^2) - 128*(2*(d*x + c)^(2/3)*b*sin((d*x + c)^(2/3)*b
+ a) - ((d*x + c)^(4/3)*b^2 - 2)*cos((d*x + c)^(2/3)*b + a))*e*f/(b^3*d) +
128*(2*(d*x + c)^(2/3)*b*sin((d*x + c)^(2/3)*b + a) - ((d*x + c)^(4/3)*b^2
- 2)*cos((d*x + c)^(2/3)*b + a))*c*f^2/(b^3*d^2) - (105*sqrt(2)*sqrt(pi)*((
(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*sqrt(I*b)) + (-(I - 1)
*cos(a) + (I + 1)*sin(a))*erf((d*x + c)^(1/3)*sqrt(-I*b)))*b^(3/2) - 16*(4*
(d*x + c)^(7/3)*b^5 - 35*(d*x + c)*b^3)*cos((d*x + c)^(2/3)*b + a) + 56*(4*
(d*x + c)^(5/3)*b^4 - 15*(d*x + c)^(1/3)*b^2)*sin((d*x + c)^(2/3)*b + a))*f
^2/(b^6*d^2))/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.52

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^{2/3}\right) \, dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/64*(8*e^2*(-I*sqrt(2)*sqrt(pi)*erf(-1/2*I*sqrt(2)*(d*x + c)^(1/3)*(I*b/a
bs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) + I*sqrt
(2)*sqrt(pi)*erf(1/2*I*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b
)))*e^(-I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + 2*(d*x + c)^(1/3)*e^(I*(d
*x + c)^(2/3)*b + I*a)/b + 2*(d*x + c)^(1/3)*e^(-I*(d*x + c)^(2/3)*b - I*a)
/b) + f^2*((sqrt(2)*sqrt(pi)*(-8*I*b^3*c^2 - 105)*erf(-1/2*I*sqrt(2)*(d*x +
 c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b^4*(I*b/abs(b) + 1)*sqrt
(abs(b))) - 2*I*(8*I*(d*x + c)^(7/3)*b^3 - 16*I*(d*x + c)^(4/3)*b^3*c + 8*I
*(d*x + c)^(1/3)*b^3*c^2 - 28*(d*x + c)^(5/3)*b^2 + 32*(d*x + c)^(2/3)*b^2*
c + 70*(-I*d*x - I*c)*b + 32*I*b*c + 105*(d*x + c)^(1/3))*e^(I*(d*x + c)^(2
/3)*b + I*a)/b^4)/d^2 - (sqrt(2)*sqrt(pi)*(-8*I*b^3*c^2 + 105)*erf(1/2*I*sq
rt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b^4*(-I*b/a
bs(b) + 1)*sqrt(abs(b))) + 2*I*(8*I*(d*x + c)^(7/3)*b^3 - 16*I*(d*x + c)^(4
/3)*b^3*c + 8*I*(d*x + c)^(1/3)*b^3*c^2 + 28*(d*x + c)^(5/3)*b^2 - 32*(d*x
+ c)^(2/3)*b^2*c + 70*(-I*d*x - I*c)*b + 32*I*b*c - 105*(d*x + c)^(1/3))*e^
(-I*(d*x + c)^(2/3)*b - I*a)/b^4)/d^2) + 16*(I*sqrt(2)*sqrt(pi)*c*erf(-1/2*
```

```
I*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b*(I*b/ab
s(b) + 1)*sqrt(abs(b))) - I*sqrt(2)*sqrt(pi)*c*erf(1/2*I*sqrt(2)*(d*x + c)^
(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(-I*b/abs(b) + 1)*sqrt(ab
s(b))) - 2*I*(I*(d*x + c)^(4/3)*b^2 - I*(d*x + c)^(1/3)*b^2*c - 2*(d*x + c)
^(2/3)*b - 2*I)*e^(I*(d*x + c)^(2/3)*b + I*a)/b^3 - 2*I*(I*(d*x + c)^(4/3)*
b^2 - I*(d*x + c)^(1/3)*b^2*c + 2*(d*x + c)^(2/3)*b - 2*I)*e^(-I*(d*x + c)^
(2/3)*b - I*a)/b^3)*e*f/d)/d
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^{2/3}\right)\,(e+f\,x)^2\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(2/3))*(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(2/3))*(e + f*x)^2, x)
```

3.213 $\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 243

$$\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\frac{3f\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^2}-\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}-\frac{3f(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}+\frac{3(de-cf)\sqrt{\frac{\pi}{2}}\cos(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^2}-\frac{3(de-cf)\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d^2}+\frac{3f(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^2}$$

```
[Out] 3*f*cos(a+b*(d*x+c)^(2/3))/b^3/d^2-3/2*(-c*f+d*e)*(d*x+c)^(1/3)*cos(a+b*(d*
x+c)^(2/3))/b/d^2-3/2*f*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(2/3))/b/d^2+3*f*(d*x
+c)^(2/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d^2+3/4*(-c*f+d*e)*cos(a)*FresnelC((d*
x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/d^2-3/4*(-c*f
+d*e)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1
/2)/b^(3/2)/d^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3514, 3466, 3435, 3433, 3432, 3460, 3377, 2718}

$$\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\frac{3\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^2}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^2}+\frac{3f\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^2}+\frac{3f(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^2}-\frac{3\sqrt[3]{c+dx}(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}-\frac{3f(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}$$

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (3*f*Cos[a + b*(c + d*x)^(2/3)])/(b^3*d^2) - (3*(d*e - c*f)*(c + d*x)^(1/3)
*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d^2) - (3*f*(c + d*x)^(4/3)*Cos[a + b*(c
+ d*x)^(2/3)])/(2*b*d^2) + (3*(d*e - c*f)*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b
]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(2*b^(3/2)*d^2) - (3*(d*e - c*f)*Sqrt[Pi/2]*
FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(2*b^(3/2)*d^2) + (3*f
*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)])/(b^2*d^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3\text{Subst}\left(\int \left((de-cf)x^2\sin\left(a+bx^2\right)+fx^5\sin\left(a+bx^2\right)\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}\\
&= \frac{(3f)\text{Subst}\left(\int x^5\sin\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}\\
&\quad+\frac{(3(de-cf))\text{Subst}\left(\int x^2\sin\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}\\
&= -\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}\\
&\quad+\frac{(3f)\text{Subst}\left(\int x^2\sin(a+bx)\,dx,x,(c+dx)^{2/3}\right)}{2d^2}\\
&\quad+\frac{(3(de-cf))\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{2bd^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2} - \frac{3f(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2} \\
&\quad + \frac{(3f)\text{Subst}\left(\int x\cos(a+bx)\,dx, x, (c+dx)^{2/3}\right)}{bd^2} \\
&\quad + \frac{(3(de-cf)\cos(a))\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{2bd^2} \\
&\quad - \frac{(3(de-cf)\sin(a))\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{2bd^2} \\
&= -\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2} - \frac{3f(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2} \\
&\quad + \frac{3(de-cf)\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^2} \\
&\quad - \frac{3(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d^2} \\
&\quad + \frac{3f(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^2} \\
&\quad - \frac{(3f)\text{Subst}\left(\int \sin(a+bx)\,dx, x, (c+dx)^{2/3}\right)}{b^2d^2} \\
&= \frac{3f\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^2} - \frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2} \\
&\quad - \frac{3f(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2} \\
&\quad + \frac{3(de-cf)\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^2} \\
&\quad - \frac{3(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d^2} \\
&\quad + \frac{3f(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.88

$$
\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx = \frac{3\left(4f\cos\left(a+b(c+dx)^{2/3}\right) - 2b^2de\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right) - 2b^2dfx\sqrt[3]{c+dx}\cos\left(a\right.\right.}{}
$$

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^(2/3)],x]
```

[Out] (3*(4*f*Cos[a + b*(c + d*x)^(2/3)] - 2*b^2*d*e*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)] - 2*b^2*d*f*x*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)] + b^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)] - b^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a] + 4*b*f*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)]))/(4*b^3*d^2)

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{-\frac{3f(dx+c)^{\frac{4}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{6f\left(\frac{(dx+c)^{\frac{2}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b^{2}}\right)}{b}+\frac{3(cf-de)(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}\cdots}{d^{2}}$
default	$\frac{-\frac{3f(dx+c)^{\frac{4}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{6f\left(\frac{(dx+c)^{\frac{2}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b^{2}}\right)}{b}+\frac{3(cf-de)(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}\cdots}{d^{2}}$
parts	$-\frac{3(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)fx}{2db}-\frac{3(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)e}{2db}+\frac{3\sqrt{2}\,\sqrt{\pi}\,\cos(a)\,\mathrm{C}\left(\frac{(dx+c)^{\frac{1}{3}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)fx}{4d\,b^{\frac{3}{2}}}+\cdots$

[In] int((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)

[Out] 3/d^2*(-1/2*f/b*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(2/3))+2*f/b*(1/2/b*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(2/3))+1/2/b^2*cos(a+b*(d*x+c)^(2/3)))+1/2*(c*f-d*e)/b*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))-1/4*(c*f-d*e)/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.65

$$\int (e+fx)\sin \left(a+b(c+dx)^{2/3}\right)\,dx = \frac{3\left(\sqrt{2}\pi(bde-bcf)\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)-\sqrt{2}\pi(bde-bcf)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\right.\right.}{}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 3/4*(sqrt(2)*pi*(b*d*e - b*c*f)*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*(d*x
+ c)^(1/3)*sqrt(b/pi)) - sqrt(2)*pi*(b*d*e - b*c*f)*sqrt(b/pi)*fresnel_sin(
sqrt(2)*(d*x + c)^(1/3)*sqrt(b/pi))*sin(a) + 4*(d*x + c)^(2/3)*b*f*sin((d*x
 + c)^(2/3)*b + a) - 2*((b^2*d*f*x + b^2*d*e)*(d*x + c)^(1/3) - 2*f)*cos((d
*x + c)^(2/3)*b + a))/(b^3*d^2)
```

Sympy [F]

$$\int (e+fx)\sin \left(a+b(c+dx)^{2/3}\right)\,dx = \int (e+fx)\sin \left(a+b(c+dx)^{\frac{2}{3}}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b*(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02

$$\int (e+fx)\sin \left(a+b(c+dx)^{2/3}\right)\,dx = -\frac{3\left(\frac{\left(\sqrt{2}\sqrt{\pi}\left(((i-1)\cos(a)+(i+1)\sin(a))\,\mathrm{erf}\left((dx+c)^{\frac{1}{3}}\sqrt{i\,b}\right)+(-(i+1)\cos(a)-(i-1)\sin(a))\,\mathrm{erf}\left((dx+c)^{\frac{1}{3}}\sqrt{-i\,b}\right)\right)b^{\frac{3}{2}}+8\,(dx+c)^{\frac{1}{3}}b^2\cos\left((dx+c\right.\right.}{b^3}\right.}{}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/16*((sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c)^(
1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*sq
rt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))*e/b^
```

```
3 - (sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c)^(1/3
)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*sqrt(
-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))*c*f/(b^
3*d) - 8*(2*(d*x + c)^(2/3)*b*sin((d*x + c)^(2/3)*b + a) - ((d*x + c)^(4/3)
*b^2 - 2)*cos((d*x + c)^(2/3)*b + a))*f/(b^3*d))/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.68

$$\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx = -\frac{3\left(e\left(-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}i\sqrt{2}(dx+c)^{\frac{1}{3}}\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(ia)}}{b\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}}+\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}i\sqrt{2}(dx+c)^{\frac{1}{3}}\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(-ia)}}{b\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}}+\frac{2(dx+c)^{\frac{1}{3}}e^{\left(i(dx+c)^{\frac{2}{3}}b+i\cdots\right.}}{b}\cdots\right.\right.}{\cdots}$$

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/8*(e*(-I*sqrt(2)*sqrt(pi)*erf(-1/2*I*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b)
 + 1)*sqrt(abs(b)))*e^(I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) + I*sqrt(2)*s
qrt(pi)*erf(1/2*I*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e
^(-I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + 2*(d*x + c)^(1/3)*e^(I*(d*x +
c)^(2/3)*b + I*a)/b + 2*(d*x + c)^(1/3)*e^(-I*(d*x + c)^(2/3)*b - I*a)/b) +
 (I*sqrt(2)*sqrt(pi)*c*erf(-1/2*I*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*
sqrt(abs(b)))*e^(I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) - I*sqrt(2)*sqrt(pi
)*c*erf(1/2*I*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I
*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) - 2*I*(I*(d*x + c)^(4/3)*b^2 - I*(d*
x + c)^(1/3)*b^2*c - 2*(d*x + c)^(2/3)*b - 2*I)*e^(I*(d*x + c)^(2/3)*b + I*
a)/b^3 - 2*I*(I*(d*x + c)^(4/3)*b^2 - I*(d*x + c)^(1/3)*b^2*c + 2*(d*x + c)
^(2/3)*b - 2*I)*e^(-I*(d*x + c)^(2/3)*b - I*a)/b^3)*f/d)/d
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^{2/3}\right)\,(e+f\,x)\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(2/3))*(e + f*x),x)
[Out] int(sin(a + b*(c + d*x)^(2/3))*(e + f*x), x)
```

3.214 $\int \sin\left(a+b(c+dx)^{2/3}\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 130

$$\int \sin\left(a+b(c+dx)^{2/3}\right) dx = -\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d}$$

```
[Out] -3/2*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))/b/d+3/4*cos(a)*FresnelC((d*x+c)^(
1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/d-3/4*FresnelS((d*x
+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(3/2)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3444, 3466, 3435, 3433, 3432}

$$\int \sin\left(a+b(c+dx)^{2/3}\right) dx = \frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d} - \frac{3\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d} - \frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}$$

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d) + (3*Sqrt[Pi/2]*Cos
[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(2*b^(3/2)*d) - (3*Sqrt[P
i/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(2*b^(3/2)*d)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3444

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Dist[k/f, Subst[Int[x^(k - 1)*(a + b
*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3\text{Subst}\left(\int x^2 \sin\left(a+bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d}\\
&= -\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{3\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{2bd}\\
&= -\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{(3\cos(a))\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{2bd}\\
&\quad - \frac{(3\sin(a))\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{2bd}
\end{aligned}$$

$$= -\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \sin\left(a+b(c+dx)^{2/3}\right)\,dx =$$

$$-\frac{3\left(2\sqrt{b}\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)-\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)+\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{\right.\right.}{4b^{3/2}d}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(2*Sqrt[b]*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)] - Sqrt[2*Pi]*Cos[
a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)] + Sqrt[2*Pi]*FresnelS[Sqrt[
b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a]))/(4*b^(3/2)*d)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{-\frac{3(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{3\sqrt{2}\sqrt{\pi}\left(\cos(a)\,C\left(\frac{(dx+c)^{\frac{1}{3}}\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\,S\left(\frac{(dx+c)^{\frac{1}{3}}\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4b^{\frac{3}{2}}}}{d}$	86
default	$\frac{-\frac{3(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{3\sqrt{2}\sqrt{\pi}\left(\cos(a)\,C\left(\frac{(dx+c)^{\frac{1}{3}}\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\,S\left(\frac{(dx+c)^{\frac{1}{3}}\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4b^{\frac{3}{2}}}}{d}$	86

```
[In] int(sin(a+b*(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*(-1/2/b*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))+1/4/b^(3/2)*2^(1/2)*Pi^(1/
2)*(cos(a)*FresnelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS
((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \sin \left(a+b(c+dx)^{2/3}\right) dx = \frac{3 \left(\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right) - \sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)\sin(a) - 2\,(dx+c$$
$$4\,b^2d$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 3/4*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*(d*x + c)^(1/3)*sqrt(
b/pi)) - sqrt(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*(d*x + c)^(1/3)*sqrt(b/p
i))*sin(a) - 2*(d*x + c)^(1/3)*b*cos((d*x + c)^(2/3)*b + a))/(b^2*d)
```

Sympy [F]

$$\int \sin \left(a+b(c+dx)^{2/3}\right) dx = \int \sin \left(a+b(c+dx)^{\frac{2}{3}}\right) dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.71

$$\int \sin \left(a+b(c+dx)^{2/3}\right) dx =$$
$$-\frac{3 \left(\sqrt{2}\sqrt{\pi}\left(((i-1)\,\cos(a)+(i+1)\,\sin(a))\,\mathrm{erf}\left((dx+c)^{\frac{1}{3}}\sqrt{i\,b}\right)+(-(i+1)\,\cos(a)-(i-1)\,\sin(a))\,\mathrm{erf}\right.\right.}{16\,b^3d}$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/16*(sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c)^(1
/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*sqr
t(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))/(b^3*
d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.32

$$\int \sin \left(a+b(c+dx)^{2/3}\right)\,dx = -\frac{3\left(-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}i\sqrt{2}(dx+c)^{\frac{1}{3}}\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(ia)}}{b\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}}+\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}i\sqrt{2}(dx+c)^{\frac{1}{3}}\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(-ia)}}{b\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}}+\frac{2(dx+c)^{\frac{1}{3}}e^{\left(i(dx+c)^{\frac{2}{3}}b+ia\right)}}{b}\right.}{8d}$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/8*(-I*sqrt(2)*sqrt(pi)*erf(-1/2*I*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) +
1)*sqrt(abs(b)))*e^(I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) + I*sqrt(2)*sqrt
(pi)*erf(1/2*I*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-
I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + 2*(d*x + c)^(1/3)*e^(I*(d*x + c)^
(2/3)*b + I*a)/b + 2*(d*x + c)^(1/3)*e^(-I*(d*x + c)^(2/3)*b - I*a)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sin \left(a+b(c+dx)^{2/3}\right)\,dx = \int \sin \left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(2/3)),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(2/3)), x)
```

3.215 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx} dx = \text{Int}\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}, x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx} dx = \int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx} dx$$

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x),x]
[Out] Defer[Int][Sin[a + b*(c + d*x)^(2/3)]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx} dx$$

Mathematica [N/A]

Not integrable

Time = 63.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x),x]
[Out] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin \left(a+b(dx+c)^{\frac{2}{3}}\right)}{fx+e}dx$$

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x)
[Out] int(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^{\frac{2}{3}}b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x, algorithm="fricas")
[Out] integral(sin((d*x + c)^(2/3)*b + a)/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{e+fx}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(f*x+e),x)
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 6.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{e+fx} \, dx = \int \frac{\sin \left(a+b\,(c+d\,x)^{2/3}\right)}{e+f\,x} \, dx$$

```
[In] int(sin(a + b*(c + d*x)^(2/3))/(e + f*x),x)
[Out] int(sin(a + b*(c + d*x)^(2/3))/(e + f*x), x)
```

3.216 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx=\mathrm{Int}\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx$$

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 58.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin \left(a+b(dx+c)^{\frac{2}{3}}\right)}{(fx+e)^2}dx$$

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x)
[Out] int(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left((dx+c)^{\frac{2}{3}}b+a\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sin((d*x + c)^(2/3)*b + a)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{\left(e+fx\right)^{2}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(f*x+e)**2,x)
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{\left(fx+e\right)^{2}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{\left(fx+e\right)^{2}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)}{{\left(e+f\,x\right)}^2}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(2/3))/(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(2/3))/(e + f*x)^2, x)
```

3.217 $\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 855

$$\int (e+fx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx=\frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3}-\frac{b^7f^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3}+\frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3}-\frac{b^3f(de-cf)(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60d^3}+\frac{b^5f^2(c+dx)^{4/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20160d^3}+\frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3}-\frac{b^3f^2(c+dx)^2\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{1008d^3}+\frac{bf^2(c+dx)^{8/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{24d^3}-\frac{b^9f^2\cos(a)\operatorname{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3}+\frac{b^3(de-cf)^2\cos(a)\operatorname{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3}+\frac{b^6f(de-cf)\operatorname{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{120d^3}+\frac{b^8f^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3}-\frac{b^2(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3}+\frac{b^4f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3}-\frac{b^6f^2(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60480d^3}$$

```
[Out] -1/120960*b^9*f^2*Ci(b/(d*x+c)^(1/3))*cos(a)/d^3+1/2*b^3*(-c*f+d*e)^2*Ci(b/
(d*x+c)^(1/3))*cos(a)/d^3+1/120*b^5*f*(-c*f+d*e)*(d*x+c)^(1/3)*cos(a+b/(d*x
+c)^(1/3))/d^3-1/120960*b^7*f^2*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(1/3))/d^3+1/
2*b*(-c*f+d*e)^2*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(1/3))/d^3-1/60*b^3*f*(-c*f+
d*e)*(d*x+c)*cos(a+b/(d*x+c)^(1/3))/d^3+1/20160*b^5*f^2*(d*x+c)^(4/3)*cos(a
+b/(d*x+c)^(1/3))/d^3+1/5*b*f*(-c*f+d*e)*(d*x+c)^(5/3)*cos(a+b/(d*x+c)^(1/3
))/d^3-1/1008*b^3*f^2*(d*x+c)^2*cos(a+b/(d*x+c)^(1/3))/d^3+1/24*b*f^2*(d*x+
c)^(8/3)*cos(a+b/(d*x+c)^(1/3))/d^3+1/120*b^6*f*(-c*f+d*e)*cos(a)*Si(b/(d*x
+c)^(1/3))/d^3+1/120*b^6*f*(-c*f+d*e)*Ci(b/(d*x+c)^(1/3))*sin(a)/d^3+1/1209
60*b^9*f^2*Si(b/(d*x+c)^(1/3))*sin(a)/d^3-1/2*b^3*(-c*f+d*e)^2*Si(b/(d*x+c)
^(1/3))*sin(a)/d^3+1/120960*b^8*f^2*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3))/d^
3-1/2*b^2*(-c*f+d*e)^2*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3))/d^3+1/120*b^4*f
*(-c*f+d*e)*(d*x+c)^(2/3)*sin(a+b/(d*x+c)^(1/3))/d^3-1/60480*b^6*f^2*(d*x+c
)*sin(a+b/(d*x+c)^(1/3))/d^3+(-c*f+d*e)^2*(d*x+c)*sin(a+b/(d*x+c)^(1/3))/d^
3-1/20*b^2*f*(-c*f+d*e)*(d*x+c)^(4/3)*sin(a+b/(d*x+c)^(1/3))/d^3+1/5040*b^4
*f^2*(d*x+c)^(5/3)*sin(a+b/(d*x+c)^(1/3))/d^3+f*(-c*f+d*e)*(d*x+c)^2*sin(a+
b/(d*x+c)^(1/3))/d^3-1/168*b^2*f^2*(d*x+c)^(7/3)*sin(a+b/(d*x+c)^(1/3))/d^3
+1/3*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^(1/3))/d^3
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used

$= \{3512, 3378, 3384, 3380, 3383\}$

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) dx = -\frac{f^2 \cos(a) \operatorname{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right) b^9}{120960d^3} + \frac{f^2 \sin(a)\operatorname{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right) b^9}{120960d^3} + \frac{f^2 \sqrt[3]{c+dx} \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) b^8}{120960d^3} - \frac{f^2 (c+dx)^{2/3} \cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) b^7}{120960d^3} + \frac{f(de-cf) \operatorname{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right) \sin(a) b^6}{120d^3} - \frac{f^2 (c+dx) \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) b^6}{60480d^3} + \frac{f(de-cf) \cos(a)\operatorname{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right) b^6}{120d^3} + \frac{f^2 (c+dx)^{4/3} \cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) b^5}{20160d^3} + \frac{f(de-cf) \sqrt[3]{c+dx} \cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) b^5}{120d^3} + \frac{f^2 (c+dx)^{5/3} \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) b^4}{5040d^3} + \frac{f(de-cf)(c+dx)^{2/3} \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) b^4}{120d^3} - \frac{f^2 (c+dx)^2 \cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) b^3}{1008d^3} - \frac{f(de-cf)(c+dx) \cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) b^3}{60d^3} + \frac{(de-cf)^2 \cos(a) \operatorname{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right) b^3}{2d^3} - \frac{(de-cf)^2 \sin(a)\operatorname{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right) b^3}{2d^3} + \frac{f^2 (c+dx)^{7/3} \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) b^2}{}$$

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (b^5*f*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(120*d^3) -
(b^7*f^2*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(120960*d^3) + (b*(d*e
 - c*f)^2*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d^3) - (b^3*f*(d*e
 - c*f)*(c + d*x)*Cos[a + b/(c + d*x)^(1/3)])/(60*d^3) + (b^5*f^2*(c + d*x)
^(4/3)*Cos[a + b/(c + d*x)^(1/3)])/(20160*d^3) + (b*f*(d*e - c*f)*(c + d*x)
^(5/3)*Cos[a + b/(c + d*x)^(1/3)])/(5*d^3) - (b^3*f^2*(c + d*x)^2*Cos[a + b
/(c + d*x)^(1/3)])/(1008*d^3) + (b*f^2*(c + d*x)^(8/3)*Cos[a + b/(c + d*x)^
(1/3)])/(24*d^3) - (b^9*f^2*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)])/(120960*
d^3) + (b^3*(d*e - c*f)^2*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)])/(2*d^3) +
(b^6*f*(d*e - c*f)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(120*d^3) + (b^8*
f^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(120960*d^3) - (b^2*(d*e -
c*f)^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(2*d^3) + (b^4*f*(d*e -
c*f)*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(120*d^3) - (b^6*f^2*(c +
d*x)*Sin[a + b/(c + d*x)^(1/3)])/(60480*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin
[a + b/(c + d*x)^(1/3)])/d^3 - (b^2*f*(d*e - c*f)*(c + d*x)^(4/3)*Sin[a + b
/(c + d*x)^(1/3)])/(20*d^3) + (b^4*f^2*(c + d*x)^(5/3)*Sin[a + b/(c + d*x)^
(1/3)])/(5040*d^3) + (f*(d*e - c*f)*(c + d*x)^2*Sin[a + b/(c + d*x)^(1/3)])
/d^3 - (b^2*f^2*(c + d*x)^(7/3)*Sin[a + b/(c + d*x)^(1/3)])/(168*d^3) + (f^
2*(c + d*x)^3*Sin[a + b/(c + d*x)^(1/3)])/(3*d^3) + (b^6*f*(d*e - c*f)*Cos[
a]*SinIntegral[b/(c + d*x)^(1/3)])/(120*d^3) + (b^9*f^2*Sin[a]*SinIntegral[
b/(c + d*x)^(1/3)])/(120960*d^3) - (b^3*(d*e - c*f)^2*Sin[a]*SinIntegral[b/
(c + d*x)^(1/3)])/(2*d^3)
```

<u>Rule 3378</u>

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

<u>Rule 3380</u>

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

<u>Rule 3383</u>

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

<u>Rule 3384</u>

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = -\frac{3\text{Subst}\left(\int \left(\frac{f^2\sin(a+bx)}{d^2x^{10}}+\frac{2f(de-cf)\sin(a+bx)}{d^2x^7}+\frac{(de-cf)^2\sin(a+bx)}{d^2x^4}\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{(3f^2)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{10}}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{(6f(de-cf))\text{Subst}\left(\int \frac{\sin(a+bx)}{x^7}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{(3(de-cf)^2)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d^3}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3d^3} - \frac{(bf^2)\,\text{Subst}\left(\int \frac{\cos(a+bx)}{x^9}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{3d^3} - \frac{(bf(de-cf))\text{Subst}\left(\int \frac{\cos(a+bx)}{x^6}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{(b(de-cf)^2)\,\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d^3}$$

$$= \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} + \frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3} + \frac{bf^2(c+dx)^{8/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{24d^3} + \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3d^3} + \frac{(b^2f^2)\operatorname{Subst}\left(\int\frac{\sin(a+bx)}{x^8}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{24d^3} + \frac{(b^2f(de-cf))\operatorname{Subst}\left(\int\frac{\sin(a+bx)}{x^5}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{5d^3} + \frac{(b^2(de-cf)^2)\operatorname{Subst}\left(\int\frac{\sin(a+bx)}{x^2}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{2d^3}$$

$$
\begin{aligned}
&= \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&+ \frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3} \\
&+ \frac{bf^2(c+dx)^{8/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{24d^3} - \frac{b^2(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&+ \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} \\
&- \frac{b^2f(de-cf)(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20d^3} \\
&+ \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{b^2f^2(c+dx)^{7/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{168d^3} \\
&+ \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3d^3} + \frac{(b^3f^2)\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^7}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{168d^3} \\
&+ \frac{(b^3f(de-cf))\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{20d^3} \\
&+ \frac{(b^3(de-cf)^2)\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&- \frac{b^3 f(de-cf)(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60d^3} \\
&+ \frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3} \\
&- \frac{b^3 f^2(c+dx)^2\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{1008d^3} + \frac{bf^2(c+dx)^{8/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{24d^3} \\
&- \frac{b^2(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&+ \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} \\
&- \frac{b^2 f(de-cf)(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20d^3} \\
&+ \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{b^2 f^2(c+dx)^{7/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{168d^3} \\
&+ \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3d^3} - \frac{(b^4 f^2)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^6}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{1008d^3} \\
&- \frac{(b^4 f(de-cf))\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{60d^3} \\
&+ \frac{(b^3(de-cf)^2\cos(a))\,\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&- \frac{(b^3(de-cf)^2\sin(a))\,\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&- \frac{b^3f(de-cf)(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60d^3} \\
&+ \frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3} \\
&- \frac{b^3f^2(c+dx)^2\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{1008d^3} + \frac{bf^2(c+dx)^{8/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{24d^3} \\
&+ \frac{b^3(de-cf)^2\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&- \frac{b^2(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&+ \frac{b^4f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} \\
&+ \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} \\
&- \frac{b^2f(de-cf)(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20d^3} \\
&+ \frac{b^4f^2(c+dx)^{5/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5040d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} \\
&- \frac{b^2f^2(c+dx)^{7/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{168d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3d^3} \\
&- \frac{b^3(de-cf)^2\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} - \frac{(b^5f^2)\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^5}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{5040d^3} \\
&- \frac{(b^5f(de-cf))\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{120d^3}
\end{aligned}
$$

$$= \frac{b^5 f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} + \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} - \frac{b^3 f(de-cf)(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60d^3} + \frac{b^5 f^2(c+dx)^{4/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20160d^3} + \frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3} - \frac{b^3 f^2(c+dx)^2\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{1008d^3} + \frac{bf^2(c+dx)^{8/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{24d^3} + \frac{b^3(de-cf)^2\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} - \frac{b^2(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} + \frac{b^4 f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} + \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{b^2 f(de-cf)(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20d^3} + \frac{b^4 f^2(c+dx)^{5/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5040d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{b^2 f^2(c+dx)^{7/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{168d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3d^3} - \frac{b^3(de-cf)^2\sin(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} + \frac{(b^6 f^2)\,\mathrm{Subst}\left(\int\frac{\sin(a+bx)}{x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{20160d^3} + \frac{(b^6 f(de-cf))\,\mathrm{Subst}\left(\int\frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{120d^3}$$

$$= \frac{b^5 f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} + \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} - \frac{b^3 f(de-cf)(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60d^3} + \frac{b^5 f^2(c+dx)^{4/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20160d^3} + \frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3} - \frac{b^3 f^2(c+dx)^2\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{1008d^3} + \frac{bf^2(c+dx)^{8/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{24d^3} + \frac{b^3(de-cf)^2\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} - \frac{b^2(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} + \frac{b^4 f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^6 f^2(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60480d^3} + \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{b^2 f(de-cf)(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20d^3} + \frac{b^4 f^2(c+dx)^{5/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5040d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{b^2 f^2(c+dx)^{7/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{168d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3d^3} - \frac{b^3(de-cf)^2\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} + \frac{(b^7 f^2)\,\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{60480d^3} + \frac{(b^6 f(de-cf)\cos(a))\,\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{120d^3} + \frac{(b^6 f(de-cf)\sin(a))\,\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{120d^3}$$

$$
\begin{aligned}
&= \frac{b^5 f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^7 f^2 (c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3} \\
&+ \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&- \frac{b^3 f(de-cf)(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60d^3} \\
&+ \frac{b^5 f^2 (c+dx)^{4/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20160d^3} \\
&+ \frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3} \\
&- \frac{b^3 f^2 (c+dx)^2\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{1008d^3} + \frac{bf^2(c+dx)^{8/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{24d^3} \\
&+ \frac{b^3(de-cf)^2\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&+ \frac{b^6 f(de-cf)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{120d^3} \\
&- \frac{b^2(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&+ \frac{b^4 f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} \\
&- \frac{b^6 f^2 (c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60480d^3} + \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} \\
&- \frac{b^2 f(de-cf)(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20d^3} \\
&+ \frac{b^4 f^2 (c+dx)^{5/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5040d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} \\
&- \frac{b^2 f^2 (c+dx)^{7/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{168d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3d^3} \\
&+ \frac{b^6 f(de-cf)\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^3(de-cf)^2\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&- \frac{(b^8 f^2)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{120960d^3}
\end{aligned}
$$

$$= \frac{b^5 f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^7 f^2 (c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3}$$
$$+ \frac{b(de-cf)^2 (c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3}$$
$$- \frac{b^3 f(de-cf)(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60d^3}$$
$$+ \frac{b^5 f^2 (c+dx)^{4/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20160d^3}$$
$$+ \frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3}$$
$$- \frac{b^3 f^2 (c+dx)^2\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{1008d^3} + \frac{bf^2 (c+dx)^{8/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{24d^3}$$
$$+ \frac{b^3 (de-cf)^2 \cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3}$$
$$+ \frac{b^6 f(de-cf)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{120d^3}$$
$$+ \frac{b^8 f^2 \sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3} - \frac{b^2 (de-cf)^2 \sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3}$$
$$+ \frac{b^4 f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3}$$
$$- \frac{b^6 f^2 (c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60480d^3} + \frac{(de-cf)^2 (c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3}$$
$$- \frac{b^2 f(de-cf)(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20d^3}$$
$$+ \frac{b^4 f^2 (c+dx)^{5/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5040d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3}$$
$$- \frac{b^2 f^2 (c+dx)^{7/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{168d^3} + \frac{f^2 (c+dx)^3\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3d^3}$$
$$+ \frac{b^6 f(de-cf)\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^3 (de-cf)^2 \sin(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3}$$
$$- \frac{(b^9 f^2)\,\mathrm{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{120960d^3}$$

$$
\begin{aligned}
&= \frac{b^5 f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^7 f^2 (c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3} \\
&+ \frac{b(de-cf)^2 (c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&- \frac{b^3 f(de-cf)(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60d^3} \\
&+ \frac{b^5 f^2 (c+dx)^{4/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20160d^3} \\
&+ \frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3} \\
&- \frac{b^3 f^2 (c+dx)^2\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{1008d^3} + \frac{bf^2 (c+dx)^{8/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{24d^3} \\
&+ \frac{b^3 (de-cf)^2 \cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&+ \frac{b^6 f(de-cf)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{120d^3} \\
&+ \frac{b^8 f^2 \sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3} - \frac{b^2 (de-cf)^2 \sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&+ \frac{b^4 f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} \\
&- \frac{b^6 f^2 (c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60480d^3} + \frac{(de-cf)^2 (c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} \\
&- \frac{b^2 f(de-cf)(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20d^3} \\
&+ \frac{b^4 f^2 (c+dx)^{5/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5040d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} \\
&- \frac{b^2 f^2 (c+dx)^{7/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{168d^3} + \frac{f^2 (c+dx)^3\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3d^3} \\
&+ \frac{b^6 f(de-cf)\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^3 (de-cf)^2 \sin(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&- \frac{(b^9 f^2 \cos(a))\,\mathrm{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{120960d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^5 f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^7 f^2 (c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3} \\
&+ \frac{b(de-cf)^2 (c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&- \frac{b^3 f(de-cf)(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60d^3} \\
&+ \frac{b^5 f^2 (c+dx)^{4/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20160d^3} \\
&+ \frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3} \\
&- \frac{b^3 f^2 (c+dx)^2\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{1008d^3} + \frac{bf^2 (c+dx)^{8/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{24d^3} \\
&- \frac{b^9 f^2 \cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3} \\
&+ \frac{b^3 (de-cf)^2 \cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&+ \frac{b^6 f(de-cf)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{120d^3} \\
&+ \frac{b^8 f^2 \sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3} - \frac{b^2 (de-cf)^2 \sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} \\
&+ \frac{b^4 f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} \\
&- \frac{b^6 f^2 (c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60480d^3} + \frac{(de-cf)^2 (c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} \\
&- \frac{b^2 f(de-cf)(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{20d^3} \\
&+ \frac{b^4 f^2 (c+dx)^{5/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5040d^3} \\
&+ \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{b^2 f^2 (c+dx)^{7/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{168d^3} \\
&+ \frac{f^2 (c+dx)^3\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3d^3} + \frac{b^6 f(de-cf)\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 929, normalized size of antiderivative = 1.09

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx =$$
$$-\frac{i\left((\cos(a)+i\sin(a))\left(60480ib^3d^2e^2\,\text{ExpIntegralEi}\left(\frac{ib}{\sqrt[3]{c+dx}}\right)+1008b^6def\,\text{ExpIntegralEi}\left(\frac{ib}{\sqrt[3]{c+dx}}\right.\right.\right.}{}$$

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] ((-1/241920*I)*((Cos[a] + I*Sin[a])*((60480*I)*b^3*d^2*e^2*ExpIntegralEi[(I
*b)/(c + d*x)^(1/3)] + 1008*b^6*d*e*f*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)]
- (120960*I)*b^3*c*d*e*f*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)] - I*b^9*f^2*E
xpIntegralEi[(I*b)/(c + d*x)^(1/3)] - 1008*b^6*c*f^2*ExpIntegralEi[(I*b)/(c
 + d*x)^(1/3)] + (60480*I)*b^3*c^2*f^2*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)]
 + (c + d*x)^(1/3)*(b^8*f^2 - I*b^7*f^2*(c + d*x)^(1/3) - 2*b^6*f^2*(c + d*
x)^(2/3) + (24*I)*b^3*f*(c + d*x)^(2/3)*(-84*d*e + 79*c*f - 5*d*f*x) + (6*I
)*b^5*f*(168*d*e - 167*c*f + d*f*x) + 24*b^4*f*(c + d*x)^(1/3)*(42*d*e - 41
*c*f + d*f*x) + 40320*(c + d*x)^(2/3)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3
*e^2 + 3*e*f*x + f^2*x^2)) + (1008*I)*b*(c + d*x)^(1/3)*(41*c^2*f^2 - 2*c*d
*f*(48*e + 7*f*x) + d^2*(60*e^2 + 24*e*f*x + 5*f^2*x^2)) - 144*b^2*(383*c^2
*f^2 - 2*c*d*f*(399*e + 16*f*x) + d^2*(420*e^2 + 42*e*f*x + 5*f^2*x^2)))*(C
os[b/(c + d*x)^(1/3)] + I*Sin[b/(c + d*x)^(1/3)])) - ((c + d*x)^(1/3)*(b^8*
f^2 + I*b^7*f^2*(c + d*x)^(1/3) - 2*b^6*f^2*(c + d*x)^(2/3) - (6*I)*b^5*f*(
168*d*e - 167*c*f + d*f*x) + 24*b^4*f*(c + d*x)^(1/3)*(42*d*e - 41*c*f + d*
f*x) + (24*I)*b^3*f*(c + d*x)^(2/3)*(84*d*e - 79*c*f + 5*d*f*x) + 40320*(c
+ d*x)^(2/3)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2)
) - (1008*I)*b*(c + d*x)^(1/3)*(41*c^2*f^2 - 2*c*d*f*(48*e + 7*f*x) + d^2*(
60*e^2 + 24*e*f*x + 5*f^2*x^2)) - 144*b^2*(383*c^2*f^2 - 2*c*d*f*(399*e + 1
6*f*x) + d^2*(420*e^2 + 42*e*f*x + 5*f^2*x^2))) + I*b^3*(-60480*d^2*e^2 + 1
008*((-I)*b^3 + 120*c)*d*e*f + (b^6 + (1008*I)*b^3*c - 60480*c^2)*f^2)*ExpI
ntegralEi[((-I)*b)/(c + d*x)^(1/3)]*(Cos[b/(c + d*x)^(1/3)] + I*Sin[b/(c +
d*x)^(1/3)]))*(Cos[a + b/(c + d*x)^(1/3)] - I*Sin[a + b/(c + d*x)^(1/3)])))
/d^3
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 936, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	Expression too large to display	936
default	Expression too large to display	936
parts	Expression too large to display	2775

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d^3*b^3*(2*b^3*d*e*f*(-1/6*sin(a+b/(d*x+c)^(1/3))/b^6*(d*x+c)^2-1/30*cos
(a+b/(d*x+c)^(1/3))/b^5*(d*x+c)^(5/3)+1/120*sin(a+b/(d*x+c)^(1/3))/b^4*(d*x
+c)^(4/3)+1/360*cos(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/720*sin(a+b/(d*x+c)^(1
/3))/b^2*(d*x+c)^(2/3)-1/720*cos(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)-1/720*S
i(b/(d*x+c)^(1/3))*cos(a)-1/720*Ci(b/(d*x+c)^(1/3))*sin(a))-2*b^3*f^2*c*(-1
/6*sin(a+b/(d*x+c)^(1/3))/b^6*(d*x+c)^2-1/30*cos(a+b/(d*x+c)^(1/3))/b^5*(d*
x+c)^(5/3)+1/120*sin(a+b/(d*x+c)^(1/3))/b^4*(d*x+c)^(4/3)+1/360*cos(a+b/(d*
x+c)^(1/3))/b^3*(d*x+c)-1/720*sin(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)-1/72
0*cos(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)-1/720*Si(b/(d*x+c)^(1/3))*cos(a)-1
/720*Ci(b/(d*x+c)^(1/3))*sin(a))-2*c*d*e*f*(-1/3*sin(a+b/(d*x+c)^(1/3))/b^3
*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x+c)^(
1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3)
)*cos(a))+b^6*f^2*(-1/9*sin(a+b/(d*x+c)^(1/3))/b^9*(d*x+c)^3-1/72*cos(a+b/(
d*x+c)^(1/3))/b^8*(d*x+c)^(8/3)+1/504*sin(a+b/(d*x+c)^(1/3))/b^7*(d*x+c)^(7
/3)+1/3024*cos(a+b/(d*x+c)^(1/3))/b^6*(d*x+c)^2-1/15120*sin(a+b/(d*x+c)^(1/
3))/b^5*(d*x+c)^(5/3)-1/60480*cos(a+b/(d*x+c)^(1/3))/b^4*(d*x+c)^(4/3)+1/18
1440*sin(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)+1/362880*cos(a+b/(d*x+c)^(1/3))/b^2
*(d*x+c)^(2/3)-1/362880*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)-1/362880*Si(
b/(d*x+c)^(1/3))*sin(a)+1/362880*Ci(b/(d*x+c)^(1/3))*cos(a))+d^2*e^2*(-1/3*
sin(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(
2/3)+1/6*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(d*x+c)^(1/3))*sin
(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))+f^2*c^2*(-1/3*sin(a+b/(d*x+c)^(1/3))/b^
3*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x+c)^
(1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3
))*cos(a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.67

$$\int (e+fx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx =$$
$$-\frac{\left(120\,b^3d^2f^2x^2+2016\,b^3cdef-1896\,b^3c^2f^2+48\,(42\,b^3d^2ef-37\,b^3cdf^2)x-(5040\,bd^2f^2x^2+60480\,bd^2\right.}{}$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] -1/120960*((120*b^3*d^2*f^2*x^2 + 2016*b^3*c*d*e*f - 1896*b^3*c^2*f^2 + 48*
(42*b^3*d^2*e*f - 37*b^3*c*d*f^2)*x - (5040*b*d^2*f^2*x^2 + 60480*b*d^2*e^2
 - 96768*b*c*d*e*f - (b^7 - 41328*b*c^2)*f^2 + 2016*(12*b*d^2*e*f - 7*b*c*d
*f^2)*x)*(d*x + c)^(2/3) - 6*(b^5*d*f^2*x + 168*b^5*d*e*f - 167*b^5*c*f^2)*
(d*x + c)^(1/3))*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) - ((60480
*b^3*d^2*e^2 - 120960*b^3*c*d*e*f - (b^9 - 60480*b^3*c^2)*f^2)*cos(a) + 100
8*(b^6*d*e*f - b^6*c*f^2)*sin(a))*cos_integral(b/(d*x + c)^(1/3)) - (40320*
d^3*f^2*x^3 + 120960*d^3*e*f*x^2 + 120960*c*d^2*e^2 - 120960*c^2*d*e*f - 2*
(b^6*c - 20160*c^3)*f^2 - 2*(b^6*d*f^2 - 60480*d^3*e^2)*x + 24*(b^4*d*f^2*x
 + 42*b^4*d*e*f - 41*b^4*c*f^2)*(d*x + c)^(2/3) - (720*b^2*d^2*f^2*x^2 + 60
480*b^2*d^2*e^2 - 114912*b^2*c*d*e*f - (b^8 - 55152*b^2*c^2)*f^2 + 288*(21*
b^2*d^2*e*f - 16*b^2*c*d*f^2)*x)*(d*x + c)^(1/3))*sin((a*d*x + a*c + (d*x +
 c)^(2/3)*b)/(d*x + c)) - (1008*(b^6*d*e*f - b^6*c*f^2)*cos(a) - (60480*b^3
*d^2*e^2 - 120960*b^3*c*d*e*f - (b^9 - 60480*b^3*c^2)*f^2)*sin(a))*sin_inte
gral(b/(d*x + c)^(1/3)))/d^3
```

Sympy [F]

$$\int (e+fx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \int (e+fx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b/(c + d*x)**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 1003, normalized size of antiderivative = 1.17

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/241920*(60480*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(
a) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 +
 2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x
 + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))
)*e^2 - 120960*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a
) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 +
2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x
+ c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))
*c*e*f/d + 60480*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos
(a) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3
+ 2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*
x + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)
))*c^2*f^2/d^2 + 1008*(((-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)^(
1/3)))*cos(a) + (Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*sin(a)
)*b^6 + 2*((d*x + c)^(1/3)*b^5 - 2*(d*x + c)*b^3 + 24*(d*x + c)^(5/3)*b)*co
s(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 2*((d*x + c)^(2/3)*b^4 - 6*(d*
x + c)^(4/3)*b^2 + 120*(d*x + c)^2)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(
1/3)))*e*f/d - 1008*(((-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)^(1/
3)))*cos(a) + (Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*
b^6 + 2*((d*x + c)^(1/3)*b^5 - 2*(d*x + c)*b^3 + 24*(d*x + c)^(5/3)*b)*cos(
((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 2*((d*x + c)^(2/3)*b^4 - 6*(d*x
+ c)^(4/3)*b^2 + 120*(d*x + c)^2)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3)))*c*f^2/d^2 - (((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos
(a) - (-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^9
 + 2*((d*x + c)^(2/3)*b^7 - 6*(d*x + c)^(4/3)*b^5 + 120*(d*x + c)^2*b^3 - 5
040*(d*x + c)^(8/3)*b)*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d
*x + c)^(1/3)*b^8 - 2*(d*x + c)*b^6 + 24*(d*x + c)^(5/3)*b^4 - 720*(d*x + c
)^(7/3)*b^2 + 40320*(d*x + c)^3)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3
)))*f^2/d^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11931 vs. 2(739) = 1478.

Time = 0.95 (sec) , antiderivative size = 11931, normalized size of antiderivative = 13.95

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 1/120960*(60480*(a^3*b^4*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(
d*x + c)^(1/3)) + a^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(
d*x + c)^(1/3)) - 3*((d*x + c)^(1/3)*a + b)*a^2*b^4*cos(a)*cos_integral(-a
+ ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 3*((d*x + c)^(
1/3)*a + b)*a^2*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x +
c)^(1/3))/(d*x + c)^(1/3) + 3*((d*x + c)^(1/3)*a + b)^2*a*b^4*cos(a)*cos_in
tegral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 3*((
d*x + c)^(1/3)*a + b)^2*a*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a +
b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3*b^4*cos(a)*
cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - ((d*
x + c)^(1/3)*a + b)^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(
d*x + c)^(1/3))/(d*x + c) + a^2*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(
1/3)) - 2*((d*x + c)^(1/3)*a + b)*a*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x +
c)^(1/3))/(d*x + c)^(1/3) + ((d*x + c)^(1/3)*a + b)^2*b^4*sin(((d*x + c)^(1
/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + a*b^4*cos(((d*x + c)^(1/3)*a
+ b)/(d*x + c)^(1/3)) - ((d*x + c)^(1/3)*a + b)*b^4*cos(((d*x + c)^(1/3)*a
+ b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 2*b^4*sin(((d*x + c)^(1/3)*a + b)/(
d*x + c)^(1/3)))*e^2/((a^3 - 3*((d*x + c)^(1/3)*a + b)*a^2/(d*x + c)^(1/3)
+ 3*((d*x + c)^(1/3)*a + b)^2*a/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3
/(d*x + c))*b) - (a^9*b^10*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)
/(d*x + c)^(1/3)) + a^9*b^10*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b
)/(d*x + c)^(1/3)) - 9*((d*x + c)^(1/3)*a + b)*a^8*b^10*cos(a)*cos_integral
(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 9*((d*x +
c)^(1/3)*a + b)*a^8*b^10*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d
*x + c)^(1/3))/(d*x + c)^(1/3) + 36*((d*x + c)^(1/3)*a + b)^2*a^7*b^10*cos(
a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/
3) + 36*((d*x + c)^(1/3)*a + b)^2*a^7*b^10*sin(a)*sin_integral(a - ((d*x +
c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 84*((d*x + c)^(1/3)*a +
b)^3*a^6*b^10*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1
/3))/(d*x + c) - 84*((d*x + c)^(1/3)*a + b)^3*a^6*b^10*sin(a)*sin_integral(
a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + 126*((d*x + c)^(1/
3)*a + b)^4*a^5*b^10*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x
+ c)^(1/3))/(d*x + c)^(4/3) + 126*((d*x + c)^(1/3)*a + b)^4*a^5*b^10*sin(a)
*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3)
- 126*((d*x + c)^(1/3)*a + b)^5*a^4*b^10*cos(a)*cos_integral(-a + ((d*x + c
```

)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) - 126*((d*x + c)^(1/3)*a + b)^5*a^4*b^10*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 84*((d*x + c)^(1/3)*a + b)^6*a^3*b^10*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 + 1008*a^9*b^7*c*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a) + a^8*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 1008*a^9*b^7*c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 84*((d*x + c)^(1/3)*a + b)^6*a^3*b^10*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 - 36*((d*x + c)^(1/3)*a + b)^7*a^2*b^10*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(7/3) - 9072*((d*x + c)^(1/3)*a + b)*a^8*b^7*c*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^(1/3) - 8*((d*x + c)^(1/3)*a + b)*a^7*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 9072*((d*x + c)^(1/3)*a + b)*a^8*b^7*c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 36*((d*x + c)^(1/3)*a + b)^7*a^2*b^10*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(7/3) + 9*((d*x + c)^(1/3)*a + b)^8*a*b^10*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(8/3) + 36288*((d*x + c)^(1/3)*a + b)^2*a^7*b^7*c*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^(2/3) + 28*((d*x + c)^(1/3)*a + b)^2*a^6*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 36288*((d*x + c)^(1/3)*a + b)^2*a^7*b^7*c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 9*((d*x + c)^(1/3)*a + b)^8*a*b^10*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(8/3) + a^7*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - ((d*x + c)^(1/3)*a + b)^9*b^10*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^3 - 84672*((d*x + c)^(1/3)*a + b)^3*a^6*b^7*c*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c) - 56*((d*x + c)^(1/3)*a + b)^3*a^5*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + 84672*((d*x + c)^(1/3)*a + b)^3*a^6*b^7*c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - ((d*x + c)^(1/3)*a + b)^9*b^10*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^3 - 7*((d*x + c)^(1/3)*a + b)*a^6*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 127008*((d*x + c)^(1/3)*a + b)^4*a^5*b^7*c*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^(4/3) + 70*((d*x + c)^(1/3)*a + b)^4*a^4*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 127008*((d*x + c)^(1/3)*a + b)^4*a^5*b^7*c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) + 21*((d*x + c)^(1/3)*a + b)^2*a^5*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 127008*((d*x + c)^(1/3)*a + b)^5*a^4*b^7*c*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^(5/3) - 56*((d*x + c)^(1/3)*a + b)^5*a^3*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 127008*((d*x + c)^(1/3)*a + b)^5*a^4*b^7*c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) - 35*((d*x + c)^(1/3)*a + b)^3*a^4*b^10*cos(((d*x + c)^(

1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - 1008*a^8*b^7*c*cos(((d*x + c)^(1/3
)*a + b)/(d*x + c)^(1/3)) - 60480*a^9*b^4*c^2*cos(a)*cos_integral(-a + ((d*
x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 84672*((d*x + c)^(1/3)*a + b)^6*a^3*
b^7*c*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*
x + c)^2 + 28*((d*x + c)^(1/3)*a + b)^6*a^2*b^10*sin(((d*x + c)^(1/3)*a + b
)/(d*x + c)^(1/3))/(d*x + c)^2 - 2*a^6*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*
x + c)^(1/3)) - 84672*((d*x + c)^(1/3)*a + b)^6*a^3*b^7*c*cos(a)*sin_integr
al(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 - 60480*a^9*b^4
*c^2*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 35*
((d*x + c)^(1/3)*a + b)^4*a^3*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1
/3))/(d*x + c)^(4/3) + 8064*((d*x + c)^(1/3)*a + b)*a^7*b^7*c*cos(((d*x + c
)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 544320*((d*x + c)^(1/3)*a
 + b)*a^8*b^4*c^2*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c
)^(1/3))/(d*x + c)^(1/3) - 36288*((d*x + c)^(1/3)*a + b)^7*a^2*b^7*c*cos_in
tegral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^(7/3)
 - 8*((d*x + c)^(1/3)*a + b)^7*a*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)
^(1/3))/(d*x + c)^(7/3) + 12*((d*x + c)^(1/3)*a + b)*a^5*b^10*sin(((d*x + c
)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 36288*((d*x + c)^(1/3)*a
+ b)^7*a^2*b^7*c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^
(1/3))/(d*x + c)^(7/3) + 544320*((d*x + c)^(1/3)*a + b)*a^8*b^4*c^2*sin(a)*
sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) -
 21*((d*x + c)^(1/3)*a + b)^5*a^2*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c
)^(1/3))/(d*x + c)^(5/3) - 28224*((d*x + c)^(1/3)*a + b)^2*a^6*b^7*c*cos(((
d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 2177280*((d*x + c)
^(1/3)*a + b)^2*a^7*b^4*c^2*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b
)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 9072*((d*x + c)^(1/3)*a + b)^8*a*b^7*c
*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c
)^(8/3) + ((d*x + c)^(1/3)*a + b)^8*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x +
 c)^(1/3))/(d*x + c)^(8/3) - 30*((d*x + c)^(1/3)*a + b)^2*a^4*b^10*sin(((d*
x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 9072*((d*x + c)^(1/3
)*a + b)^8*a*b^7*c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c
)^(1/3))/(d*x + c)^(8/3) - 2177280*((d*x + c)^(1/3)*a + b)^2*a^7*b^4*c^2*si
n(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2
/3) + 7*((d*x + c)^(1/3)*a + b)^6*a*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x +
 c)^(1/3))/(d*x + c)^2 - 6*a^5*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(
1/3)) + 56448*((d*x + c)^(1/3)*a + b)^3*a^5*b^7*c*cos(((d*x + c)^(1/3)*a +
b)/(d*x + c)^(1/3))/(d*x + c) + 5080320*((d*x + c)^(1/3)*a + b)^3*a^6*b^4*c
^2*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x +
 c) - 1008*((d*x + c)^(1/3)*a + b)^9*b^7*c*cos_integral(-a + ((d*x + c)^(1/
3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^3 + 40*((d*x + c)^(1/3)*a + b)^
3*a^3*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + 1008*a^
7*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 1008*((d*x + c)^(1/3
)*a + b)^9*b^7*c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^
(1/3))/(d*x + c)^3 + 5080320*((d*x + c)^(1/3)*a + b)^3*a^6*b^4*c^2*sin(a)*s
in_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - ((d*x

+ c)^(1/3)*a + b)^7*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(7/3) + 30*((d*x + c)^(1/3)*a + b)*a^4*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 70560*((d*x + c)^(1/3)*a + b)^4*a^4*b^7*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 7620480*((d*x + c)^(1/3)*a + b)^4*a^5*b^4*c^2*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 30*((d*x + c)^(1/3)*a + b)^4*a^2*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 7056*((d*x + c)^(1/3)*a + b)*a^6*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 7620480*((d*x + c)^(1/3)*a + b)^4*a^5*b^4*c^2*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 60*((d*x + c)^(1/3)*a + b)^2*a^3*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 56448*((d*x + c)^(1/3)*a + b)^5*a^3*b^7*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 7620480*((d*x + c)^(1/3)*a + b)^5*a^4*b^4*c^2*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 12*((d*x + c)^(1/3)*a + b)^5*a*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 21168*((d*x + c)^(1/3)*a + b)^2*a^5*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 7620480*((d*x + c)^(1/3)*a + b)^5*a^4*b^4*c^2*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 60*((d*x + c)^(1/3)*a + b)^3*a^2*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - 28224*((d*x + c)^(1/3)*a + b)^6*a^2*b^7*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 + 2016*a^6*b^7*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 5080320*((d*x + c)^(1/3)*a + b)^6*a^3*b^4*c^2*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 - 2*((d*x + c)^(1/3)*a + b)^6*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 + 24*a^4*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 35280*((d*x + c)^(1/3)*a + b)^3*a^4*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - 60480*a^8*b^4*c^2*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 5080320*((d*x + c)^(1/3)*a + b)^6*a^3*b^4*c^2*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 - 30*((d*x + c)^(1/3)*a + b)^4*a*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) + 8064*((d*x + c)^(1/3)*a + b)^7*a*b^7*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(7/3) - 12096*((d*x + c)^(1/3)*a + b)*a^5*b^7*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 2177280*((d*x + c)^(1/3)*a + b)^7*a^2*b^4*c^2*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(7/3) - 96*((d*x + c)^(1/3)*a + b)*a^3*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 35280*((d*x + c)^(1/3)*a + b)^4*a^3*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) + 483840*((d*x + c)^(1/3)*a + b)*a^7*b^4*c^2*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 2177280*((d*x + c)^(1/3)*a + b)^7*a^2*b^4*c^2*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(7/3) + 6*((d*x + c)^(1/3)*a + b)^5*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) - 1008*((d*x + c)^(1/3)*a + b)^8*b^7*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(8/3) + 30240*((d*x + c)^(1/3)*a + b)^2*a^4*b^7*c*cos(((d*x + c)^(1

/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 544320*((d*x + c)^(1/3)*a + b
)^8*a*b^4*c^2*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1
/3))/(d*x + c)^(8/3) + 144*((d*x + c)^(1/3)*a + b)^2*a^2*b^10*sin(((d*x + c
)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 21168*((d*x + c)^(1/3)*a
+ b)^5*a^2*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/
3) - 1693440*((d*x + c)^(1/3)*a + b)^2*a^6*b^4*c^2*sin(((d*x + c)^(1/3)*a +
b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 544320*((d*x + c)^(1/3)*a + b)^8*a*b
^4*c^2*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*
x + c)^(8/3) + 120*a^3*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) -
40320*((d*x + c)^(1/3)*a + b)^3*a^3*b^7*c*cos(((d*x + c)^(1/3)*a + b)/(d*x
+ c)^(1/3))/(d*x + c) - 60480*a^7*b^4*c^2*cos(((d*x + c)^(1/3)*a + b)/(d*x
+ c)^(1/3)) + 60480*((d*x + c)^(1/3)*a + b)^9*b^4*c^2*cos(a)*cos_integral(-
a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^3 - 96*((d*x + c)^(1
/3)*a + b)^3*a*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)
+ 7056*((d*x + c)^(1/3)*a + b)^6*a*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x +
c)^(1/3))/(d*x + c)^2 - 6048*a^5*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x +
c)^(1/3)) + 3386880*((d*x + c)^(1/3)*a + b)^3*a^5*b^4*c^2*sin(((d*x + c)^(1
/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + 60480*((d*x + c)^(1/3)*a + b)^9*b^4
*c^2*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x
+ c)^3 - 360*((d*x + c)^(1/3)*a + b)*a^2*b^10*cos(((d*x + c)^(1/3)*a + b)/(
d*x + c)^(1/3))/(d*x + c)^(1/3) + 30240*((d*x + c)^(1/3)*a + b)^4*a^2*b^7*c
*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) + 423360*((d*
x + c)^(1/3)*a + b)*a^6*b^4*c^2*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)
)/(d*x + c)^(1/3) + 24*((d*x + c)^(1/3)*a + b)^4*b^10*sin(((d*x + c)^(1/3)*
a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 1008*((d*x + c)^(1/3)*a + b)^7*b^
7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(7/3) + 30240*((
d*x + c)^(1/3)*a + b)*a^4*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)
)/(d*x + c)^(1/3) - 4233600*((d*x + c)^(1/3)*a + b)^4*a^4*b^4*c^2*sin(((d*x
+ c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) + 360*((d*x + c)^(1/3)*
a + b)^2*a*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3
) - 12096*((d*x + c)^(1/3)*a + b)^5*a*b^7*c*cos(((d*x + c)^(1/3)*a + b)/(d*
x + c)^(1/3))/(d*x + c)^(5/3) - 1270080*((d*x + c)^(1/3)*a + b)^2*a^5*b^4*c
^2*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 60480*((d
*x + c)^(1/3)*a + b)^2*a^3*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3
))/(d*x + c)^(2/3) + 3386880*((d*x + c)^(1/3)*a + b)^5*a^3*b^4*c^2*sin(((d*
x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) - 120*((d*x + c)^(1/3)
*a + b)^3*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + 201
6*((d*x + c)^(1/3)*a + b)^6*b^7*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3))/(d*x + c)^2 - 24192*a^4*b^7*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3)) + 2116800*((d*x + c)^(1/3)*a + b)^3*a^4*b^4*c^2*cos(((d*x + c)^(1/3)*a
+ b)/(d*x + c)^(1/3))/(d*x + c) - 720*a^2*b^10*sin(((d*x + c)^(1/3)*a + b)/
(d*x + c)^(1/3)) + 60480*((d*x + c)^(1/3)*a + b)^3*a^2*b^7*c*sin(((d*x + c)
^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - 1693440*((d*x + c)^(1/3)*a + b)^
6*a^2*b^4*c^2*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 + 12
0960*a^6*b^4*c^2*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 96768*((d*x

+ c)^(1/3)*a + b)*a^3*b^7*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(
d*x + c)^(1/3) - 2116800*((d*x + c)^(1/3)*a + b)^4*a^3*b^4*c^2*cos(((d*x +
c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) + 1440*((d*x + c)^(1/3)*a
+ b)*a*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) -
30240*((d*x + c)^(1/3)*a + b)^4*a*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x +
c)^(1/3))/(d*x + c)^(4/3) + 483840*((d*x + c)^(1/3)*a + b)^7*a*b^4*c^2*sin(
((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(7/3) - 725760*((d*x + c
)^(1/3)*a + b)*a^5*b^4*c^2*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*
x + c)^(1/3) - 145152*((d*x + c)^(1/3)*a + b)^2*a^2*b^7*c*cos(((d*x + c)^(1
/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 1270080*((d*x + c)^(1/3)*a +
b)^5*a^2*b^4*c^2*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/
3) - 720*((d*x + c)^(1/3)*a + b)^2*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x +
c)^(1/3))/(d*x + c)^(2/3) + 6048*((d*x + c)^(1/3)*a + b)^5*b^7*c*sin(((d*x
+ c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) - 60480*((d*x + c)^(1/3)
*a + b)^8*b^4*c^2*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(8
/3) + 1814400*((d*x + c)^(1/3)*a + b)^2*a^4*b^4*c^2*sin(((d*x + c)^(1/3)*a
+ b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 5040*a*b^10*cos(((d*x + c)^(1/3)*a
+ b)/(d*x + c)^(1/3)) + 96768*((d*x + c)^(1/3)*a + b)^3*a*b^7*c*cos(((d*x +
c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - 423360*((d*x + c)^(1/3)*a + b
)^6*a*b^4*c^2*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 + 12
0960*a^3*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2419200*((d*x
+ c)^(1/3)*a + b)^3*a^3*b^4*c^2*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3
))/(d*x + c) + 5040*((d*x + c)^(1/3)*a + b)*b^10*cos(((d*x + c)^(1/3)*a + b
)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 24192*((d*x + c)^(1/3)*a + b)^4*b^7*c*
cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) + 60480*((d*x
+ c)^(1/3)*a + b)^7*b^4*c^2*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d
x + c)^(7/3) - 362880((d*x + c)^(1/3)*a + b)*a^2*b^7*c*sin(((d*x + c)^(1/
3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 1814400*((d*x + c)^(1/3)*a + b
)^4*a^2*b^4*c^2*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3
) + 362880*((d*x + c)^(1/3)*a + b)^2*a*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d
*x + c)^(1/3))/(d*x + c)^(2/3) - 725760*((d*x + c)^(1/3)*a + b)^5*a*b^4*c^2
*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 40320*b^10*
sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 120960*((d*x + c)^(1/3)*a +
b)^3*b^7*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + 120960*
((d*x + c)^(1/3)*a + b)^6*b^4*c^2*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3))/(d*x + c)^2)*f^2/((a^9*d^2 - 9*((d*x + c)^(1/3)*a + b)*a^8*d^2/(d*x + c
)^(1/3) + 36*((d*x + c)^(1/3)*a + b)^2*a^7*d^2/(d*x + c)^(2/3) - 84*((d*x +
c)^(1/3)*a + b)^3*a^6*d^2/(d*x + c) + 126*((d*x + c)^(1/3)*a + b)^4*a^5*d^
2/(d*x + c)^(4/3) - 126*((d*x + c)^(1/3)*a + b)^5*a^4*d^2/(d*x + c)^(5/3) +
84*((d*x + c)^(1/3)*a + b)^6*a^3*d^2/(d*x + c)^2 - 36*((d*x + c)^(1/3)*a +
b)^7*a^2*d^2/(d*x + c)^(7/3) + 9*((d*x + c)^(1/3)*a + b)^8*a*d^2/(d*x + c)
^(8/3) - ((d*x + c)^(1/3)*a + b)^9*d^2/(d*x + c)^3)*b) + 1008*(a^6*b^7*cos_
integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a) - a^6*b^7*cos
(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 6*((d*x + c
)^(1/3)*a + b)*a^5*b^7*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^

(1/3))*sin(a)/(d*x + c)^(1/3) + 6*((d*x + c)^(1/3)*a + b)*a^5*b^7*cos(a)*si
n_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 1
5*((d*x + c)^(1/3)*a + b)^2*a^4*b^7*cos_integral(-a + ((d*x + c)^(1/3)*a +
b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^(2/3) - 15*((d*x + c)^(1/3)*a + b)^2*a
^4*b^7*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*
x + c)^(2/3) - 20*((d*x + c)^(1/3)*a + b)^3*a^3*b^7*cos_integral(-a + ((d*x
 + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c) + 20*((d*x + c)^(1/3)*
a + b)^3*a^3*b^7*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^
(1/3))/(d*x + c) + 15*((d*x + c)^(1/3)*a + b)^4*a^2*b^7*cos_integral(-a + (
(d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^(4/3) - 15*((d*x +
 c)^(1/3)*a + b)^4*a^2*b^7*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/
(d*x + c)^(1/3))/(d*x + c)^(4/3) - 6*((d*x + c)^(1/3)*a + b)^5*a*b^7*cos_in
tegral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^(5/3)
 + 6*((d*x + c)^(1/3)*a + b)^5*a*b^7*cos(a)*sin_integral(a - ((d*x + c)^(1/
3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) - a^5*b^7*cos(((d*x + c)^(1/3)*a
 + b)/(d*x + c)^(1/3)) - 120*a^6*b^4*c*cos(a)*cos_integral(-a + ((d*x + c)^
(1/3)*a + b)/(d*x + c)^(1/3)) + ((d*x + c)^(1/3)*a + b)^6*b^7*cos_integral(
-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^2 - ((d*x +
c)^(1/3)*a + b)^6*b^7*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x
+ c)^(1/3))/(d*x + c)^2 - 120*a^6*b^4*c*sin(a)*sin_integral(a - ((d*x + c)^
(1/3)*a + b)/(d*x + c)^(1/3)) + 5*((d*x + c)^(1/3)*a + b)*a^4*b^7*cos(((d*x
 + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 720*((d*x + c)^(1/3)*
a + b)*a^5*b^4*c*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)
^(1/3))/(d*x + c)^(1/3) + 720*((d*x + c)^(1/3)*a + b)*a^5*b^4*c*sin(a)*sin_
integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 10*
((d*x + c)^(1/3)*a + b)^2*a^3*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3))/(d*x + c)^(2/3) - 1800*((d*x + c)^(1/3)*a + b)^2*a^4*b^4*c*cos(a)*cos_i
ntegral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 180
0*((d*x + c)^(1/3)*a + b)^2*a^4*b^4*c*sin(a)*sin_integral(a - ((d*x + c)^(1
/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 10*((d*x + c)^(1/3)*a + b)^3*
a^2*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + 2400*((d*x
 + c)^(1/3)*a + b)^3*a^3*b^4*c*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a
+ b)/(d*x + c)^(1/3))/(d*x + c) + a^4*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x
+ c)^(1/3)) + 2400*((d*x + c)^(1/3)*a + b)^3*a^3*b^4*c*sin(a)*sin_integral(
a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - 5*((d*x + c)^(1/3)
*a + b)^4*a*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3
) - 1800*((d*x + c)^(1/3)*a + b)^4*a^2*b^4*c*cos(a)*cos_integral(-a + ((d*x
 + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 4*((d*x + c)^(1/3)*a
+ b)*a^3*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) -
 1800*((d*x + c)^(1/3)*a + b)^4*a^2*b^4*c*sin(a)*sin_integral(a - ((d*x + c
)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) + ((d*x + c)^(1/3)*a + b)^5
*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 720*((d
*x + c)^(1/3)*a + b)^5*a*b^4*c*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a
+ b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 6*((d*x + c)^(1/3)*a + b)^2*a^2*b^7
*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 720*((d*x +

c)^(1/3)*a + b)^5*a*b^4*c*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 2*a^3*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 120*((d*x + c)^(1/3)*a + b)^6*b^4*c*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 - 4*((d*x + c)^(1/3)*a + b)^3*a*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - 120*a^5*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 120*((d*x + c)^(1/3)*a + b)^6*b^4*c*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 - 6*((d*x + c)^(1/3)*a + b)*a^2*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + ((d*x + c)^(1/3)*a + b)^4*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) + 600*((d*x + c)^(1/3)*a + b)*a^4*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 6*((d*x + c)^(1/3)*a + b)^2*a*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 1200*((d*x + c)^(1/3)*a + b)^2*a^3*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 2*((d*x + c)^(1/3)*a + b)^3*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - 120*a^4*b^4*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 6*a^2*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 1200*((d*x + c)^(1/3)*a + b)^3*a^2*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + 480*((d*x + c)^(1/3)*a + b)*a^3*b^4*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 12*((d*x + c)^(1/3)*a + b)*a*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 600*((d*x + c)^(1/3)*a + b)^4*a*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 720*((d*x + c)^(1/3)*a + b)^2*a^2*b^4*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 6*((d*x + c)^(1/3)*a + b)^2*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 120*((d*x + c)^(1/3)*a + b)^5*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) - 24*a*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 480*((d*x + c)^(1/3)*a + b)^3*a*b^4*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + 240*a^3*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 24*((d*x + c)^(1/3)*a + b)*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 120*((d*x + c)^(1/3)*a + b)^4*b^4*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 720*((d*x + c)^(1/3)*a + b)*a^2*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 720*((d*x + c)^(1/3)*a + b)^2*a*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 120*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 240*((d*x + c)^(1/3)*a + b)^3*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c))*e*f/((a^6 - 6*((d*x + c)^(1/3)*a + b)*a^5/(d*x + c)^(1/3) + 15*((d*x + c)^(1/3)*a + b)^2*a^4/(d*x + c)^(2/3) - 20*((d*x + c)^(1/3)*a + b)^3*a^3/(d*x + c) + 15*((d*x + c)^(1/3)*a + b)^4*a^2/(d*x + c)^(4/3) - 6*((d*x + c)^(1/3)*a + b)^5*a/(d*x + c)^(5/3) + ((d*x + c)^(1/3)*a + b)^6/(d*x + c)^2)*b*d))/d

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \int \sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)\,(e+f\,x)^2\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/3))*(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^(1/3))*(e + f*x)^2, x)
```

3.218 $\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 419

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \frac{b^5 f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3 f(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^2} + \frac{bf(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2} + \frac{b^3(de-cf)\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{b^6 f\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{240d^2} - \frac{b^2(de-cf)\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{b^4 f(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2} - \frac{b^2 f(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{40d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{b^6 f\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} - \frac{b^3(de-cf)\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}$$

```
[Out] 1/2*b^3*(-c*f+d*e)*Ci(b/(d*x+c)^(1/3))*cos(a)/d^2+1/240*b^5*f*(d*x+c)^(1/3)
*cos(a+b/(d*x+c)^(1/3))/d^2+1/2*b*(-c*f+d*e)*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^
(1/3))/d^2-1/120*b^3*f*(d*x+c)*cos(a+b/(d*x+c)^(1/3))/d^2+1/10*b*f*(d*x+c)^
(5/3)*cos(a+b/(d*x+c)^(1/3))/d^2+1/240*b^6*f*cos(a)*Si(b/(d*x+c)^(1/3))/d^2
+1/240*b^6*f*Ci(b/(d*x+c)^(1/3))*sin(a)/d^2-1/2*b^3*(-c*f+d*e)*Si(b/(d*x+c)
^(1/3))*sin(a)/d^2-1/2*b^2*(-c*f+d*e)*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3))/
d^2+1/240*b^4*f*(d*x+c)^(2/3)*sin(a+b/(d*x+c)^(1/3))/d^2+(-c*f+d*e)*(d*x+c)
*sin(a+b/(d*x+c)^(1/3))/d^2-1/40*b^2*f*(d*x+c)^(4/3)*sin(a+b/(d*x+c)^(1/3))
/d^2+1/2*f*(d*x+c)^2*sin(a+b/(d*x+c)^(1/3))/d^2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

= {3512, 3378, 3384, 3380, 3383}

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx = \frac{b^6 f\sin(a)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b^6 f\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b^5 f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b^4 f(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b^3\cos(a)(de-cf)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3\sin(a)(de-cf)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3 f(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^2} - \frac{b^2\sqrt[3]{c+dx}(de-cf)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^2 f(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{40d^2} + \frac{(c+dx)(de-cf)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2} + \frac{b(c+dx)^{2/3}(de-cf)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{bf(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2}$$

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (b^5*f*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(240*d^2) + (b*(d*e - c*
f)*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d^2) - (b^3*f*(c + d*x)*C
os[a + b/(c + d*x)^(1/3)])/(120*d^2) + (b*f*(c + d*x)^(5/3)*Cos[a + b/(c +
d*x)^(1/3)])/(10*d^2) + (b^3*(d*e - c*f)*Cos[a]*CosIntegral[b/(c + d*x)^(1/
3)])/(2*d^2) + (b^6*f*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(240*d^2) - (b
^2*(d*e - c*f)*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(2*d^2) + (b^4*f
*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(240*d^2) + ((d*e - c*f)*(c +
d*x)*Sin[a + b/(c + d*x)^(1/3)])/d^2 - (b^2*f*(c + d*x)^(4/3)*Sin[a + b/(c
+ d*x)^(1/3)])/(40*d^2) + (f*(c + d*x)^2*Sin[a + b/(c + d*x)^(1/3)])/(2*d^2
) + (b^6*f*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(240*d^2) - (b^3*(d*e - c
*f)*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)])/(2*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = -\frac{3\text{Subst}\left(\int \left(\frac{f\sin(a+bx)}{dx^7}+\frac{(de-cf)\sin(a+bx)}{dx^4}\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{(3f)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^7}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^2}-\frac{(3(de-cf))\text{Subst}\left(\int \frac{\sin(a+bx)}{x^4}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^2}$$

$$= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{(bf)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^6}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{(b(de-cf))\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^2}$$

$$= \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}+\frac{bf(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{(b^2f)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^5}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{10d^2} + \frac{(b^2(de-cf))\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{2d^2}$$

$$= \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{bf(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2} - \frac{b^2(de-cf)\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2} - \frac{b^2f(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{40d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{(b^3f)\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^4}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{40d^2} + \frac{(b^3(de-cf))\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{2d^2}$$

$$= \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3f(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^2} + \frac{bf(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2} - \frac{b^2(de-cf)\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2} - \frac{b^2f(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{40d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{(b^4f)\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^3}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{120d^2} + \frac{(b^3(de-cf)\cos(a))\,\text{Subst}\left(\int\frac{\cos(bx)}{x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{(b^3(de-cf)\sin(a))\,\text{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{2d^2}$$

$$= \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3f(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^2} + \frac{bf(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2} + \frac{b^3(de-cf)\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^2(de-cf)\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{b^4f(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2} - \frac{b^2f(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{40d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3(de-cf)\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{(b^5f)\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{240d^2}$$

$$= \frac{b^5f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3f(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^2} + \frac{bf(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2} + \frac{b^3(de-cf)\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^2(de-cf)\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{b^4f(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2} - \frac{b^2f(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{40d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3(de-cf)\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{(b^6f)\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{240d^2}$$

$$
\begin{aligned}
&= \frac{b^5 f \sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} \\
&\quad - \frac{b^3 f(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^2} + \frac{bf(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2} \\
&\quad + \frac{b^3(de-cf)\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} \\
&\quad - \frac{b^2(de-cf)\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{b^4 f(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} \\
&\quad + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2} - \frac{b^2 f(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{40d^2} \\
&\quad + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3(de-cf)\sin(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} \\
&\quad + \frac{(b^6 f\cos(a))\,\mathrm{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{240d^2} \\
&\quad + \frac{(b^6 f\sin(a))\,\mathrm{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{240d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^5 f \sqrt[3]{c+dx} \cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b(de-cf)(c+dx)^{2/3} \cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} \\
&\quad - \frac{b^3 f(c+dx) \cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^2} + \frac{bf(c+dx)^{5/3} \cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2} \\
&\quad + \frac{b^3(de-cf)\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} \\
&\quad + \frac{b^6 f\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{240d^2} - \frac{b^2(de-cf)\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} \\
&\quad + \frac{b^4 f(c+dx)^{2/3} \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2} \\
&\quad - \frac{b^2 f(c+dx)^{4/3} \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{40d^2} + \frac{f(c+dx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} \\
&\quad + \frac{b^6 f\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} - \frac{b^3(de-cf)\sin(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.29

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$

$$= \frac{e\sqrt[3]{c+dx}\cos\left(\frac{b}{\sqrt[3]{c+dx}}\right)\left(b\sqrt[3]{c+dx}\cos(a)-b^2\sin(a)+2(c+dx)^{2/3}\sin(a)\right)}{2d}$$

$$+ \frac{f\sqrt[3]{c+dx}\cos\left(\frac{b}{\sqrt[3]{c+dx}}\right)\left(b^5\cos(a)-120bc\sqrt[3]{c+dx}\cos(a)-2b^3(c+dx)^{2/3}\cos(a)+24b(c+dx)^{4/3}\text{co}\right.}{}$$

$$+ \frac{e\sqrt[3]{c+dx}\left(-b^2\cos(a)+2(c+dx)^{2/3}\cos(a)-b\sqrt[3]{c+dx}\sin(a)\right)\sin\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}$$

$$+ \frac{f\sqrt[3]{c+dx}\left(120b^2c\cos(a)+b^4\sqrt[3]{c+dx}\cos(a)-240c(c+dx)^{2/3}\cos(a)-6b^2(c+dx)\cos(a)+120(c+d\right.}{}$$

$$+ \frac{b^3e\left(\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)-\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{2d}$$

$$+ \frac{b^3f\left(-120c\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+b^3\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)+b^3\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+da}}\right.\right.}{240d^2}$$

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (e*(c + d*x)^(1/3)*Cos[b/(c + d*x)^(1/3)]*(b*(c + d*x)^(1/3)*Cos[a] - b^2*S
in[a] + 2*(c + d*x)^(2/3)*Sin[a]))/(2*d) + (f*(c + d*x)^(1/3)*Cos[b/(c + d*
x)^(1/3)]*(b^5*Cos[a] - 120*b*c*(c + d*x)^(1/3)*Cos[a] - 2*b^3*(c + d*x)^(2
/3)*Cos[a] + 24*b*(c + d*x)^(4/3)*Cos[a] + 120*b^2*c*Sin[a] + b^4*(c + d*x)
^(1/3)*Sin[a] - 240*c*(c + d*x)^(2/3)*Sin[a] - 6*b^2*(c + d*x)*Sin[a] + 120
*(c + d*x)^(5/3)*Sin[a]))/(240*d^2) + (e*(c + d*x)^(1/3)*(-(b^2*Cos[a]) + 2
*(c + d*x)^(2/3)*Cos[a] - b*(c + d*x)^(1/3)*Sin[a])*Sin[b/(c + d*x)^(1/3)])
/(2*d) + (f*(c + d*x)^(1/3)*(120*b^2*c*Cos[a] + b^4*(c + d*x)^(1/3)*Cos[a]
- 240*c*(c + d*x)^(2/3)*Cos[a] - 6*b^2*(c + d*x)*Cos[a] + 120*(c + d*x)^(5/
3)*Cos[a] - b^5*Sin[a] + 120*b*c*(c + d*x)^(1/3)*Sin[a] + 2*b^3*(c + d*x)^(
2/3)*Sin[a] - 24*b*(c + d*x)^(4/3)*Sin[a])*Sin[b/(c + d*x)^(1/3)])/(240*d^2
) + (b^3*e*(Cos[a]*CosIntegral[b/(c + d*x)^(1/3)] - Sin[a]*SinIntegral[b/(c
 + d*x)^(1/3)]))/(2*d) + (b^3*f*(-120*c*Cos[a]*CosIntegral[b/(c + d*x)^(1/3
)] + b^3*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a] + b^3*Cos[a]*SinIntegral[b/(
c + d*x)^(1/3)] + 120*c*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)]))/(240*d^2)
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.93

method	result
derivativedivides	$-\frac{3b^3\left(-cf\left(-\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)}{3b^3}-\frac{\cos\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{2}{3}}}{6b^2}+\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{1}{3}}}{6b}+\frac{\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a)}{6}\right.\right.}{}$
default	$-\frac{3b^3\left(-cf\left(-\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)}{3b^3}-\frac{\cos\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{2}{3}}}{6b^2}+\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{1}{3}}}{6b}+\frac{\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a)}{6}\right.\right.}{}$
parts	Expression too large to display

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d^2*b^3*(-c*f*(-1/3*sin(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d*x+
c)^(1/3))/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1/6*
Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))+d*e*(-1/3*sin(a+
b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)+1
/6*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(d*x+c)^(1/3))*sin(a)-1/
6*Ci(b/(d*x+c)^(1/3))*cos(a))+f*b^3*(-1/6*sin(a+b/(d*x+c)^(1/3))/b^6*(d*x+c
)^2-1/30*cos(a+b/(d*x+c)^(1/3))/b^5*(d*x+c)^(5/3)+1/120*sin(a+b/(d*x+c)^(1/
3))/b^4*(d*x+c)^(4/3)+1/360*cos(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/720*sin(a+
b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)-1/720*cos(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(
1/3)-1/720*Si(b/(d*x+c)^(1/3))*cos(a)-1/720*Ci(b/(d*x+c)^(1/3))*sin(a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.62

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$
$$=\frac{\left((dx+c)^{\frac{1}{3}}b^5f-2\,b^3dfx-2\,b^3cf+24\,(bdfx+5\,bde-4\,bcf)(dx+c)^{\frac{2}{3}}\right)\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)+(b^6f\sin}{}$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/240*(((d*x + c)^(1/3)*b^5*f - 2*b^3*d*f*x - 2*b^3*c*f + 24*(b*d*f*x + 5*b
*d*e - 4*b*c*f)*(d*x + c)^(2/3))*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x
 + c)) + (b^6*f*sin(a) + 120*(b^3*d*e - b^3*c*f)*cos(a))*cos_integral(b/(d*
x + c)^(1/3)) + ((d*x + c)^(2/3)*b^4*f + 120*d^2*f*x^2 + 240*d^2*e*x + 240*
```

```
c*d*e - 120*c^2*f - 6*(b^2*d*f*x + 20*b^2*d*e - 19*b^2*c*f)*(d*x + c)^(1/3)
)*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) + (b^6*f*cos(a) - 120*(b
^3*d*e - b^3*c*f)*sin(a))*sin_integral(b/(d*x + c)^(1/3)))/d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**(1/3)),x)
[Out] Integral((e + f*x)*sin(a + b/(c + d*x)**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.09

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$

$$=\frac{120\left(\left(\left(\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)+\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos(a)+\left(i\,\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)-i\,\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\sin(a)\right)b^3+2\,(dx}{}$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
[Out] 1/480*(120*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) +
(I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 + 2*(d
*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x + c)
^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*e -
 120*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (I*Ei(
I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 + 2*(d*x + c
)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x + c)^(1/3)
*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*c*f/d + (
((-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (Ei(I*b
/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^6 + 2*((d*x + c)^(1
/3)*b^5 - 2*(d*x + c)*b^3 + 24*(d*x + c)^(5/3)*b)*cos(((d*x + c)^(1/3)*a +
b)/(d*x + c)^(1/3)) + 2*((d*x + c)^(2/3)*b^4 - 6*(d*x + c)^(4/3)*b^2 + 120*
(d*x + c)^2)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*f/d)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3727 vs. 2(357) = 714.

Time = 0.66 (sec) , antiderivative size = 3727, normalized size of antiderivative = 8.89

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
[Out] 1/240*(120*(a^3*b^4*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x +
 c)^(1/3)) + a^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x +
 c)^(1/3)) - 3*((d*x + c)^(1/3)*a + b)*a^2*b^4*cos(a)*cos_integral(-a + ((d
*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 3*((d*x + c)^(1/3)*
a + b)*a^2*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1
/3))/(d*x + c)^(1/3) + 3*((d*x + c)^(1/3)*a + b)^2*a*b^4*cos(a)*cos_integra
l(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 3*((d*x +
 c)^(1/3)*a + b)^2*a*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d
*x + c)^(1/3))/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3*b^4*cos(a)*cos_i
ntegral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - ((d*x + c
)^(1/3)*a + b)^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x +
 c)^(1/3))/(d*x + c) + a^2*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))
 - 2*((d*x + c)^(1/3)*a + b)*a*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1
/3))/(d*x + c)^(1/3) + ((d*x + c)^(1/3)*a + b)^2*b^4*sin(((d*x + c)^(1/3)*a
 + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + a*b^4*cos(((d*x + c)^(1/3)*a + b)/
(d*x + c)^(1/3)) - ((d*x + c)^(1/3)*a + b)*b^4*cos(((d*x + c)^(1/3)*a + b)/
(d*x + c)^(1/3))/(d*x + c)^(1/3) - 2*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x +
 c)^(1/3)))*e/((a^3 - 3*((d*x + c)^(1/3)*a + b)*a^2/(d*x + c)^(1/3) + 3*((d
*x + c)^(1/3)*a + b)^2*a/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3/(d*x +
 c))*b) + (a^6*b^7*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3
))*sin(a) - a^6*b^7*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x +
c)^(1/3)) - 6*((d*x + c)^(1/3)*a + b)*a^5*b^7*cos_integral(-a + ((d*x + c)^
(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^(1/3) + 6*((d*x + c)^(1/3)*a
 + b)*a^5*b^7*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3))/(d*x + c)^(1/3) + 15*((d*x + c)^(1/3)*a + b)^2*a^4*b^7*cos_integral(-a
+ ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^(2/3) - 15*((d*
x + c)^(1/3)*a + b)^2*a^4*b^7*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a +
b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 20*((d*x + c)^(1/3)*a + b)^3*a^3*b^7*
cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)
 + 20*((d*x + c)^(1/3)*a + b)^3*a^3*b^7*cos(a)*sin_integral(a - ((d*x + c)^
(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + 15*((d*x + c)^(1/3)*a + b)^4*a^2*
b^7*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x
+ c)^(4/3) - 15*((d*x + c)^(1/3)*a + b)^4*a^2*b^7*cos(a)*sin_integral(a - (
(d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 6*((d*x + c)^(1/3
)*a + b)^5*a*b^7*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))
```

```
*sin(a)/(d*x + c)^(5/3) + 6*((d*x + c)^(1/3)*a + b)^5*a*b^7*cos(a)*sin_inte
gral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) - a^5*b^7
*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 120*a^6*b^4*c*cos(a)*cos_in
tegral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + ((d*x + c)^(1/3)*a +
 b)^6*b^7*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)
/(d*x + c)^2 - ((d*x + c)^(1/3)*a + b)^6*b^7*cos(a)*sin_integral(a - ((d*x
+ c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 - 120*a^6*b^4*c*sin(a)*sin_i
ntegral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 5*((d*x + c)^(1/3)*a
 + b)*a^4*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3)
+ 720*((d*x + c)^(1/3)*a + b)*a^5*b^4*c*cos(a)*cos_integral(-a + ((d*x + c)
^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 720*((d*x + c)^(1/3)*a + b
)*a^5*b^4*c*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)
)/(d*x + c)^(1/3) - 10*((d*x + c)^(1/3)*a + b)^2*a^3*b^7*cos(((d*x + c)^(1/
3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 1800*((d*x + c)^(1/3)*a + b)^2
*a^4*b^4*c*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)
)/(d*x + c)^(2/3) - 1800*((d*x + c)^(1/3)*a + b)^2*a^4*b^4*c*sin(a)*sin_int
egral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 10*((d
*x + c)^(1/3)*a + b)^3*a^2*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))
/(d*x + c) + 2400*((d*x + c)^(1/3)*a + b)^3*a^3*b^4*c*cos(a)*cos_integral(-
a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + a^4*b^7*sin(((d*x
+ c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 2400*((d*x + c)^(1/3)*a + b)^3*a^3*b^4
*c*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x +
c) - 5*((d*x + c)^(1/3)*a + b)^4*a*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c
)^(1/3))/(d*x + c)^(4/3) - 1800*((d*x + c)^(1/3)*a + b)^4*a^2*b^4*c*cos(a)*
cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3)
- 4*((d*x + c)^(1/3)*a + b)*a^3*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(
1/3))/(d*x + c)^(1/3) - 1800*((d*x + c)^(1/3)*a + b)^4*a^2*b^4*c*sin(a)*sin
_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) + ((
d*x + c)^(1/3)*a + b)^5*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d
*x + c)^(5/3) + 720*((d*x + c)^(1/3)*a + b)^5*a*b^4*c*cos(a)*cos_integral(-
a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 6*((d*x + c)
^(1/3)*a + b)^2*a^2*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x +
 c)^(2/3) + 720*((d*x + c)^(1/3)*a + b)^5*a*b^4*c*sin(a)*sin_integral(a - (
(d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 2*a^3*b^7*cos(((d
*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 120*((d*x + c)^(1/3)*a + b)^6*b^4*c
*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c
)^2 - 4*((d*x + c)^(1/3)*a + b)^3*a*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x +
c)^(1/3))/(d*x + c) - 120*a^5*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(
1/3)) - 120*((d*x + c)^(1/3)*a + b)^6*b^4*c*sin(a)*sin_integral(a - ((d*x +
 c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 - 6*((d*x + c)^(1/3)*a + b)*a
^2*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + ((d*x
 + c)^(1/3)*a + b)^4*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x
+ c)^(4/3) + 600*((d*x + c)^(1/3)*a + b)*a^4*b^4*c*sin(((d*x + c)^(1/3)*a +
 b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 6*((d*x + c)^(1/3)*a + b)^2*a*b^7*co
s(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 1200*((d*x + c
```

```
)^(1/3)*a + b)^2*a^3*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*
x + c)^(2/3) - 2*((d*x + c)^(1/3)*a + b)^3*b^7*cos(((d*x + c)^(1/3)*a + b)/
(d*x + c)^(1/3))/(d*x + c) - 120*a^4*b^4*c*cos(((d*x + c)^(1/3)*a + b)/(d*x
 + c)^(1/3)) - 6*a^2*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 120
0*((d*x + c)^(1/3)*a + b)^3*a^2*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)
^(1/3))/(d*x + c) + 480*((d*x + c)^(1/3)*a + b)*a^3*b^4*c*cos(((d*x + c)^(1
/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 12*((d*x + c)^(1/3)*a + b)*a*
b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 600*((d*
x + c)^(1/3)*a + b)^4*a*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/
(d*x + c)^(4/3) - 720*((d*x + c)^(1/3)*a + b)^2*a^2*b^4*c*cos(((d*x + c)^(1
/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 6*((d*x + c)^(1/3)*a + b)^2*b
^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 120*((d*x
 + c)^(1/3)*a + b)^5*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*
x + c)^(5/3) - 24*a*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 480*
((d*x + c)^(1/3)*a + b)^3*a*b^4*c*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3))/(d*x + c) + 240*a^3*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))
+ 24*((d*x + c)^(1/3)*a + b)*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3
))/(d*x + c)^(1/3) - 120*((d*x + c)^(1/3)*a + b)^4*b^4*c*cos(((d*x + c)^(1/
3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 720*((d*x + c)^(1/3)*a + b)*a^
2*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 720*
((d*x + c)^(1/3)*a + b)^2*a*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3))/(d*x + c)^(2/3) + 120*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))
- 240*((d*x + c)^(1/3)*a + b)^3*b^4*c*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)
^(1/3))/(d*x + c))*f/((a^6 - 6*((d*x + c)^(1/3)*a + b)*a^5/(d*x + c)^(1/3)
+ 15*((d*x + c)^(1/3)*a + b)^2*a^4/(d*x + c)^(2/3) - 20*((d*x + c)^(1/3)*a
+ b)^3*a^3/(d*x + c) + 15*((d*x + c)^(1/3)*a + b)^4*a^2/(d*x + c)^(4/3) - 6
*((d*x + c)^(1/3)*a + b)^5*a/(d*x + c)^(5/3) + ((d*x + c)^(1/3)*a + b)^6/(d
*x + c)^2)*b*d))/d
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{1/3}}\right)\,\left(e+f\,x\right)\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/3))*(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^(1/3))*(e + f*x), x)
```

3.219 $\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 136

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx=\frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}+\frac{b^3\cos(a)\operatorname{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}-\frac{b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d}-\frac{b^3\sin(a)\operatorname{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}$$

```
[Out] 1/2*b^3*Ci(b/(d*x+c)^(1/3))*cos(a)/d+1/2*b*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(1
/3))/d-1/2*b^3*Si(b/(d*x+c)^(1/3))*sin(a)/d-1/2*b^2*(d*x+c)^(1/3)*sin(a+b/(
d*x+c)^(1/3))/d+(d*x+c)*sin(a+b/(d*x+c)^(1/3))/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3442, 3378, 3384, 3380, 3383}

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \frac{b^3\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} - \frac{b^3\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} - \frac{b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} + \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}$$

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d) + (b^3*Cos[a]*CosInteg
ral[b/(c + d*x)^(1/3)])/(2*d) - (b^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1
/3)])/(2*d) + ((c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/d - (b^3*Sin[a]*SinInt
egral[b/(c + d*x)^(1/3)])/(2*d)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d} \\
&= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} - \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d} \\
&= \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} \\
&\quad + \frac{b^2\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2d} \\
&= \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} - \frac{b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} \\
&\quad + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} + \frac{b^3\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2d} \\
&= \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} - \frac{b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} \\
&\quad + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} + \frac{(b^3\cos(a))\,\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2d} \\
&\quad - \frac{(b^3\sin(a))\,\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2d}
\end{aligned}
$$

$$= \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{b^3\cos(a)\operatorname{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} - \frac{b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} - \frac{b^3\sin(a)\operatorname{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx = \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) + b^3\cos(a)\operatorname{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right) + 2c\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) + 2dx\sin}{2d}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)],x]
[Out] (b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)] + b^3*Cos[a]*CosIntegral[b/(c
 + d*x)^(1/3)] + 2*c*Sin[a + b/(c + d*x)^(1/3)] + 2*d*x*Sin[a + b/(c + d*x)
^(1/3)] - b^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)] - b^3*Sin[a]*SinIn
tegral[b/(c + d*x)^(1/3)])/(2*d)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{3b^3\left(-\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)}{3b^3}-\frac{\cos\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{2}{3}}}{6b^2}+\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{1}{3}}}{6b}+\frac{\operatorname{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a)}{6}-\operatorname{Ci}\left(\right.\right.}{d}$
default	$-\frac{3b^3\left(-\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)}{3b^3}-\frac{\cos\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{2}{3}}}{6b^2}+\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{1}{3}}}{6b}+\frac{\operatorname{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a)}{6}-\operatorname{Ci}\left(\right.\right.}{d}$

```
[In] int(sin(a+b/(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d*b^3*(-1/3*sin(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3)
)/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(d*
x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.89

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \frac{b^3\cos(a)\operatorname{Ci}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)-b^3\sin(a)\operatorname{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)+(dx+c)^{\frac{2}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)-\left((dx+c)^{\frac{1}{3}}b^2-2\,dx\right.}{2\,d}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*cos(a)*cos_integral(b/(d*x + c)^(1/3)) - b^3*sin(a)*sin_integral(b
/(d*x + c)^(1/3)) + (d*x + c)^(2/3)*b*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)
/(d*x + c)) - ((d*x + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin((a*d*x + a*c + (d*x +
 c)^(2/3)*b)/(d*x + c)))/d
```

Sympy [F]

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(1/3)),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \frac{\left(\left(\operatorname{Ei}\left(\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)+\operatorname{Ei}\left(-\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos(a)+\left(i\operatorname{Ei}\left(\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)-i\operatorname{Ei}\left(-\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)\right)\sin(a)\right)b^3+2\,(dx+c)^{\frac{2}{3}}}{4\,d}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

[Out] 1/4*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 + 2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(114) = 228$.

Time = 0.51 (sec) , antiderivative size = 663, normalized size of antiderivative = 4.88

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \text{Too large to display}$$

[In] integrate(sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")

[Out] 1/2*(a^3*b^4*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + a^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 3*((d*x + c)^(1/3)*a + b)*a^2*b^4*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 3*((d*x + c)^(1/3)*a + b)*a^2*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 3*((d*x + c)^(1/3)*a + b)^2*a*b^4*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 3*((d*x + c)^(1/3)*a + b)^2*a*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3*b^4*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - ((d*x + c)^(1/3)*a + b)^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + a^2*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x + c)^(1/3)*a + b)*a*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + ((d*x + c)^(1/3)*a + b)^2*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + a*b^4*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - ((d*x + c)^(1/3)*a + b)*b^4*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 2*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))/((a^3 - 3*((d*x + c)^(1/3)*a + b)*a^2/(d*x + c)^(1/3) + 3*((d*x + c)^(1/3)*a + b)^2*a/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3/(d*x + c))*b*d)

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{1/3}}\right) \,d x$$

[In] int(sin(a + b/(c + d*x)^(1/3)),x)

[Out] int(sin(a + b/(c + d*x)^(1/3)), x)

3.220 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 434

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx = -\frac{3\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{f} + \frac{\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)}{f} + \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)}{f} + \frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)}{f} - \frac{3\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f} - \frac{\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{f} + \frac{\cos\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f} + \frac{\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f}$$

```
[Out] -3*cos(a)*Si(b/(d*x+c)^(1/3))/f-cos(a+(-1)^(1/3)*b*f^(1/3)/(-c*f+d*e)^(1/3)
)*Si((-1)^(1/3)*b*f^(1/3)/(-c*f+d*e)^(1/3)-b/(d*x+c)^(1/3))/f+cos(a-b*f^(1/
3)/(-c*f+d*e)^(1/3))*Si(b*f^(1/3)/(-c*f+d*e)^(1/3)+b/(d*x+c)^(1/3))/f+cos(a
-(-1)^(2/3)*b*f^(1/3)/(-c*f+d*e)^(1/3))*Si((-1)^(2/3)*b*f^(1/3)/(-c*f+d*e)^
(1/3)+b/(d*x+c)^(1/3))/f-3*Ci(b/(d*x+c)^(1/3))*sin(a)/f+Ci(b*f^(1/3)/(-c*f+
d*e)^(1/3)+b/(d*x+c)^(1/3))*sin(a-b*f^(1/3)/(-c*f+d*e)^(1/3))/f+Ci((-1)^(1/
3)*b*f^(1/3)/(-c*f+d*e)^(1/3)-b/(d*x+c)^(1/3))*sin(a+(-1)^(1/3)*b*f^(1/3)/(
-c*f+d*e)^(1/3))/f+Ci((-1)^(2/3)*b*f^(1/3)/(-c*f+d*e)^(1/3)+b/(d*x+c)^(1/3)
)*sin(a-(-1)^(2/3)*b*f^(1/3)/(-c*f+d*e)^(1/3))/f
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3512, 3384, 3380, 3383, 3426}

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx = \frac{\sin\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{f}b}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f} + \frac{\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{f} + \frac{\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{f}b}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f} - \frac{3\sin(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f} - \frac{\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{f} + \frac{\cos\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{\sqrt[3]{f}b}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f} + \frac{\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{f}b}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f} - \frac{3\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f}$$

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x),x]
```

```
[Out] (-3*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/f + (CosIntegral[(b*f^(1/3))/(d*
e - c*f)^(1/3) + b/(c + d*x)^(1/3)]*Sin[a - (b*f^(1/3))/(d*e - c*f)^(1/3)])
/f + (CosIntegral[((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3) - b/(c + d*x)^(1
/3)]*Sin[a + ((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3)])/f + (CosIntegral[((
-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3) + b/(c + d*x)^(1/3)]*Sin[a - ((-1)^(
2/3)*b*f^(1/3))/(d*e - c*f)^(1/3)])/f - (3*Cos[a]*SinIntegral[b/(c + d*x)^(
1/3)])/f - (Cos[a + ((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3)]*SinIntegral[(
(-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3) - b/(c + d*x)^(1/3)])/f + (Cos[a -
(b*f^(1/3))/(d*e - c*f)^(1/3)]*SinIntegral[(b*f^(1/3))/(d*e - c*f)^(1/3) +
```

```
b/(c + d*x)^(1/3)])/f + (Cos[a - ((-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3)]*
SinIntegral[((-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3) + b/(c + d*x)^(1/3)])/
f
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = -\frac{3\text{Subst}\left(\int \left(\frac{d\sin(a+bx)}{fx}+\frac{d(-de+cf)x^2\sin(a+bx)}{f(f+(de-cf)x^3)}\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{f}+\frac{(3(de-cf))\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{f+(de-cf)x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$= \frac{(3(de-cf))\text{Subst}\left(\int \left(\frac{\sin(a+bx)}{3(de-cf)^{2/3}\left(\sqrt[3]{f}+\sqrt[3]{de-cf}x\right)}+\frac{\sin(a+bx)}{3(de-cf)^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{f}+\sqrt[3]{de-cf}x\right)}+\frac{}{3(de}\right.\right.}{f}$$

$$-\frac{(3\cos(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$-\frac{(3\sin(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$= -\frac{3\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{f}-\frac{3\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f}$$

$$+\frac{\sqrt[3]{de-cf}\,\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt[3]{f}+\sqrt[3]{de-cf}x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$+\frac{\sqrt[3]{de-cf}\,\text{Subst}\left(\int \frac{\sin(a+bx)}{-\sqrt[3]{-1}\sqrt[3]{f}+\sqrt[3]{de-cf}x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$+\frac{\sqrt[3]{de-cf}\,\text{Subst}\left(\int \frac{\sin(a+bx)}{(-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{de-cf}x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$= -\frac{3\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{f} - \frac{3\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f}$$

$$+ \frac{\left(\sqrt[3]{de-cf}\cos\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\right)\mathrm{Subst}\left(\int \frac{\sin\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+bx\right)}{\sqrt[3]{f}+\sqrt[3]{de-cf}x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$- \frac{\left(\sqrt[3]{de-cf}\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\right)\mathrm{Subst}\left(\int \frac{\sin\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-bx\right)}{-\sqrt[3]{-1}\sqrt[3]{f}+\sqrt[3]{de-cf}x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$+ \frac{\left(\sqrt[3]{de-cf}\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\right)\mathrm{Subst}\left(\int \frac{\sin\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+bx\right)}{(-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{de-cf}x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$+ \frac{\left(\sqrt[3]{de-cf}\sin\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\right)\mathrm{Subst}\left(\int \frac{\cos\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+bx\right)}{\sqrt[3]{f}+\sqrt[3]{de-cf}x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$+ \frac{\left(\sqrt[3]{de-cf}\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\right)\mathrm{Subst}\left(\int \frac{\cos\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-bx\right)}{-\sqrt[3]{-1}\sqrt[3]{f}+\sqrt[3]{de-cf}x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$+ \frac{\left(\sqrt[3]{de-cf}\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\right)\mathrm{Subst}\left(\int \frac{\cos\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+bx\right)}{(-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{de-cf}x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$= -\frac{3\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{f} + \frac{\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)}{f} + \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)}{f} + \frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)}{f} - \frac{3\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f} - \frac{\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{f} + \frac{\cos\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f} + \frac{\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 25.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.39

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx = \frac{i\left(\left(-3\,\text{ExpIntegralEi}\left(-\frac{ib}{\sqrt[3]{c+dx}}\right)+\text{RootSum}\left[de-cf+f\#1^3\&,e^{-\frac{ib}{\#1}}\,\text{ExpIntegralEi}\left(-ib\left(\frac{1}{\sqrt[3]{c+dx}}\right.\right.\right.\right.\right.}{}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x),x]
```

```
[Out] ((I/2)*((-3*ExpIntegralEi[((-I)*b)/(c + d*x)^(1/3)] + RootSum[d*e - c*f + f
*#1^3 & , ExpIntegralEi[(-I)*b*((c + d*x)^(-1/3) - #1^(-1))]/E^((I*b)/#1) &
 ])*(Cos[a] - I*Sin[a]) + (3*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)] - RootSum
[d*e - c*f + f*#1^3 & , E^((I*b)/#1)*ExpIntegralEi[I*b*((c + d*x)^(-1/3) -
#1^(-1))] & ])*(Cos[a] + I*Sin[a])))/f
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.99 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.36

method	result
derivativedivides	$-3b^3\left(\frac{\operatorname{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\cos(a)+\operatorname{Ci}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a)}{f\,b^3}-\frac{\sum_{\_R1=\operatorname{RootOf}\left((cf-de)\_Z^3+(-3acf+3ade)\_Z^2+\left(3a^2cf-3a^2de\right.\right.}}{}\right.$
default	$-3b^3\left(\frac{\operatorname{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\cos(a)+\operatorname{Ci}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a)}{f\,b^3}-\frac{\sum_{\_R1=\operatorname{RootOf}\left((cf-de)\_Z^3+(-3acf+3ade)\_Z^2+\left(3a^2cf-3a^2de\right.\right.}}{}\right.$

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] -3*b^3*(1/f/b^3*(Si(b/(d*x+c)^(1/3))*cos(a)+Ci(b/(d*x+c)^(1/3))*sin(a))-1/3
/f/b^3*sum(-Si(-b/(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b/(d*x+c)^(1/3)-_R1+a)*s
in(_R1),_R1=RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*
d*e)*_Z-a^3*c*f+a^3*d*e-f*b^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.26

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx = \frac{i\operatorname{Ei}\left(\frac{-2i\,(dx+c)^{\frac{2}{3}}b-\left(\frac{i\,b^3f}{de-cf}\right)^{\frac{1}{3}}\left(dx-\sqrt{3}(-i\,dx-i\,c)+c\right)}{2\,(dx+c)}\right)e^{\left(\frac{1}{2}\left(\frac{i\,b^3f}{de-cf}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,a\right)}-i\operatorname{Ei}\left(\frac{2i\,(dx+c)^{\frac{2}{3}}b-\left(-\frac{i\,b^3f}{de-cf}\right)^{\frac{1}{3}}\left(dx-\right.}{2\,(dx+c)}\right.}{}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(I*Ei(1/2*(-2*I*(d*x + c)^(2/3)*b - (I*b^3*f/(d*e - c*f))^(1/3)*(d*x -
sqrt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(I*b^3*f/(d*e - c*f))^(1/3)*
(I*sqrt(3) + 1) - I*a) - I*Ei(1/2*(2*I*(d*x + c)^(2/3)*b - (-I*b^3*f/(d*e -
 c*f))^(1/3)*(d*x - sqrt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(-I*b^3*
f/(d*e - c*f))^(1/3)*(I*sqrt(3) + 1) + I*a) + I*Ei(1/2*(-2*I*(d*x + c)^(2/3
)*b - (I*b^3*f/(d*e - c*f))^(1/3)*(d*x - sqrt(3)*(I*d*x + I*c) + c))/(d*x +
```

```
c))*e^(1/2*(I*b^3*f/(d*e - c*f))^(1/3)*(-I*sqrt(3) + 1) - I*a) - I*Ei(1/2*
(2*I*(d*x + c)^(2/3)*b - (-I*b^3*f/(d*e - c*f))^(1/3)*(d*x - sqrt(3)*(I*d*x
 + I*c) + c))/(d*x + c))*e^(1/2*(-I*b^3*f/(d*e - c*f))^(1/3)*(-I*sqrt(3) +
1) + I*a) - I*Ei((I*(d*x + c)^(2/3)*b + (-I*b^3*f/(d*e - c*f))^(1/3)*(d*x +
 c))/(d*x + c))*e^(I*a - (-I*b^3*f/(d*e - c*f))^(1/3)) + I*Ei((-I*(d*x + c)
^(2/3)*b + (I*b^3*f/(d*e - c*f))^(1/3)*(d*x + c))/(d*x + c))*e^(-I*a - (I*b
^3*f/(d*e - c*f))^(1/3)) - 6*cos_integral(b/(d*x + c)^(1/3))*sin(a) - 6*cos
(a)*sin_integral(b/(d*x + c)^(1/3)))/f
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(f*x+e),x)
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e + f*x), x)
```

Maxima [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e), x)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{e+f\,x}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^(1/3))/(e + f*x), x)
```

3.221 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 566

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx = -\frac{bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\operatorname{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}} - \frac{(-1)^{2/3}bd\cos\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}} + \frac{\sqrt[3]{-1}bd\cos\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{bd\sin\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\operatorname{Si}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}} - \frac{(-1)^{2/3}bd\sin\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\operatorname{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}} - \frac{\sqrt[3]{-1}bd\sin\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$

```
[Out] -1/3*b*d*Ci(b*f^(1/3)/(c*f-d*e)^(1/3)-b/(d*x+c)^(1/3))*cos(a+b*f^(1/3)/(c*f
-d*e)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)+1/3*(-1)^(1/3)*b*d*Ci((-1)^(1/3)*b*f^(
1/3)/(c*f-d*e)^(1/3)+b/(d*x+c)^(1/3))*cos(a-(-1)^(1/3)*b*f^(1/3)/(c*f-d*e)^
(1/3))/f^(2/3)/(c*f-d*e)^(4/3)-1/3*(-1)^(2/3)*b*d*Ci((-1)^(2/3)*b*f^(1/3)/(
c*f-d*e)^(1/3)-b/(d*x+c)^(1/3))*cos(a+(-1)^(2/3)*b*f^(1/3)/(c*f-d*e)^(1/3))
/f^(2/3)/(c*f-d*e)^(4/3)-1/3*b*d*Si(b*f^(1/3)/(c*f-d*e)^(1/3)-b/(d*x+c)^(1/
3))*sin(a+b*f^(1/3)/(c*f-d*e)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)-1/3*(-1)^(1/3)
*b*d*Si((-1)^(1/3)*b*f^(1/3)/(c*f-d*e)^(1/3)+b/(d*x+c)^(1/3))*sin(a-(-1)^(1
/3)*b*f^(1/3)/(c*f-d*e)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)-1/3*(-1)^(2/3)*b*d*S
i((-1)^(2/3)*b*f^(1/3)/(c*f-d*e)^(1/3)-b/(d*x+c)^(1/3))*sin(a+(-1)^(2/3)*b*
f^(1/3)/(c*f-d*e)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)+(d*x+c)*sin(a+b/(d*x+c)^(1
/3))/(-c*f+d*e)/(f*x+e)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3512, 3422, 3415, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx$$
$$= -\frac{bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}$$
$$-\frac{(-1)^{2/3}bd\cos\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}$$
$$+\frac{\sqrt[3]{-1}bd\cos\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{f}b}{\sqrt[3]{cf-de}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}$$
$$-\frac{bd\sin\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{Si}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}$$
$$-\frac{(-1)^{2/3}bd\sin\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}$$
$$-\frac{\sqrt[3]{-1}bd\sin\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{f}b}{\sqrt[3]{cf-de}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}$$
$$+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)(de-cf)}$$

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

```
[Out] -1/3*(b*d*Cos[a + (b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*CosIntegral[(b*f^(1/3))
/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(f^(2/3)*(-(d*e) + c*f)^(4/3))
- ((-1)^(2/3)*b*d*Cos[a + ((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*CosI
ntegral[((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(
3*f^(2/3)*(-(d*e) + c*f)^(4/3)) + ((-1)^(1/3)*b*d*Cos[a - ((-1)^(1/3)*b*f^(
1/3))/(-(d*e) + c*f)^(1/3)]*CosIntegral[((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*
f)^(1/3) + b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3)) + ((c + d*x
)*Sin[a + b/(c + d*x)^(1/3)])/((d*e - c*f)*(e + f*x)) - (b*d*Sin[a + (b*f^(
1/3))/(-(d*e) + c*f)^(1/3)]*SinIntegral[(b*f^(1/3))/(-(d*e) + c*f)^(1/3) -
```

```
b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3)) - ((-1)^(2/3)*b*d*Sin[
a + ((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*SinIntegral[((-1)^(2/3)*b*
f^(1/3))/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*
f)^(4/3)) - ((-1)^(1/3)*b*d*Sin[a - ((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*f)^(
1/3)]*SinIntegral[((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3) + b/(c + d*x)
^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = -\frac{3\text{Subst}\left(\int \frac{x^2 \sin(a+bx)}{\left(\frac{f}{d}+\left(e-\frac{cf}{d}\right)x^3\right)^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{\frac{f}{d}+\left(e-\frac{cf}{d}\right)x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de-cf}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)}$$

$$- \frac{b\text{Subst}\left(\int \left(\frac{d\cos(a+bx)}{3f^{2/3}\left(\sqrt[3]{f}-\sqrt[3]{-de+cf}\,x\right)} + \frac{d\cos(a+bx)}{3f^{2/3}\left(\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{-de+cf}\,x\right)} + \frac{d\cos(a+bx)}{3f^{2/3}\left(\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{-\right.}\right.\right.}{de-cf}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt[3]{f}-\sqrt[3]{-de+cf}\,x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(de-cf)}$$

$$- \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{-de+cf}\,x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(de-cf)}$$

$$- \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{-de+cf}\,x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(de-cf)}$$

$$
\begin{aligned}
&= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)} \\
&- \frac{\left(bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-bx\right)}{\sqrt[3]{f}-\sqrt[3]{-de+cf}x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(de-cf)} \\
&- \frac{\left(bd\cos\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}+bx\right)}{\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{-de+cf}x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(de-cf)} \\
&- \frac{\left(bd\cos\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-bx\right)}{\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{-de+cf}x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(de-cf)} \\
&- \frac{\left(bd\sin\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-bx\right)}{\sqrt[3]{f}-\sqrt[3]{-de+cf}x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(de-cf)} \\
&+ \frac{\left(bd\sin\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}+bx\right)}{\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{-de+cf}x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(de-cf)} \\
&- \frac{\left(bd\sin\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-bx\right)}{\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{-de+cf}x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(de-cf)}
\end{aligned}
$$

$$= -\frac{bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$
$$-\frac{(-1)^{2/3}bd\cos\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$
$$+\frac{\sqrt[3]{-1}bd\cos\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$
$$+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)}$$
$$-\frac{bd\sin\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{Si}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$
$$-\frac{(-1)^{2/3}bd\sin\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$
$$-\frac{\sqrt[3]{-1}bd\sin\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.94 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.55

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx$$
$$= \frac{(\cos(a)+i\sin(a))\left(bd(e+fx)\text{RootSum}\left[de-cf+f\#1^3\&,\frac{\text{ExpIntegralEi}\left(\frac{ib}{\sqrt[3]{c+dx}}\right)-e^{\frac{ib}{\#1}}\text{ExpIntegralEi}\left(ib\left(\frac{}{\sqrt[3]{}}\right.\right.}{\#1}\right.\right.}{}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

```
[Out] ((Cos[a] + I*Sin[a])*(b*d*(e + f*x)*RootSum[d*e - c*f + f*#1^3 & , (ExpInte
gralEi[(I*b)/(c + d*x)^(1/3)] - E^((I*b)/#1)*ExpIntegralEi[I*b*((c + d*x)^(
-1/3) - #1^(-1))])/#1 & ] + (c + d*x)*((3*I)*f*Cos[b/(c + d*x)^(1/3)] - 3*f
*Sin[b/(c + d*x)^(1/3)])) + I*(-3*c*f - 3*d*f*x + b*d*(e + f*x)*RootSum[d*e
```

```
 - c*f + f*#1^3 & , (ExpIntegralEi[((-I)*b)/(c + d*x)^(1/3)] - ExpIntegralE
i[(-I)*b*((c + d*x)^(-1/3) - #1^(-1))]/E^((I*b)/#1))/#1 & ]*((-I)*Cos[b/(c
+ d*x)^(1/3)] + Sin[b/(c + d*x)^(1/3)]))*(Cos[a + b/(c + d*x)^(1/3)] - I*Si
n[a + b/(c + d*x)^(1/3)]))/(6*f*(-(d*e) + c*f)*(e + f*x))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.08 (sec) , antiderivative size = 1554, normalized size of antiderivative = 2.75

method	result	size
derivativedivides	Expression too large to display	1554
default	Expression too large to display	1554

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3*d*b^3*(a^2*(sin(a+b/(d*x+c)^(1/3))*(1/3/f/b^3*(a+b/(d*x+c)^(1/3))-1/3*a/
f/b^3)/(a^3*c*f-a^3*d*e-3*a^2*c*f*(a+b/(d*x+c)^(1/3))+3*a^2*d*e*(a+b/(d*x+c
)^(1/3))+3*a*c*f*(a+b/(d*x+c)^(1/3))^2-3*a*d*e*(a+b/(d*x+c)^(1/3))^2-c*f*(a
+b/(d*x+c)^(1/3))^3+d*e*(a+b/(d*x+c)^(1/3))^3+f*b^3)-2/9/f/b^3*sum(1/(_R1^2
*c*f-_R1^2*d*e-2*_R1*a*c*f+2*_R1*a*d*e+a^2*c*f-a^2*d*e)*(-Si(-b/(d*x+c)^(1/
3)+_R1-a)*cos(_R1)+Ci(b/(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf((c*f-d*e)
*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-f*b^
3))-1/9/f/b^3*sum(1/(-_RR1*c*f+_RR1*d*e+a*c*f-a*d*e)*(Si(-b/(d*x+c)^(1/3)+_
RR1-a)*sin(_RR1)+Ci(b/(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf((c*f-d*e
)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-f*b
^3)))+sin(a+b/(d*x+c)^(1/3))*(-2/3*a/f/b^3*(a+b/(d*x+c)^(1/3))^2+2/3*a^2/f/
b^3*(a+b/(d*x+c)^(1/3)))/(a^3*c*f-a^3*d*e-3*a^2*c*f*(a+b/(d*x+c)^(1/3))+3*a
^2*d*e*(a+b/(d*x+c)^(1/3))+3*a*c*f*(a+b/(d*x+c)^(1/3))^2-3*a*d*e*(a+b/(d*x+
c)^(1/3))^2-c*f*(a+b/(d*x+c)^(1/3))^3+d*e*(a+b/(d*x+c)^(1/3))^3+f*b^3)+2/9*
a/f/b^3*sum((_R1+a)/(_R1^2*c*f-_R1^2*d*e-2*_R1*a*c*f+2*_R1*a*d*e+a^2*c*f-a^
2*d*e)*(-Si(-b/(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b/(d*x+c)^(1/3)-_R1+a)*sin(
_R1)),_R1=RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*
e)*_Z-a^3*c*f+a^3*d*e-f*b^3))+2/9*a/f/b^3*sum(_RR1/(-_RR1*c*f+_RR1*d*e+a*c*
f-a*d*e)*(Si(-b/(d*x+c)^(1/3)+_RR1-a)*sin(_RR1)+Ci(b/(d*x+c)^(1/3)-_RR1+a)*
cos(_RR1)),_RR1=RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*
a^2*d*e)*_Z-a^3*c*f+a^3*d*e-f*b^3))+sin(a+b/(d*x+c)^(1/3))*(2/3*a/f/b^3*(a+
b/(d*x+c)^(1/3))^2-a^2/f/b^3*(a+b/(d*x+c)^(1/3))+1/3*(a^3*c*f-a^3*d*e+b^3*f
)/f/b^3/(c*f-d*e))/(a^3*c*f-a^3*d*e-3*a^2*c*f*(a+b/(d*x+c)^(1/3))+3*a^2*d*e
*(a+b/(d*x+c)^(1/3))+3*a*c*f*(a+b/(d*x+c)^(1/3))^2-3*a*d*e*(a+b/(d*x+c)^(1/
3))^2-c*f*(a+b/(d*x+c)^(1/3))^3+d*e*(a+b/(d*x+c)^(1/3))^3+f*b^3)-2/9*a/f/b^
3*sum(_R1/(_R1^2*c*f-_R1^2*d*e-2*_R1*a*c*f+2*_R1*a*d*e+a^2*c*f-a^2*d*e)*(-S
i(-b/(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b/(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=
RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*
```

```
c*f+a^3*d*e-f*b^3))+1/9/f/b^3*sum((2*_RR1^2*a*c*f-2*_RR1^2*a*d*e-3*_RR1*a^2
*c*f+3*_RR1*a^2*d*e+a^3*c*f-a^3*d*e+b^3*f)/(c*f-d*e)/(_RR1^2*c*f-_RR1^2*d*e
-2*_RR1*a*c*f+2*_RR1*a*d*e+a^2*c*f-a^2*d*e)*(Si(-b/(d*x+c)^(1/3)+_RR1-a)*si
n(_RR1)+Ci(b/(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf((c*f-d*e)*_Z^3+(-
3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-f*b^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 798, normalized size of antiderivative = 1.41

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx =$$

$$-\frac{\left(\frac{i b^3 f}{de-cf}\right)^{\frac{1}{3}}\left(-i\,dfx-i\,de+\sqrt{3}(dfx+de)\right)\mathrm{Ei}\left(\frac{-2i\,(dx+c)^{\frac{2}{3}}b-\left(\frac{i b^3 f}{de-cf}\right)^{\frac{1}{3}}\left(dx-\sqrt{3}(-i\,dx-i\,c)+c\right)}{2\,(dx+c)}\right)e^{\left(\frac{1}{2}\left(\frac{i b^3 f}{de-cf}\right)^{\frac{1}{3}}\left(i\sqrt{3}\right.\right.}}{}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/12*((I*b^3*f/(d*e - c*f))^(1/3)*(-I*d*f*x - I*d*e + sqrt(3)*(d*f*x + d*e
))*Ei(1/2*(-2*I*(d*x + c)^(2/3)*b - (I*b^3*f/(d*e - c*f))^(1/3)*(d*x - sqrt
(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(I*b^3*f/(d*e - c*f))^(1/3)*(I*s
qrt(3) + 1) - I*a) + (-I*b^3*f/(d*e - c*f))^(1/3)*(I*d*f*x + I*d*e - sqrt(3
)*(d*f*x + d*e))*Ei(1/2*(2*I*(d*x + c)^(2/3)*b - (-I*b^3*f/(d*e - c*f))^(1/
3)*(d*x - sqrt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(-I*b^3*f/(d*e - c
*f))^(1/3)*(I*sqrt(3) + 1) + I*a) + (I*b^3*f/(d*e - c*f))^(1/3)*(-I*d*f*x -
 I*d*e - sqrt(3)*(d*f*x + d*e))*Ei(1/2*(-2*I*(d*x + c)^(2/3)*b - (I*b^3*f/(
d*e - c*f))^(1/3)*(d*x - sqrt(3)*(I*d*x + I*c) + c))/(d*x + c))*e^(1/2*(I*b
^3*f/(d*e - c*f))^(1/3)*(-I*sqrt(3) + 1) - I*a) + (-I*b^3*f/(d*e - c*f))^(1
/3)*(I*d*f*x + I*d*e + sqrt(3)*(d*f*x + d*e))*Ei(1/2*(2*I*(d*x + c)^(2/3)*b
 - (-I*b^3*f/(d*e - c*f))^(1/3)*(d*x - sqrt(3)*(I*d*x + I*c) + c))/(d*x + c
))*e^(1/2*(-I*b^3*f/(d*e - c*f))^(1/3)*(-I*sqrt(3) + 1) + I*a) - 2*(-I*b^3*
f/(d*e - c*f))^(1/3)*(I*d*f*x + I*d*e)*Ei((I*(d*x + c)^(2/3)*b + (-I*b^3*f/
(d*e - c*f))^(1/3)*(d*x + c))/(d*x + c))*e^(I*a - (-I*b^3*f/(d*e - c*f))^(1
/3)) - 2*(I*b^3*f/(d*e - c*f))^(1/3)*(-I*d*f*x - I*d*e)*Ei((-I*(d*x + c)^(2
/3)*b + (I*b^3*f/(d*e - c*f))^(1/3)*(d*x + c))/(d*x + c))*e^(-I*a - (I*b^3*
f/(d*e - c*f))^(1/3)) - 12*(d*f*x + c*f)*sin((a*d*x + a*c + (d*x + c)^(2/3)
*b)/(d*x + c)))/(d*e^2*f - c*e*f^2 + (d*e*f^2 - c*f^3)*x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(f*x+e)**2,x)
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e)^2, x)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(e+f\,x)^2}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^(1/3))/(e + f*x)^2, x)
```

3.222 $\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 630

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}-\frac{8b^3f^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}+\frac{bf(de-cf)(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}+\frac{2bf^2(c+dx)^{7/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{21d^3}+\frac{b^3f(de-cf)\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}-\frac{16b^{9/2}f^2\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{315d^3}+\frac{2b^{3/2}(de-cf)^2\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^3}+\frac{2b^{3/2}(de-cf)^2\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d^3}+\frac{16b^{9/2}f^2\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{315d^3}+\frac{16b^4f^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}-\frac{b^2f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}+\frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}-\frac{4b^2f^2(c+dx)^{5/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{105d^3}+\frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{3d^3}-\frac{b^3f(de-cf)\sin(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$

```
[Out] 1/2*b^3*f*(-c*f+d*e)*Ci(b/(d*x+c)^(2/3))*cos(a)/d^3+2*b*(-c*f+d*e)^2*(d*x+c
)^(1/3)*cos(a+b/(d*x+c)^(2/3))/d^3-8/315*b^3*f^2*(d*x+c)*cos(a+b/(d*x+c)^(2
/3))/d^3+1/2*b*f*(-c*f+d*e)*(d*x+c)^(4/3)*cos(a+b/(d*x+c)^(2/3))/d^3+2/21*b
*f^2*(d*x+c)^(7/3)*cos(a+b/(d*x+c)^(2/3))/d^3-1/2*b^3*f*(-c*f+d*e)*Si(b/(d*
x+c)^(2/3))*sin(a)/d^3+16/315*b^4*f^2*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(2/3))/
d^3-1/2*b^2*f*(-c*f+d*e)*(d*x+c)^(2/3)*sin(a+b/(d*x+c)^(2/3))/d^3+(-c*f+d*e
)^2*(d*x+c)*sin(a+b/(d*x+c)^(2/3))/d^3-4/105*b^2*f^2*(d*x+c)^(5/3)*sin(a+b/
(d*x+c)^(2/3))/d^3+f*(-c*f+d*e)*(d*x+c)^2*sin(a+b/(d*x+c)^(2/3))/d^3+1/3*f^
2*(d*x+c)^3*sin(a+b/(d*x+c)^(2/3))/d^3-16/315*b^(9/2)*f^2*cos(a)*FresnelC(b
^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*Pi^(1/2)/d^3+2*b^(3/2)*(-c*f
+d*e)^2*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*Pi^
(1/2)/d^3+2*b^(3/2)*(-c*f+d*e)^2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^
```

```
(1/3))*sin(a)*2^(1/2)*Pi^(1/2)/d^3+16/315*b^(9/2)*f^2*FresnelS(b^(1/2)*2^(1
/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)*2^(1/2)*Pi^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules

used = {3514, 3490, 3468, 3469, 3434, 3433, 3432, 3460, 3378, 3384, 3380, 3383, 3435}

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \frac{2\sqrt{2\pi}b^{3/2}\sin(a)(de-cf)^2 \operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^3} + \frac{2\sqrt{2\pi}b^{3/2}\cos(a)(de-cf)^2 \operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{16\sqrt{2\pi}b^{9/2}f^2\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{315d^3} + \frac{16\sqrt{2\pi}b^{9/2}f^2\sin(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{315d^3} + \frac{16b^4f^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3} + \frac{b^3f\cos(a)(de-cf)\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3} - \frac{b^3f\sin(a)(de-cf)\operatorname{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3} - \frac{8b^3f^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3} - \frac{b^2f(c+dx)^{2/3}(de-cf)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3} - \frac{4b^2f^2(c+dx)^{5/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{105d^3} + \frac{f(c+dx)^2(de-cf)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{(c+dx)(de-cf)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{bf(c+dx)^{4/3}(de-cf)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3} + \frac{2b\sqrt[3]{c+dx}(de-cf)^2\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{3d^3} + \frac{2bf^2(c+dx)^{7/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{21d^3}$$

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(d*e - c*f)^2*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/d^3 - (8*b^3
*f^2*(c + d*x)*Cos[a + b/(c + d*x)^(2/3)])/(315*d^3) + (b*f*(d*e - c*f)*(c
+ d*x)^(4/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*d^3) + (2*b*f^2*(c + d*x)^(7/3)
*Cos[a + b/(c + d*x)^(2/3)])/(21*d^3) + (b^3*f*(d*e - c*f)*Cos[a]*CosIntegr
al[b/(c + d*x)^(2/3)])/(2*d^3) - (16*b^(9/2)*f^2*Sqrt[2*Pi]*Cos[a]*FresnelC
[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/(315*d^3) + (2*b^(3/2)*(d*e - c*f)^
2*Sqrt[2*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/d^3 + (
2*b^(3/2)*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^
(1/3)]*Sin[a])/d^3 + (16*b^(9/2)*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)]*Sin[a])/(315*d^3) + (16*b^4*f^2*(c + d*x)^(1/3)*Sin[a +
 b/(c + d*x)^(2/3)])/(315*d^3) - (b^2*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a +
 b/(c + d*x)^(2/3)])/(2*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin[a + b/(c + d*x)
^(2/3)])/d^3 - (4*b^2*f^2*(c + d*x)^(5/3)*Sin[a + b/(c + d*x)^(2/3)])/(105*
d^3) + (f*(d*e - c*f)*(c + d*x)^2*Sin[a + b/(c + d*x)^(2/3)])/d^3 + (f^2*(c
 + d*x)^3*Sin[a + b/(c + d*x)^(2/3)])/(3*d^3) - (b^3*f*(d*e - c*f)*Sin[a]*S
inIntegral[b/(c + d*x)^(2/3)])/(2*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{3\text{Subst}\left(\int \left((de-cf)^2x^2\sin\left(a+\frac{b}{x^2}\right)-2f(-de+cf)x^5\sin\left(a+\frac{b}{x^2}\right)+f^2x^8\sin\left(a+\frac{b}{x^2}\right)\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^3}$$

$$= \frac{(3f^2)\,\text{Subst}\left(\int x^8\sin\left(a+\frac{b}{x^2}\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^3} + \frac{(6f(de-cf))\text{Subst}\left(\int x^5\sin\left(a+\frac{b}{x^2}\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^3} + \frac{(3(de-cf)^2)\,\text{Subst}\left(\int x^2\sin\left(a+\frac{b}{x^2}\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^3}$$

$$= -\frac{(3f^2)\,\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^{10}}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{(3f(de-cf))\text{Subst}\left(\int \frac{\sin(a+bx)}{x^4}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{d^3} - \frac{(3(de-cf)^2)\,\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^4}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^3}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{3d^3} - \frac{(2bf^2)\,\text{Subst}\left(\int \frac{\cos(a+bx^2)}{x^8}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{3d^3} - \frac{(bf(de-cf))\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{d^3} - \frac{(2b(de-cf)^2)\,\text{Subst}\left(\int \frac{\cos(a+bx^2)}{x^2}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{bf(de-cf)(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3} + \frac{2bf^2(c+dx)^{7/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{21d^3} + \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{3d^3} + \frac{(4b^2f^2)\,\text{Subst}\left(\int\frac{\sin(a+bx^2)}{x^6}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{21d^3} + \frac{(b^2f(de-cf))\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^2}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{2d^3} + \frac{(4b^2(de-cf)^2)\,\text{Subst}\left(\int\sin\left(a+bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{bf(de-cf)(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3} + \frac{2bf^2(c+dx)^{7/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{21d^3} - \frac{b^2f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3} + \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{4b^2f^2(c+dx)^{5/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{105d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{3d^3} + \frac{(8b^3f^2)\,\text{Subst}\left(\int\frac{\cos(a+bx^2)}{x^4}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{105d^3} + \frac{(b^3f(de-cf))\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{2d^3} + \frac{(4b^2(de-cf)^2\cos(a))\,\text{Subst}\left(\int\sin\left(bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^3} + \frac{(4b^2(de-cf)^2\sin(a))\,\text{Subst}\left(\int\cos\left(bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{8b^3f^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}$$
$$+ \frac{bf(de-cf)(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3} + \frac{2bf^2(c+dx)^{7/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{21d^3}$$
$$+ \frac{2b^{3/2}(de-cf)^2\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^3}$$
$$+ \frac{2b^{3/2}(de-cf)^2\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d^3}$$
$$- \frac{b^2f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$
$$+ \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{4b^2f^2(c+dx)^{5/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{105d^3}$$
$$+ \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{3d^3}$$
$$- \frac{(16b^4f^2)\,\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{315d^3}$$
$$+ \frac{(b^3f(de-cf)\cos(a))\,\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d^3}$$
$$- \frac{(b^3f(de-cf)\sin(a))\,\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{8b^3f^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}$$
$$+ \frac{bf(de-cf)(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3} + \frac{2bf^2(c+dx)^{7/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{21d^3}$$
$$+ \frac{b^3f(de-cf)\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$
$$+ \frac{2b^{3/2}(de-cf)^2\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^3}$$
$$+ \frac{2b^{3/2}(de-cf)^2\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d^3}$$
$$+ \frac{16b^4f^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3} - \frac{b^2f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$
$$+ \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{4b^2f^2(c+dx)^{5/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{105d^3}$$
$$+ \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}$$
$$+ \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{3d^3} - \frac{b^3f(de-cf)\sin(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$
$$- \frac{(32b^5f^2)\,\text{Subst}\left(\int\cos\left(a+bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{315d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{8b^3f^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}$$
$$+ \frac{bf(de-cf)(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3} + \frac{2bf^2(c+dx)^{7/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{21d^3}$$
$$+ \frac{b^3f(de-cf)\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$
$$+ \frac{2b^{3/2}(de-cf)^2\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^3}$$
$$+ \frac{2b^{3/2}(de-cf)^2\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d^3}$$
$$+ \frac{16b^4f^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3} - \frac{b^2f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$
$$+ \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{4b^2f^2(c+dx)^{5/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{105d^3}$$
$$+ \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}$$
$$+ \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{3d^3} - \frac{b^3f(de-cf)\sin(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$
$$- \frac{(32b^5f^2\cos(a))\,\text{Subst}\left(\int\cos\left(bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{315d^3}$$
$$+ \frac{(32b^5f^2\sin(a))\,\text{Subst}\left(\int\sin\left(bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{315d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{8b^3f^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}$$
$$+ \frac{bf(de-cf)(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3} + \frac{2bf^2(c+dx)^{7/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{21d^3}$$
$$+ \frac{b^3f(de-cf)\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$
$$- \frac{16b^{9/2}f^2\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{315d^3}$$
$$+ \frac{2b^{3/2}(de-cf)^2\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^3}$$
$$+ \frac{2b^{3/2}(de-cf)^2\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d^3}$$
$$+ \frac{16b^{9/2}f^2\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{315d^3} + \frac{16b^4f^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}$$
$$- \frac{b^2f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$
$$+ \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{4b^2f^2(c+dx)^{5/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{105d^3}$$
$$+ \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}$$
$$+ \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{3d^3} - \frac{b^3f(de-cf)\sin(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 613, normalized size of antiderivative = 0.97

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx = \frac{ie^{-ia}\left(e^{-\frac{ib}{(c+dx)^{2/3}}}\sqrt[3]{c+dx}\left(32b^4f^2+16ib^3f^2(c+dx)^{2/3}+3b^2f\sqrt[3]{c+dx}(-105de+97cf\right.\right.}{}$$

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] ((I/1260)*(((c + d*x)^(1/3)*(32*b^4*f^2 + (16*I)*b^3*f^2*(c + d*x)^(2/3) +
3*b^2*f*(c + d*x)^(1/3)*(-105*d*e + 97*c*f - 8*d*f*x) - (15*I)*b*(84*d^2*e^
2 + 21*d*e*f*(-7*c + d*x) + f^2*(67*c^2 - 13*c*d*x + 4*d^2*x^2)) + 210*(c +
 d*x)^(2/3)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))
))/E^((I*b)/(c + d*x)^(2/3)) - E^(I*(2*a + b/(c + d*x)^(2/3)))*(c + d*x)^(1
/3)*(32*b^4*f^2 - (16*I)*b^3*f^2*(c + d*x)^(2/3) + 3*b^2*f*(c + d*x)^(1/3)*
(-105*d*e + 97*c*f - 8*d*f*x) + (15*I)*b*(84*d^2*e^2 + 21*d*e*f*(-7*c + d*x
) + f^2*(67*c^2 - 13*c*d*x + 4*d^2*x^2)) + 210*(c + d*x)^(2/3)*(c^2*f^2 - c
*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))) + 4*(-1)^(1/4)*b^(3/2)
*E^((2*I)*a)*((315*I)*d^2*e^2 - (630*I)*c*d*e*f + (8*b^3 + (315*I)*c^2)*f^2
)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b])/(c + d*x)^(1/3)] - 4*(-1)^(1/4)*b^(3/2
)*(315*d^2*e^2 - 630*c*d*e*f + ((8*I)*b^3 + 315*c^2)*f^2)*Sqrt[Pi]*Erfi[((-
1)^(3/4)*Sqrt[b])/(c + d*x)^(1/3)] + (315*I)*b^3*f*(-(d*e) + c*f)*ExpIntegr
alEi[((-I)*b)/(c + d*x)^(2/3)] + (315*I)*b^3*E^((2*I)*a)*f*(-(d*e) + c*f)*E
xpIntegralEi[(I*b)/(c + d*x)^(2/3)]))/(d^3*E^(I*a))
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{(cf-de)^2(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2(cf-de)^2b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)^{\frac{1}{3}}}\right)\right.\right.}{}$
default	$\frac{(cf-de)^2(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2(cf-de)^2b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)^{\frac{1}{3}}}\right)\right.\right.}{}$
parts	Expression too large to display

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^3*(1/3*(c*f-d*e)^2*(d*x+c)*sin(a+b/(d*x+c)^(2/3))-2/3*(c*f-d*e)^2*b*(-(
d*x+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*Fresne
lS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))+sin(a)*FresnelC(b^(1/2)*2^(1/2)/
Pi^(1/2)/(d*x+c)^(1/3))))-1/3*f*(c*f-d*e)*(d*x+c)^2*sin(a+b/(d*x+c)^(2/3))+
2/3*f*(c*f-d*e)*b*(-1/4*(d*x+c)^(4/3)*cos(a+b/(d*x+c)^(2/3))-1/2*b*(-1/2*(d
*x+c)^(2/3)*sin(a+b/(d*x+c)^(2/3))+b*(1/2*cos(a)*Ci(b/(d*x+c)^(2/3))-1/2*si
n(a)*Si(b/(d*x+c)^(2/3)))))+1/9*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^(2/3))-2/9*f^
```

```
2*b*(-1/7*(d*x+c)^(7/3)*cos(a+b/(d*x+c)^(2/3))-2/7*b*(-1/5*(d*x+c)^(5/3)*si
n(a+b/(d*x+c)^(2/3))+2/5*b*(-1/3*(d*x+c)*cos(a+b/(d*x+c)^(2/3))-2/3*b*(-(d*
x+c)^(1/3)*sin(a+b/(d*x+c)^(2/3))+b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC
(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi
^(1/2)/(d*x+c)^(1/3)))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.73

$$\int (e+fx)^2 \sin \left(a + \frac{b}{(c+dx)^{2/3}}\right) dx = \frac{315 \left(b^3 def - b^3 cf^2\right) \cos (a) \operatorname{Ci}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right) - 4\sqrt{2}(8\pi b^4 f^2 \cos(a) - 315\pi(bd^2e^2 - 2bcdef}{}$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 1/630*(315*(b^3*d*e*f - b^3*c*f^2)*cos(a)*cos_integral(b/(d*x + c)^(2/3)) -
 4*sqrt(2)*(8*pi*b^4*f^2*cos(a) - 315*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f
^2)*sin(a))*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3)) + 4*
sqrt(2)*(8*pi*b^4*f^2*sin(a) + 315*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)
*cos(a))*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3)) - 315*(
b^3*d*e*f - b^3*c*f^2)*sin(a)*sin_integral(b/(d*x + c)^(2/3)) - (16*b^3*d*f
^2*x + 16*b^3*c*f^2 - 15*(4*b*d^2*f^2*x^2 + 84*b*d^2*e^2 - 147*b*c*d*e*f +
67*b*c^2*f^2 + (21*b*d^2*e*f - 13*b*c*d*f^2)*x)*(d*x + c)^(1/3))*cos((a*d*x
 + a*c + (d*x + c)^(1/3)*b)/(d*x + c)) + (210*d^3*f^2*x^3 + 630*d^3*e*f*x^2
 + 32*(d*x + c)^(1/3)*b^4*f^2 + 630*d^3*e^2*x + 630*c*d^2*e^2 - 630*c^2*d*e
*f + 210*c^3*f^2 - 3*(8*b^2*d*f^2*x + 105*b^2*d*e*f - 97*b^2*c*f^2)*(d*x +
c)^(2/3))*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)))/d^3
```

Sympy [F]

$$\int (e+fx)^2 \sin \left(a + \frac{b}{(c+dx)^{2/3}}\right) dx = \int (e+fx)^2 \sin \left(a + \frac{b}{(c+dx)^{\frac{2}{3}}}\right) dx$$

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b/(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 1260, normalized size of antiderivative = 2.00

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] 1/1260*(630*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^2*c
os(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^(4/3)*sqrt(
(d*x + c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + (((I + 1
)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(pi)*(erf(sqr
t(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/(d
*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) -
1))*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^(4/3))*e^2/((d*
x + c)^(1/3)*b) - 1260*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-
4/3))*b^2*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^
(4/3)*sqrt((d*x + c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3))
 + (((I + 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(p
i)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(
sqrt(I*b/(d*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)
^(2/3))) - 1))*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^(4/3
))*c*e*f/((d*x + c)^(1/3)*b*d) + 630*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqr
t((d*x + c)^(-4/3))*b^2*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt
(2)*(d*x + c)^(4/3)*sqrt((d*x + c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d
*x + c)^(2/3)) + (((I + 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) -
(I - 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*
sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(
-I*b/(d*x + c)^(2/3))) - 1))*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt(
(d*x + c)^(4/3))*c^2*f^2/((d*x + c)^(1/3)*b*d^2) + 315*(((Ei(I*b/(d*x + c)^
(2/3)) + Ei(-I*b/(d*x + c)^(2/3)))*cos(a) + (I*Ei(I*b/(d*x + c)^(2/3)) - I*
Ei(-I*b/(d*x + c)^(2/3)))*sin(a))*b^3 + 2*(d*x + c)^(4/3)*b*cos(((d*x + c)^
(2/3)*a + b)/(d*x + c)^(2/3)) - 2*((d*x + c)^(2/3)*b^2 - 2*(d*x + c)^2)*sin
(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)))*e*f/d - 315*(((Ei(I*b/(d*x + c)^
(2/3)) + Ei(-I*b/(d*x + c)^(2/3)))*cos(a) + (I*Ei(I*b/(d*x + c)^(2/3)) - I*
Ei(-I*b/(d*x + c)^(2/3)))*sin(a))*b^3 + 2*(d*x + c)^(4/3)*b*cos(((d*x + c)^
(2/3)*a + b)/(d*x + c)^(2/3)) - 2*((d*x + c)^(2/3)*b^2 - 2*(d*x + c)^2)*sin
(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)))*c*f^2/d^2 - 2*sqrt(2)*(8*((-(I -
 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(s
qrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I + 1)*sqrt(pi)*(erf(sqrt(I*b/
(d*x + c)^(2/3))) - 1) + (I - 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3)))
- 1))*sin(a))*b^5*(b^2/(d*x + c)^(4/3))^(1/4) + 2*(4*sqrt(2)*(d*x + c)^(4/3
)*sqrt((d*x + c)^(-4/3))*b^4 - 15*sqrt(2)*(d*x + c)^(8/3)*sqrt((d*x + c)^(-
```

```
4/3))*b^2)*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) - (16*sqrt(2)*(d*x
+ c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^5 - 12*sqrt(2)*(d*x + c)^2*sqrt((d*x +
c)^(-4/3))*b^3 + 105*sqrt(2)*(d*x + c)^(10/3)*sqrt((d*x + c)^(-4/3))*b)*sin
(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)))*sqrt((d*x + c)^(4/3))*f^2/((d*x
+ c)^(1/3)*b*d^2))/d
```

Giac [F]

$$\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int (fx+e)^2 \sin \left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)\,dx$$

[In] `integrate((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")`

[Out] `integrate((f*x + e)^2*sin(a + b/(d*x + c)^(2/3)), x)`

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin \left(a+\frac{b}{(c+d\,x)^{2/3}}\right)\,(e+f\,x)^2\,dx$$

[In] `int(sin(a + b/(c + d*x)^(2/3))*(e + f*x)^2,x)`

[Out] `int(sin(a + b/(c + d*x)^(2/3))*(e + f*x)^2, x)`

3.223 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 318

$$\begin{aligned}\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx &= \frac{2b(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} \\ &+ \frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{b^3f\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} \\ &+ \frac{2b^{3/2}(de-cf)\sqrt{2\pi}\cos(a)\,\mathrm{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^2} \\ &+ \frac{2b^{3/2}(de-cf)\sqrt{2\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d^2} \\ &- \frac{b^2f(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} \\ &+ \frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^2} - \frac{b^3f\sin(a)\mathrm{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}\end{aligned}$$

```
[Out] 1/4*b^3*f*Ci(b/(d*x+c)^(2/3))*cos(a)/d^2+2*b*(-c*f+d*e)*(d*x+c)^(1/3)*cos(a
+b/(d*x+c)^(2/3))/d^2+1/4*b*f*(d*x+c)^(4/3)*cos(a+b/(d*x+c)^(2/3))/d^2-1/4*
b^3*f*Si(b/(d*x+c)^(2/3))*sin(a)/d^2-1/4*b^2*f*(d*x+c)^(2/3)*sin(a+b/(d*x+c
)^(2/3))/d^2+(-c*f+d*e)*(d*x+c)*sin(a+b/(d*x+c)^(2/3))/d^2+1/2*f*(d*x+c)^2*
sin(a+b/(d*x+c)^(2/3))/d^2+2*b^(3/2)*(-c*f+d*e)*cos(a)*FresnelS(b^(1/2)*2^(
1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*Pi^(1/2)/d^2+2*b^(3/2)*(-c*f+d*e)*Fres
nelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)*2^(1/2)*Pi^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3514, 3490, 3468, 3469, 3434, 3433, 3432, 3460, 3378, 3384, 3380, 3383}

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{2\sqrt{2\pi}b^{3/2}\sin(a)(de-cf)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^2} + \frac{2\sqrt{2\pi}b^{3/2}\cos(a)(de-cf)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^2} + \frac{b^3f\cos(a)\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} - \frac{b^3f\sin(a)\operatorname{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} - \frac{b^2f(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{(c+dx)(de-cf)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{2b\sqrt[3]{c+dx}(de-cf)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^2} + \frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}$$

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/d^2 + (b*f*(c
+ d*x)^(4/3)*Cos[a + b/(c + d*x)^(2/3)])/(4*d^2) + (b^3*f*Cos[a]*CosIntegra
l[b/(c + d*x)^(2/3)])/(4*d^2) + (2*b^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*Fr
esnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/d^2 + (2*b^(3/2)*(d*e - c*f)*
Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/d^2 - (b^
2*f*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)])/(4*d^2) + ((d*e - c*f)*(c +
 d*x)*Sin[a + b/(c + d*x)^(2/3)])/d^2 + (f*(c + d*x)^2*Sin[a + b/(c + d*x)^
(2/3)])/(2*d^2) - (b^3*f*Sin[a]*SinIntegral[b/(c + d*x)^(2/3)])/(4*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3\text{Subst}\left(\int \left((de-cf)x^2\sin\left(a+\frac{b}{x^2}\right)+fx^5\sin\left(a+\frac{b}{x^2}\right)\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}\\
&= \frac{(3f)\text{Subst}\left(\int x^5\sin\left(a+\frac{b}{x^2}\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}\\
&\quad+\frac{(3(de-cf))\text{Subst}\left(\int x^2\sin\left(a+\frac{b}{x^2}\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}\\
&= -\frac{(3f)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^4}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{2d^2}-\frac{(3(de-cf))\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^4}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^2}\\
&= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^2}\\
&\quad-\frac{(bf)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{2d^2}\\
&\quad-\frac{(2b(de-cf))\text{Subst}\left(\int \frac{\cos(a+bx^2)}{x^2}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^2}
\end{aligned}$$

$$= \frac{2b(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^2} + \frac{(b^2f)\operatorname{Subst}\left(\int\frac{\sin(a+bx)}{x^2}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{(4b^2(de-cf))\operatorname{Subst}\left(\int\sin\left(a+bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^2}$$

$$= \frac{2b(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} - \frac{b^2f(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^2} + \frac{(b^3f)\operatorname{Subst}\left(\int\frac{\cos(a+bx)}{x}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{(4b^2(de-cf)\cos(a))\operatorname{Subst}\left(\int\sin\left(bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^2} + \frac{(4b^2(de-cf)\sin(a))\operatorname{Subst}\left(\int\cos\left(bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^2}$$

$$= \frac{2b(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{2b^{3/2}(de-cf)\sqrt{2\pi}\cos(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^2} + \frac{2b^{3/2}(de-cf)\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d^2} - \frac{b^2f(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^2} + \frac{(b^3f\cos(a))\operatorname{Subst}\left(\int\frac{\cos(bx)}{x}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{4d^2} - \frac{(b^3f\sin(a))\operatorname{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{4d^2}$$

$$=\frac{2b(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2}+\frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}+\frac{b^3f\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}+\frac{2b^{3/2}(de-cf)\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^2}+\frac{2b^{3/2}(de-cf)\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d^2}-\frac{b^2f(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}+\frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^2}-\frac{b^3f\sin(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.19

$$\int(e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\frac{8bde\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)-7bcf\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+bdfx\sqrt[3]{c+dx}\cos\left(\right.}{}$$

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (8*b*d*e*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)] - 7*b*c*f*(c + d*x)^(1/
3)*Cos[a + b/(c + d*x)^(2/3)] + b*d*f*x*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)
^(2/3)] + b^3*f*Cos[a]*CosIntegral[b/(c + d*x)^(2/3)] + 8*b^(3/2)*(d*e - c*
f)*Sqrt[2*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)] + 8*b^(
3/2)*d*e*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] -
 8*b^(3/2)*c*f*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Si
n[a] + 4*c*d*e*Sin[a + b/(c + d*x)^(2/3)] - 2*c^2*f*Sin[a + b/(c + d*x)^(2/
3)] + 4*d^2*e*x*Sin[a + b/(c + d*x)^(2/3)] + 2*d^2*f*x^2*Sin[a + b/(c + d*x
)^(2/3)] - b^2*f*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)] - b^3*f*Sin[a]*
SinIntegral[b/(c + d*x)^(2/3)])/(4*d^2)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)+2(cf-de)b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)^{\frac{1}{3}}}\right)\right.\right.}{}$
default	$\frac{-(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)+2(cf-de)b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)^{\frac{1}{3}}}\right)\right.\right.}{}$
parts	Expression too large to display

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^2*(-1/3*(c*f-d*e)*(d*x+c)*sin(a+b/(d*x+c)^(2/3))+2/3*(c*f-d*e)*b*(-(d*x
+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(
b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^
(1/2)/(d*x+c)^(1/3))))+1/6*f*(d*x+c)^2*sin(a+b/(d*x+c)^(2/3))-1/3*f*b*(-1/4
*(d*x+c)^(4/3)*cos(a+b/(d*x+c)^(2/3))-1/2*b*(-1/2*(d*x+c)^(2/3)*sin(a+b/(d*
x+c)^(2/3))+b*(1/2*cos(a)*Ci(b/(d*x+c)^(2/3))-1/2*sin(a)*Si(b/(d*x+c)^(2/3)
)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.78

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\frac{b^3f\cos(a)\,\mathrm{Ci}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-b^3f\sin(a)\,\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right)+8\sqrt{2}\pi(bde-bcf)\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\left(\right.}{}$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*f*cos(a)*cos_integral(b/(d*x + c)^(2/3)) - b^3*f*sin(a)*sin_integr
al(b/(d*x + c)^(2/3)) + 8*sqrt(2)*pi*(b*d*e - b*c*f)*sqrt(b/pi)*cos(a)*fres
nel_sin(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3)) + 8*sqrt(2)*pi*(b*d*e - b*c*f)*
sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3))*sin(a) + (b*d*f*
x + 8*b*d*e - 7*b*c*f)*(d*x + c)^(1/3)*cos((a*d*x + a*c + (d*x + c)^(1/3)*b
)/(d*x + c)) + (2*d^2*f*x^2 + 4*d^2*e*x - (d*x + c)^(2/3)*b^2*f + 4*c*d*e -
 2*c^2*f)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)))/d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)dx$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.84

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\text{Too large to display}$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] 1/8*(4*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^2*cos(((
d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^(4/3)*sqrt((d*x
+ c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + (((I + 1)*sqr
t(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(pi)*(erf(sqrt(-I*
b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x +
c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*s
in(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^(4/3))*e/((d*x + c)^
(1/3)*b) - 4*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^2*
cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^(4/3)*sqrt
((d*x + c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + (((I +
1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(pi)*(erf(sq
rt(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/(
d*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) -
 1))*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^(4/3))*c*f/((d
*x + c)^(1/3)*b*d) + (((Ei(I*b/(d*x + c)^(2/3)) + Ei(-I*b/(d*x + c)^(2/3)))
*cos(a) + (I*Ei(I*b/(d*x + c)^(2/3)) - I*Ei(-I*b/(d*x + c)^(2/3)))*sin(a))*
```

```
b^3 + 2*(d*x + c)^(4/3)*b*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) - 2*
((d*x + c)^(2/3)*b^2 - 2*(d*x + c)^2)*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)
^(2/3)))*f/d)/d
```

Giac [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)\,dx$$

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)\,(e+f\,x)\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(2/3))*(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))*(e + f*x), x)
```

3.224 $\int \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 141

$$\int \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} + \frac{2b^{3/2}\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d} + \frac{2b^{3/2}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}$$

```
[Out] 2*b*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))/d+(d*x+c)*sin(a+b/(d*x+c)^(2/3))/d
+2*b^(3/2)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*
Pi^(1/2)/d+2*b^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a
)*2^(1/2)*Pi^(1/2)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used

$= \{3444, 3490, 3468, 3469, 3434, 3433, 3432\}$

$$\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx = \frac{2\sqrt{2\pi}b^{3/2}\sin(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d} + \frac{2\sqrt{2\pi}b^{3/2}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} + \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}$$

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/d + (2*b^(3/2)*Sqrt[2*Pi]*
Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/d + (2*b^(3/2)*Sqrt[
2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/d + ((c + d*x)
*Sin[a + b/(c + d*x)^(2/3)])/d
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3444

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Dist[k/f, Subst[Int[x^(k - 1)*(a + b
*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
```

```
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3\text{Subst}\left(\int x^2 \sin\left(a+\frac{b}{x^2}\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d}\\
&= -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}\\
&= \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} - \frac{(2b)\text{Subst}\left(\int \frac{\cos(a+bx^2)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}\\
&= \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}\\
&\quad + \frac{(4b^2)\,\text{Subst}\left(\int \sin\left(a+bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}\\
&= \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}\\
&\quad + \frac{(4b^2\cos(a))\,\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}\\
&\quad + \frac{(4b^2\sin(a))\,\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}
\end{aligned}$$

$$= \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} + \frac{2b^{3/2}\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d} + \frac{2b^{3/2}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx = \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+2b^{3/2}\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+2b^{3/2}\sqrt{2\pi}\,\text{Fre}}{d}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)] + 2*b^(3/2)*Sqrt[2*Pi]*Cos[
a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)] + 2*b^(3/2)*Sqrt[2*Pi]*Fr
esnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] + c*Sin[a + b/(c + d*x)
^(2/3)] + d*x*Sin[a + b/(c + d*x)^(2/3)])/d
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\text{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)^{\frac{1}{3}}}\right)+\sin(a)\,\text{C}\left(\frac{\sqrt{b}\,\sqrt{}}{\sqrt{\pi}\,(dx}\right.\right.\right.}{d}$
default	$\frac{(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\text{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)^{\frac{1}{3}}}\right)+\sin(a)\,\text{C}\left(\frac{\sqrt{b}\,\sqrt{}}{\sqrt{\pi}\,(dx}\right.\right.\right.}{d}$

```
[In] int(sin(a+b/(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*(1/3*(d*x+c)*sin(a+b/(d*x+c)^(2/3))-2/3*b*(-(d*x+c)^(1/3)*cos(a+b/(d*x+
c)^(2/3))-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2
)/(d*x+c)^(1/3))+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int \sin \left(a + \frac{b}{(c+dx)^{2/3}}\right) dx = \frac{2\sqrt{2}\pi b\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right) + 2\sqrt{2}\pi b\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right)\sin(a) + 2(dx+c)^{\frac{1}{3}}b\cos\left(\frac{ad\ldots}{}\right.}{d}$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] (2*sqrt(2)*pi*b*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/pi)/(d*x + c)^
(1/3)) + 2*sqrt(2)*pi*b*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/(d*x + c)
^(1/3))*sin(a) + 2*(d*x + c)^(1/3)*b*cos((a*d*x + a*c + (d*x + c)^(1/3)*b)/
(d*x + c)) + (d*x + c)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)))/d
```

Sympy [F]

$$\int \sin \left(a + \frac{b}{(c+dx)^{2/3}}\right) dx = \int \sin \left(a + \frac{b}{(c+dx)^{\frac{2}{3}}}\right) dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.55

$$\int \sin \left(a + \frac{b}{(c+dx)^{2/3}}\right) dx = \frac{\sqrt{2}\left(2\sqrt{2}(dx+c)^{\frac{2}{3}}\sqrt{\frac{1}{(dx+c)^{\frac{4}{3}}}}b^2\cos\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right) + \sqrt{2}(dx+c)^{\frac{4}{3}}\sqrt{\frac{1}{(dx+c)^{\frac{4}{3}}}}b\sin\left(\frac{(dx+c)^{\frac{2}{3}}a\ldots}{(dx+c)^{\frac{2}{3}}}\right.\right.}{}$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^2*cos(((d*x
 + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^(4/3)*sqrt((d*x + c
```

```
)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + (((I + 1)*sqrt(p
i)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(pi)*(erf(sqrt(-I*b/(
d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^
(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(
a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^(4/3))/((d*x + c)^(1/3)
*b*d)
```

Giac [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(2/3)),x)
[Out] int(sin(a + b/(c + d*x)^(2/3)), x)
```

3.225 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$$

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x),x]
[Out] Defer[Int][Sin[a + b/(c + d*x)^(2/3)]/(e + f*x), x]
```

Rubi steps

$$\text{integral}=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 63.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x),x]
[Out] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{fx+e}dx$$

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x)
[Out] int(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x, algorithm="fricas")
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{e+fx}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(f*x+e),x)
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{fx+e}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 6.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx} \, dx=\int \frac{\sin \left(a+\frac{b}{{\left(c+d\,x\right)}^{2/3}}\right)}{e+f\,x} \,d x$$

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))/(e + f*x), x)
```

3.226 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2} dx = \text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}, x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2} dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2} dx$$

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 46.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(fx+e)^{2}}dx$$

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x)
[Out] int(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(fx+e)^{2}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(f^2*x^2 + 2*e*f*
x + e^2), x)
```

Sympy [N/A]

Not integrable

Time = 22.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin \left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{(e+fx)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(f*x+e)**2,x)
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin \left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin \left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(fx+e)^2}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{{\left(e+f\,x\right)}^2}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^(2/3))/(e + f*x)^2, x)
```

3.227 $\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 289

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{2160e\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d\sqrt[3]{c+dx}} - \frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d} + \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{2160e\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d} - \frac{360e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d} + \frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}$$

```
[Out] 2160*e*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^7/d/(d*x+c)^(1/3)-1080*e*
(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^5/d+90*e*(d*x+c)*(
e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^3/d-3*e*(d*x+c)^(5/3)*(e*(d*x+c))
^(1/3)*cos(a+b*(d*x+c)^(1/3))/b/d+2160*e*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^
(1/3))/b^6/d-360*e*(d*x+c)^(2/3)*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^(1/3))/b
^4/d+18*e*(d*x+c)^(4/3)*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3512, 15, 3377, 2718}

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{2160e\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d\sqrt[3]{c+dx}} + \frac{2160e\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d} - \frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d} - \frac{360e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d} + \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} + \frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} - \frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

```
[In] Int[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (2160*e*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^7*d*(c + d*x)^(1
/3)) - (1080*e*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3
)])/(b^5*d) + (90*e*(c + d*x)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)
])/(b^3*d) - (3*e*(c + d*x)^(5/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(
1/3)])/(b*d) + (2160*e*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^6
*d) - (360*e*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)]
)/(b^4*d) + (18*e*(c + d*x)^(4/3)*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(
1/3)])/(b^2*d)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3\text{Subst}\left(\int x^2(ex^3)^{4/3}\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d}\\
&= \frac{\left(3e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^6\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d\sqrt[3]{c+dx}}\\
&= -\frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}\\
&\quad+\frac{\left(18e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^5\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd\sqrt[3]{c+dx}}\\
&= -\frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}\\
&\quad+\frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}\\
&\quad-\frac{\left(90e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^4\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{c+dx}}\\
&= \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}\\
&\quad-\frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}\\
&\quad+\frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}\\
&\quad-\frac{\left(360e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^3\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^3d\sqrt[3]{c+dx}}
\end{aligned}
$$

$$= \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{360e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d} + \frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} + \frac{\left(1080e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}}$$

$$= -\frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d} + \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{360e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d} + \frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} + \frac{\left(2160e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^5d\sqrt[3]{c+dx}}$$

$$= -\frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d} + \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{2160e\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d} - \frac{360e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d} + \frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} - \frac{\left(2160e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^6d\sqrt[3]{c+dx}}$$

$$= \frac{2160e\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d\sqrt[3]{c+dx}} - \frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d} + \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{2160e\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d} - \frac{360e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d} + \frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.78

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{3(e(c+dx))^{4/3}\left(-\cos\left(b\sqrt[3]{c+dx}\right)\left(\left(-720+360b^2(c+dx)^{2/3}-30b^4(c+dx)^{4/3}+b^6(c+\right.\right.\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*(e*(c + d*x))^(4/3)*(-(Cos[b*(c + d*x)^(1/3)]*((-720 + 360*b^2*(c + d*x)
^(2/3) - 30*b^4*(c + d*x)^(4/3) + b^6*(c + d*x)^2)*Cos[a] - 6*b*(120*(c + d
*x)^(1/3) - 20*b^2*(c + d*x) + b^4*(c + d*x)^(5/3))*Sin[a])) + (6*b*(120*(c
 + d*x)^(1/3) - 20*b^2*(c + d*x) + b^4*(c + d*x)^(5/3))*Cos[a] + (-720 + 36
0*b^2*(c + d*x)^(2/3) - 30*b^4*(c + d*x)^(4/3) + b^6*(c + d*x)^2)*Sin[a])*S
in[b*(c + d*x)^(1/3)]))/(b^7*d*(c + d*x)^(4/3))
```

Maple [F]

$$\int (dex+ce)^{\frac{4}{3}}\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)dx$$

```
[In] int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.72 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.81

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{3\left(\left(30\,b^4d^2ex^2+60\,b^4cdex+30\,b^4c^2e-(b^6d^2ex^2+2\,b^6cdex+(b^6c^2-720)e)(dx+c)^{\frac{2}{3}}-\right.\right.}{}$$

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((30*b^4*d^2*e*x^2 + 60*b^4*c*d*e*x + 30*b^4*c^2*e - (b^6*d^2*e*x^2 + 2*b
^6*c*d*e*x + (b^6*c^2 - 720)*e)*(d*x + c)^(2/3) - 360*(b^2*d*e*x + b^2*c*e)
*(d*x + c)^(1/3))*(d*e*x + c*e)^(1/3)*cos((d*x + c)^(1/3)*b + a) + 6*(120*b
*d*e*x + 120*b*c*e - 20*(b^3*d*e*x + b^3*c*e)*(d*x + c)^(2/3) + (b^5*d^2*e*
x^2 + 2*b^5*c*d*e*x + b^5*c^2*e)*(d*x + c)^(1/3))*(d*e*x + c*e)^(1/3)*sin((
d*x + c)^(1/3)*b + a))/(b^7*d^2*x + b^7*c*d)
```

Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\text{Timed out}$$

[In] integrate((d*e*x+c*e)**(4/3)*sin(a+b*(d*x+c)**(1/3)),x)

[Out] Timed out

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.61

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{3\left(3\left(\left(\Gamma\left(6,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(6,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(6,i\,(dx+c)^{\frac{1}{3}}b\right)+\Gamma\left(6,-i\,(dx+\right.\right.\right.\right.}{}$$

[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")

[Out] 3/2*(3*((gamma(6, I*b*conjugate((d*x + c)^(1/3))) + gamma(6, -I*b*conjugate((d*x + c)^(1/3))) + gamma(6, I*(d*x + c)^(1/3)*b) + gamma(6, -I*(d*x + c)^(1/3)*b))*cos(a) + (-I*gamma(6, I*b*conjugate((d*x + c)^(1/3))) + I*gamma(6, -I*b*conjugate((d*x + c)^(1/3))) - I*gamma(6, I*(d*x + c)^(1/3)*b) + I*gamma(6, -I*(d*x + c)^(1/3)*b))*sin(a))*e - 2*(b^6*d^2*e*x^2 + 2*b^6*c*d*e*x + b^6*c^2*e)*cos((d*x + c)^(1/3)*b + a))*e^(1/3)/(b^7*d)

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.68

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=$$

$$-\frac{3\left(d^2e\left(\frac{\left(b^6c^2e^7-2\,(dex+ce)b^6ce^6+(dex+ce)^2b^6e^5+12\,(dex+ce)^{\frac{1}{3}}b^4ce^6|e|^{\frac{2}{3}}-30\,(dex+ce)^{\frac{4}{3}}b^4e^5|e|^{\frac{2}{3}}+360\,(dex+ce)^{\frac{2}{3}}b^2e^5|e|^{\frac{4}{3}}-720\,e^7\right)\cos\left(\right.}{b^7d^2e^6|e|^{\frac{2}{3}}}\right.\right.}{}$$

[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")

```
[Out] -3*(d^2*e*((b^6*c^2*e^7 - 2*(d*e*x + c*e)*b^6*c*e^6 + (d*e*x + c*e)^2*b^6*e
^5 + 12*(d*e*x + c*e)^(1/3)*b^4*c*e^6*abs(e)^(2/3) - 30*(d*e*x + c*e)^(4/3)
*b^4*e^5*abs(e)^(2/3) + 360*(d*e*x + c*e)^(2/3)*b^2*e^5*abs(e)^(4/3) - 720*
e^7)*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b^7*d^2*e^6*abs(e)^
(2/3)) + 6*((d*e*x + c*e)^(2/3)*b^5*c*e^5*abs(e)^(4/3) - (d*e*x + c*e)^(5/3
)*b^5*e^4*abs(e)^(4/3) - 2*b^3*c*e^7 + 20*(d*e*x + c*e)*b^3*e^6 - 120*(d*e*
x + c*e)^(1/3)*b*e^6*abs(e)^(2/3))*sin((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^
(2/3))/e)/(b^7*d^2*e^6*abs(e)^(2/3))) + c^2*e^2*cos((a*e + (d*e*x + c*e)^(1
/3)*b*abs(e)^(2/3))/e)/(b*abs(e)^(2/3)) - 2*c*((b^3*c*e^4 - (d*e*x + c*e)*b
^3*e^3 + 6*(d*e*x + c*e)^(1/3)*b*e^3*abs(e)^(2/3))*cos((a*e + (d*e*x + c*e)
^(1/3)*b*abs(e)^(2/3))/e)/(b^4*e^2*abs(e)^(2/3)) + 3*((d*e*x + c*e)^(2/3)*b
^2*e^2*abs(e)^(4/3) - 2*e^4)*sin((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))
/e)/(b^4*e^2*abs(e)^(2/3))))/d
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{4/3}\sin \left(a+b\sqrt[3]{c+dx}\right)\,dx=\int \sin \left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{4/3} \,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(4/3),x)
[Out] int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(4/3), x)
```

3.228 $\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 202

$$\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{72(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d(c+dx)^{2/3}}-\frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}-\frac{72(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}}+\frac{12\sqrt[3]{c+dx}(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}$$

```
[Out] 36*(e*(d*x+c))^(2/3)*cos(a+b*(d*x+c)^(1/3))/b^3/d-72*(e*(d*x+c))^(2/3)*cos(
a+b*(d*x+c)^(1/3))/b^5/d/(d*x+c)^(2/3)-3*(d*x+c)^(2/3)*(e*(d*x+c))^(2/3)*co
s(a+b*(d*x+c)^(1/3))/b/d-72*(e*(d*x+c))^(2/3)*sin(a+b*(d*x+c)^(1/3))/b^4/d/
(d*x+c)^(1/3)+12*(d*x+c)^(1/3)*(e*(d*x+c))^(2/3)*sin(a+b*(d*x+c)^(1/3))/b^2
/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3512, 15, 3377, 2718}

$$\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=-\frac{72(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d(c+dx)^{2/3}}-\frac{72(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}}+\frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{12\sqrt[3]{c+dx}(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

```
[In] Int[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (36*(e*(c + d*x))^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (72*(e*(c + d
*x))^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5*d*(c + d*x)^(2/3)) - (3*(c + d*
x)^(2/3)*(e*(c + d*x))^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d) - (72*(e*(c
+ d*x))^(2/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^4*d*(c + d*x)^(1/3)) + (12*(c
+ d*x)^(1/3)*(e*(c + d*x))^(2/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
```

```
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{3\text{Subst}\left(\int x^2(ex^3)^{2/3}\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{\left(3(e(c+dx))^{2/3}\right)\text{Subst}\left(\int x^4\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d(c+dx)^{2/3}}$$

$$= -\frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{\left(12(e(c+dx))^{2/3}\right)\text{Subst}\left(\int x^3\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd(c+dx)^{2/3}}$$

$$= -\frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{12\sqrt[3]{c+dx}(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} - \frac{\left(36(e(c+dx))^{2/3}\right)\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d(c+dx)^{2/3}}$$

$$= \frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{12\sqrt[3]{c+dx}(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} - \frac{\left(72(e(c+dx))^{2/3}\right)\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^3d(c+dx)^{2/3}}$$

$$= \frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{72(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}} + \frac{12\sqrt[3]{c+dx}(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} + \frac{(72(e(c+dx))^{2/3})\,\mathrm{Subst}\left(\int\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^4d(c+dx)^{2/3}}$$

$$= \frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{72(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d(c+dx)^{2/3}} - \frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{72(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}} + \frac{12\sqrt[3]{c+dx}(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.55

$$\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{3(e(c+dx))^{2/3}\left(\left(24-12b^2(c+dx)^{2/3}+b^4(c+dx)^{4/3}\right)\cos\left(a+b\sqrt[3]{c+dx}\right)-4b\left(-6\sqrt[3]{c+dx}+b^2(c+d\right.\right.}{b^5d(c+dx)^{2/3}}$$

```
[In] Integrate[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(2/3)*((24 - 12*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(4/3)
)*Cos[a + b*(c + d*x)^(1/3)] - 4*b*(-6*(c + d*x)^(1/3) + b^2*(c + d*x))*Sin
[a + b*(c + d*x)^(1/3)]))/(b^5*d*(c + d*x)^(2/3))
```

Maple [F]

$$\int (dex+ce)^{\frac{2}{3}} \sin\left(a+b(dx+c)^{\frac{1}{3}}\right) dx$$

[In] int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x)

[Out] int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x)

Fricas [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.71

$$\int (ce+dex)^{2/3} \sin\left(a +b\sqrt[3]{c+dx}\right) dx = \frac{3\left(\left(12\,b^2dx+12\,b^2c-(b^4dx+b^4c)(dx+c)^{\frac{2}{3}}-24\,(dx+c)^{\frac{1}{3}}\right)(dex+ce)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{1}{3}}b\right.}{b^5d^2x+b}$$

[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")

[Out] 3*((12*b^2*d*x + 12*b^2*c - (b^4*d*x + b^4*c)*(d*x + c)^(2/3) - 24*(d*x + c
)^(1/3))*(d*e*x + c*e)^(2/3)*cos((d*x + c)^(1/3)*b + a) - 4*(d*e*x + c*e)^(
2/3)*(6*(d*x + c)^(2/3)*b - (b^3*d*x + b^3*c)*(d*x + c)^(1/3))*sin((d*x + c
)^(1/3)*b + a))/(b^5*d^2*x + b^5*c*d)

Sympy [F]

$$\int (ce+dex)^{2/3} \sin\left(a+b\sqrt[3]{c+dx}\right) dx = \int (e(c+dx))^{\frac{2}{3}} \sin\left(a+b\sqrt[3]{c+dx}\right) dx$$

[In] integrate((d*e*x+c*e)**(2/3)*sin(a+b*(d*x+c)**(1/3)),x)

[Out] Integral((e*(c + d*x))**(2/3)*sin(a + b*(c + d*x)**(1/3)), x)

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int (ce+dex)^{2/3} \sin\left(a+b\sqrt[3]{c+dx}\right) dx =$$

$$-\frac{3\left((b^4dx+b^4c)(dx+c)^{\frac{1}{3}}e^{\frac{2}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)+\left(3\left(\Gamma\left(3,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(3,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(3\right.\right.\right.\right.}{}$$

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*((b^4*d*x + b^4*c)*(d*x + c)^(1/3)*e^(2/3)*cos((d*x + c)^(1/3)*b + a) +
(3*(gamma(3, I*b*conjugate((d*x + c)^(1/3))) + gamma(3, -I*b*conjugate((d*x
 + c)^(1/3))) + gamma(3, I*(d*x + c)^(1/3)*b) + gamma(3, -I*(d*x + c)^(1/3)
*b))*cos(a) - 4*(b^3*d*x + b^3*c)*sin((d*x + c)^(1/3)*b + a) - 3*(I*gamma(3
, I*b*conjugate((d*x + c)^(1/3))) - I*gamma(3, -I*b*conjugate((d*x + c)^(1/
3))) + I*gamma(3, I*(d*x + c)^(1/3)*b) - I*gamma(3, -I*(d*x + c)^(1/3)*b))*
sin(a))*e^(2/3))/(b^5*d)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.42

$$\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{3\left(c\left(\frac{(dex+ce)^{\frac{1}{3}}e\cos\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b|e|^{\frac{2}{3}}}-\frac{e^2\sin\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b^2|e|^{\frac{4}{3}}}\right)-\frac{\left((dex+ce)^{\frac{1}{3}}b^4ce^4|e|^{\frac{2}{3}}-(dex+ce)^{\frac{4}{3}}b^4e^3|e|^{\frac{2}{3}}+12\,(dex+ce)^{\frac{2}{3}}b\cdots\right.}{b^5e^2|e|^{\frac{4}{3}}}\cdots}{d\cdots}$$

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3*(c*((d*e*x + c*e)^(1/3)*e*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))
/e)/(b*abs(e)^(2/3)) - e^2*sin((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e
)/(b^2*abs(e)^(4/3))) - (((d*e*x + c*e)^(1/3)*b^4*c*e^4*abs(e)^(2/3) - (d*e
*x + c*e)^(4/3)*b^4*e^3*abs(e)^(2/3) + 12*(d*e*x + c*e)^(2/3)*b^2*e^3*abs(e
)^(4/3) - 24*e^5)*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b^5*e^
2*abs(e)^(4/3)) - (b^3*c*e^5 - 4*(d*e*x + c*e)*b^3*e^4 + 24*(d*e*x + c*e)^(
1/3)*b*e^4*abs(e)^(2/3))*sin((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/
(b^5*e^2*abs(e)^(4/3)))/e)/d
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \int \sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{2/3}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(2/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(2/3), x)
```

3.229 $\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 160

$$\begin{aligned}\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx &= \frac{18\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} \\ &\quad-\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} \\ &\quad-\frac{18\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}} \\ &\quad+\frac{9\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}\end{aligned}$$

```
[Out] 18*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^3/d-3*(d*x+c)^(2/3)*(e*(d*x+c
))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b/d-18*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^(1
/3))/b^4/d/(d*x+c)^(1/3)+9*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^
(1/3))/b^2/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used

$= \{3512, 15, 3377, 2717\}$

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{18\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}} + \frac{18\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} + \frac{9\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} - \frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

```
[In] Int[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (18*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (3*(c + d*x)^
(2/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d) - (18*(e*(c + d
*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^4*d*(c + d*x)^(1/3)) + (9*(c + d*
x)^(1/3)*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{3\text{Subst}\left(\int x^2 \sqrt[3]{ex^3} \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^3 \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d\sqrt[3]{c+dx}}$$

$$= -\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{\left(9\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^2 \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd\sqrt[3]{c+dx}}$$

$$= -\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{9\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} - \frac{\left(18\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{c+dx}}$$

$$= \frac{18\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{9\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} - \frac{\left(18\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^3d\sqrt[3]{c+dx}}$$

$$= \frac{18\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{18\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}} + \frac{9\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx =$$
$$-\frac{3\sqrt[3]{e(c+dx)}\left(\left(-6b\sqrt[3]{c+dx}+b^3(c+dx)\right)\cos\left(a+b\sqrt[3]{c+dx}\right)-3\left(-2+b^2(c+dx)^{2/3}\right)\sin\left(a+b\sqrt[3]{c\cdots}\right.\right.}{b^4d\sqrt[3]{c+dx}}$$

```
[In] Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(1/3)*((-6*b*(c + d*x)^(1/3) + b^3*(c + d*x))*Cos[a + b*(
c + d*x)^(1/3)] - 3*(-2 + b^2*(c + d*x)^(2/3))*Sin[a + b*(c + d*x)^(1/3)]))
/(b^4*d*(c + d*x)^(1/3))
```

Maple [F]

$$\int (dex+ce)^{\frac{1}{3}}\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)dx$$

```
[In] int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.80

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\frac{3\left(\left(6\,bdx+6\,bc-(b^3dx+b^3c)(dx+c)^{\frac{2}{3}}\right)(dex+ce)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)+3\,(dex+ce)^{\frac{1}{3}}\left((b^2dx+b^2c\cdots\right.\right.}{b^4d^2x+b^4cd}$$

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((6*b*d*x + 6*b*c - (b^3*d*x + b^3*c)*(d*x + c)^(2/3))*(d*e*x + c*e)^(1/3
)*cos((d*x + c)^(1/3)*b + a) + 3*(d*e*x + c*e)^(1/3)*((b^2*d*x + b^2*c)*(d*
x + c)^(1/3) - 2*(d*x + c)^(2/3))*sin((d*x + c)^(1/3)*b + a))/(b^4*d^2*x +
b^4*c*d)
```

Sympy [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int \sqrt[3]{e\left(c+dx\right)}\sin\left(a+b\sqrt[3]{c+dx}\right)dx$$

```
[In] integrate((d*e*x+c*e)**(1/3)*sin(a+b*(d*x+c)**(1/3)),x)
[Out] Integral((e*(c + d*x))**(1/3)*sin(a + b*(c + d*x)**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=$$

$$-\frac{3\left(4\left(b^3dx+b^3c\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-3\left(-i\,\Gamma\left(3,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(3,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)-i\,\Gamma\left(3,i\,(\right.\right.\right.}{}$$

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
[Out] -3/4*(4*(b^3*d*x + b^3*c)*cos((d*x + c)^(1/3)*b + a) - 3*(-I*gamma(3, I*b*c
onjugate((d*x + c)^(1/3))) + I*gamma(3, -I*b*conjugate((d*x + c)^(1/3))) -
I*gamma(3, I*(d*x + c)^(1/3)*b) + I*gamma(3, -I*(d*x + c)^(1/3)*b))*cos(a)
+ 3*(gamma(3, I*b*conjugate((d*x + c)^(1/3))) + gamma(3, -I*b*conjugate((d*
x + c)^(1/3))) + gamma(3, I*(d*x + c)^(1/3)*b) + gamma(3, -I*(d*x + c)^(1/3
)*b))*sin(a))*e^(1/3)/(b^4*d)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.20

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=$$

$$-\frac{3\left(\frac{ce\cos\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b|e|^{\frac{2}{3}}}-\frac{\frac{\left(b^3ce^4-(dex+ce)b^3e^3+6\,(dex+ce)^{\frac{1}{3}}be^3|e|^{\frac{2}{3}}\right)\cos\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b^4e^2|e|^{\frac{2}{3}}}+\frac{3\left((dex+ce)^{\frac{2}{3}}b^2e^2|e|^{\frac{4}{3}}-2\,e^4\right)\sin\left(\right.}{b^4e^2|e|^{\frac{2}{3}}}}{e}\right.}{d}$$

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3*(c*e*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b*abs(e)^(2/3))
- ((b^3*c*e^4 - (d*e*x + c*e)*b^3*e^3 + 6*(d*e*x + c*e)^(1/3)*b*e^3*abs(e)^
(2/3))*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b^4*e^2*abs(e)^(2
/3)) + 3*((d*e*x + c*e)^(2/3)*b^2*e^2*abs(e)^(4/3) - 2*e^4)*sin((a*e + (d*e
*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b^4*e^2*abs(e)^(2/3)))/e)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int \sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{1/3}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(1/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(1/3), x)
```

3.230 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx=-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}}+\frac{3\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}$$

```
[Out] -3*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d/(e*(d*x+c))^(1/3)+3*(d*x+c)^(1/
3)*sin(a+b*(d*x+c)^(1/3))/b^2/d/(e*(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3512, 15, 3377, 2717}

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx=\frac{3\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}}$$

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(1/3)) +
 (3*(c + d*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d*(e*(c + d*x))^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3\text{Subst}\left(\int \frac{x^2 \sin(a+bx)}{\sqrt[3]{ex^3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d} \\
&= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d\sqrt[3]{e(c+dx)}} \\
&= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}} + \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}} \\
&= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}} + \frac{3\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx = \frac{-3b(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)+3\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*b*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)] + 3*(c + d*x)^(1/3)*Sin[a
+ b*(c + d*x)^(1/3)])/(b^2*d*(e*(c + d*x))^(1/3))
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{1}{3}}}dx$$

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3\left((dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}b\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-(dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^2d^2ex+b^2cde}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

```
[Out] -3*((d*e*x + c*e)^(2/3)*(d*x + c)^(2/3)*b*cos((d*x + c)^(1/3)*b + a) - (d*e
*x + c*e)^(2/3)*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a))/(b^2*d^2*e*x +
b^2*c*d*e)
```

Sympy [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx=\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{e\left(c+dx\right)}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(1/3),x)
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(1/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.52

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx= -\frac{3\left(\left(-i\,\Gamma\left(2,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(2,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)-i\,\Gamma\left(2,i\,(dx+c)^{\frac{1}{3}}b\right)+i\,\Gamma\left(2,-i\,(dx+c)^{\frac{1}{3}}b\right)\right)c\right.}{4}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
[Out] -3/4*((-I*gamma(2, I*b*conjugate((d*x + c)^(1/3))) + I*gamma(2, -I*b*conjug
ate((d*x + c)^(1/3))) - I*gamma(2, I*(d*x + c)^(1/3)*b) + I*gamma(2, -I*(d*
x + c)^(1/3)*b))*cos(a) - (gamma(2, I*b*conjugate((d*x + c)^(1/3))) + gamma
(2, -I*b*conjugate((d*x + c)^(1/3))) + gamma(2, I*(d*x + c)^(1/3)*b) + gamm
a(2, -I*(d*x + c)^(1/3)*b))*sin(a))/(b^2*d*e^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx=-\frac{3\left(\frac{(dex+ce)^{\frac{1}{3}}e\cos\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b|e|^{\frac{2}{3}}}-\frac{e^2\sin\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b^2|e|^{\frac{4}{3}}}\right)}{de}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
[Out] -3*((d*e*x + c*e)^(1/3)*e*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)
/(b*abs(e)^(2/3)) - e^2*sin((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(
b^2*abs(e)^(4/3)))/(d*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx=\int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{(c\,e+d\,e\,x)^{1/3}}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(1/3),x)
[Out] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(1/3), x)
```

3.231 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 42

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

```
[Out] -3*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d/(e*(d*x+c))^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3512, 15, 2718}

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
[Out] (-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{(ex^3)^{2/3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}\\ &= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}}\\ &= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
[Out] (-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(2/3))
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{2}{3}}}dx$$

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx=-\frac{3\,(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{bd^2ex+bcde}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
[Out] -3*(d*e*x + c*e)^(1/3)*(d*x + c)^(2/3)*cos((d*x + c)^(1/3)*b + a)/(b*d^2*e*
x + b*c*d*e)
```

Sympy [F]

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{\left(e\,(c+dx)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(2/3),x)
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(2/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx=-\frac{3\,\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{bde^{\frac{2}{3}}}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
[Out] -3*cos((d*x + c)^(1/3)*b + a)/(b*d*e^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3\cos\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{bd|e|^{\frac{2}{3}}}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
[Out] -3*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b*d*abs(e)^(2/3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx = \int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{(c\,e+d\,e\,x)^{2/3}}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(2/3),x)
[Out] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(2/3), x)
```

3.232 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 120

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx = \frac{3b\sqrt[3]{c+dx}\cos(a)\operatorname{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3b\sqrt[3]{c+dx}\sin(a)\operatorname{Si}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$

```
[Out] 3*b*(d*x+c)^(1/3)*Ci(b*(d*x+c)^(1/3))*cos(a)/d/e/(e*(d*x+c))^(1/3)-3*b*(d*x
+c)^(1/3)*Si(b*(d*x+c)^(1/3))*sin(a)/d/e/(e*(d*x+c))^(1/3)-3*sin(a+b*(d*x+c
)^(1/3))/d/e/(e*(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3512, 15, 3378, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx = \frac{3b\cos(a)\sqrt[3]{c+dx}\operatorname{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3b\sin(a)\sqrt[3]{c+dx}\operatorname{Si}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (3*b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b*(c + d*x)^(1/3)])/(d*e*(e*(c + d*
x))^(1/3)) - (3*Sin[a + b*(c + d*x)^(1/3)])/(d*e*(e*(c + d*x))^(1/3)) - (3*
b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b*(c + d*x)^(1/3)])/(d*e*(e*(c + d*x))
^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{3\text{Subst}\left(\int \frac{x^2 \sin(a+bx)}{(ex^3)^{4/3}} \, dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{\left(3\sqrt[3]{c+dx}\right) \text{Subst}\left(\int \frac{\sin(a+bx)}{x^2} \, dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$= -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} + \frac{\left(3b\sqrt[3]{c+dx}\right) \text{Subst}\left(\int \frac{\cos(a+bx)}{x} \, dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$= -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} + \frac{\left(3b\sqrt[3]{c+dx}\cos(a)\right) \text{Subst}\left(\int \frac{\cos(bx)}{x} \, dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{\left(3b\sqrt[3]{c+dx}\sin(a)\right) \text{Subst}\left(\int \frac{\sin(bx)}{x} \, dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$= \frac{3b\sqrt[3]{c+dx}\cos(a)\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3b\sqrt[3]{c+dx}\sin(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}} \, dx = -\frac{3\left(-b\sqrt[3]{c+dx}\cos(a)\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right) + \sin\left(a+b\sqrt[3]{c+dx}\right) + b\sqrt[3]{c+dx}\sin(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)\right)}{de\sqrt[3]{e(c+dx)}}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (-3*(-(b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b*(c + d*x)^(1/3)]) + Sin[a + b
*(c + d*x)^(1/3)] + b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b*(c + d*x)^(1/3)]
))/(d*e*(e*(c + d*x))^(1/3))
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{4}{3}}}dx$$

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
[Out] integral((d*e*x + c*e)^(2/3)*sin((d*x + c)^(1/3)*b + a)/(d^2*e^2*x^2 + 2*c*
d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e\,(c+dx))^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(4/3),x)
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(4/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx = \frac{3\left(\left(\Gamma\left(-1,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(-1,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(-1,i\,(dx+c)^{\frac{1}{3}}b\right)+\Gamma\right.\right.}{}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
[Out] 3/4*((gamma(-1, I*b*conjugate((d*x + c)^(1/3))) + gamma(-1, -I*b*conjugate(
(d*x + c)^(1/3))) + gamma(-1, I*(d*x + c)^(1/3)*b) + gamma(-1, -I*(d*x + c)
^(1/3)*b))*cos(a) + (-I*gamma(-1, I*b*conjugate((d*x + c)^(1/3))) + I*gamma
(-1, -I*b*conjugate((d*x + c)^(1/3))) - I*gamma(-1, I*(d*x + c)^(1/3)*b) +
I*gamma(-1, -I*(d*x + c)^(1/3)*b))*sin(a))*b/(d*e^(4/3))
```

Giac [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(d*e*x + c*e)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{(c\,e+d\,e\,x)^{4/3}}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(4/3),x)
[Out] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(4/3), x)
```

3.233 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 175

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx = -\frac{3b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b^2(c+dx)^{2/3}\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)\sin(a)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b^2(c+dx)^{2/3}\cos(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}$$

```
[Out] -3/2*b*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(1/3))/d/e/(e*(d*x+c))^(2/3)-3/2*b^2*(
d*x+c)^(2/3)*cos(a)*Si(b*(d*x+c)^(1/3))/d/e/(e*(d*x+c))^(2/3)-3/2*b^2*(d*x+
c)^(2/3)*Ci(b*(d*x+c)^(1/3))*sin(a)/d/e/(e*(d*x+c))^(2/3)-3/2*sin(a+b*(d*x+
c)^(1/3))/d/e/(e*(d*x+c))^(2/3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used

= {3512, 15, 3378, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx = -\frac{3b^2\sin(a)(c+dx)^{2/3}\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b^2\cos(a)(c+dx)^{2/3}\text{Si}\left(b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}$$

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (-3*b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(2*d*e*(e*(c + d*x))^(2/3
)) - (3*b^2*(c + d*x)^(2/3)*CosIntegral[b*(c + d*x)^(1/3)]*Sin[a])/(2*d*e*(
e*(c + d*x))^(2/3)) - (3*Sin[a + b*(c + d*x)^(1/3)])/(2*d*e*(e*(c + d*x))^(
2/3)) - (3*b^2*(c + d*x)^(2/3)*Cos[a]*SinIntegral[b*(c + d*x)^(1/3)])/(2*d*
e*(e*(c + d*x))^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{(ex^3)^{5/3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d} \\
&= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \sqrt[3]{c+dx}\right)}{de(e(c+dx))^{2/3}} \\
&= -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} + \frac{\left(3b(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} \\
&= -\frac{3b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} \\
&\quad - \frac{\left(3b^2(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, \sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} \\
&= -\frac{3b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} \\
&\quad - \frac{\left(3b^2(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} \\
&\quad - \frac{\left(3b^2(c+dx)^{2/3}\sin(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} \\
&= -\frac{3b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b^2(c+dx)^{2/3}\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)\sin(a)}{2de(e(c+dx))^{2/3}} \\
&\quad - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b^2(c+dx)^{2/3}\cos(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.66

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx = -\frac{3\left(b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)+b^2(c+dx)^{2/3}\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)\sin(a)+\sin\left(a+b\sqrt[3]{c+dx}\right)+\right.}{2de(e(c+dx))^{2/3}}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (-3*(b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)] + b^2*(c + d*x)^(2/3)*Cos
Integral[b*(c + d*x)^(1/3)]*Sin[a] + Sin[a + b*(c + d*x)^(1/3)] + b^2*(c +
d*x)^(2/3)*Cos[a]*SinIntegral[b*(c + d*x)^(1/3)]))/(2*d*e*(e*(c + d*x))^(2/
3))
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{5}{3}}}dx$$

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(1/3)*sin((d*x + c)^(1/3)*b + a)/(d^2*e^2*x^2 + 2*c*
d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\left(e\left(c+dx\right)\right)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(5/3),x)
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(5/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.74

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx = -\frac{3\left(\left(-i\,\Gamma\left(-2, i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(-2,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)-i\,\Gamma\left(-2,i\,(dx+c)^{\frac{1}{3}}b\right)+i\,\Gamma\left(-2,-i\,(dx+c)^{\frac{1}{3}}b\right)\right.\right.}{}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
[Out] -3/4*((-I*gamma(-2, I*b*conjugate((d*x + c)^(1/3))) + I*gamma(-2, -I*b*conj
ugate((d*x + c)^(1/3))) - I*gamma(-2, I*(d*x + c)^(1/3)*b) + I*gamma(-2, -I
*(d*x + c)^(1/3)*b))*cos(a) - (gamma(-2, I*b*conjugate((d*x + c)^(1/3))) +
gamma(-2, -I*b*conjugate((d*x + c)^(1/3))) + gamma(-2, I*(d*x + c)^(1/3)*b)
 + gamma(-2, -I*(d*x + c)^(1/3)*b))*sin(a))*b^2/(d*e^(5/3))
```

Giac [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(d*e*x + c*e)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx=\int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{(c\,e+d\,e\,x)^{5/3}}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(5/3),x)
[Out] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(5/3), x)
```

3.234 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 267

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx=\frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}-\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}+\frac{b^4\sqrt[3]{c+dx}\,\mathrm{CosIntegral}\left(b\sqrt[3]{c+dx}\right)\sin(a)}{8de^2\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}}+\frac{b^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}+\frac{b^4\sqrt[3]{c+dx}\cos(a)\mathrm{Si}\left(b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}$$

```
[Out] 1/8*b^3*cos(a+b*(d*x+c)^(1/3))/d/e^2/(e*(d*x+c))^(1/3)-1/4*b*cos(a+b*(d*x+c
)^(1/3))/d/e^2/(d*x+c)^(2/3)/(e*(d*x+c))^(1/3)+1/8*b^4*(d*x+c)^(1/3)*cos(a)
*Si(b*(d*x+c)^(1/3))/d/e^2/(e*(d*x+c))^(1/3)+1/8*b^4*(d*x+c)^(1/3)*Ci(b*(d*
x+c)^(1/3))*sin(a)/d/e^2/(e*(d*x+c))^(1/3)-3/4*sin(a+b*(d*x+c)^(1/3))/d/e^2
/(d*x+c)/(e*(d*x+c))^(1/3)+1/8*b^2*sin(a+b*(d*x+c)^(1/3))/d/e^2/(d*x+c)^(1/
3)/(e*(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3512, 15, 3378, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx = \frac{b^4\sin(a)\sqrt[3]{c+dx}\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}} + \frac{b^4\cos(a)\sqrt[3]{c+dx}\text{Si}\left(b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}} + \frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}} + \frac{b^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}} - \frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}$$

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (b^3*Cos[a + b*(c + d*x)^(1/3)])/(8*d*e^2*(e*(c + d*x))^(1/3)) - (b*Cos[a +
 b*(c + d*x)^(1/3)])/(4*d*e^2*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)) + (b^4*(
c + d*x)^(1/3)*CosIntegral[b*(c + d*x)^(1/3)]*Sin[a])/(8*d*e^2*(e*(c + d*x)
)^(1/3)) - (3*Sin[a + b*(c + d*x)^(1/3)])/(4*d*e^2*(c + d*x)*(e*(c + d*x))^
(1/3)) + (b^2*Sin[a + b*(c + d*x)^(1/3)])/(8*d*e^2*(c + d*x)^(1/3)*(e*(c +
d*x))^(1/3)) + (b^4*(c + d*x)^(1/3)*Cos[a]*SinIntegral[b*(c + d*x)^(1/3)])/
(8*d*e^2*(e*(c + d*x))^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{(ex^3)^{7/3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}\\
&= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^5}\,dx, x, \sqrt[3]{c+dx}\right)}{de^2\sqrt[3]{e(c+dx)}}\\
&= -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}} + \frac{\left(3b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^4}\,dx, x, \sqrt[3]{c+dx}\right)}{4de^2\sqrt[3]{e(c+dx)}}\\
&= -\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}}\\
&\quad - \frac{\left(b^2\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \sqrt[3]{c+dx}\right)}{4de^2\sqrt[3]{e(c+dx)}}\\
&= -\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}}\\
&\quad + \frac{b^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} - \frac{\left(b^3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}
\end{aligned}$$

$$=\frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}-\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}}+\frac{b^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}+\frac{\left(b^4\sqrt[3]{c+dx}\right)\text{Subst}\left(\int\frac{\sin(a+bx)}{x}dx,x,\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}$$

$$=\frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}-\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}}+\frac{b^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}+\frac{\left(b^4\sqrt[3]{c+dx}\cos(a)\right)\text{Subst}\left(\int\frac{\sin(bx)}{x}dx,x,\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}+\frac{\left(b^4\sqrt[3]{c+dx}\sin(a)\right)\text{Subst}\left(\int\frac{\cos(bx)}{x}dx,x,\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}$$

$$=\frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}-\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}+\frac{b^4\sqrt[3]{c+dx}\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)\sin(a)}{8de^2\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}}+\frac{b^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}+\frac{b^4\sqrt[3]{c+dx}\cos(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.69

$$\int\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}dx=\frac{b^3c\cos\left(a+b\sqrt[3]{c+dx}\right)+b^3dx\cos\left(a+b\sqrt[3]{c+dx}\right)-2b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c\right.}}{}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (b^3*c*Cos[a + b*(c + d*x)^(1/3)] + b^3*d*x*Cos[a + b*(c + d*x)^(1/3)] - 2*
b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)] + b^4*(c + d*x)^(4/3)*CosInteg
ral[b*(c + d*x)^(1/3)]*Sin[a] - 6*Sin[a + b*(c + d*x)^(1/3)] + b^2*(c + d*x
)^(2/3)*Sin[a + b*(c + d*x)^(1/3)] + b^4*(c + d*x)^(4/3)*Cos[a]*SinIntegral
[b*(c + d*x)^(1/3)])/(8*d*e*(e*(c + d*x))^(4/3))
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{7}{3}}}dx$$

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx = \int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="fricas")
[Out] integral((d*e*x + c*e)^(2/3)*sin((d*x + c)^(1/3)*b + a)/(d^3*e^3*x^3 + 3*c*
d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(7/3),x)
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.48

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx = \frac{3\left(\left(-i\,\Gamma\left(-4,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(-4,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)-i\,\Gamma\left(-4,i\,(dx+c)^{\frac{1}{3}}b\right)\right.\right.}{}$$

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="maxima")
[Out] 3/4*((-I*gamma(-4, I*b*conjugate((d*x + c)^(1/3))) + I*gamma(-4, -I*b*conju
gate((d*x + c)^(1/3))) - I*gamma(-4, I*(d*x + c)^(1/3)*b) + I*gamma(-4, -I*
(d*x + c)^(1/3)*b))*cos(a) - (gamma(-4, I*b*conjugate((d*x + c)^(1/3))) + g
amma(-4, -I*b*conjugate((d*x + c)^(1/3))) + gamma(-4, I*(d*x + c)^(1/3)*b)
+ gamma(-4, -I*(d*x + c)^(1/3)*b))*sin(a))*b^4/(d*e^(7/3))
```

Giac [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx = \int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="giac")

[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(d*e*x + c*e)^(7/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx = \int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{(c\,e+d\,e\,x)^{7/3}}\,dx$$

[In] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(7/3),x)

[Out] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(7/3), x)

3.235 $\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 267

$$\begin{aligned}\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx &= \frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d}\\ &- \frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}\\ &- \frac{45e\sqrt{\pi}\sqrt[3]{e(c+dx)}\cos(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{8\sqrt{2}b^{7/2}d\sqrt[3]{c+dx}}\\ &+ \frac{45e\sqrt{\pi}\sqrt[3]{e(c+dx)}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{8\sqrt{2}b^{7/2}d\sqrt[3]{c+dx}}\\ &+ \frac{15e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d}\end{aligned}$$

```
[Out] 45/8*e*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(2/3))/b^3/d-3/2*e*(d*x+c)^(4/3)*(
e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(2/3))/b/d+15/4*e*(d*x+c)^(2/3)*(e*(d*x+c)
)^(1/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d-45/16*e*(e*(d*x+c))^(1/3)*cos(a)*Fresn
elC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)/d/(d*x+c)^(1/3
)*2^(1/2)+45/16*e*(e*(d*x+c))^(1/3)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/
Pi^(1/2))*sin(a)*Pi^(1/2)/b^(7/2)/d/(d*x+c)^(1/3)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3516, 3498, 3496, 3466, 3467, 3435, 3433, 3432}

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx = -\frac{45\sqrt{\pi}e\cos(a)\sqrt[3]{e(c+dx)}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{8\sqrt{2}b^{7/2}d\sqrt[3]{c+dx}} + \frac{45\sqrt{\pi}e\sin(a)\sqrt[3]{e(c+dx)}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{8\sqrt{2}b^{7/2}d\sqrt[3]{c+dx}} + \frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d} + \frac{15e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d} - \frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}$$

```
[In] Int[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (45*e*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(8*b^3*d) - (3*e*(c +
 d*x)^(4/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d) - (45*e
*Sqrt[Pi]*(e*(c + d*x))^(1/3)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^
(1/3)])/(8*Sqrt[2]*b^(7/2)*d*(c + d*x)^(1/3)) + (45*e*Sqrt[Pi]*(e*(c + d*x)
)^(1/3)*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(8*Sqrt[2]*b^(
7/2)*d*(c + d*x)^(1/3)) + (15*e*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)*Sin[a +
 b*(c + d*x)^(2/3)])/(4*b^2*d)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (ex)^{4/3}\sin\left(a+bx^{2/3}\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{\left(e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^{4/3}\sin\left(a+bx^{2/3}\right)\,dx, x, c+dx\right)}{d\sqrt[3]{c+dx}}$$

$$= \frac{\left(3e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^{6}\sin\left(a+bx^{2}\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d\sqrt[3]{c+dx}}$$

$$= -\frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{\left(15e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^4\cos\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{2bd\sqrt[3]{c+dx}}$$

$$= -\frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{15e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d} - \frac{\left(45e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^2\sin\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{4b^2d\sqrt[3]{c+dx}}$$

$$= \frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d} - \frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{15e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d} - \frac{\left(45e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int\cos\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{8b^3d\sqrt[3]{c+dx}}$$

$$= \frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d} - \frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{15e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d} - \frac{\left(45e\sqrt[3]{e(c+dx)}\cos(a)\right)\text{Subst}\left(\int\cos\left(bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{8b^3d\sqrt[3]{c+dx}} + \frac{\left(45e\sqrt[3]{e(c+dx)}\sin(a)\right)\text{Subst}\left(\int\sin\left(bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{8b^3d\sqrt[3]{c+dx}}$$

$$= \frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d} - \frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} - \frac{45e\sqrt{\pi}\sqrt[3]{e(c+dx)}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{8\sqrt{2}b^{7/2}d\sqrt[3]{c+dx}} + \frac{45e\sqrt{\pi}\sqrt[3]{e(c+dx)}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{8\sqrt{2}b^{7/2}d\sqrt[3]{c+dx}} + \frac{15e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.66

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx = -\frac{3(e(c+dx))^{4/3}\left(15\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right) - 15\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a) + 2\right.}{16b^{7/2}d(c+d}$$

```
[In] Integrate[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(4/3)*(15*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(
c + d*x)^(1/3)] - 15*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)
]*Sin[a] + 2*Sqrt[b]*((c + d*x)^(1/3)*(-15 + 4*b^2*(c + d*x)^(4/3))*Cos[a +
 b*(c + d*x)^(2/3)] - 10*b*(c + d*x)*Sin[a + b*(c + d*x)^(2/3)])))/(16*b^(7
/2)*d*(c + d*x)^(4/3))
```

Maple [F]

$$\int (dex+ce)^{\frac{4}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)dx$$

```
[In] int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

Fricas [F]

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\int (dex+ce)^{\frac{4}{3}}\sin\left((dx+c)^{\frac{2}{3}}b+a\right)\,dx$$

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
[Out] integral((d*e*x + c*e)^(4/3)*sin((d*x + c)^(2/3)*b + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\text{Timed out}$$

```
[In] integrate((d*e*x+c*e)**(4/3)*sin(a+b*(d*x+c)**(2/3)),x)
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.45

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\frac{3\left(\left(\left(\left(i\,\Gamma\left(\frac{7}{2},-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)-i\,\Gamma\left(\frac{7}{2},i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(\frac{7}{4}\pi+\frac{7}{3}\arctan\left(0,dx+c\right)\right)+\right.\right.\right.}{}$$

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
[Out] 3/8*(((I*gamma(7/2, -I*b*conjugate((d*x + c)^(2/3))) - I*gamma(7/2, I*(d*x
+ c)^(2/3)*b))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (-I*gamma(7/2, I*b*c
onjugate((d*x + c)^(2/3))) + I*gamma(7/2, -I*(d*x + c)^(2/3)*b))*cos(-7/4*p
i + 7/3*arctan2(0, d*x + c)) - (gamma(7/2, -I*b*conjugate((d*x + c)^(2/3)))
 + gamma(7/2, I*(d*x + c)^(2/3)*b))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) +
 (gamma(7/2, I*b*conjugate((d*x + c)^(2/3))) + gamma(7/2, -I*(d*x + c)^(2/3
)*b))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*cos(a) - ((gamma(7/2, -I*b*co
njugate((d*x + c)^(2/3))) + gamma(7/2, I*(d*x + c)^(2/3)*b))*cos(7/4*pi + 7
/3*arctan2(0, d*x + c)) + (gamma(7/2, I*b*conjugate((d*x + c)^(2/3))) + gam
ma(7/2, -I*(d*x + c)^(2/3)*b))*cos(-7/4*pi + 7/3*arctan2(0, d*x + c)) - (-I
*gamma(7/2, -I*b*conjugate((d*x + c)^(2/3))) + I*gamma(7/2, I*(d*x + c)^(2/
3)*b))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) - (-I*gamma(7/2, I*b*conjugate
((d*x + c)^(2/3))) + I*gamma(7/2, -I*(d*x + c)^(2/3)*b))*sin(-7/4*pi + 7/3*
arctan2(0, d*x + c)))*sin(a))*sqrt((d*x + c)^(2/3)*b)*e^(4/3)/((d*x + c)^(1
/3)*b^4*d)
```

Giac [F(-2)]

Exception generated.

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx = \text{Exception raised: TypeError}$$

[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx = \int \sin\left(a+b\,(c+d\,x)^{2/3}\right)\,(c\,e+d\,e\,x)^{4/3}\,dx$$

[In] int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(4/3),x)

[Out] int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(4/3), x)

3.236 $\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=-\frac{3\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}-\frac{9\sqrt{\pi}(e(c+dx))^{2/3}\cos(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{4\sqrt{2}b^{5/2}d(c+dx)^{2/3}}-\frac{9\sqrt{\pi}(e(c+dx))^{2/3}\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{4\sqrt{2}b^{5/2}d(c+dx)^{2/3}}+\frac{9(e(c+dx))^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d\sqrt[3]{c+dx}}$$

```
[Out] -3/2*(d*x+c)^(1/3)*(e*(d*x+c))^(2/3)*cos(a+b*(d*x+c)^(2/3))/b/d+9/4*(e*(d*x
+c))^(2/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d/(d*x+c)^(1/3)-9/8*(e*(d*x+c))^(2/3)
*cos(a)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)/d
/(d*x+c)^(2/3)*2^(1/2)-9/8*(e*(d*x+c))^(2/3)*FresnelC((d*x+c)^(1/3)*b^(1/2)
*2^(1/2)/Pi^(1/2))*sin(a)*Pi^(1/2)/b^(5/2)/d/(d*x+c)^(2/3)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3516, 3498, 3496, 3466, 3467, 3434, 3433, 3432}

$$\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx = -\frac{9\sqrt{\pi}\sin(a)(e(c+dx))^{2/3}\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{4\sqrt{2}b^{5/2}d(c+dx)^{2/3}} - \frac{9\sqrt{\pi}\cos(a)(e(c+dx))^{2/3}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{4\sqrt{2}b^{5/2}d(c+dx)^{2/3}} + \frac{9(e(c+dx))^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d\sqrt[3]{c+dx}} - \frac{3\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}$$

```
[In] Int[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(e*(c + d*x))^(2/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d)
 - (9*Sqrt[Pi]*(e*(c + d*x))^(2/3)*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c +
d*x)^(1/3)])/(4*Sqrt[2]*b^(5/2)*d*(c + d*x)^(2/3)) - (9*Sqrt[Pi]*(e*(c + d*
x))^(2/3)*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(4*Sqrt[2]*b
^(5/2)*d*(c + d*x)^(2/3)) + (9*(e*(c + d*x))^(2/3)*Sin[a + b*(c + d*x)^(2/3
)])/(4*b^2*d*(c + d*x)^(1/3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (ex)^{2/3}\sin\left(a+bx^{2/3}\right)\,dx, x, c+dx\right)}{d} \\
&= \frac{(e(c+dx))^{2/3}\text{Subst}\left(\int x^{2/3}\sin\left(a+bx^{2/3}\right)\,dx, x, c+dx\right)}{d(c+dx)^{2/3}} \\
&= \frac{\left(3(e(c+dx))^{2/3}\right)\text{Subst}\left(\int x^4\sin\left(a+bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d(c+dx)^{2/3}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} \\
&\quad + \frac{\left(9(e(c+dx))^{2/3}\right)\text{Subst}\left(\int x^2\cos\left(a+bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{2bd(c+dx)^{2/3}} \\
&= -\frac{3\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} \\
&\quad + \frac{9(e(c+dx))^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d\sqrt[3]{c+dx}} \\
&\quad - \frac{\left(9(e(c+dx))^{2/3}\right)\text{Subst}\left(\int \sin\left(a+bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{4b^2d(c+dx)^{2/3}} \\
&= -\frac{3\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} \\
&\quad + \frac{9(e(c+dx))^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d\sqrt[3]{c+dx}} \\
&\quad - \frac{\left(9(e(c+dx))^{2/3}\cos(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{4b^2d(c+dx)^{2/3}} \\
&\quad - \frac{\left(9(e(c+dx))^{2/3}\sin(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{4b^2d(c+dx)^{2/3}} \\
&= -\frac{3\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} \\
&\quad - \frac{9\sqrt{\pi}(e(c+dx))^{2/3}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{4\sqrt{2}b^{5/2}d(c+dx)^{2/3}} \\
&\quad - \frac{9\sqrt{\pi}(e(c+dx))^{2/3}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{4\sqrt{2}b^{5/2}d(c+dx)^{2/3}} \\
&\quad + \frac{9(e(c+dx))^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d\sqrt[3]{c+dx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.70

$$
\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx =
$$

$$
-\frac{3(e(c+dx))^{2/3}\left(3\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)+3\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)+2\sqrt{\cdots}\right.}{8b^{5/2}d(c+dx)^{2/3}}
$$

```
[In] Integrate[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(2/3)*(3*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)^(1/3)] + 3*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*
Sin[a] + 2*Sqrt[b]*(2*b*(c + d*x)*Cos[a + b*(c + d*x)^(2/3)] - 3*(c + d*x)^
(1/3)*Sin[a + b*(c + d*x)^(2/3)])))/(8*b^(5/2)*d*(c + d*x)^(2/3))
```

Maple [F]

$$\int (dex+ce)^{\frac{2}{3}} \sin\left(a+b(dx+c)^{\frac{2}{3}}\right) dx$$

```
[In] int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x)
[Out] int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

Fricas [F]

$$\int (ce+dex)^{2/3} \sin\left(a+b(c+dx)^{2/3}\right) \, dx = \int (dex+ce)^{\frac{2}{3}} \sin\left((dx+c)^{\frac{2}{3}}b+a\right) dx$$

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
[Out] integral((d*e*x + c*e)^(2/3)*sin((d*x + c)^(2/3)*b + a), x)
```

Sympy [F]

$$\int (ce+dex)^{2/3} \sin\left(a+b(c+dx)^{2/3}\right) \, dx = \int (e(c+dx))^{\frac{2}{3}} \sin\left(a+b(c+dx)^{\frac{2}{3}}\right) dx$$

```
[In] integrate((d*e*x+c*e)**(2/3)*sin(a+b*(d*x+c)**(2/3)),x)
[Out] Integral((e*(c + d*x))**(2/3)*sin(a + b*(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.87

$$\int (ce+dex)^{2/3} \sin\left(a+b(c+dx)^{2/3}\right) \, dx =$$

$$-\frac{3\left(3\,(dx+c)^{\frac{2}{3}}\left(\left(\left(\Gamma\left(\frac{3}{2},-i\,\overline{b(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(\frac{3}{2},i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(\frac{3}{4}\pi+\arctan\left(0,dx+c\right)\right)+\left(\Gamma\left(\frac{3}{2},i\right.\right.\right.\right.\right.}{}$$

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/16*(3*(d*x + c)^(2/3)*(((gamma(3/2, -I*b*conjugate((d*x + c)^(2/3))) + g
amma(3/2, I*(d*x + c)^(2/3)*b))*cos(3/4*pi + arctan2(0, d*x + c)) + (gamma(
3/2, I*b*conjugate((d*x + c)^(2/3))) + gamma(3/2, -I*(d*x + c)^(2/3)*b))*co
s(-3/4*pi + arctan2(0, d*x + c)) + (I*gamma(3/2, -I*b*conjugate((d*x + c)^(
2/3))) - I*gamma(3/2, I*(d*x + c)^(2/3)*b))*sin(3/4*pi + arctan2(0, d*x + c
)) + (I*gamma(3/2, I*b*conjugate((d*x + c)^(2/3))) - I*gamma(3/2, -I*(d*x +
 c)^(2/3)*b))*sin(-3/4*pi + arctan2(0, d*x + c)))*cos(a) + ((I*gamma(3/2, -
I*b*conjugate((d*x + c)^(2/3))) - I*gamma(3/2, I*(d*x + c)^(2/3)*b))*cos(3/
4*pi + arctan2(0, d*x + c)) + (-I*gamma(3/2, I*b*conjugate((d*x + c)^(2/3))
) + I*gamma(3/2, -I*(d*x + c)^(2/3)*b))*cos(-3/4*pi + arctan2(0, d*x + c))
- (gamma(3/2, -I*b*conjugate((d*x + c)^(2/3))) + gamma(3/2, I*(d*x + c)^(2/
3)*b))*sin(3/4*pi + arctan2(0, d*x + c)) + (gamma(3/2, I*b*conjugate((d*x +
 c)^(2/3))) + gamma(3/2, -I*(d*x + c)^(2/3)*b))*sin(-3/4*pi + arctan2(0, d*
x + c)))*sin(a))*sqrt((d*x + c)^(2/3)*b)*e^(2/3) + 8*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*e^(2/3)*cos((d*x + c)^(2/3)*b + a))/(b^3*d^2*x + b^3*c*d)
```

Giac [F(-2)]

Exception generated.

$$\int (ce+dex)^{2/3} \sin \left(a+b(c+dx)^{2/3}\right) \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{2/3} \sin \left(a+b(c+dx)^{2/3}\right) \, dx = \int \sin \left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{2/3} \,d x$$

```
[In] int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(2/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(2/3), x)
```

3.237 $\int \sqrt[3]{ce+dex} \sin\left(a+b(c+dx)^{2/3}\right) dx$

Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \sqrt[3]{ce+dex} \sin\left(a+b(c+dx)^{2/3}\right) dx = -\frac{3\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{3\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{2b^2d\sqrt[3]{c+dx}}$$

```
[Out] -3/2*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(2/3))/b/d+3/2*(e*(d*x
+c))^(1/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d/(d*x+c)^(1/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3516, 3462, 3460, 3377, 2717}

$$\int \sqrt[3]{ce+dex} \sin\left(a+b(c+dx)^{2/3}\right) dx = \frac{3\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{2b^2d\sqrt[3]{c+dx}} - \frac{3\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}$$

```
[In] Int[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d)
 + (3*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(2/3)])/(2*b^2*d*(c + d*x)^(1
/3))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt[3]{ex}\sin\left(a+bx^{2/3}\right)\,dx,x,c+dx\right)}{d}\\
&= \frac{\sqrt[3]{e(c+dx)}\text{Subst}\left(\int \sqrt[3]{x}\sin\left(a+bx^{2/3}\right)\,dx,x,c+dx\right)}{d\sqrt[3]{c+dx}}\\
&= \frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx,x,(c+dx)^{2/3}\right)}{2d\sqrt[3]{c+dx}}\\
&= -\frac{3\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}\\
&\quad+\frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \cos(a+bx)\,dx,x,(c+dx)^{2/3}\right)}{2bd\sqrt[3]{c+dx}}
\end{aligned}
$$

$$= -\frac{3\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{3\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{2b^2d\sqrt[3]{c+dx}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx = -\frac{3\sqrt[3]{e(c+dx)}\left(b(c+dx)^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)-\sin\left(a+b(c+dx)^{2/3}\right)\right)}{2b^2d\sqrt[3]{c+dx}}$$

```
[In] Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(2/3)],x]
[Out] (-3*(e*(c + d*x))^(1/3)*(b*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(2/3)] - Sin
[a + b*(c + d*x)^(2/3)]))/(2*b^2*d*(c + d*x)^(1/3))
```

Maple [F]

$$\int (dex+ce)^{\frac{1}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)dx$$

```
[In] int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x)
[Out] int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.67 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx = -\frac{3\left((bdx+bc)(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{2}{3}}b+a\right)-(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\sin\left((dx+c)^{\frac{2}{3}}b+a\right)\right)}{2\left(b^2d^2x+b^2cd\right)}$$

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
[Out] -3/2*((b*d*x + b*c)*(d*e*x + c*e)^(1/3)*(d*x + c)^(1/3)*cos((d*x + c)^(2/3)
*b + a) - (d*e*x + c*e)^(1/3)*(d*x + c)^(2/3)*sin((d*x + c)^(2/3)*b + a))/(
b^2*d^2*x + b^2*c*d)
```

Sympy [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx = \int \sqrt[3]{e\left(c+dx\right)}\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)dx$$

```
[In] integrate((d*e*x+c*e)**(1/3)*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(1/3)*sin(a + b*(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.45

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx =$$

$$-\frac{3\left(\left(-i\,\Gamma\left(2, i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+i\,\Gamma\left(2,-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)-i\,\Gamma\left(2, i\,(dx+c)^{\frac{2}{3}}b\right)+i\,\Gamma\left(2,-i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos(}{8\,b}$$

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/8*((-I*gamma(2, I*b*conjugate((d*x + c)^(2/3))) + I*gamma(2, -I*b*conjug
ate((d*x + c)^(2/3))) - I*gamma(2, I*(d*x + c)^(2/3)*b) + I*gamma(2, -I*(d*
x + c)^(2/3)*b))*cos(a) - (gamma(2, I*b*conjugate((d*x + c)^(2/3))) + gamma
(2, -I*b*conjugate((d*x + c)^(2/3))) + gamma(2, I*(d*x + c)^(2/3)*b) + gamm
a(2, -I*(d*x + c)^(2/3)*b))*sin(a))*e^(1/3)/(b^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\int \sin\left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{1/3}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(1/3),x)
[Out] int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(1/3), x)
```

3.238 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 44

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx=-\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

```
[Out] -3/2*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))/b/d/(e*(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3516, 3462, 3460, 2718}

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx=-\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(1/3))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{\sqrt[3]{ex}}\,dx, x, c+dx\right)}{d} \\
&= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{\sqrt[3]{x}}\,dx, x, c+dx\right)}{d\sqrt[3]{e(c+dx)}} \\
&= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \sin(a+bx)\,dx, x, (c+dx)^{2/3}\right)}{2d\sqrt[3]{e(c+dx)}} \\
&= -\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
[Out] (-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(1/3))
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(dex+ce)^{\frac{1}{3}}}dx$$

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3\,(dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{2}{3}}b+a\right)}{2\,(bd^2ex+bcde)}$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/2*(d*e*x + c*e)^(2/3)*(d*x + c)^(1/3)*cos((d*x + c)^(2/3)*b + a)/(b*d^2*
e*x + b*c*d*e)
```

Sympy [F]

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx = \int \frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{\sqrt[3]{e\,(c+dx)}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(1/3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(1/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3\,\cos\left((dx+c)^{\frac{2}{3}}b+a\right)}{2\,bde^{\frac{1}{3}}}$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/2*cos((d*x + c)^(2/3)*b + a)/(b*d*e^(1/3))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx = \int \frac{\sin\left(a+b\,(c+d\,x)^{2/3}\right)}{(c\,e+d\,e\,x)^{1/3}}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(1/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(1/3), x)
```

3.239 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 133

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}d(e(c+dx))^{2/3}} + \frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{\sqrt{b}d(e(c+dx))^{2/3}}$$

```
[Out] 3/2*(d*x+c)^(2/3)*cos(a)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*2
^(1/2)*Pi^(1/2)/d/(e*(d*x+c))^(2/3)/b^(1/2)+3/2*(d*x+c)^(2/3)*FresnelC((d*x
+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/d/(e*(d*x+c))^(
2/3)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3516, 3498, 3464, 3434, 3433, 3432}

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}\sin(a)(c+dx)^{2/3}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}d(e(c+dx))^{2/3}} + \frac{3\sqrt{\frac{\pi}{2}}\cos(a)(c+dx)^{2/3}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}d(e(c+dx))^{2/3}}$$

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (3*Sqrt[Pi/2]*(c + d*x)^(2/3)*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^
(1/3)])/(Sqrt[b]*d*(e*(c + d*x))^(2/3)) + (3*Sqrt[Pi/2]*(c + d*x)^(2/3)*Fre
snelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(Sqrt[b]*d*(e*(c + d*x))^
(2/3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3464

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)], x_Symbol] :> Dist[2/n, Subst[I
nt[Sin[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
n/2 - 1]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{(ex)^{2/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)^{2/3}\text{Subst}\left(\int \frac{\sin(a+bx^{2/3})}{x^{2/3}}\,dx, x, c+dx\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \sin\left(a+bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{\left(3(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}} + \frac{\left(3(c+dx)^{2/3}\sin(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}d(e(c+dx))^{2/3}} + \frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{\sqrt{b}d(e(c+dx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.72

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\left(\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right) + \text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+\right.\right.}{\sqrt{b}d(e(c+dx))^{2/3}}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (3*Sqrt[Pi/2]*(c + d*x)^(2/3)*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)
^(1/3)] + FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a]))/(Sqrt[b]*d*
(e*(c + d*x))^(2/3))
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(dex+ce)^{\frac{2}{3}}}dx$$

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

Fricas [F]

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\, dx = \int \frac{\sin \left((dx+c)^{\frac{2}{3}}b+a\right)}{(dex+ce)^{\frac{2}{3}}}\, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
[Out] integral(sin((d*x + c)^(2/3)*b + a)/(d*e*x + c*e)^(2/3), x)
```

Sympy [F]

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\, dx = \int \frac{\sin \left(a+b(c+dx)^{\frac{2}{3}}\right)}{(e\,(c+dx))^{\frac{2}{3}}}\, dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(2/3),x)
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(2/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.66

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\, dx = \frac{3\left(\left(\left(-i\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-i\,b\overline{(dx+c)^{\frac{2}{3}}}}\right)-1\right)+i\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{i\,(dx+c)^{\frac{2}{3}}b}\right)-\right.\right.\right.\right.}{}$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
[Out] 3/8*(((-I*sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(2/3)))) - 1) + I*sqr
t(pi)*(erf(sqrt(I*(d*x + c)^(2/3)*b)) - 1))*cos(1/4*pi + 1/3*arctan2(0, d*x
 + c)) + (I*sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(2/3)))) - 1) - I*sq
rt(pi)*(erf(sqrt(-I*(d*x + c)^(2/3)*b)) - 1))*cos(-1/4*pi + 1/3*arctan2(0,
d*x + c)) + (sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(2/3)))) - 1) + sq
rt(pi)*(erf(sqrt(I*(d*x + c)^(2/3)*b)) - 1))*sin(1/4*pi + 1/3*arctan2(0, d*
x + c)) - (sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(2/3)))) - 1) + sqrt(
pi)*(erf(sqrt(-I*(d*x + c)^(2/3)*b)) - 1))*sin(-1/4*pi + 1/3*arctan2(0, d*x
 + c)))*cos(a) + ((sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(2/3)))) - 1
) + sqrt(pi)*(erf(sqrt(I*(d*x + c)^(2/3)*b)) - 1))*cos(1/4*pi + 1/3*arctan2
(0, d*x + c)) + (sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(2/3)))) - 1) +
```

```
 sqrt(pi)*(erf(sqrt(-I*(d*x + c)^(2/3)*b)) - 1))*cos(-1/4*pi + 1/3*arctan2(
0, d*x + c)) + (I*sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(2/3)))) - 1)
 - I*sqrt(pi)*(erf(sqrt(I*(d*x + c)^(2/3)*b)) - 1))*sin(1/4*pi + 1/3*arctan
2(0, d*x + c)) + (I*sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(2/3)))) - 1
) - I*sqrt(pi)*(erf(sqrt(-I*(d*x + c)^(2/3)*b)) - 1))*sin(-1/4*pi + 1/3*arc
tan2(0, d*x + c)))*sin(a))*sqrt((d*x + c)^(2/3)*b)/((d*x + c)^(1/3)*b*d*e^(
2/3))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin \left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)}{{\left(c\,e+d\,e\,x\right)}^{2/3}}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(2/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(2/3), x)
```

3.240 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 168

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx = \frac{3\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\cos(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{de\sqrt[3]{e(c+dx)}}$$

```
[Out] -3*sin(a+b*(d*x+c)^(2/3))/d/e/(e*(d*x+c))^(1/3)+3*(d*x+c)^(1/3)*cos(a)*Fres
nelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/e/(
e*(d*x+c))^(1/3)-3*(d*x+c)^(1/3)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^
(1/2))*sin(a)*b^(1/2)*2^(1/2)*Pi^(1/2)/d/e/(e*(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3516, 3498, 3496, 3468, 3435, 3433, 3432}

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx = \frac{3\sqrt{2\pi}\sqrt{b}\cos(a)\sqrt[3]{c+dx}\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sqrt{2\pi}\sqrt{b}\sin(a)\sqrt[3]{c+dx}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{de\sqrt[3]{e(c+dx)}}$$

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)^(1/3)])/(d*e*(e*(c + d*x))^(1/3)) - (3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x)
^(1/3)*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(d*e*(e*(c + d*
x))^(1/3)) - (3*Sin[a + b*(c + d*x)^(2/3)])/(d*e*(e*(c + d*x))^(1/3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
```

```
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sin(a+bx^{2/3})}{(ex)^{4/3}}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\sqrt[3]{c+dx}\text{Subst}\left(\int \frac{\sin(a+bx^{2/3})}{x^{4/3}}\,dx, x, c+dx\right)}{de\sqrt[3]{e(c+dx)}}$$
$$= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^2}\,dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$
$$= -\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{de\sqrt[3]{e(c+dx)}} + \frac{\left(6b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$
$$= -\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{de\sqrt[3]{e(c+dx)}} + \frac{\left(6b\sqrt[3]{c+dx}\cos(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{\left(6b\sqrt[3]{c+dx}\sin(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$
$$= \frac{3\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{de\sqrt[3]{e(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.79

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx = -\frac{3\left(-\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right) + \sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin\right.}{de\sqrt[3]{e(c+dx)}}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
[Out] (-3*(-(Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi
]*(c + d*x)^(1/3)]) + Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(1/3)*FresnelS[Sqrt[b]*S
qrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a] + Sin[a + b*(c + d*x)^(2/3)]))/(d*e*(e*(c
 + d*x))^(1/3))
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(dex+ce)^{\frac{4}{3}}}dx$$

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
[Out] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
[Out] integral((d*e*x + c*e)^(2/3)*sin((d*x + c)^(2/3)*b + a)/(d^2*e^2*x^2 + 2*c*
d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{(e\,(c+dx))^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(4/3),x)
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(4/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.26

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx =$$

$$-\frac{3\left(\left(\left(\left(-i\,\Gamma\left(-\frac{1}{2},-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+i\,\Gamma\left(-\frac{1}{2},i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(\frac{1}{4}\,\pi+\frac{1}{3}\,\arctan\left(0,dx+c\right)\right)+\left(i\,\Gamma\left(-\frac{1}{2},i\,b\right.\right.\right.\right.}{}$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

```
[Out] -3/8*(((-I*gamma(-1/2, -I*b*conjugate((d*x + c)^(2/3))) + I*gamma(-1/2, I*(
d*x + c)^(2/3)*b))*cos(1/4*pi + 1/3*arctan2(0, d*x + c)) + (I*gamma(-1/2, I
*b*conjugate((d*x + c)^(2/3))) - I*gamma(-1/2, -I*(d*x + c)^(2/3)*b))*cos(-
1/4*pi + 1/3*arctan2(0, d*x + c)) - (gamma(-1/2, -I*b*conjugate((d*x + c)^(
2/3))) + gamma(-1/2, I*(d*x + c)^(2/3)*b))*sin(1/4*pi + 1/3*arctan2(0, d*x
+ c)) + (gamma(-1/2, I*b*conjugate((d*x + c)^(2/3))) + gamma(-1/2, -I*(d*x
+ c)^(2/3)*b))*sin(-1/4*pi + 1/3*arctan2(0, d*x + c)))*cos(a) + ((gamma(-1/
2, -I*b*conjugate((d*x + c)^(2/3))) + gamma(-1/2, I*(d*x + c)^(2/3)*b))*cos
(1/4*pi + 1/3*arctan2(0, d*x + c)) + (gamma(-1/2, I*b*conjugate((d*x + c)^(
2/3))) + gamma(-1/2, -I*(d*x + c)^(2/3)*b))*cos(-1/4*pi + 1/3*arctan2(0, d*
x + c)) + (-I*gamma(-1/2, -I*b*conjugate((d*x + c)^(2/3))) + I*gamma(-1/2,
I*(d*x + c)^(2/3)*b))*sin(1/4*pi + 1/3*arctan2(0, d*x + c)) + (-I*gamma(-1/
2, I*b*conjugate((d*x + c)^(2/3))) + I*gamma(-1/2, -I*(d*x + c)^(2/3)*b))*s
in(-1/4*pi + 1/3*arctan2(0, d*x + c)))*sin(a))*sqrt((d*x + c)^(2/3)*b)/((d*
x + c)^(1/3)*d*e^(4/3))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)}{{\left(c\,e+d\,e\,x\right)}^{4/3}}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(4/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(4/3), x)
```

3.241 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 126

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \frac{3b(c+dx)^{2/3}\cos(a)\,\text{CosIntegral}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b(c+dx)^{2/3}\sin(a)\text{Si}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}$$

```
[Out] 3/2*b*(d*x+c)^(2/3)*Ci(b*(d*x+c)^(2/3))*cos(a)/d/e/(e*(d*x+c))^(2/3)-3/2*b*
(d*x+c)^(2/3)*Si(b*(d*x+c)^(2/3))*sin(a)/d/e/(e*(d*x+c))^(2/3)-3/2*sin(a+b*
(d*x+c)^(2/3))/d/e/(e*(d*x+c))^(2/3)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3516, 3462, 3460, 3378, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \frac{3b\cos(a)(c+dx)^{2/3}\,\text{CosIntegral}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b\sin(a)(c+dx)^{2/3}\text{Si}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}$$

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (3*b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b*(c + d*x)^(2/3)])/(2*d*e*(e*(c +
d*x))^(2/3)) - (3*Sin[a + b*(c + d*x)^(2/3)])/(2*d*e*(e*(c + d*x))^(2/3)) -
 (3*b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b*(c + d*x)^(2/3)])/(2*d*e*(e*(c +
 d*x))^(2/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{(ex)^{5/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)^{2/3}\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{x^{5/3}}\,dx, x, c+dx\right)}{de(e(c+dx))^{2/3}}$$

$$= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, (c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}$$

$$= -\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} + \frac{\left(3b(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, (c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}$$

$$= -\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} + \frac{\left(3b(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, (c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} - \frac{\left(3b(c+dx)^{2/3}\sin(a)\right)\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, (c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}$$

$$= \frac{3b(c+dx)^{2/3}\cos(a)\,\text{CosIntegral}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b(c+dx)^{2/3}\sin(a)\text{Si}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = -\frac{3\left(-b(c+dx)^{2/3}\cos(a)\,\text{CosIntegral}\left(b(c+dx)^{2/3}\right) + \sin\left(a+b(c+dx)^{2/3}\right) + b(c+dx)^{2/3}\sin(a)\text{Si}\left(b(c+dx\right.\right.}{2de(e(c+dx))^{2/3}}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (-3*(-(b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b*(c + d*x)^(2/3)]) + Sin[a + b
*(c + d*x)^(2/3)] + b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b*(c + d*x)^(2/3)]
))/(2*d*e*(e*(c + d*x))^(2/3))
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(dex+ce)^{\frac{5}{3}}}dx$$

[In] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)

[Out] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)

Fricas [F]

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")

[Out] integral((d*e*x + c*e)^(1/3)*sin((d*x + c)^(2/3)*b + a)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)

Sympy [F]

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{(e\,(c+dx))^{\frac{5}{3}}}\,dx$$

[In] integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(5/3),x)

[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(5/3), x)

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \frac{3\left(\left(\Gamma\left(-1, i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(-1,-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(-1, i\,(dx+c)^{\frac{2}{3}}b\right)+\right.\right.}{}$$

[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")

[Out] 3/8*((gamma(-1, I*b*conjugate((d*x + c)^(2/3))) + gamma(-1, -I*b*conjugate((d*x + c)^(2/3))) + gamma(-1, I*(d*x + c)^(2/3)*b) + gamma(-1, -I*(d*x + c)^(2/3)*b))*cos(a) + (-I*gamma(-1, I*b*conjugate((d*x + c)^(2/3))) + I*gamma(-1, -I*b*conjugate((d*x + c)^(2/3))) - I*gamma(-1, I*(d*x + c)^(2/3)*b) + I*gamma(-1, -I*(d*x + c)^(2/3)*b))*sin(a))*b/(d*e^(5/3))

Giac [F(-2)]

Exception generated.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left(a+b\,(c+d\,x)^{2/3}\right)}{(c\,e+d\,e\,x)^{5/3}}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(5/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(5/3), x)
```

3.242 $\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 247

$$
\begin{aligned}
\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = & -\frac{b^3\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} \\
& +\frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} \\
& -\frac{b^4\sqrt[3]{e(c+dx)}\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{8d\sqrt[3]{c+dx}} \\
& -\frac{b^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} \\
& +\frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} \\
& -\frac{b^4\sqrt[3]{e(c+dx)}\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}}
\end{aligned}
$$

```
[Out] -1/8*b^3*(e*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(1/3))/d+1/4*b*(d*x+c)^(2/3)*(e*
(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(1/3))/d-1/8*b^4*(e*(d*x+c))^(1/3)*cos(a)*Si
(b/(d*x+c)^(1/3))/d/(d*x+c)^(1/3)-1/8*b^4*(e*(d*x+c))^(1/3)*Ci(b/(d*x+c)^(1
/3))*sin(a)/d/(d*x+c)^(1/3)-1/8*b^2*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*sin(a+b
/(d*x+c)^(1/3))/d+3/4*(d*x+c)*(e*(d*x+c))^(1/3)*sin(a+b/(d*x+c)^(1/3))/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3512, 15, 3378, 3384, 3380, 3383}

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx = -\frac{b^4\sin(a)\sqrt[3]{e(c+dx)}\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}} - \frac{b^4\cos(a)\sqrt[3]{e(c+dx)}\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}} - \frac{b^3\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} - \frac{b^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} + \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d}$$

```
[In] Int[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] -1/8*(b^3*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/d + (b*(c + d*x)^
(2/3)*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(4*d) - (b^4*(e*(c +
d*x))^(1/3)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(8*d*(c + d*x)^(1/3)) -
(b^2*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(8*d)
+ (3*(c + d*x)*(e*(c + d*x))^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(4*d) - (b^4
*(e*(c + d*x))^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(8*d*(c + d*x)^
(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3\text{Subst}\left(\int \frac{\sqrt[3]{\frac{e}{x^3}}\sin(a+bx)}{x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d} \\
&= -\frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^5}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d\sqrt[3]{c+dx}} \\
&= \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} \\
&\quad - \frac{\left(3b\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{4d\sqrt[3]{c+dx}}
\end{aligned}
$$

$$= \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} + \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} + \frac{\left(b^2\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int\frac{\sin(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{4d\sqrt[3]{c+dx}}$$

$$= \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{b^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} + \frac{\left(b^3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int\frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}}$$

$$= -\frac{b^3\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{b^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{\left(b^4\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int\frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}}$$

$$= -\frac{b^3\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{b^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{\left(b^4\sqrt[3]{e(c+dx)}\cos(a)\right)\text{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}} - \frac{\left(b^4\sqrt[3]{e(c+dx)}\sin(a)\right)\text{Subst}\left(\int\frac{\cos(bx)}{x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}}$$

$$= -\frac{b^3\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{b^4\sqrt[3]{e(c+dx)}\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{8d\sqrt[3]{c+dx}} - \frac{b^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{b^4\sqrt[3]{e(c+dx)}\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.84

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=$$
$$-\frac{\sqrt[3]{e(c+dx)}\left(-2bc\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)-2bdx\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)-\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] -1/8*((e*(c + d*x))^(1/3)*(-2*b*c*Cos[a + b/(c + d*x)^(1/3)] - 2*b*d*x*Cos[
a + b/(c + d*x)^(1/3)] + b^3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)] + b
^4*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a] - 6*c*(c + d*x)^(1/3)*Sin[a + b/(c
 + d*x)^(1/3)] - 6*d*x*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)] + b^2*(c
+ d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)] + b^4*Cos[a]*SinIntegral[b/(c + d*x
)^(1/3)]))/(d*(c + d*x)^(1/3))
```

Maple [F]

$$\int (dex+ce)^{\frac{1}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)dx$$

```
[In] int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x)
```

```
[Out] int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x)
```

Fricas [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int (dex+ce)^{\frac{1}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)dx$$

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(1/3)*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c
)), x)
```

Sympy [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int \sqrt[3]{e\,(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx$$

```
[In] integrate((d*e*x+c*e)**(1/3)*sin(a+b/(d*x+c)**(1/3)),x)
[Out] Integral((e*(c + d*x))**(1/3)*sin(a + b/(c + d*x)**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.52

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=$$
$$-\frac{3\left(\left(-i\,\Gamma\left(-4,i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+i\,\Gamma\left(-4,-i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)-i\,\Gamma\left(-4,\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(-4,-\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos\left(a\right)}{4\,d}$$

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
[Out] -3/4*((-I*gamma(-4, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(-4, -I*b*con
jugate((d*x + c)^(-1/3))) - I*gamma(-4, I*b/(d*x + c)^(1/3)) + I*gamma(-4,
-I*b/(d*x + c)^(1/3)))*cos(a) - (gamma(-4, I*b*conjugate((d*x + c)^(-1/3)))
 + gamma(-4, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(-4, I*b/(d*x + c)^(1
/3)) + gamma(-4, -I*b/(d*x + c)^(1/3)))*sin(a))*b^4*e^(1/3)/d
```

Giac [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int (dex+ce)^{\frac{1}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)dx$$

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(1/3)*sin(a + b/(d*x + c)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int \sin\left(a+\frac{b}{\left(c+d\,x\right)^{1/3}}\right)\,\left(c\,e+d\,e\,x\right)^{1/3}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/3))*(c*e + d*e*x)^(1/3),x)
[Out] int(sin(a + b/(c + d*x)^(1/3))*(c*e + d*e*x)^(1/3), x)
```

3.243 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 168

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx = \frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b^2\sqrt[3]{c+dx}\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b^2\sqrt[3]{c+dx}\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

```
[Out] 3/2*b*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(1/3))/d/(e*(d*x+c))^(1/3)+3/2*b^2*(d*x
+c)^(1/3)*cos(a)*Si(b/(d*x+c)^(1/3))/d/(e*(d*x+c))^(1/3)+3/2*b^2*(d*x+c)^(1
/3)*Ci(b/(d*x+c)^(1/3))*sin(a)/d/(e*(d*x+c))^(1/3)+3/2*(d*x+c)*sin(a+b/(d*x
+c)^(1/3))/d/(e*(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3512, 15, 3378, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx = \frac{3b^2\sin(a)\sqrt[3]{c+dx}\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b^2\cos(a)\sqrt[3]{c+dx}\,\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (3*b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d*(e*(c + d*x))^(1/3))
+ (3*b^2*(c + d*x)^(1/3)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(2*d*(e*(c
+ d*x))^(1/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/(2*d*(e*(c + d*x)
)^(1/3)) + (3*b^2*(c + d*x)^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(2
*d*(e*(c + d*x))^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt[3]{\frac{e}{x^3}}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d\sqrt[3]{e(c+dx)}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} - \frac{\left(3b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

$$= \frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{\left(3b^2\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

$$= \frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{\left(3b^2\sqrt[3]{c+dx}\cos(a)\right)\text{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{\left(3b^2\sqrt[3]{c+dx}\sin(a)\right)\text{Subst}\left(\int\frac{\cos(bx)}{x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

$$= \frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b^2\sqrt[3]{c+dx}\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b^2\sqrt[3]{c+dx}\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx = \frac{3\left(b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^2\sqrt[3]{c+dx}\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)+c\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\cdots\right.}{2d\sqrt[3]{e(c+dx)}}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (3*(b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)] + b^2*(c + d*x)^(1/3)*CosI
ntegral[b/(c + d*x)^(1/3)]*Sin[a] + c*Sin[a + b/(c + d*x)^(1/3)] + d*x*Sin[
a + b/(c + d*x)^(1/3)] + b^2*(c + d*x)^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)
^(1/3)]))/(2*d*(e*(c + d*x))^(1/3))
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}dx$$

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))/(d*e*x + c*e)^(1/
3), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{e\,(c+dx)}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(1/3),x)
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(1/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx =$$

$$-\frac{3\left((dx+c)^{\frac{1}{3}}\left(\left(-i\,\Gamma\left(-1,i\,b\frac{1}{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(-1,-i\,b\frac{1}{(dx+c)^{\frac{1}{3}}}\right)-i\,\Gamma\left(-1,\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(-1,-\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right.\right.\right.\right.}{}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/8*((d*x + c)^(1/3)*((-I*gamma(-1, I*b*conjugate((d*x + c)^(-1/3))) + I*g
amma(-1, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(-1, I*b/(d*x + c)^(1/3
)) + I*gamma(-1, -I*b/(d*x + c)^(1/3)))*cos(a) - (gamma(-1, I*b*conjugate((
d*x + c)^(-1/3))) + gamma(-1, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(-1,
 I*b/(d*x + c)^(1/3)) + gamma(-1, -I*b/(d*x + c)^(1/3)))*sin(a))*b^2 - 4*(d
*x + c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))/((d*x + c)^(1/3)*d*e^
(1/3))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{1/3}}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(1/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(1/3), x)
```

3.244 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 116

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3b(c+dx)^{2/3}\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3b(c+dx)^{2/3}\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

```
[Out] -3*b*(d*x+c)^(2/3)*Ci(b/(d*x+c)^(1/3))*cos(a)/d/(e*(d*x+c))^(2/3)+3*b*(d*x+
c)^(2/3)*Si(b/(d*x+c)^(1/3))*sin(a)/d/(e*(d*x+c))^(2/3)+3*(d*x+c)*sin(a+b/(
d*x+c)^(1/3))/d/(e*(d*x+c))^(2/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3512, 15, 3378, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3b\cos(a)(c+dx)^{2/3}\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3b\sin(a)(c+dx)^{2/3}\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (-3*b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)])/(d*(e*(c + d*x
))^(2/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/(d*(e*(c + d*x))^(2/3)
) + (3*b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)])/(d*(e*(c +
d*x))^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{2/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} - \frac{\left(3b(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} - \frac{\left(3b(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{\left(3b(c+dx)^{2/3}\sin(a)\right)\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

$$= -\frac{3b(c+dx)^{2/3}\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3b(c+dx)^{2/3}\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx = \frac{3\left(-b(c+dx)^{2/3}\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right) + (c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right.\right.}{d(e(c+dx))^{2/3}}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
[Out] (3*(-(b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)]) + (c + d*x)*
Sin[a + b/(c + d*x)^(1/3)] + b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b/(c + d*
x)^(1/3)]))/(d*(e*(c + d*x))^(2/3))
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}dx$$

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))/(d*e*x + c*e)^(2/
3), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e\,(c+dx))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(2/3),x)
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(2/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.34

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx =$$

$$-\frac{3\left(\left(\left(\mathrm{Ei}\left(i\,b\frac{1}{(dx+c)^{\frac{1}{3}}}\right)+\mathrm{Ei}\left(-i\,b\frac{1}{(dx+c)^{\frac{1}{3}}}\right)+\mathrm{Ei}\left(\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)+\mathrm{Ei}\left(-\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos(a)+\left(i\,\mathrm{Ei}\left(i\,b\frac{1}{(dx+c)^{\frac{1}{3}}}\right)-\right.\right.\right.}{4\,d}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
```

```
[Out] -3/4*(((Ei(I*b*conjugate((d*x + c)^(-1/3))) + Ei(-I*b*conjugate((d*x + c)^(
-1/3))) + Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (I*E
i(I*b*conjugate((d*x + c)^(-1/3))) - I*Ei(-I*b*conjugate((d*x + c)^(-1/3)))
 + I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b*e^(1/3
) - 4*(d*x + c)^(1/3)*e^(1/3)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))
/(d*e)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{2/3}}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(2/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(2/3), x)
```

3.245 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 45

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx=\frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde\sqrt[3]{e(c+dx)}}$$

```
[Out] 3*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(1/3))/b/d/e/(e*(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3512, 15, 2718}

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx=\frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde\sqrt[3]{e(c+dx)}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*e*(e*(c + d*x))^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{4/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d} \\
&= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}} \\
&= \frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde\sqrt[3]{e(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx = \frac{3(c+dx)^{4/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bd(e(c+dx))^{4/3}}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
[Out] (3*(c + d*x)^(4/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(4/3))
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}dx$$

[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)

[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx = \frac{3\,(dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)}{bd^2e^2x+bcde^2}$$

[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")

[Out] 3*(d*e*x + c*e)^(2/3)*(d*x + c)^(1/3)*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))/(b*d^2*e^2*x + b*c*d*e^2)

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e\,(c+dx))^{\frac{4}{3}}}\,dx$$

[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(4/3),x)

[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(4/3), x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx = \frac{3\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{bde^{\frac{4}{3}}}$$

[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")

[Out] 3*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(b*d*e^(4/3))

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{4/3}}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(4/3),x)
[Out] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(4/3), x)
```

3.246 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 91

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}}-\frac{3(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de(e(c+dx))^{2/3}}$$

```
[Out] 3*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(1/3))/b/d/e/(e*(d*x+c))^(2/3)-3*(d*x+c)^(2
/3)*sin(a+b/(d*x+c)^(1/3))/b^2/d/e/(e*(d*x+c))^(2/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3512, 15, 3377, 2717}

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}}-\frac{3(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de(e(c+dx))^{2/3}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*e*(e*(c + d*x))^(2/3))
- (3*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(b^2*d*e*(e*(c + d*x))^(2/
3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{5/3} x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d} \\
&= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de(e(c+dx))^{2/3}} \\
&= \frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}} - \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}} \\
&= \frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}} - \frac{3(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de(e(c+dx))^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3(c+dx)^{5/3}\left(\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}}-\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2}\right)}{d(e(c+dx))^{5/3}}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
[Out] (3*(c + d*x)^(5/3)*(Cos[a + b/(c + d*x)^(1/3)]/(b*(c + d*x)^(1/3)) - Sin[a
+ b/(c + d*x)^(1/3)]/b^2))/(d*(e*(c + d*x))^(5/3))
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{5}{3}}}dx$$

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3\left((dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)-(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\sin\left(\frac{a\right.}{}\right.}{b^2d^2e^2x+b^2cde^2}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
[Out] 3*((d*e*x + c*e)^(1/3)*(d*x + c)^(1/3)*b*cos((a*d*x + a*c + (d*x + c)^(2/3)
*b)/(d*x + c)) - (d*e*x + c*e)^(1/3)*(d*x + c)^(2/3)*sin((a*d*x + a*c + (d*
x + c)^(2/3)*b)/(d*x + c)))/(b^2*d^2*e^2*x + b^2*c*d*e^2)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\left(e\left(c+dx\right)\right)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(5/3),x)
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(5/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.88

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx =$$

$$-\frac{3\left(4\,b^2\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right) - (dx+c)^{\frac{2}{3}}\left(\left(-i\,\Gamma\left(3, i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right) + i\,\Gamma\left(3, -i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right) - i\,\Gamma\left(3, \frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right) + i\right.\right.\right.}{8\,(}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
[Out] -3/8*(4*b^2*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - (d*x + c)^(2/3)*
((-I*gamma(3, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(3, -I*b*conjugate(
(d*x + c)^(-1/3))) - I*gamma(3, I*b/(d*x + c)^(1/3)) + I*gamma(3, -I*b/(d*x
 + c)^(1/3)))*cos(a) - (gamma(3, I*b*conjugate((d*x + c)^(-1/3))) + gamma(3
, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(3, I*b/(d*x + c)^(1/3)) + gamma
(3, -I*b/(d*x + c)^(1/3)))*sin(a)))/((d*x + c)^(2/3)*b^2*d*e^(5/3))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{5/3}}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(5/3),x)
[Out] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(5/3), x)
```

3.247 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 172

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx = -\frac{18\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}}+\frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} -\frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}+\frac{18\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2\sqrt[3]{e(c+dx)}}$$

```
[Out] -18*cos(a+b/(d*x+c)^(1/3))/b^3/d/e^2/(e*(d*x+c))^(1/3)+3*cos(a+b/(d*x+c)^(1
/3))/b/d/e^2/(d*x+c)^(2/3)/(e*(d*x+c))^(1/3)-9*sin(a+b/(d*x+c)^(1/3))/b^2/d
/e^2/(d*x+c)^(1/3)/(e*(d*x+c))^(1/3)+18*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3)
)/b^4/d/e^2/(e*(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3512, 15, 3377, 2717}

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx = \frac{18\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2\sqrt[3]{e(c+dx)}} -\frac{18\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}}-\frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}+\frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (-18*Cos[a + b/(c + d*x)^(1/3)])/(b^3*d*e^2*(e*(c + d*x))^(1/3)) + (3*Cos[a
 + b/(c + d*x)^(1/3)])/(b*d*e^2*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)) - (9*S
in[a + b/(c + d*x)^(1/3)])/(b^2*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3))
+ (18*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(b^4*d*e^2*(e*(c + d*x))^
(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{7/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}\\
&= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^3\sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de^2\sqrt[3]{e(c+dx)}}\\
&= \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{\left(9\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^2\cos(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{bde^2\sqrt[3]{e(c+dx)}}
\end{aligned}
$$

$$= \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} + \frac{\left(18\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{e(c+dx)}}$$

$$= -\frac{18\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} + \frac{\left(18\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}}$$

$$= -\frac{18\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} + \frac{18\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2\sqrt[3]{e(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx = \frac{(c+dx)^{2/3}\left(3b\sqrt[3]{c+dx}\left(-6c-6dx+b^2\sqrt[3]{c+dx}\right)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+9(c\right.}{b^4d(e(c+dx))^{7/3}}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] ((c + d*x)^(2/3)*(3*b*(c + d*x)^(1/3)*(-6*c - 6*d*x + b^2*(c + d*x)^(1/3))*
Cos[a + b/(c + d*x)^(1/3)] + 9*(c + d*x)*(-b^2 + 2*(c + d*x)^(2/3))*Sin[a +
 b/(c + d*x)^(1/3)]))/(b^4*d*(e*(c + d*x))^(7/3))
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{7}{3}}}dx$$

[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)

[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)

Fricas [A] (verification not implemented)

none

Time = 0.78 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.93

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx = \frac{3\left(\left((dx+c)^{\frac{1}{3}}b^3-6\,bdx-6\,bc\right)(dex+ce)^{\frac{2}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)-3\,(dex+c\cdots\right.}{b^4d^3e^3x^2+2\,b^4cd^2e^3x+b^4c\cdots}$$

[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="fricas")

[Out] 3*(((d*x + c)^(1/3)*b^3 - 6*b*d*x - 6*b*c)*(d*e*x + c*e)^(2/3)*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) - 3*(d*e*x + c*e)^(2/3)*((d*x + c)^(2/3)*b^2 - 2*(d*x + c)^(4/3))*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)))/(b^4*d^3*e^3*x^2 + 2*b^4*c*d^2*e^3*x + b^4*c^2*d*e^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx = \text{Timed out}$$

[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(7/3),x)

[Out] Timed out

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.17 (sec) , antiderivative size = 1389, normalized size of antiderivative = 8.08

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="maxima")
```

```
[Out] -3/16*(2*(cos(a)^2 + sin(a)^2)*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1
/3)) - 2*(b^4*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2*sin(a) + b^4*s
in(a)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2)*cos((2*(d*x + c)^(1/3
)*a + b)/(d*x + c)^(1/3)) + 2*(b^4*cos(a)*cos(((d*x + c)^(1/3)*a + b)/(d*x
+ c)^(1/3))^2 + b^4*cos(a)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2)*
sin((2*(d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + ((((-I*gamma(5, I*b*conjug
ate((d*x + c)^(-1/3))) + I*gamma(5, -I*b*conjugate((d*x + c)^(-1/3))) - I*g
amma(5, I*b/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x + c)^(1/3)))*cos(a)^3 -
 (gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, -I*b*conjugate((d*x
+ c)^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamma(5, -I*b/(d*x + c)^(1/
3)))*cos(a)^2*sin(a) + (-I*gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + I*ga
mma(5, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(5, I*b/(d*x + c)^(1/3))
+ I*gamma(5, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2 - (gamma(5, I*b*conjuga
te((d*x + c)^(-1/3))) + gamma(5, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(
5, I*b/(d*x + c)^(1/3)) + gamma(5, -I*b/(d*x + c)^(1/3)))*sin(a)^3)*d*x + (
(-I*gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(5, -I*b*conjugate((
d*x + c)^(-1/3))) - I*gamma(5, I*b/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x
+ c)^(1/3)))*cos(a)^3 - (gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(
5, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamm
a(5, -I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) + (-I*gamma(5, I*b*conjugate((d
*x + c)^(-1/3))) + I*gamma(5, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(5
, I*b/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2
- (gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, -I*b*conjugate((d*x
 + c)^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamma(5, -I*b/(d*x + c)^(1
/3)))*sin(a)^3)*c)*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2 + (((-I*g
amma(5, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(5, -I*b*conjugate((d*x +
 c)^(-1/3))) - I*gamma(5, I*b/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x + c)^
(1/3)))*cos(a)^3 - (gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, -I
*b*conjugate((d*x + c)^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamma(5,
-I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) + (-I*gamma(5, I*b*conjugate((d*x +
c)^(-1/3))) + I*gamma(5, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(5, I*b
/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2 - (ga
mma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, -I*b*conjugate((d*x + c)
^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamma(5, -I*b/(d*x + c)^(1/3)))
```

```
*sin(a)^3)*d*x + ((-I*gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(5
, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(5, I*b/(d*x + c)^(1/3)) + I*g
amma(5, -I*b/(d*x + c)^(1/3)))*cos(a)^3 - (gamma(5, I*b*conjugate((d*x + c)
^(-1/3))) + gamma(5, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, I*b/(d*x
+ c)^(1/3)) + gamma(5, -I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) + (-I*gamma(5
, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(5, -I*b*conjugate((d*x + c)^(-
1/3))) - I*gamma(5, I*b/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x + c)^(1/3))
)*cos(a)*sin(a)^2 - (gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, -
I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamma(5,
 -I*b/(d*x + c)^(1/3)))*sin(a)^3)*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^
(1/3))^2)*(d*x + c)^(1/3))/((((cos(a)^2 + sin(a)^2)*b^4*d^2*e^2*x + (cos(a)
^2 + sin(a)^2)*b^4*c*d*e^2)*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2
+ ((cos(a)^2 + sin(a)^2)*b^4*d^2*e^2*x + (cos(a)^2 + sin(a)^2)*b^4*c*d*e^2)
*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2)*(d*x + c)^(1/3)*e^(1/3))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{7/3}}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(7/3),x)
[Out] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(7/3), x)
```

3.248 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 217

$$
\begin{aligned}
\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx = &-\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}} \\
&+\frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}}+\frac{72(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^5de^2(e(c+dx))^{2/3}} \\
&-\frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}}+\frac{72\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2(e(c+dx))^{2/3}}
\end{aligned}
$$

```
[Out] -36*cos(a+b/(d*x+c)^(1/3))/b^3/d/e^2/(e*(d*x+c))^(2/3)+3*cos(a+b/(d*x+c)^(1
/3))/b/d/e^2/(d*x+c)^(2/3)/(e*(d*x+c))^(2/3)+72*(d*x+c)^(2/3)*cos(a+b/(d*x+
c)^(1/3))/b^5/d/e^2/(e*(d*x+c))^(2/3)-12*sin(a+b/(d*x+c)^(1/3))/b^2/d/e^2/(
d*x+c)^(1/3)/(e*(d*x+c))^(2/3)+72*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3))/b^4/
d/e^2/(e*(d*x+c))^(2/3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3512, 15, 3377, 2718}

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx = \frac{72(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^5de^2(e(c+dx))^{2/3}} + \frac{72\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2(e(c+dx))^{2/3}} - \frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}} - \frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(8/3),x]
```

```
[Out] (-36*Cos[a + b/(c + d*x)^(1/3)])/(b^3*d*e^2*(e*(c + d*x))^(2/3)) + (3*Cos[a
 + b/(c + d*x)^(1/3)])/(b*d*e^2*(c + d*x)^(2/3)*(e*(c + d*x))^(2/3)) + (72*
(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(b^5*d*e^2*(e*(c + d*x))^(2/3))
 - (12*Sin[a + b/(c + d*x)^(1/3)])/(b^2*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))
^(2/3)) + (72*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(b^4*d*e^2*(e*(c
+ d*x))^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
```

```
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{8/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int x^4\sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} - \frac{\left(12(c+dx)^{2/3}\right)\text{Subst}\left(\int x^3\cos(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{bde^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} - \frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} + \frac{\left(36(c+dx)^{2/3}\right)\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{b^2de^2(e(c+dx))^{2/3}}$$

$$= -\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} - \frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} + \frac{\left(72(c+dx)^{2/3}\right)\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}}$$

$$= -\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} - \frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} + \frac{72\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2(e(c+dx))^{2/3}} - \frac{\left(72(c+dx)^{2/3}\right)\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{b^4de^2(e(c+dx))^{2/3}}$$

$$= -\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} + \frac{72(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^5de^2(e(c+dx))^{2/3}} - \frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} + \frac{72\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2(e(c+dx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.52

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx = \frac{(c+dx)^{4/3}\left(3(b^4-12b^2(c+dx)^{2/3}+24(c+dx)^{4/3})\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+12b\right.}{b^5d(e(c+dx))^{8/3}}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(8/3),x]
```

```
[Out] ((c + d*x)^(4/3)*(3*(b^4 - 12*b^2*(c + d*x)^(2/3) + 24*(c + d*x)^(4/3))*Cos
[a + b/(c + d*x)^(1/3)] + 12*b*(6*c + 6*d*x - b^2*(c + d*x)^(1/3))*Sin[a +
b/(c + d*x)^(1/3)]))/(b^5*d*(e*(c + d*x))^(8/3))
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}dx$$

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.78 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.83

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx = \frac{3\left(\left((dx+c)^{\frac{1}{3}}b^4-12\,b^2dx-12\,b^2c+24\,(dx+c)^{\frac{5}{3}}\right)(dex+ce)^{\frac{1}{3}}\cos\left(\frac{adx+ac+(dx}{dx+c}\right.\right.}{b^5d^3e^3x^2+2}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x, algorithm="fricas")
```

```
[Out] 3*(((d*x + c)^(1/3)*b^4 - 12*b^2*d*x - 12*b^2*c + 24*(d*x + c)^(5/3))*(d*e*
x + c*e)^(1/3)*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) - 4*((d*x +
 c)^(2/3)*b^3 - 6*(b*d*x + b*c)*(d*x + c)^(1/3))*(d*e*x + c*e)^(1/3)*sin((a
*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)))/(b^5*d^3*e^3*x^2 + 2*b^5*c*d^2*
e^3*x + b^5*c^2*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a + \frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(8/3),x)
```

```
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.67 (sec) , antiderivative size = 1943, normalized size of antiderivative = 8.95

$$\int \frac{\sin\left(a + \frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x, algorithm="maxima")
```

```
[Out] -3/20*(2*((cos(a)^2 + sin(a)^2)*b^5*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(
1/3)) - (b^5*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2*sin(a) + b^5*si
n(a)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2)*cos((2*(d*x + c)^(1/3)
*a + b)/(d*x + c)^(1/3)) + (b^5*cos(a)*cos(((d*x + c)^(1/3)*a + b)/(d*x + c
)^(1/3))^2 + b^5*cos(a)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2)*sin
((2*(d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*(d*x + c)^(1/3)*e^(1/3) - ((((
gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, -I*b*conjugate((d*x +
c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(1/3)
))*cos(a)^3 + (-I*gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(6, -I
*b*conjugate((d*x + c)^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) + I*gamma
(6, -I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) + (gamma(6, I*b*conjugate((d*x +
 c)^(-1/3))) + gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, I*b/(d
*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2 + (-I*gamm
a(6, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x + c)
^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c)^(1/
3)))*sin(a)^3)*d^2*x^2 + 2*((gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + ga
```

```
mma(6, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)) +
gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^3 + (-I*gamma(6, I*b*conjugate((d*x
+ c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(6, I
*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) + (
gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, -I*b*conjugate((d*x +
c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(1/3)
))*cos(a)*sin(a)^2 + (-I*gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + I*gamm
a(6, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) +
I*gamma(6, -I*b/(d*x + c)^(1/3)))*sin(a)^3)*c*d*x + ((gamma(6, I*b*conjugat
e((d*x + c)^(-1/3))) + gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(6
, I*b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^3 + (-I*gam
ma(6, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x + c
)^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c)^(1
/3)))*cos(a)^2*sin(a) + (gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + gamma(
6, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)) + gamm
a(6, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2 + (-I*gamma(6, I*b*conjugate((d
*x + c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(6
, I*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c)^(1/3)))*sin(a)^3)*c^2)*c
os(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2 + (((gamma(6, I*b*conjugate((
d*x + c)^(-1/3))) + gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, I
*b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^3 + (-I*gamma(
6, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x + c)^(
-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c)^(1/3)
))*cos(a)^2*sin(a) + (gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + gamma(6,
-I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)) + gamma(6
, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2 + (-I*gamma(6, I*b*conjugate((d*x
+ c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(6, I
*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c)^(1/3)))*sin(a)^3)*d^2*x^2 +
 2*((gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, -I*b*conjugate((d
*x + c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^
(1/3)))*cos(a)^3 + (-I*gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(
6, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) + I*
gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) + (gamma(6, I*b*conjugate((
d*x + c)^(-1/3))) + gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, I
*b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2 + (-I
*gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x
 + c)^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c
)^(1/3)))*sin(a)^3)*c*d*x + ((gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + g
amma(6, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)) +
 gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^3 + (-I*gamma(6, I*b*conjugate((d*x
 + c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(6,
I*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) +
(gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, -I*b*conjugate((d*x +
 c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(1/3
)))*cos(a)*sin(a)^2 + (-I*gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + I*gam
```

```
ma(6, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) +
 I*gamma(6, -I*b/(d*x + c)^(1/3)))*sin(a)^3)*c^2)*sin(((d*x + c)^(1/3)*a +
b)/(d*x + c)^(1/3))^2)*e^(1/3))/(((cos(a)^2 + sin(a)^2)*b^5*d^3*e^3*x^2 + 2
*(cos(a)^2 + sin(a)^2)*b^5*c*d^2*e^3*x + (cos(a)^2 + sin(a)^2)*b^5*c^2*d*e^
3)*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2 + ((cos(a)^2 + sin(a)^2)*
b^5*d^3*e^3*x^2 + 2*(cos(a)^2 + sin(a)^2)*b^5*c*d^2*e^3*x + (cos(a)^2 + sin
(a)^2)*b^5*c^2*d*e^3)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{8/3}}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(8/3),x)
[Out] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(8/3), x)
```

3.249 $\int (ce+dex)^{4/3} \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right) dx$

Optimal result

Integrand size = 27, antiderivative size = 299

$$\begin{aligned}
\int (ce+dex)^{4/3} \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = & -\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} \\
& +\frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} \\
& -\frac{8b^{7/2}e\sqrt{2\pi}\sqrt[3]{e(c+dx)}\cos(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}} \\
& -\frac{8b^{7/2}e\sqrt{2\pi}\sqrt[3]{e(c+dx)}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{35d\sqrt[3]{c+dx}} \\
& -\frac{4b^2e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} \\
& +\frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{7d}
\end{aligned}$$

```
[Out] -8/35*b^3*e*(e*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(2/3))/d+6/35*b*e*(d*x+c)^(4/
3)*(e*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(2/3))/d-4/35*b^2*e*(d*x+c)^(2/3)*(e*(
d*x+c))^(1/3)*sin(a+b/(d*x+c)^(2/3))/d+3/7*e*(d*x+c)^2*(e*(d*x+c))^(1/3)*si
n(a+b/(d*x+c)^(2/3))/d-8/35*b^(7/2)*e*(e*(d*x+c))^(1/3)*cos(a)*FresnelS(b^(
1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*Pi^(1/2)/d/(d*x+c)^(1/3)-8/35*
b^(7/2)*e*(e*(d*x+c))^(1/3)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3)
)*sin(a)*2^(1/2)*Pi^(1/2)/d/(d*x+c)^(1/3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3516, 3498, 3496, 3490, 3468, 3469, 3434, 3433, 3432}

$$\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx = -\frac{8\sqrt{2\pi}b^{7/2}e\sin(a)\sqrt[3]{e(c+dx)}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}} - \frac{8\sqrt{2\pi}b^{7/2}e\cos(a)\sqrt[3]{e(c+dx)}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}} - \frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} - \frac{4b^2e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{7d} + \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d}$$

```
[In] Int[(c*e + d*e*x)^(4/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (-8*b^3*e*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/(35*d) + (6*b*e*(
c + d*x)^(4/3)*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/(35*d) - (8*
b^(7/2)*e*Sqrt[2*Pi]*(e*(c + d*x))^(1/3)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)])/(35*d*(c + d*x)^(1/3)) - (8*b^(7/2)*e*Sqrt[2*Pi]*(e*(c
 + d*x))^(1/3)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(35*d
*(c + d*x)^(1/3)) - (4*b^2*e*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)*Sin[a + b/
(c + d*x)^(2/3)])/(35*d) + (3*e*(c + d*x)^2*(e*(c + d*x))^(1/3)*Sin[a + b/(
c + d*x)^(2/3)])/(7*d)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (ex)^{4/3}\sin\left(a+\frac{b}{x^{2/3}}\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{\left(e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^{4/3}\sin\left(a+\frac{b}{x^{2/3}}\right)\,dx, x, c+dx\right)}{d\sqrt[3]{c+dx}}$$

$$= \frac{\left(3e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^6\sin\left(a+\frac{b}{x^2}\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d\sqrt[3]{c+dx}}$$

$$= -\frac{\left(3e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^8}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d\sqrt[3]{c+dx}}$$

$$= \frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{7d} - \frac{\left(6be\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\cos(a+bx^2)}{x^6}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{7d\sqrt[3]{c+dx}}$$

$$= \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{7d} + \frac{\left(12b^2e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}}$$

$$= \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} - \frac{4b^2e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{7d} + \frac{\left(8b^3e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\cos(a+bx^2)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}}$$

$$= -\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} - \frac{4b^2e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{7d} - \frac{\left(16b^4e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int\sin\left(a+bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}}$$

$$= -\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} - \frac{4b^2e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{7d} - \frac{\left(16b^4e\sqrt[3]{e(c+dx)}\cos(a)\right)\text{Subst}\left(\int\sin\left(bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}} - \frac{\left(16b^4e\sqrt[3]{e(c+dx)}\sin(a)\right)\text{Subst}\left(\int\cos\left(bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}}$$

$$
\begin{aligned}
&= -\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} \\
&+ \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} \\
&- \frac{8b^{7/2}e\sqrt{2\pi}\sqrt[3]{e(c+dx)}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}} \\
&- \frac{8b^{7/2}e\sqrt{2\pi}\sqrt[3]{e(c+dx)}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{35d\sqrt[3]{c+dx}} \\
&- \frac{4b^2e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} \\
&+ \frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{7d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.79

$$
\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx = \frac{(e(c+dx))^{4/3}\left(\frac{\cos\left(\frac{b}{(c+dx)^{2/3}}\right)(-8b^3\cos(a)+6b(c+dx)^{4/3}\cos(a)-4b^2(c+dx)^{2/3}\sin(a)+15(c+dx)^2\sin(a))}{c+dx} - \right.}{}
$$

```
[In] Integrate[(c*e + d*e*x)^(4/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] ((e*(c + d*x))^(4/3)*((Cos[b/(c + d*x)^(2/3)]*(-8*b^3*Cos[a] + 6*b*(c + d*x
)^(4/3)*Cos[a] - 4*b^2*(c + d*x)^(2/3)*Sin[a] + 15*(c + d*x)^2*Sin[a]))/(c
+ d*x) - (8*b^(7/2)*Sqrt[2*Pi]*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d
*x)^(1/3)] + FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a]))/(c + d
*x)^(4/3) + ((-4*b^2*(c + d*x)^(2/3)*Cos[a] + 15*(c + d*x)^2*Cos[a] + 8*b^3
*Sin[a] - 6*b*(c + d*x)^(4/3)*Sin[a])*Sin[b/(c + d*x)^(2/3)])/(c + d*x)))/(
35*d)
```

Maple [F]

$$\int (dex+ce)^{\frac{4}{3}} \sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right) dx$$

```
[In] int((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x)
[Out] int((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

Fricas [F]

$$\int (ce+dex)^{4/3} \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \int (dex+ce)^{\frac{4}{3}} \sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right) dx$$

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
[Out] integral((d*e*x + c*e)^(4/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c
)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{4/3} \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)**(4/3)*sin(a+b/(d*x+c)**(2/3)),x)
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 1120, normalized size of antiderivative = 3.75

$$\int (ce+dex)^{4/3} \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \text{Too large to display}$$

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
[Out] -3/8*((((-I*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-7/2, -I
*b/(d*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (I*gamma(-7/2,
 -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-7/2, I*b/(d*x + c)^(2/3)))*co
s(-7/4*pi + 7/3*arctan2(0, d*x + c)) + (gamma(-7/2, I*b*conjugate((d*x + c)
```

^(-2/3))) + gamma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) - (gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*cos(a) - ((gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7/2, -I*b/(d*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4*pi + 7/3*arctan2(0, d*x + c)) - (-I*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) - (-I*gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-7/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*sin(a))*d^2*e^(4/3)*x^2 + 2*(((-I*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-7/2, -I*b/(d*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (I*gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4*pi + 7/3*arctan2(0, d*x + c)) + (gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) - (gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*cos(a) - ((gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7/2, -I*b/(d*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4*pi + 7/3*arctan2(0, d*x + c)) - (-I*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) - (-I*gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-7/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*sin(a))*c*d*e^(4/3)*x + (((-I*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-7/2, -I*b/(d*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (I*gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4*pi + 7/3*arctan2(0, d*x + c)) + (gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) - (gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*cos(a) - ((gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7/2, -I*b/(d*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4*pi + 7/3*arctan2(0, d*x + c)) - (-I*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) - (-I*gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-7/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*sin(a))*c^2*e^(4/3))*(d*x + c)^(1/3)*(b/(d*x + c)^(2/3))^(7/2)/d

Giac [F(-2)]

Exception generated.

$$\int (ce + dex)^{4/3} \sin\left(a + \frac{b}{(c + dx)^{2/3}}\right) dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,6,1,0,0,0]%%%}+%%%{-2,[0,3,1,1,1,0]%%%}+%%%{1,[0,0,1,2,2
,0]%%%
```

Mupad [F(-1)]

Timed out.

$$\int (ce + dex)^{4/3} \sin\left(a + \frac{b}{(c + dx)^{2/3}}\right) dx = \int \sin\left(a + \frac{b}{(c + d\,x)^{2/3}}\right) (c\,e + d\,e\,x)^{4/3}\, dx$$

```
[In] int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(4/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(4/3), x)
```

3.250 $\int (ce+dex)^{2/3} \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 262

$$\int (ce+dex)^{2/3} \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right) \, dx = \frac{2b\sqrt[3]{c+dx}(e(c+dx))^{2/3} \cos \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} + \frac{4\sqrt{2}b^{5/2}\sqrt{\pi}(e(c+dx))^{2/3} \cos(a) \operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}} - \frac{4\sqrt{2}b^{5/2}\sqrt{\pi}(e(c+dx))^{2/3} \operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right) \sin(a)}{5d(c+dx)^{2/3}} - \frac{4b^2(e(c+dx))^{2/3} \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d\sqrt[3]{c+dx}} + \frac{3(c+dx)(e(c+dx))^{2/3} \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d}$$

```
[Out] 2/5*b*(d*x+c)^(1/3)*(e*(d*x+c))^(2/3)*cos(a+b/(d*x+c)^(2/3))/d-4/5*b^2*(e*(
d*x+c))^(2/3)*sin(a+b/(d*x+c)^(2/3))/d/(d*x+c)^(1/3)+3/5*(d*x+c)*(e*(d*x+c)
)^(2/3)*sin(a+b/(d*x+c)^(2/3))/d+4/5*b^(5/2)*(e*(d*x+c))^(2/3)*cos(a)*Fresn
elC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*Pi^(1/2)/d/(d*x+c)^(2/3
)-4/5*b^(5/2)*(e*(d*x+c))^(2/3)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(
1/3))*sin(a)*2^(1/2)*Pi^(1/2)/d/(d*x+c)^(2/3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3516, 3498, 3496, 3490, 3468, 3469, 3435, 3433, 3432}

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\frac{4\sqrt{2}\sqrt{\pi}b^{5/2}\cos(a)(e(c+dx))^{2/3}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}-\frac{4\sqrt{2}\sqrt{\pi}b^{5/2}\sin(a)(e(c+dx))^{2/3}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}-\frac{4b^2(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d\sqrt[3]{c+dx}}+\frac{3(c+dx)(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d}+\frac{2b\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d}$$

```
[In] Int[(c*e + d*e*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(c + d*x)^(1/3)*(e*(c + d*x))^(2/3)*Cos[a + b/(c + d*x)^(2/3)])/(5*d)
+ (4*Sqrt[2]*b^(5/2)*Sqrt[Pi]*(e*(c + d*x))^(2/3)*Cos[a]*FresnelC[(Sqrt[b]*
Sqrt[2/Pi])/(c + d*x)^(1/3)])/(5*d*(c + d*x)^(2/3)) - (4*Sqrt[2]*b^(5/2)*Sq
rt[Pi]*(e*(c + d*x))^(2/3)*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*S
in[a])/(5*d*(c + d*x)^(2/3)) - (4*b^2*(e*(c + d*x))^(2/3)*Sin[a + b/(c + d*
x)^(2/3)])/(5*d*(c + d*x)^(1/3)) + (3*(c + d*x)*(e*(c + d*x))^(2/3)*Sin[a +
 b/(c + d*x)^(2/3)])/(5*d)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (ex)^{2/3}\sin\left(a+\frac{b}{x^{2/3}}\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{2/3}\text{Subst}\left(\int x^{2/3}\sin\left(a+\frac{b}{x^{2/3}}\right)\,dx,x,c+dx\right)}{d(c+dx)^{2/3}}$$

$$= \frac{\left(3(e(c+dx))^{2/3}\right)\text{Subst}\left(\int x^4\sin\left(a+\frac{b}{x^2}\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d(c+dx)^{2/3}}$$

$$= -\frac{\left(3(e(c+dx))^{2/3}\right)\text{Subst}\left(\int\frac{\sin(a+bx^2)}{x^6}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d(c+dx)^{2/3}}$$

$$= \frac{3(c+dx)(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} - \frac{\left(6b(e(c+dx))^{2/3}\right)\text{Subst}\left(\int\frac{\cos(a+bx^2)}{x^4}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}$$

$$= \frac{2b\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} + \frac{3(c+dx)(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} + \frac{\left(4b^2(e(c+dx))^{2/3}\right)\text{Subst}\left(\int\frac{\sin(a+bx^2)}{x^2}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}$$

$$= \frac{2b\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} - \frac{4b^2(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d\sqrt[3]{c+dx}} + \frac{3(c+dx)(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} + \frac{\left(8b^3(e(c+dx))^{2/3}\right)\text{Subst}\left(\int\cos\left(a+bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}$$

$$= \frac{2b\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} - \frac{4b^2(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d\sqrt[3]{c+dx}} + \frac{3(c+dx)(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} + \frac{\left(8b^3(e(c+dx))^{2/3}\cos(a)\right)\text{Subst}\left(\int\cos\left(bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}} - \frac{\left(8b^3(e(c+dx))^{2/3}\sin(a)\right)\text{Subst}\left(\int\sin\left(bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}$$

$$
\begin{aligned}
&= \frac{2b\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} \\
&+ \frac{4\sqrt{2}b^{5/2}\sqrt{\pi}(e(c+dx))^{2/3}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}} \\
&- \frac{4\sqrt{2}b^{5/2}\sqrt{\pi}(e(c+dx))^{2/3}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{5d(c+dx)^{2/3}} \\
&- \frac{4b^2(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d\sqrt[3]{c+dx}} \\
&+ \frac{3(c+dx)(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.87

$$
\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{(e(c+dx))^{2/3}\left(2bc\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+2bdx\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+4b^{5/2}\sqrt{2\pi}\cos(a)\,\text{Fr}\right.}{}
$$

```
[In] Integrate[(c*e + d*e*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] ((e*(c + d*x))^(2/3)*(2*b*c*Cos[a + b/(c + d*x)^(2/3)] + 2*b*d*x*Cos[a + b/
(c + d*x)^(2/3)] + 4*b^(5/2)*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]
)/(c + d*x)^(1/3)] - 4*b^(5/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c
+ d*x)^(1/3)]*Sin[a] - 4*b^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)] + 3
*c*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)] + 3*d*x*(c + d*x)^(2/3)*Sin[a
 + b/(c + d*x)^(2/3)]))/(5*d*(c + d*x)^(2/3))
```

Maple [F]

$$
\int (dex+ce)^{\frac{2}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)dx
$$

```
[In] int((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

Fricas [F]

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int (dex+ce)^{\frac{2}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)\,dx$$

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(2/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c
)), x)
```

Sympy [F]

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int (e(c+dx))^{\frac{2}{3}}\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)\,dx$$

```
[In] integrate((d*e*x+c*e)**(2/3)*sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(2/3)*sin(a + b/(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.77 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.86

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\text{Too large to display}$$

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/8*((((-I*gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-5/2, -I
*b/(d*x + c)^(2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x + c)) + (I*gamma(-5/2,
 -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-5/2, I*b/(d*x + c)^(2/3)))*co
s(-5/4*pi + 5/3*arctan2(0, d*x + c)) + (gamma(-5/2, I*b*conjugate((d*x + c)
^(-2/3))) + gamma(-5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arctan2(0,
d*x + c)) - (gamma(-5/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-5/2, I*
b/(d*x + c)^(2/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x + c)))*cos(a) - ((gamm
a(-5/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-5/2, -I*b/(d*x + c)^(2/3)
))*cos(5/4*pi + 5/3*arctan2(0, d*x + c)) + (gamma(-5/2, -I*b*conjugate((d*x
 + c)^(-2/3))) + gamma(-5/2, I*b/(d*x + c)^(2/3)))*cos(-5/4*pi + 5/3*arctan
2(0, d*x + c)) - (-I*gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma
(-5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arctan2(0, d*x + c)) - (-I*g
```

```
amma(-5/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-5/2, I*b/(d*x + c)^
(2/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x + c)))*sin(a))*d*e^(2/3)*x + (((-I
*gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-5/2, -I*b/(d*x + c
)^(2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x + c)) + (I*gamma(-5/2, -I*b*conju
gate((d*x + c)^(-2/3))) - I*gamma(-5/2, I*b/(d*x + c)^(2/3)))*cos(-5/4*pi +
 5/3*arctan2(0, d*x + c)) + (gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))) +
 gamma(-5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arctan2(0, d*x + c)) -
 (gamma(-5/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-5/2, I*b/(d*x + c)
^(2/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x + c)))*cos(a) - ((gamma(-5/2, I*b
*conjugate((d*x + c)^(-2/3))) + gamma(-5/2, -I*b/(d*x + c)^(2/3)))*cos(5/4*
pi + 5/3*arctan2(0, d*x + c)) + (gamma(-5/2, -I*b*conjugate((d*x + c)^(-2/3
))) + gamma(-5/2, I*b/(d*x + c)^(2/3)))*cos(-5/4*pi + 5/3*arctan2(0, d*x +
c)) - (-I*gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-5/2, -I*b
/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arctan2(0, d*x + c)) - (-I*gamma(-5/2,
-I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-5/2, I*b/(d*x + c)^(2/3)))*sin
(-5/4*pi + 5/3*arctan2(0, d*x + c)))*sin(a))*c*e^(2/3))*(d*x + c)^(2/3)*(b/
(d*x + c)^(2/3))^(5/2)/d
```

Giac [F]

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int (dex+ce)^{\frac{2}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)\,dx$$

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(2/3)*sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)\,(c\,e+d\,e\,x)^{2/3}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(2/3),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(2/3), x)
```

3.251 $\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 168

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\frac{3b\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d}+\frac{3b^2\sqrt[3]{e(c+dx)}\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)\sin(a)}{4d\sqrt[3]{c+dx}}+\frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d}+\frac{3b^2\sqrt[3]{e(c+dx)}\cos(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}}$$

```
[Out] 3/4*b*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(2/3))/d+3/4*b^2*(e*(
d*x+c))^(1/3)*cos(a)*Si(b/(d*x+c)^(2/3))/d/(d*x+c)^(1/3)+3/4*b^2*(e*(d*x+c)
)^(1/3)*Ci(b/(d*x+c)^(2/3))*sin(a)/d/(d*x+c)^(1/3)+3/4*(d*x+c)*(e*(d*x+c))^
(1/3)*sin(a+b/(d*x+c)^(2/3))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3516, 3462, 3460, 3378, 3384, 3380, 3383}

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{3b^2\sin(a)\sqrt[3]{e(c+dx)}\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}} + \frac{3b^2\cos(a)\sqrt[3]{e(c+dx)}\,\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}} + \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} + \frac{3b\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d}$$

```
[In] Int[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (3*b*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/(4*d)
+ (3*b^2*(e*(c + d*x))^(1/3)*CosIntegral[b/(c + d*x)^(2/3)]*Sin[a])/(4*d*(c
 + d*x)^(1/3)) + (3*(c + d*x)*(e*(c + d*x))^(1/3)*Sin[a + b/(c + d*x)^(2/3)
])/(4*d) + (3*b^2*(e*(c + d*x))^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)^(2/3)]
)/(4*d*(c + d*x)^(1/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt[3]{ex}\sin\left(a+\frac{b}{x^{2/3}}\right)\,dx, x, c+dx\right)}{d}\\
&= \frac{\sqrt[3]{e(c+dx)}\,\text{Subst}\left(\int \sqrt[3]{x}\sin\left(a+\frac{b}{x^{2/3}}\right)\,dx, x, c+dx\right)}{d\sqrt[3]{c+dx}}\\
&= -\frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{c+dx}}\\
&= \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} - \frac{\left(3b\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}}\\
&= \frac{3b\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d}\\
&\quad+\frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d}\\
&\quad+\frac{\left(3b^2\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}}
\end{aligned}
$$

$$= \frac{3b\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} + \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} + \frac{\left(3b^2\sqrt[3]{e(c+dx)}\cos(a)\right)\mathrm{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}} + \frac{\left(3b^2\sqrt[3]{e(c+dx)}\sin(a)\right)\mathrm{Subst}\left(\int\frac{\cos(bx)}{x}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}}$$

$$= \frac{3b\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} + \frac{3b^2\sqrt[3]{e(c+dx)}\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)\sin(a)}{4d\sqrt[3]{c+dx}} + \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} + \frac{3b^2\sqrt[3]{e(c+dx)}\cos(a)\mathrm{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.67

$$\int\sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx = \frac{3\sqrt[3]{e(c+dx)}\left(b(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+b^2\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)\sin(a)+(c-$$

$$4d\sqrt[3]{c+dx}$$

```
[In] Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (3*(e*(c + d*x))^(1/3)*(b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(2/3)] + b^2*
CosIntegral[b/(c + d*x)^(2/3)]*Sin[a] + (c + d*x)^(4/3)*Sin[a + b/(c + d*x)
^(2/3)] + b^2*Cos[a]*SinIntegral[b/(c + d*x)^(2/3)]))/(4*d*(c + d*x)^(1/3))
```

Maple [F]

$$\int(dex+ce)^{\frac{1}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)dx$$

```
[In] int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

Fricas [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int (dex+ce)^{\frac{1}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)\,dx$$

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
[Out] integral((d*e*x + c*e)^(1/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c
)), x)
```

Sympy [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sqrt[3]{e\,(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)\,dx$$

```
[In] integrate((d*e*x+c*e)**(1/3)*sin(a+b/(d*x+c)**(2/3)),x)
[Out] Integral((e*(c + d*x))**(1/3)*sin(a + b/(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.77

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\frac{3\left(\left(-i\,\Gamma\left(-2,i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\,\Gamma\left(-2,-i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)-i\,\Gamma\left(-2,\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)+i\,\Gamma\left(-2,-\frac{i}{(dx\cdots}\right.\right.\right.}{}$$

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
[Out] 3/8*((-I*gamma(-2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-2, -I*b*conj
ugate((d*x + c)^(-2/3))) - I*gamma(-2, I*b/(d*x + c)^(2/3)) + I*gamma(-2, -
I*b/(d*x + c)^(2/3)))*cos(a) - (gamma(-2, I*b*conjugate((d*x + c)^(-2/3)))
+ gamma(-2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-2, I*b/(d*x + c)^(2/
3)) + gamma(-2, -I*b/(d*x + c)^(2/3)))*sin(a))*b^2*e^(1/3)/d
```

Giac [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int (dex+ce)^{\frac{1}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)\,dx$$

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(1/3)*sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)\,(c\,e+d\,e\,x)^{1/3}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(1/3),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(1/3), x)
```

3.252 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 122

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3b\sqrt[3]{c+dx}\cos(a)\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b\sqrt[3]{c+dx}\sin(a)\operatorname{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

```
[Out] -3/2*b*(d*x+c)^(1/3)*Ci(b/(d*x+c)^(2/3))*cos(a)/d/(e*(d*x+c))^(1/3)+3/2*b*(
d*x+c)^(1/3)*Si(b/(d*x+c)^(2/3))*sin(a)/d/(e*(d*x+c))^(1/3)+3/2*(d*x+c)*sin
(a+b/(d*x+c)^(2/3))/d/(e*(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3516, 3462, 3460, 3378, 3384, 3380, 3383}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3b\cos(a)\sqrt[3]{c+dx}\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b\sin(a)\sqrt[3]{c+dx}\operatorname{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b/(c + d*x)^(2/3)])/(2*d*(e*(c + d
*x))^(1/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(2/3)])/(2*d*(e*(c + d*x))^(
1/3)) + (3*b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b/(c + d*x)^(2/3)])/(2*d*(e
*(c + d*x))^(1/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{\sqrt[3]{ex}}\,dx, x, c+dx\right)}{d}\\
&= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{\sqrt[3]{x}}\,dx, x, c+dx\right)}{d\sqrt[3]{e(c+dx)}}\\
&= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}\\
&= \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} - \frac{\left(3b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}\\
&= \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} - \frac{\left(3b\sqrt[3]{c+dx}\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}\\
&\quad + \frac{\left(3b\sqrt[3]{c+dx}\sin(a)\right)\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}\\
&= -\frac{3b\sqrt[3]{c+dx}\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}\\
&\quad + \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b\sqrt[3]{c+dx}\sin(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

$$
\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx = \frac{3\left(-b\sqrt[3]{c+dx}\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right) + (c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) + b\sqrt[3]{c\right.}{2d\sqrt[3]{e(c+dx)}}
$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
[Out] (3*(-(b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b/(c + d*x)^(2/3)]) + (c + d*x)*
Sin[a + b/(c + d*x)^(2/3)] + b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b/(c + d*
x)^(2/3)]))/(2*d*(e*(c + d*x))^(1/3))
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}dx$$

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(d*e*x + c*e)^(1/
3), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{\sqrt[3]{e\,(c+dx)}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(1/3),x)
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(1/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx =$$

$$-\frac{3\left(\left(\Gamma\left(-1, i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(-1, -i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(-1, \frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(-1, -\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos(a)+\left(-i\,\Gamma\right.\right.}{8\,de^{\frac{1}{3}}}$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/8*((gamma(-1, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-1, -I*b*conjugat
e((d*x + c)^(-2/3))) + gamma(-1, I*b/(d*x + c)^(2/3)) + gamma(-1, -I*b/(d*x
 + c)^(2/3)))*cos(a) + (-I*gamma(-1, I*b*conjugate((d*x + c)^(-2/3))) + I*g
amma(-1, -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-1, I*b/(d*x + c)^(2/3
)) + I*gamma(-1, -I*b/(d*x + c)^(2/3)))*sin(a))*b/(d*e^(1/3))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(c\,e+d\,e\,x)^{1/3}}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(1/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(1/3), x)
```

3.253 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 164

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3\sqrt{b}\sqrt{2\pi}(c+dx)^{2/3}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3\sqrt{b}\sqrt{2\pi}(c+dx)^{2/3}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d(e(c+dx))^{2/3}} + \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}}$$

```
[Out] 3*(d*x+c)*sin(a+b/(d*x+c)^(2/3))/d/(e*(d*x+c))^(2/3)-3*(d*x+c)^(2/3)*cos(a)
*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*b^(1/2)*2^(1/2)*Pi^(1/2)/
d/(e*(d*x+c))^(2/3)+3*(d*x+c)^(2/3)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+
c)^(1/3))*sin(a)*b^(1/2)*2^(1/2)*Pi^(1/2)/d/(e*(d*x+c))^(2/3)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {3516, 3498, 3496, 3440, 3468, 3435, 3433, 3432}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3\sqrt{2\pi}\sqrt{b}\cos(a)(c+dx)^{2/3}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3\sqrt{2\pi}\sqrt{b}\sin(a)(c+dx)^{2/3}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (-3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(2/3)*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])
/(c + d*x)^(1/3)])/(d*(e*(c + d*x))^(2/3)) + (3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x
)^(2/3)*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(d*(e*(c + d
*x))^(2/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(2/3)])/(d*(e*(c + d*x))^(2/
3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
```

```
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{2/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)^{2/3}\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{2/3}}\,dx, x, c+dx\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \sin\left(a+\frac{b}{x^2}\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}}$$

$$= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}} - \frac{\left(6b(c+dx)^{2/3}\right)\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}} - \frac{\left(6b(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{\left(6b(c+dx)^{2/3}\sin(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

$$= -\frac{3\sqrt{b}\sqrt{2\pi}(c+dx)^{2/3}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3\sqrt{b}\sqrt{2\pi}(c+dx)^{2/3}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d(e(c+dx))^{2/3}} + \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx = \frac{3\left(-\sqrt{b}\sqrt{2\pi}(c+dx)^{2/3}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+\sqrt{b}\sqrt{2\pi}(c+dx)^{2/3}\,\text{Fresnel}\right.}{d(e(c+dx))^{2/3}}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
[Out] (3*(-(Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(2/3)*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)]) + Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(2/3)*FresnelS[(Sqrt[b]
*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] + (c + d*x)*Sin[a + b/(c + d*x)^(2/3)]
))/(d*(e*(c + d*x))^(2/3))
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}dx$$

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(d*e*x + c*e)^(2/
3), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{\left(e\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(2/3),x)

[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(2/3), x)

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.34

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3\,(dx+c)^{\frac{1}{3}}\left(\left(\left(-i\,\Gamma\left(-\frac{1}{2},i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\,\Gamma\left(-\frac{1}{2},-\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(\frac{1}{4}\,\pi+\frac{1}{3}\,\arctan\left(0,dx+c\right)\right)+\left(i\,\Gamma\left(-\right.\right.\right.\right.}{}$$

[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")

[Out] -3/8*(d*x + c)^(1/3)*(((-I*gamma(-1/2, I*b*conjugate((d*x + c)^(-2/3))) + I *gamma(-1/2, -I*b/(d*x + c)^(2/3)))*cos(1/4*pi + 1/3*arctan2(0, d*x + c)) + (I*gamma(-1/2, -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-1/2, I*b/(d*x + c)^(2/3)))*cos(-1/4*pi + 1/3*arctan2(0, d*x + c)) + (gamma(-1/2, I*b*conj ugate((d*x + c)^(-2/3))) + gamma(-1/2, -I*b/(d*x + c)^(2/3)))*sin(1/4*pi + 1/3*arctan2(0, d*x + c)) - (gamma(-1/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-1/2, I*b/(d*x + c)^(2/3)))*sin(-1/4*pi + 1/3*arctan2(0, d*x + c)))* cos(a) - ((gamma(-1/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-1/2, -I*b/ (d*x + c)^(2/3)))*cos(1/4*pi + 1/3*arctan2(0, d*x + c)) + (gamma(-1/2, -I*b *conjugate((d*x + c)^(-2/3))) + gamma(-1/2, I*b/(d*x + c)^(2/3)))*cos(-1/4* pi + 1/3*arctan2(0, d*x + c)) - (-I*gamma(-1/2, I*b*conjugate((d*x + c)^(-2 /3))) + I*gamma(-1/2, -I*b/(d*x + c)^(2/3)))*sin(1/4*pi + 1/3*arctan2(0, d* x + c)) - (-I*gamma(-1/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-1/2, I*b/(d*x + c)^(2/3)))*sin(-1/4*pi + 1/3*arctan2(0, d*x + c)))*sin(a))*sqrt (b/(d*x + c)^(2/3))/(d*e^(2/3))

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(c\,e+d\,e\,x)^{2/3}}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(2/3),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(2/3), x)
```

3.254 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 141

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = -\frac{3\sqrt{\pi}\sqrt[3]{c+dx}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}} - \frac{3\sqrt{\pi}\sqrt[3]{c+dx}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}}$$

```
[Out] -3/2*(d*x+c)^(1/3)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*
Pi^(1/2)/d/e/(e*(d*x+c))^(1/3)*2^(1/2)/b^(1/2)-3/2*(d*x+c)^(1/3)*FresnelC(b
^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)*Pi^(1/2)/d/e/(e*(d*x+c))^(1/3
)*2^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3516, 3498, 3464, 3434, 3433, 3432}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = -\frac{3\sqrt{\pi}\sin(a)\sqrt[3]{c+dx}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}} - \frac{3\sqrt{\pi}\cos(a)\sqrt[3]{c+dx}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (-3*Sqrt[Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)
^(1/3)])/(Sqrt[2]*Sqrt[b]*d*e*(e*(c + d*x))^(1/3)) - (3*Sqrt[Pi]*(c + d*x)^
(1/3)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(Sqrt[2]*Sqrt[
b]*d*e*(e*(c + d*x))^(1/3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3464

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)], x_Symbol] :> Dist[2/n, Subst[I
nt[Sin[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
n/2 - 1]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{4/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\sqrt[3]{c+dx}\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{4/3}}\,dx, x, c+dx\right)}{de\sqrt[3]{e(c+dx)}}$$

$$= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \sin\left(a+bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$= -\frac{\left(3\sqrt[3]{c+dx}\cos(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{\left(3\sqrt[3]{c+dx}\sin(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$= -\frac{3\sqrt{\pi}\sqrt[3]{c+dx}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}} - \frac{3\sqrt{\pi}\sqrt[3]{c+dx}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = -\frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{4/3}\left(\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)\right)}{\sqrt{b}d(e(c+dx))^{4/3}}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (-3*Sqrt[Pi/2]*(c + d*x)^(4/3)*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d
*x)^(1/3)] + FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a]))/(Sqrt[
b]*d*(e*(c + d*x))^(4/3))
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}dx$$

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(2/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c
))/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{(e\,(c+dx))^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(4/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(4/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.45

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/8*(((-I*sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(-2/3)))) - 1) + I*sqr
t(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1/3*arctan2(0, d*
x + c)) + (I*sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(-2/3)))) - 1) - I
*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/3*arctan2(0
, d*x + c)) - (sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(-2/3)))) - 1) +
sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/3*arctan2(0,
 d*x + c)) + (sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(-2/3)))) - 1) +
sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1))*sin(-1/4*pi + 1/3*arctan2(0,
 d*x + c)))*cos(a) - ((sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(-2/3))))
 - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1/3*ar
ctan2(0, d*x + c)) + (sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(-2/3))))
 - 1) + sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/3*ar
ctan2(0, d*x + c)) - (I*sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(-2/3)))
) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/3
*arctan2(0, d*x + c)) - (I*sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(-2/
3)))) - 1) - I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1))*sin(-1/4*pi +
 1/3*arctan2(0, d*x + c)))*sin(a))/((d*x + c)^(1/3)*d*e^(4/3)*sqrt(b/(d*x +
 c)^(2/3)))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(c\,e+d\,e\,x)^{4/3}}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(4/3),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(4/3), x)
```

3.255 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde(e(c+dx))^{2/3}}$$

```
[Out] 3/2*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(2/3))/b/d/e/(e*(d*x+c))^(2/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3516, 3462, 3460, 2718}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde(e(c+dx))^{2/3}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (3*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e*(e*(c + d*x))^(2/3)
)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{5/3}}\,dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)^{2/3}\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{5/3}}\,dx, x, c+dx\right)}{de(e(c+dx))^{2/3}} \\
&= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2de(e(c+dx))^{2/3}} \\
&= \frac{3(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde(e(c+dx))^{2/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3(c+dx)^{5/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bd(e(c+dx))^{5/3}}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
[Out] (3*(c + d*x)^(5/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(5/3))
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{5}{3}}}dx$$

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3\,(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{2\,(bd^2e^2x+bcde^2)}$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
[Out] 3/2*(d*e*x + c*e)^(1/3)*(d*x + c)^(2/3)*cos((a*d*x + a*c + (d*x + c)^(1/3)*
b)/(d*x + c))/(b*d^2*e^2*x + b*c*d*e^2)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{(e\,(c+dx))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(5/3),x)
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(5/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx = \frac{3\cos\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)}{2\,bde^{\frac{5}{3}}}$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
[Out] 3/2*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3))/(b*d*e^(5/3))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(c\,e+d\,e\,x)^{5/3}}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(5/3),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(5/3), x)
```

3.256 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx = \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} - \frac{3\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2b^2de^2\sqrt[3]{e(c+dx)}}$$

```
[Out] 3/2*cos(a+b/(d*x+c)^(2/3))/b/d/e^2/(d*x+c)^(1/3)/(e*(d*x+c))^(1/3)-3/2*(d*x
+c)^(1/3)*sin(a+b/(d*x+c)^(2/3))/b^2/d/e^2/(e*(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3516, 3462, 3460, 3377, 2717}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx = \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} - \frac{3\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2b^2de^2\sqrt[3]{e(c+dx)}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (3*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))^(1/
3)) - (3*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)])/(2*b^2*d*e^2*(e*(c + d
*x))^(1/3))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{7/3}}\,dx, x, c+dx\right)}{d}\\
&= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{7/3}}\,dx, x, c+dx\right)}{de^2\sqrt[3]{e(c+dx)}}\\
&= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2de^2\sqrt[3]{e(c+dx)}}\\
&= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} - \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{e(c+dx)}}\\
&= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} - \frac{3\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2b^2de^2\sqrt[3]{e(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.76

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx = -\frac{3(c+dx)^{5/3}\left(-b\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{2b^2d(e(c+dx))^{7/3}}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(5/3)*(-(b*Cos[a + b/(c + d*x)^(2/3)]) + (c + d*x)^(2/3)*Sin[
a + b/(c + d*x)^(2/3)]))/(2*b^2*d*(e*(c + d*x))^(7/3))
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{7}{3}}}dx$$

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.72 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.40

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx = \frac{3\left((dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)-(dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}\sin\left(\frac{adx\cdots}{}\right.\right.}{2\left(b^2d^3e^3x^2+2\,b^2cd^2e^3x+b^2c^2de^3\right)}$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x, algorithm="fricas")
```

```
[Out] 3/2*((d*e*x + c*e)^(2/3)*(d*x + c)^(2/3)*b*cos((a*d*x + a*c + (d*x + c)^(1/
3)*b)/(d*x + c)) - (d*e*x + c*e)^(2/3)*(d*x + c)^(4/3)*sin((a*d*x + a*c + (
d*x + c)^(1/3)*b)/(d*x + c)))/(b^2*d^3*e^3*x^2 + 2*b^2*c*d^2*e^3*x + b^2*c^
2*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx=\text{Timed out}$$

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(7/3),x)
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx=\frac{3\left(\left(-i\,\Gamma\left(2,i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\,\Gamma\left(2,-i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)-i\,\Gamma\left(2,\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)+i\,\Gamma\left(2,-\frac{}{(dx}\right.\right.\right.}{}$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x, algorithm="maxima")
[Out] 3/8*((-I*gamma(2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(2, -I*b*conjug
ate((d*x + c)^(-2/3))) - I*gamma(2, I*b/(d*x + c)^(2/3)) + I*gamma(2, -I*b/
(d*x + c)^(2/3)))*cos(a) - (gamma(2, I*b*conjugate((d*x + c)^(-2/3))) + gam
ma(2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(2, I*b/(d*x + c)^(2/3)) + g
amma(2, -I*b/(d*x + c)^(2/3)))*sin(a))/(b^2*d*e^(7/3))
```

Giac [F]

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx=\int \frac{\sin \left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{{(c\,e+d\,e\,x)}^{7/3}}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(7/3),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(7/3), x)
```

3.257 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 237

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx = \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} + \frac{9\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}} + \frac{9\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{4b^{5/2}de^2(e(c+dx))^{2/3}} - \frac{9\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}}$$

```
[Out] 3/2*cos(a+b/(d*x+c)^(2/3))/b/d/e^2/(d*x+c)^(1/3)/(e*(d*x+c))^(2/3)-9/4*(d*x
+c)^(1/3)*sin(a+b/(d*x+c)^(2/3))/b^2/d/e^2/(e*(d*x+c))^(2/3)+9/8*(d*x+c)^(2
/3)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*Pi^(1/2
)/b^(5/2)/d/e^2/(e*(d*x+c))^(2/3)+9/8*(d*x+c)^(2/3)*FresnelC(b^(1/2)*2^(1/2
)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)*2^(1/2)*Pi^(1/2)/b^(5/2)/d/e^2/(e*(d*x+c))
^(2/3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3516, 3498, 3496, 3490, 3466, 3467, 3434, 3433, 3432}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx = \frac{9\sqrt{\frac{\pi}{2}}\sin(a)(c+dx)^{2/3}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}} + \frac{9\sqrt{\frac{\pi}{2}}\cos(a)(c+dx)^{2/3}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}} - \frac{9\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}} + \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(8/3),x]
```

```
[Out] (3*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))^(2/
3)) + (9*Sqrt[Pi/2]*(c + d*x)^(2/3)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c
 + d*x)^(1/3)])/(4*b^(5/2)*d*e^2*(e*(c + d*x))^(2/3)) + (9*Sqrt[Pi/2]*(c +
d*x)^(2/3)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(4*b^(5/2
)*d*e^2*(e*(c + d*x))^(2/3)) - (9*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)
])/(4*b^2*d*e^2*(e*(c + d*x))^(2/3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
```

```
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{8/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)^{2/3}\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{8/3}}\,dx, x, c+dx\right)}{de^2(e(c+dx))^{2/3}}$$

$$= \frac{\left(3(c+dx)^{2/3}\right) \text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^6} \, dx, x, \sqrt[3]{c+dx}\right)}{de^2(e(c+dx))^{2/3}}$$

$$= -\frac{\left(3(c+dx)^{2/3}\right) \text{Subst}\left(\int x^4 \sin\left(a+bx^2\right) \, dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} - \frac{\left(9(c+dx)^{2/3}\right) \text{Subst}\left(\int x^2 \cos\left(a+bx^2\right) \, dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2bde^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} - \frac{9\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}} + \frac{\left(9(c+dx)^{2/3}\right) \text{Subst}\left(\int \sin\left(a+bx^2\right) \, dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{4b^2de^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} - \frac{9\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}} + \frac{\left(9(c+dx)^{2/3}\cos(a)\right) \text{Subst}\left(\int \sin\left(bx^2\right) \, dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{4b^2de^2(e(c+dx))^{2/3}} + \frac{\left(9(c+dx)^{2/3}\sin(a)\right) \text{Subst}\left(\int \cos\left(bx^2\right) \, dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{4b^2de^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} + \frac{9\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}} + \frac{9\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{4b^{5/2}de^2(e(c+dx))^{2/3}} - \frac{9\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.70

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx = \frac{(c+dx)^{5/3}\left(9\sqrt{2\pi}(c+dx)\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+9\sqrt{2\pi}(c+dx)\,\text{Fresne}\ldots\right.}{8b^{5/2}\ldots}$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(8/3),x]
```

```
[Out] ((c + d*x)^(5/3)*(9*Sqrt[2*Pi]*(c + d*x)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)] + 9*Sqrt[2*Pi]*(c + d*x)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/
(c + d*x)^(1/3)]*Sin[a] + 6*Sqrt[b]*(2*b*Cos[a + b/(c + d*x)^(2/3)] - 3*(c
+ d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)])))/(8*b^(5/2)*d*(e*(c + d*x))^(8/3)
)
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}dx$$

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(1/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c
))/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(8/3),x)
```

```
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.72

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx = \frac{3\,(dx+c)^{\frac{1}{3}}\left(\left(\left(\left(i\,\Gamma\left(\frac{5}{2},i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)-i\,\Gamma\left(\frac{5}{2},-\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(\frac{5}{4}\,\pi+\frac{5}{3}\,\arctan\left(0,d\right.\right.\right.\right.\right.}{}$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x, algorithm="maxima")
```

```
[Out] 3/8*(d*x + c)^(1/3)*(((I*gamma(5/2, I*b*conjugate((d*x + c)^(-2/3))) - I*ga
mma(5/2, -I*b/(d*x + c)^(2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x + c)) + (-I
*gamma(5/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(5/2, I*b/(d*x + c)^
(2/3)))*cos(-5/4*pi + 5/3*arctan2(0, d*x + c)) + (gamma(5/2, I*b*conjugate(
(d*x + c)^(-2/3))) + gamma(5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arc
tan2(0, d*x + c)) - (gamma(5/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(5
/2, I*b/(d*x + c)^(2/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x + c)))*cos(a) +
((gamma(5/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(5/2, -I*b/(d*x + c)^(
2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x + c)) + (gamma(5/2, -I*b*conjugate((
d*x + c)^(-2/3))) + gamma(5/2, I*b/(d*x + c)^(2/3)))*cos(-5/4*pi + 5/3*arct
an2(0, d*x + c)) + (-I*gamma(5/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamm
a(5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arctan2(0, d*x + c)) + (-I*g
amma(5/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(5/2, I*b/(d*x + c)^(2
/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x + c)))*sin(a))*e^(1/3)/((d^3*e^3*x^2
 + 2*c*d^2*e^3*x + c^2*d*e^3)*(b/(d*x + c)^(2/3))^(5/2))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(c\,e+d\,e\,x)^{8/3}}\,dx$$

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(8/3),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(8/3), x)
```

3.258 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 277

$$
\begin{aligned}
\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx = & -\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}} \\
& + \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} + \frac{45\sqrt{\pi}\sqrt[3]{c+dx}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}} \\
& - \frac{45\sqrt{\pi}\sqrt[3]{c+dx}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}} - \frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}
\end{aligned}
$$

```
[Out] -45/8*cos(a+b/(d*x+c)^(2/3))/b^3/d/e^3/(e*(d*x+c))^(1/3)+3/2*cos(a+b/(d*x+c
)^(2/3))/b/d/e^3/(d*x+c)^(4/3)/(e*(d*x+c))^(1/3)-15/4*sin(a+b/(d*x+c)^(2/3)
)/b^2/d/e^3/(d*x+c)^(2/3)/(e*(d*x+c))^(1/3)+45/16*(d*x+c)^(1/3)*cos(a)*Fres
nelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*Pi^(1/2)/b^(7/2)/d/e^3/(e*(d*x
+c))^(1/3)*2^(1/2)-45/16*(d*x+c)^(1/3)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d
*x+c)^(1/3))*sin(a)*Pi^(1/2)/b^(7/2)/d/e^3/(e*(d*x+c))^(1/3)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3516, 3498, 3496, 3490, 3466, 3467, 3435, 3433, 3432}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx = \frac{45\sqrt{\pi}\cos(a)\sqrt[3]{c+dx}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}} - \frac{45\sqrt{\pi}\sin(a)\sqrt[3]{c+dx}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}} - \frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}} - \frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}}$$

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(10/3),x]
```

```
[Out] (-45*Cos[a + b/(c + d*x)^(2/3)])/(8*b^3*d*e^3*(e*(c + d*x))^(1/3)) + (3*Cos
[a + b/(c + d*x)^(2/3)])/(2*b*d*e^3*(c + d*x)^(4/3)*(e*(c + d*x))^(1/3)) +
(45*Sqrt[Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)
^(1/3)])/(8*Sqrt[2]*b^(7/2)*d*e^3*(e*(c + d*x))^(1/3)) - (45*Sqrt[Pi]*(c +
d*x)^(1/3)*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(8*Sqrt[2
]*b^(7/2)*d*e^3*(e*(c + d*x))^(1/3)) - (15*Sin[a + b/(c + d*x)^(2/3)])/(4*b
^2*d*e^3*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
```

```
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{10/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{10/3}}\,dx, x, c+dx\right)}{de^3\sqrt[3]{e(c+dx)}}$$

$$=\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int\frac{\sin\left(a+\frac{b}{x^2}\right)}{x^8}\,dx,x,\sqrt[3]{c+dx}\right)}{de^3\sqrt[3]{e(c+dx)}}$$

$$=-\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^6\sin\left(a+bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{de^3\sqrt[3]{e(c+dx)}}$$

$$=\frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}}-\frac{\left(15\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^4\cos\left(a+bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{2bde^3\sqrt[3]{e(c+dx)}}$$

$$=\frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}}-\frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}+\frac{\left(45\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^2\sin\left(a+bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{4b^2de^3\sqrt[3]{e(c+dx)}}$$

$$=-\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}}+\frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}}-\frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}+\frac{\left(45\sqrt[3]{c+dx}\right)\text{Subst}\left(\int\cos\left(a+bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}}$$

$$=-\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}}+\frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}}-\frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}+\frac{\left(45\sqrt[3]{c+dx}\cos(a)\right)\text{Subst}\left(\int\cos\left(bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}}-\frac{\left(45\sqrt[3]{c+dx}\sin(a)\right)\text{Subst}\left(\int\sin\left(bx^2\right)\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}}$$

$$
= -\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} + \frac{45\sqrt{\pi}\sqrt[3]{c+dx}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}} - \frac{45\sqrt{\pi}\sqrt[3]{c+dx}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}} - \frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}
$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.69

$$
\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx = \frac{(e(c+dx))^{2/3}\left(45\sqrt{2\pi}(c+dx)^{5/3}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right) - 45\sqrt{2\pi}(c+dx)\right.}{}
$$

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(10/3),x]
```

```
[Out] ((e*(c + d*x))^(2/3)*(45*Sqrt[2*Pi]*(c + d*x)^(5/3)*Cos[a]*FresnelC[(Sqrt[b
]*Sqrt[2/Pi])/(c + d*x)^(1/3)] - 45*Sqrt[2*Pi]*(c + d*x)^(5/3)*FresnelS[(Sq
rt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] - 6*Sqrt[b]*((-4*b^2 + 15*(c + d*
x)^(4/3))*Cos[a + b/(c + d*x)^(2/3)] + 10*b*(c + d*x)^(2/3)*Sin[a + b/(c +
d*x)^(2/3)])))/(16*b^(7/2)*d*e^4*(c + d*x)^(7/3))
```

Maple [F]

$$
\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{10}{3}}}dx
$$

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{10}{3}}}\,dx$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(2/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c
))/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4
*e^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(10/3),x)
```

```
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.46

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx = \frac{3\left(\left(\left(\left(i\,\Gamma\left(\frac{7}{2}, i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right) - i\,\Gamma\left(\frac{7}{2}, -\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(\frac{7}{4}\,\pi + \frac{7}{3}\,\arctan\left(0, dx+c\right)\right)\right.\right.\right.}{}$$

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x, algorithm="maxima")
```

```
[Out] 3/8*(((I*gamma(7/2, I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(7/2, -I*b/(d
*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (-I*gamma(7/2, -I*b
*conjugate((d*x + c)^(-2/3))) + I*gamma(7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4
*pi + 7/3*arctan2(0, d*x + c)) + (gamma(7/2, I*b*conjugate((d*x + c)^(-2/3)
)) + gamma(7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)
) - (gamma(7/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(7/2, I*b/(d*x + c
)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*cos(a) + ((gamma(7/2, I*b
*conjugate((d*x + c)^(-2/3))) + gamma(7/2, -I*b/(d*x + c)^(2/3)))*cos(7/4*p
```

```
i + 7/3*arctan2(0, d*x + c)) + (gamma(7/2, -I*b*conjugate((d*x + c)^(-2/3))
) + gamma(7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4*pi + 7/3*arctan2(0, d*x + c))
 + (-I*gamma(7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(7/2, -I*b/(d*x
 + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) + (-I*gamma(7/2, -I*b*c
onjugate((d*x + c)^(-2/3))) + I*gamma(7/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*p
i + 7/3*arctan2(0, d*x + c)))*sin(a))/((d^3*e^(10/3)*x^2 + 2*c*d^2*e^(10/3)
*x + c^2*d*e^(10/3))*(d*x + c)^(1/3)*(b/(d*x + c)^(2/3))^(7/2))
```

Giac [F]

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx=\int \frac{\sin \left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{10}{3}}}\,dx$$

[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x, algorithm="giac")

[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(10/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx=\int \frac{\sin \left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(c\,e+d\,e\,x)^{10/3}}\,dx$$

[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(10/3),x)

[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(10/3), x)

3.259 $\int (ex)^m \sin(a + b(c + dx)^n) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (ex)^m \sin(a + b(c + dx)^n) \, dx = \text{Int}((ex)^m \sin(a + b(c + dx)^n), x)$$

```
[Out] Unintegrable((e*x)^m*sin(a+b*(d*x+c)^n),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m \sin(a + b(c + dx)^n) \, dx = \int (ex)^m \sin(a + b(c + dx)^n) \, dx$$

```
[In] Int[(e*x)^m*Sin[a + b*(c + d*x)^n],x]
[Out] Defer[Int][(e*x)^m*Sin[a + b*(c + d*x)^n], x]
```

Rubi steps

$$\text{integral} = \int (ex)^m \sin(a + b(c + dx)^n) \, dx$$

Mathematica [N/A]

Not integrable

Time = 10.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m \sin \left(a+b(c+dx)^n\right) \, dx=\int (ex)^m \sin \left(a+b(c+dx)^n\right) \, dx$$

```
[In] Integrate[(e*x)^m*Sin[a + b*(c + d*x)^n],x]
[Out] Integrate[(e*x)^m*Sin[a + b*(c + d*x)^n], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (ex)^m \sin \left(a+b(dx+c)^n\right) dx$$

```
[In] int((e*x)^m*sin(a+b*(d*x+c)^n),x)
[Out] int((e*x)^m*sin(a+b*(d*x+c)^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m \sin \left(a+b(c+dx)^n\right) \, dx=\int (ex)^m \sin \left((dx+c)^n b+a\right) \, dx$$

```
[In] integrate((e*x)^m*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
[Out] integral((e*x)^m*sin((d*x + c)^n*b + a), x)
```

Sympy [N/A]

Not integrable

Time = 8.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (ex)^m \sin \left(a+b(c+dx)^n\right) \, dx=\int (ex)^m \sin \left(a+b(c+dx)^n\right) dx$$

```
[In] integrate((e*x)**m*sin(a+b*(d*x+c)**n),x)
[Out] Integral((e*x)**m*sin(a + b*(c + d*x)**n), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m \sin \left(a+b(c+dx)^n\right) \, dx=\int (ex)^m \sin \left((dx+c)^n b+a\right) \, dx$$

```
[In] integrate((e*x)^m*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
[Out] integrate((e*x)^m*sin((d*x + c)^n*b + a), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m \sin \left(a+b(c+dx)^n\right) \, dx=\int (ex)^m \sin \left((dx+c)^n b+a\right) \, dx$$

```
[In] integrate((e*x)^m*sin(a+b*(d*x+c)^n),x, algorithm="giac")
[Out] integrate((e*x)^m*sin((d*x + c)^n*b + a), x)
```

Mupad [N/A]

Not integrable

Time = 6.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m \sin \left(a+b(c+dx)^n\right) \, dx=\int \sin \left(a+b\,{\left(c+d\,x\right)}^n\right)\,{\left(e\,x\right)}^m \,d x$$

```
[In] int(sin(a + b*(c + d*x)^n)*(e*x)^m,x)
[Out] int(sin(a + b*(c + d*x)^n)*(e*x)^m, x)
```

3.260 $\int x^3 \sin(a + b(c + dx)^n) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 503

$$\begin{aligned}
\int x^3 \sin(a + b(c + dx)^n) \, dx = & -\frac{ic^3 e^{ia}(c + dx) (-ib(c + dx)^n)^{-1/n} \Gamma\left(\frac{1}{n}, -ib(c + dx)^n\right)}{2d^4 n} \\
& + \frac{ic^3 e^{-ia}(c + dx) (ib(c + dx)^n)^{-1/n} \Gamma\left(\frac{1}{n}, ib(c + dx)^n\right)}{2d^4 n} \\
& + \frac{3ic^2 e^{ia}(c + dx)^2 (-ib(c + dx)^n)^{-2/n} \Gamma\left(\frac{2}{n}, -ib(c + dx)^n\right)}{2d^4 n} \\
& - \frac{3ic^2 e^{-ia}(c + dx)^2 (ib(c + dx)^n)^{-2/n} \Gamma\left(\frac{2}{n}, ib(c + dx)^n\right)}{2d^4 n} \\
& - \frac{3ice^{ia}(c + dx)^3 (-ib(c + dx)^n)^{-3/n} \Gamma\left(\frac{3}{n}, -ib(c + dx)^n\right)}{2d^4 n} \\
& + \frac{3ice^{-ia}(c + dx)^3 (ib(c + dx)^n)^{-3/n} \Gamma\left(\frac{3}{n}, ib(c + dx)^n\right)}{2d^4 n} \\
& + \frac{ie^{ia}(c + dx)^4 (-ib(c + dx)^n)^{-4/n} \Gamma\left(\frac{4}{n}, -ib(c + dx)^n\right)}{2d^4 n} \\
& - \frac{ie^{-ia}(c + dx)^4 (ib(c + dx)^n)^{-4/n} \Gamma\left(\frac{4}{n}, ib(c + dx)^n\right)}{2d^4 n}
\end{aligned}$$

```
[Out] -1/2*I*c^3*exp(I*a)*(d*x+c)*GAMMA(1/n,-I*b*(d*x+c)^n)/d^4/n/((-I*b*(d*x+c)^
n)^(1/n))+1/2*I*c^3*(d*x+c)*GAMMA(1/n,I*b*(d*x+c)^n)/d^4/exp(I*a)/n/((I*b*(
d*x+c)^n)^(1/n))+3/2*I*c^2*exp(I*a)*(d*x+c)^2*GAMMA(2/n,-I*b*(d*x+c)^n)/d^4
/n/((-I*b*(d*x+c)^n)^(2/n))-3/2*I*c^2*(d*x+c)^2*GAMMA(2/n,I*b*(d*x+c)^n)/d^
4/exp(I*a)/n/((I*b*(d*x+c)^n)^(2/n))-3/2*I*c*exp(I*a)*(d*x+c)^3*GAMMA(3/n,-
I*b*(d*x+c)^n)/d^4/n/((-I*b*(d*x+c)^n)^(3/n))+3/2*I*c*(d*x+c)^3*GAMMA(3/n,I
*b*(d*x+c)^n)/d^4/exp(I*a)/n/((I*b*(d*x+c)^n)^(3/n))+1/2*I*exp(I*a)*(d*x+c)
^4*GAMMA(4/n,-I*b*(d*x+c)^n)/d^4/n/((-I*b*(d*x+c)^n)^(4/n))-1/2*I*(d*x+c)^4
*GAMMA(4/n,I*b*(d*x+c)^n)/d^4/exp(I*a)/n/((I*b*(d*x+c)^n)^(4/n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3514, 3446, 2239, 3504, 2250}

$$\int x^3 \sin \left(a+b(c+dx)^n\right) dx = -\frac{ie^{ia}c^3(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^4n} + \frac{ie^{-ia}c^3(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^4n} + \frac{3ie^{ia}c^2(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)}{2d^4n} - \frac{3ie^{-ia}c^2(c+dx)^2\left(ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},ib(c+dx)^n\right)}{2d^4n} + \frac{ie^{ia}(c+dx)^4\left(-ib(c+dx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},-ib(c+dx)^n\right)}{2d^4n} - \frac{3ie^{ia}c(c+dx)^3\left(-ib(c+dx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-ib(c+dx)^n\right)}{2d^4n} + \frac{3ie^{-ia}c(c+dx)^3\left(ib(c+dx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},ib(c+dx)^n\right)}{2d^4n} - \frac{ie^{-ia}(c+dx)^4\left(ib(c+dx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},ib(c+dx)^n\right)}{2d^4n}$$

```
[In] Int[x^3*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((-1/2*I)*c^3*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d^4*n*(
(-I)*b*(c + d*x)^n)^n^(-1)) + ((I/2)*c^3*(c + d*x)*Gamma[n^(-1), I*b*(c + d
*x)^n])/(d^4*E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) + (((3*I)/2)*c^2*E^(I*a)*(
c + d*x)^2*Gamma[2/n, (-I)*b*(c + d*x)^n])/(d^4*n*((-I)*b*(c + d*x)^n)^(2/n
)) - (((3*I)/2)*c^2*(c + d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(d^4*E^(I*a)*n
*(I*b*(c + d*x)^n)^(2/n)) - (((3*I)/2)*c*E^(I*a)*(c + d*x)^3*Gamma[3/n, (-I
)*b*(c + d*x)^n])/(d^4*n*((-I)*b*(c + d*x)^n)^(3/n)) + (((3*I)/2)*c*(c + d*
x)^3*Gamma[3/n, I*b*(c + d*x)^n])/(d^4*E^(I*a)*n*(I*b*(c + d*x)^n)^(3/n)) +
 ((I/2)*E^(I*a)*(c + d*x)^4*Gamma[4/n, (-I)*b*(c + d*x)^n])/(d^4*n*((-I)*b*
(c + d*x)^n)^(4/n)) - ((I/2)*(c + d*x)^4*Gamma[4/n, I*b*(c + d*x)^n])/(d^4*
E^(I*a)*n*(I*b*(c + d*x)^n)^(4/n))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-c^3 \sin \left(a+bx^n\right)+3c^2 x \sin \left(a+bx^n\right)-3cx^2 \sin \left(a+bx^n\right)+x^3 \sin \left(a+bx^n\right)\right) \, dx,x,c+dx\right)}{d^4} \\
&= \frac{\text{Subst}\left(\int x^3 \sin \left(a+bx^n\right) \, dx,x,c+dx\right)}{d^4}-\frac{(3c)\text{Subst}\left(\int x^2 \sin \left(a+bx^n\right) \, dx,x,c+dx\right)}{d^4} \\
&\quad +\frac{\left(3c^2\right) \text{Subst}\left(\int x \sin \left(a+bx^n\right) \, dx,x,c+dx\right)}{d^4}-\frac{c^3\text{Subst}\left(\int \sin \left(a+bx^n\right) \, dx,x,c+dx\right)}{d^4} \\
&= \frac{i\text{Subst}\left(\int e^{-ia-ibx^n} x^3 \, dx,x,c+dx\right)}{2d^4}-\frac{i\text{Subst}\left(\int e^{ia+ibx^n} x^3 \, dx,x,c+dx\right)}{2d^4} \\
&\quad -\frac{(3ic)\text{Subst}\left(\int e^{-ia-ibx^n} x^2 \, dx,x,c+dx\right)}{2d^4}+\frac{(3ic)\text{Subst}\left(\int e^{ia+ibx^n} x^2 \, dx,x,c+dx\right)}{2d^4} \\
&\quad +\frac{\left(3ic^2\right) \text{Subst}\left(\int e^{-ia-ibx^n} x \, dx,x,c+dx\right)}{2d^4}-\frac{\left(3ic^2\right) \text{Subst}\left(\int e^{ia+ibx^n} x \, dx,x,c+dx\right)}{2d^4} \\
&\quad -\frac{\left(ic^3\right) \text{Subst}\left(\int e^{-ia-ibx^n} \, dx,x,c+dx\right)}{2d^4}+\frac{\left(ic^3\right) \text{Subst}\left(\int e^{ia+ibx^n} \, dx,x,c+dx\right)}{2d^4}
\end{aligned}$$

$$= -\frac{ic^3e^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^4n} + \frac{ic^3e^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^4n} + \frac{3ic^2e^{ia}(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)}{2d^4n} - \frac{3ic^2e^{-ia}(c+dx)^2\left(ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},ib(c+dx)^n\right)}{2d^4n} - \frac{3ice^{ia}(c+dx)^3\left(-ib(c+dx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-ib(c+dx)^n\right)}{2d^4n} + \frac{3ice^{-ia}(c+dx)^3\left(ib(c+dx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},ib(c+dx)^n\right)}{2d^4n} + \frac{ie^{ia}(c+dx)^4\left(-ib(c+dx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},-ib(c+dx)^n\right)}{2d^4n} - \frac{ie^{-ia}(c+dx)^4\left(ib(c+dx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},ib(c+dx)^n\right)}{2d^4n}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.76

$$\int x^3\sin\left(a+b(c+dx)^n\right)\,dx = -\frac{ie^{-ia}(c+dx)\left(-c^3(ib(c+dx)^n)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)+e^{2ia}(-ib(c+dx)^n)^{-4/n}\left(c^3(-ib(c+dx)^n)^{3/n}\Gamma\right.\right.}{}$$

```
[In] Integrate[x^3*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((-1/2*I)*(c + d*x)*(-((c^3*Gamma[n^(-1), I*b*(c + d*x)^n])/(I*b*(c + d*x)^
n)^n^(-1)) + (E^((2*I)*a)*(c^3*((-I)*b*(c + d*x)^n)^(3/n)*Gamma[n^(-1), (-I
)*b*(c + d*x)^n] - (c + d*x)*(3*c^2*((-I)*b*(c + d*x)^n)^(2/n)*Gamma[2/n, (
-I)*b*(c + d*x)^n] - (c + d*x)*(3*c*((-I)*b*(c + d*x)^n)^n^(-1)*Gamma[3/n,
(-I)*b*(c + d*x)^n] - (c + d*x)*Gamma[4/n, (-I)*b*(c + d*x)^n]))))/((-I)*b*
(c + d*x)^n)^(4/n) + ((c + d*x)*(3*c^2*(I*b*(c + d*x)^n)^(2/n)*Gamma[2/n, I
*b*(c + d*x)^n] - (c + d*x)*(3*c*(I*b*(c + d*x)^n)^n^(-1)*Gamma[3/n, I*b*(c
 + d*x)^n] - (c + d*x)*Gamma[4/n, I*b*(c + d*x)^n])))/(I*b*(c + d*x)^n)^(4/
n)))/(d^4*E^(I*a)*n)
```

Maple [F]

$$\int x^3 \sin \left(a+b(dx+c)^n\right) dx$$

```
[In] int(x^3*sin(a+b*(d*x+c)^n),x)
[Out] int(x^3*sin(a+b*(d*x+c)^n),x)
```

Fricas [F]

$$\int x^3 \sin \left(a+b(c+dx)^n\right) \, dx = \int x^3 \sin \left((dx+c)^n b+a\right) \, dx$$

```
[In] integrate(x^3*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
[Out] integral(x^3*sin((d*x + c)^n*b + a), x)
```

Sympy [F]

$$\int x^3 \sin \left(a+b(c+dx)^n\right) \, dx = \int x^3 \sin \left(a+b(c+dx)^n\right) dx$$

```
[In] integrate(x**3*sin(a+b*(d*x+c)**n),x)
[Out] Integral(x**3*sin(a + b*(c + d*x)**n), x)
```

Maxima [F]

$$\int x^3 \sin \left(a+b(c+dx)^n\right) \, dx = \int x^3 \sin \left((dx+c)^n b+a\right) \, dx$$

```
[In] integrate(x^3*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
[Out] integrate(x^3*sin((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int x^3 \sin(a + b(c + dx)^n) \, dx = \int x^3 \sin((dx + c)^n b + a) \, dx$$

```
[In] integrate(x^3*sin(a+b*(d*x+c)^n),x, algorithm="giac")
[Out] integrate(x^3*sin((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sin(a + b(c + dx)^n) \, dx = \int x^3 \sin(a + b(c + dx)^n) \, dx$$

```
[In] int(x^3*sin(a + b*(c + d*x)^n),x)
[Out] int(x^3*sin(a + b*(c + d*x)^n), x)
```

3.261 $\int x^2 \sin(a + b(c + dx)^n) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 369

$$\int x^2 \sin(a + b(c + dx)^n) \, dx = \frac{ic^2 e^{ia}(c + dx)(-ib(c + dx)^n)^{-1/n} \Gamma\left(\frac{1}{n}, -ib(c + dx)^n\right)}{2d^3 n} - \frac{ic^2 e^{-ia}(c + dx)(ib(c + dx)^n)^{-1/n} \Gamma\left(\frac{1}{n}, ib(c + dx)^n\right)}{2d^3 n} - \frac{ice^{ia}(c + dx)^2(-ib(c + dx)^n)^{-2/n} \Gamma\left(\frac{2}{n}, -ib(c + dx)^n\right)}{d^3 n} + \frac{ice^{-ia}(c + dx)^2(ib(c + dx)^n)^{-2/n} \Gamma\left(\frac{2}{n}, ib(c + dx)^n\right)}{d^3 n} + \frac{ie^{ia}(c + dx)^3(-ib(c + dx)^n)^{-3/n} \Gamma\left(\frac{3}{n}, -ib(c + dx)^n\right)}{2d^3 n} - \frac{ie^{-ia}(c + dx)^3(ib(c + dx)^n)^{-3/n} \Gamma\left(\frac{3}{n}, ib(c + dx)^n\right)}{2d^3 n}$$

```
[Out] 1/2*I*c^2*exp(I*a)*(d*x+c)*GAMMA(1/n,-I*b*(d*x+c)^n)/d^3/n/((-I*b*(d*x+c)^n
)^(1/n))-1/2*I*c^2*(d*x+c)*GAMMA(1/n,I*b*(d*x+c)^n)/d^3/exp(I*a)/n/((I*b*(d
*x+c)^n)^(1/n))-I*c*exp(I*a)*(d*x+c)^2*GAMMA(2/n,-I*b*(d*x+c)^n)/d^3/n/((-I
*b*(d*x+c)^n)^(2/n))+I*c*(d*x+c)^2*GAMMA(2/n,I*b*(d*x+c)^n)/d^3/exp(I*a)/n/
((I*b*(d*x+c)^n)^(2/n))+1/2*I*exp(I*a)*(d*x+c)^3*GAMMA(3/n,-I*b*(d*x+c)^n)/
d^3/n/((-I*b*(d*x+c)^n)^(3/n))-1/2*I*(d*x+c)^3*GAMMA(3/n,I*b*(d*x+c)^n)/d^3
/exp(I*a)/n/((I*b*(d*x+c)^n)^(3/n))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3514, 3446, 2239, 3504, 2250}

$$\begin{aligned}\int x^2 \sin \left(a+b(c+dx)^n\right) dx = & \frac{ie^{ia}c^2(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^3n} \\ & - \frac{ie^{-ia}c^2(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^3n} \\ & + \frac{ie^{ia}(c+dx)^3\left(-ib(c+dx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-ib(c+dx)^n\right)}{2d^3n} \\ & - \frac{ie^{ia}c(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)}{d^3n} \\ & + \frac{ie^{-ia}c(c+dx)^2\left(ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},ib(c+dx)^n\right)}{d^3n} \\ & - \frac{ie^{-ia}(c+dx)^3\left(ib(c+dx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},ib(c+dx)^n\right)}{2d^3n}\end{aligned}$$

```
[In] Int[x^2*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((I/2)*c^2*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d^3*n*((-I
)*b*(c + d*x)^n)^n^(-1)) - ((I/2)*c^2*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)
^n])/(d^3*E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) - (I*c*E^(I*a)*(c + d*x)^2*Ga
mma[2/n, (-I)*b*(c + d*x)^n])/(d^3*n*((-I)*b*(c + d*x)^n)^(2/n)) + (I*c*(c
+ d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(d^3*E^(I*a)*n*(I*b*(c + d*x)^n)^(2/n
)) + ((I/2)*E^(I*a)*(c + d*x)^3*Gamma[3/n, (-I)*b*(c + d*x)^n])/(d^3*n*((-I
)*b*(c + d*x)^n)^(3/n)) - ((I/2)*(c + d*x)^3*Gamma[3/n, I*b*(c + d*x)^n])/(
d^3*E^(I*a)*n*(I*b*(c + d*x)^n)^(3/n))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(c^2 \sin(a+bx^n) - 2cx\sin(a+bx^n) + x^2\sin(a+bx^n)\right)\,dx, x, c+dx\right)}{d^3} \\
&= \frac{\text{Subst}\left(\int x^2\sin(a+bx^n)\,dx, x, c+dx\right)}{d^3} - \frac{(2c)\text{Subst}\left(\int x\sin(a+bx^n)\,dx, x, c+dx\right)}{d^3} \\
&\quad + \frac{c^2\text{Subst}\left(\int \sin(a+bx^n)\,dx, x, c+dx\right)}{d^3} \\
&= \frac{i\text{Subst}\left(\int e^{-ia-ibx^n}x^2\,dx, x, c+dx\right)}{2d^3} - \frac{i\text{Subst}\left(\int e^{ia+ibx^n}x^2\,dx, x, c+dx\right)}{2d^3} \\
&\quad - \frac{(ic)\text{Subst}\left(\int e^{-ia-ibx^n}x\,dx, x, c+dx\right)}{d^3} + \frac{(ic)\text{Subst}\left(\int e^{ia+ibx^n}x\,dx, x, c+dx\right)}{d^3} \\
&\quad + \frac{(ic^2)\,\text{Subst}\left(\int e^{-ia-ibx^n}\,dx, x, c+dx\right)}{2d^3} - \frac{(ic^2)\,\text{Subst}\left(\int e^{ia+ibx^n}\,dx, x, c+dx\right)}{2d^3}
\end{aligned}$$

$$= \frac{ic^2e^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^3n} - \frac{ic^2e^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^3n} - \frac{ice^{ia}(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)}{d^3n} + \frac{ice^{-ia}(c+dx)^2\left(ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},ib(c+dx)^n\right)}{d^3n} + \frac{ie^{ia}(c+dx)^3\left(-ib(c+dx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-ib(c+dx)^n\right)}{2d^3n} - \frac{ie^{-ia}(c+dx)^3\left(ib(c+dx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},ib(c+dx)^n\right)}{2d^3n}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.78

$$\int x^2 \sin\left(a+b(c+dx)^n\right)\,dx$$

$$= \frac{ie^{-ia}(c+dx)\left(e^{2ia}(-ib(c+dx)^n)^{-3/n}\left(c^2(-ib(c+dx)^n)^{2/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)-(c+dx)\left(2c(-ib(c+d\right.\right.\right.}{}$$

```
[In] Integrate[x^2*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((I/2)*(c + d*x)*((E^((2*I)*a)*(c^2*((-I)*b*(c + d*x)^n)^(2/n)*Gamma[n^(-1)
, (-I)*b*(c + d*x)^n] - (c + d*x)*(2*c*((-I)*b*(c + d*x)^n)^n^(-1)*Gamma[2/
n, (-I)*b*(c + d*x)^n] - (c + d*x)*Gamma[3/n, (-I)*b*(c + d*x)^n])))/((-I)*
b*(c + d*x)^n)^(3/n) + (-(c^2*(I*b*(c + d*x)^n)^(2/n)*Gamma[n^(-1), I*b*(c
+ d*x)^n]) + (c + d*x)*(2*c*(I*b*(c + d*x)^n)^n^(-1)*Gamma[2/n, I*b*(c + d*
x)^n] - (c + d*x)*Gamma[3/n, I*b*(c + d*x)^n]))/(I*b*(c + d*x)^n)^(3/n)))/(
d^3*E^(I*a)*n)
```

Maple [F]

$$\int x^2 \sin\left(a+b(dx+c)^n\right)dx$$

```
[In] int(x^2*sin(a+b*(d*x+c)^n),x)
```

```
[Out] int(x^2*sin(a+b*(d*x+c)^n),x)
```

Fricas [F]

$$\int x^2 \sin(a + b(c + dx)^n) \, dx = \int x^2 \sin((dx + c)^n b + a) \, dx$$

```
[In] integrate(x^2*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
[Out] integral(x^2*sin((d*x + c)^n*b + a), x)
```

Sympy [F]

$$\int x^2 \sin(a + b(c + dx)^n) \, dx = \int x^2 \sin(a + b(c + dx)^n) \, dx$$

```
[In] integrate(x**2*sin(a+b*(d*x+c)**n),x)
[Out] Integral(x**2*sin(a + b*(c + d*x)**n), x)
```

Maxima [F]

$$\int x^2 \sin(a + b(c + dx)^n) \, dx = \int x^2 \sin((dx + c)^n b + a) \, dx$$

```
[In] integrate(x^2*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
[Out] integrate(x^2*sin((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int x^2 \sin(a + b(c + dx)^n) \, dx = \int x^2 \sin((dx + c)^n b + a) \, dx$$

```
[In] integrate(x^2*sin(a+b*(d*x+c)^n),x, algorithm="giac")
[Out] integrate(x^2*sin((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sin (a + b(c + dx)^n) \, dx = \int x^2 \, \sin (a + b \, (c + d \, x)^n) \, dx$$

```
[In] int(x^2*sin(a + b*(c + d*x)^n),x)
[Out] int(x^2*sin(a + b*(c + d*x)^n), x)
```

3.262 $\int x \sin \left(a+b(c+d x)^n\right) d x$

Optimal result

Integrand size = 14, antiderivative size = 243

$$
\begin{aligned}
\int x \sin \left(a+b(c+d x)^n\right) d x= & -\frac{i c e^{i a}(c+d x)\left(-i b(c+d x)^n\right)^{-1 / n} \Gamma\left(\frac{1}{n},-i b(c+d x)^n\right)}{2 d^2 n} \\
& +\frac{i c e^{-i a}(c+d x)\left(i b(c+d x)^n\right)^{-1 / n} \Gamma\left(\frac{1}{n}, i b(c+d x)^n\right)}{2 d^2 n} \\
& +\frac{i e^{i a}(c+d x)^2\left(-i b(c+d x)^n\right)^{-2 / n} \Gamma\left(\frac{2}{n},-i b(c+d x)^n\right)}{2 d^2 n} \\
& -\frac{i e^{-i a}(c+d x)^2\left(i b(c+d x)^n\right)^{-2 / n} \Gamma\left(\frac{2}{n}, i b(c+d x)^n\right)}{2 d^2 n}
\end{aligned}
$$

```
[Out] -1/2*I*c*exp(I*a)*(d*x+c)*GAMMA(1/n,-I*b*(d*x+c)^n)/d^2/n/((-I*b*(d*x+c)^n)
^(1/n))+1/2*I*c*(d*x+c)*GAMMA(1/n,I*b*(d*x+c)^n)/d^2/exp(I*a)/n/((I*b*(d*x+
c)^n)^(1/n))+1/2*I*exp(I*a)*(d*x+c)^2*GAMMA(2/n,-I*b*(d*x+c)^n)/d^2/n/((-I*
b*(d*x+c)^n)^(2/n))-1/2*I*(d*x+c)^2*GAMMA(2/n,I*b*(d*x+c)^n)/d^2/exp(I*a)/n
/((I*b*(d*x+c)^n)^(2/n))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used

= {3514, 3446, 2239, 3504, 2250}

$$\int x \sin\left(a+b(c+dx)^n\right)\,dx = \frac{ie^{ia}(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)}{2d^2n} - \frac{ie^{ia}c(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^2n} + \frac{ie^{-ia}c(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^2n} - \frac{ie^{-ia}(c+dx)^2\left(ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},ib(c+dx)^n\right)}{2d^2n}$$

```
[In] Int[x*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((-1/2*I)*c*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d^2*n*((-
I)*b*(c + d*x)^n)^n^(-1)) + ((I/2)*c*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^
n])/(d^2*E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) + ((I/2)*E^(I*a)*(c + d*x)^2*G
amma[2/n, (-I)*b*(c + d*x)^n])/(d^2*n*((-I)*b*(c + d*x)^n)^(2/n)) - ((I/2)*
(c + d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(d^2*E^(I*a)*n*(I*b*(c + d*x)^n)^(
2/n))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int(-c\sin(a+bx^n)+x\sin(a+bx^n))\,dx,x,c+dx\right)}{d^2}\\
&= \frac{\text{Subst}\left(\int x\sin(a+bx^n)\,dx,x,c+dx\right)}{d^2}-\frac{c\text{Subst}\left(\int\sin(a+bx^n)\,dx,x,c+dx\right)}{d^2}\\
&= \frac{i\text{Subst}\left(\int e^{-ia-ibx^n}x\,dx,x,c+dx\right)}{2d^2}-\frac{i\text{Subst}\left(\int e^{ia+ibx^n}x\,dx,x,c+dx\right)}{2d^2}\\
&\quad-\frac{(ic)\text{Subst}\left(\int e^{-ia-ibx^n}\,dx,x,c+dx\right)}{2d^2}+\frac{(ic)\text{Subst}\left(\int e^{ia+ibx^n}\,dx,x,c+dx\right)}{2d^2}\\
&= -\frac{ice^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^2n}\\
&\quad+\frac{ice^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^2n}\\
&\quad+\frac{ie^{ia}(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)}{2d^2n}\\
&\quad-\frac{ie^{-ia}(c+dx)^2\left(ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},ib(c+dx)^n\right)}{2d^2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.79

$$\int x\sin\left(a+b(c+dx)^n\right)\,dx$$
$$=\frac{(c+dx)\left(\left(-ib(c+dx)^n\right)^{-2/n}\left(c\left(-ib(c+dx)^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)-(c+dx)\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)\right)(-i\,\text{co}}{}$$

```
[In] Integrate[x*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((c + d*x)*(((c*((-I)*b*(c + d*x)^n)^n^(-1)*Gamma[n^(-1), (-I)*b*(c + d*x)^
n] - (c + d*x)*Gamma[2/n, (-I)*b*(c + d*x)^n])*((-I)*Cos[a] + Sin[a]))/((-I
)*b*(c + d*x)^n)^(2/n) + ((c*(I*b*(c + d*x)^n)^n^(-1)*Gamma[n^(-1), I*b*(c
+ d*x)^n] - (c + d*x)*Gamma[2/n, I*b*(c + d*x)^n])*(I*Cos[a] + Sin[a]))/(I*
b*(c + d*x)^n)^(2/n)))/(2*d^2*n)
```

Maple [F]

$$\int x \sin \left(a + b(dx + c)^n\right) dx$$

```
[In] int(x*sin(a+b*(d*x+c)^n),x)
[Out] int(x*sin(a+b*(d*x+c)^n),x)
```

Fricas [F]

$$\int x \sin \left(a + b(c + dx)^n\right) \, dx = \int x \sin \left((dx + c)^n b + a\right) \, dx$$

```
[In] integrate(x*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
[Out] integral(x*sin((d*x + c)^n*b + a), x)
```

Sympy [F]

$$\int x \sin \left(a + b(c + dx)^n\right) \, dx = \int x \sin \left(a + b(c + dx)^n\right) dx$$

```
[In] integrate(x*sin(a+b*(d*x+c)**n),x)
[Out] Integral(x*sin(a + b*(c + d*x)**n), x)
```

Maxima [F]

$$\int x \sin \left(a + b(c + dx)^n\right) \, dx = \int x \sin \left((dx + c)^n b + a\right) \, dx$$

```
[In] integrate(x*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
[Out] integrate(x*sin((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int x \sin (a + b(c + dx)^n) \, dx = \int x \sin ((dx + c)^n b + a) \, dx$$

```
[In] integrate(x*sin(a+b*(d*x+c)^n),x, algorithm="giac")
[Out] integrate(x*sin((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \sin (a + b(c + dx)^n) \, dx = \int x \sin (a + b (c + d x)^n) \, dx$$

```
[In] int(x*sin(a + b*(c + d*x)^n),x)
[Out] int(x*sin(a + b*(c + d*x)^n), x)
```

3.263 $\int \sin(a+b(c+dx)^n)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 117

$$\int \sin(a+b(c+dx)^n)\,dx = \frac{ie^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2dn} - \frac{ie^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2dn}$$

```
[Out] 1/2*I*exp(I*a)*(d*x+c)*GAMMA(1/n,-I*b*(d*x+c)^n)/d/n/((-I*b*(d*x+c)^n)^(1/n
))-1/2*I*(d*x+c)*GAMMA(1/n,I*b*(d*x+c)^n)/d/exp(I*a)/n/((I*b*(d*x+c)^n)^(1/
n))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3446, 2239}

$$\int \sin(a+b(c+dx)^n)\,dx = \frac{ie^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2dn} - \frac{ie^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2dn}$$

```
[In] Int[Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((I/2)*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d*n*((-I)*b*(c
 + d*x)^n)^n^(-1)) - ((I/2)*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^n])/(d*E^
(I*a)*n*(I*b*(c + d*x)^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}i\int e^{-ia-ib(c+dx)^n}\,dx - \frac{1}{2}i\int e^{ia+ib(c+dx)^n}\,dx\\ &= \frac{ie^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2dn}\\ &\quad - \frac{ie^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2dn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\begin{aligned}&\int \sin\left(a+b(c+dx)^n\right)\,dx\\ &= -\frac{i(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)\left(\cos(a)-i\sin(a)\right)}{2dn}\\ &\quad + \frac{i(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)\left(\cos(a)+i\sin(a)\right)}{2dn}\end{aligned}$$

```
[In] Integrate[Sin[a + b*(c + d*x)^n],x]
[Out] ((-1/2*I)*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^n]*(Cos[a] - I*Sin[a]))/(d*
n*(I*b*(c + d*x)^n)^n^(-1)) + ((I/2)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*
x)^n]*(Cos[a] + I*Sin[a]))/(d*n*((-I)*b*(c + d*x)^n)^n^(-1))
```

Maple [F]

$$\int \sin \left(a + b(dx + c)^n\right) dx$$

```
[In] int(sin(a+b*(d*x+c)^n),x)
[Out] int(sin(a+b*(d*x+c)^n),x)
```

Fricas [F]

$$\int \sin \left(a + b(c + dx)^n\right)\, dx = \int \sin \left((dx + c)^n b + a\right)\, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^n),x, algorithm="fricas")
[Out] integral(sin((d*x + c)^n*b + a), x)
```

Sympy [F]

$$\int \sin \left(a + b(c + dx)^n\right)\, dx = \int \sin \left(a + b(c + dx)^n\right) dx$$

```
[In] integrate(sin(a+b*(d*x+c)**n),x)
[Out] Integral(sin(a + b*(c + d*x)**n), x)
```

Maxima [F]

$$\int \sin \left(a + b(c + dx)^n\right)\, dx = \int \sin \left((dx + c)^n b + a\right)\, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^n),x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int \sin\left(a + b(c + dx)^n\right) \, dx = \int \sin\left((dx + c)^n b + a\right) \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^n),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a + b(c + dx)^n\right) \, dx = \int \sin\left(a + b\,(c + d\,x)^n\right) \, dx$$

```
[In] int(sin(a + b*(c + d*x)^n),x)
[Out] int(sin(a + b*(c + d*x)^n), x)
```

3.264 $\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx = \operatorname{Int}\left(\frac{\sin(a+b(c+dx)^n)}{x},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^n)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx = \int \frac{\sin(a+b(c+dx)^n)}{x}\,dx$$

```
[In] Int[Sin[a + b*(c + d*x)^n]/x,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^n]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin(a+b(c+dx)^n)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx=\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx$$

```
[In] Integrate[Sin[a + b*(c + d*x)^n]/x,x]
[Out] Integrate[Sin[a + b*(c + d*x)^n]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+b(dx+c)^n)}{x}dx$$

```
[In] int(sin(a+b*(d*x+c)^n)/x,x)
[Out] int(sin(a+b*(d*x+c)^n)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx=\int \frac{\sin((dx+c)^nb+a)}{x}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^n)/x,x, algorithm="fricas")
[Out] integral(sin((d*x + c)^n*b + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx=\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)**n)/x,x)
[Out] Integral(sin(a + b*(c + d*x)**n)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx = \int \frac{\sin((dx+c)^n b+a)}{x}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^n)/x,x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^n*b + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx = \int \frac{\sin((dx+c)^n b+a)}{x}\,dx$$

```
[In] integrate(sin(a+b*(d*x+c)^n)/x,x, algorithm="giac")
[Out] integrate(sin((d*x + c)^n*b + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 6.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx = \int \frac{\sin(a+b\,(c+d\,x)^n)}{x}\,dx$$

```
[In] int(sin(a + b*(c + d*x)^n)/x,x)
[Out] int(sin(a + b*(c + d*x)^n)/x, x)
```

3.265 $\int \frac{\sin(a+b(c+dx)^n)}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sin(a+b(c+dx)^n)}{x^2} dx = \operatorname{Int}\left(\frac{\sin(a+b(c+dx)^n)}{x^2}, x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^n)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin(a+b(c+dx)^n)}{x^2} dx = \int \frac{\sin(a+b(c+dx)^n)}{x^2} dx$$

```
[In] Int[Sin[a + b*(c + d*x)^n]/x^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^n]/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sin(a+b(c+dx)^n)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin (a + b(c + dx)^n)}{x^2} \, dx = \int \frac{\sin (a + b(c + dx)^n)}{x^2} \, dx$$

```
[In] Integrate[Sin[a + b*(c + d*x)^n]/x^2,x]
[Out] Integrate[Sin[a + b*(c + d*x)^n]/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin (a + b(dx + c)^n)}{x^2} dx$$

```
[In] int(sin(a+b*(d*x+c)^n)/x^2,x)
[Out] int(sin(a+b*(d*x+c)^n)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin (a + b(c + dx)^n)}{x^2} \, dx = \int \frac{\sin ((dx + c)^n b + a)}{x^2} \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^n)/x^2,x, algorithm="fricas")
[Out] integral(sin((d*x + c)^n*b + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 7.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sin (a + b(c + dx)^n)}{x^2} \, dx = \int \frac{\sin (a + b(c + dx)^n)}{x^2} \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)**n)/x**2,x)
[Out] Integral(sin(a + b*(c + d*x)**n)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin (a + b(c + dx)^n)}{x^2} \, dx = \int \frac{\sin \left((dx + c)^n b + a \right)}{x^2} \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^n)/x^2,x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^n*b + a)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin (a + b(c + dx)^n)}{x^2} \, dx = \int \frac{\sin \left((dx + c)^n b + a \right)}{x^2} \, dx$$

```
[In] integrate(sin(a+b*(d*x+c)^n)/x^2,x, algorithm="giac")
[Out] integrate(sin((d*x + c)^n*b + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin (a + b(c + dx)^n)}{x^2} \, dx = \int \frac{\sin \left(a + b \left(c + d\, x \right)^n \right)}{x^2} \, dx$$

```
[In] int(sin(a + b*(c + d*x)^n)/x^2,x)
[Out] int(sin(a + b*(c + d*x)^n)/x^2, x)
```

3.266 $\int x^3(a+b\sin(c+d(f+gx)^n))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 519

$$\begin{aligned}
&\int x^3(a+b\sin(c+d(f+gx)^n))\,dx\\
&=\frac{ax^4}{4}-\frac{ibe^{ic}f^3(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^4n}\\
&\quad+\frac{ibe^{-ic}f^3(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2g^4n}\\
&\quad+\frac{3ibe^{ic}f^2(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^4n}\\
&\quad-\frac{3ibe^{-ic}f^2(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{2g^4n}\\
&\quad-\frac{3ibe^{ic}f(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)}{2g^4n}\\
&\quad+\frac{3ibe^{-ic}f(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},id(f+gx)^n\right)}{2g^4n}\\
&\quad+\frac{ibe^{ic}(f+gx)^4\left(-id(f+gx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},-id(f+gx)^n\right)}{2g^4n}\\
&\quad-\frac{ibe^{-ic}(f+gx)^4\left(id(f+gx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},id(f+gx)^n\right)}{2g^4n}
\end{aligned}$$

```
[Out] 1/4*a*x^4-1/2*I*b*exp(I*c)*f^3*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^4/n/((-I
*d*(g*x+f)^n)^(1/n))+1/2*I*b*f^3*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/
g^4/n/((I*d*(g*x+f)^n)^(1/n))+3/2*I*b*exp(I*c)*f^2*(g*x+f)^2*GAMMA(2/n,-I*d
*(g*x+f)^n)/g^4/n/((-I*d*(g*x+f)^n)^(2/n))-3/2*I*b*f^2*(g*x+f)^2*GAMMA(2/n,
I*d*(g*x+f)^n)/exp(I*c)/g^4/n/((I*d*(g*x+f)^n)^(2/n))-3/2*I*b*exp(I*c)*f*(g
*x+f)^3*GAMMA(3/n,-I*d*(g*x+f)^n)/g^4/n/((-I*d*(g*x+f)^n)^(3/n))+3/2*I*b*f*
```

```
(g*x+f)^3*GAMMA(3/n,I*d*(g*x+f)^n)/exp(I*c)/g^4/n/((I*d*(g*x+f)^n)^(3/n))+1
/2*I*b*exp(I*c)*(g*x+f)^4*GAMMA(4/n,-I*d*(g*x+f)^n)/g^4/n/((-I*d*(g*x+f)^n)
^(4/n))-1/2*I*b*(g*x+f)^4*GAMMA(4/n,I*d*(g*x+f)^n)/exp(I*c)/g^4/n/((I*d*(g*
x+f)^n)^(4/n))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {14, 3514, 3446, 2239, 3504, 2250}

$$\begin{aligned}
&\int x^3(a+b\sin(c+d(f+gx)^n))\,dx \\
&=\frac{ax^4}{4}-\frac{ibe^{ic}f^3(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^4n} \\
&+\frac{ibe^{-ic}f^3(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2g^4n} \\
&+\frac{3ibe^{ic}f^2(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^4n} \\
&-\frac{3ibe^{-ic}f^2(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{2g^4n} \\
&+\frac{ibe^{ic}(f+gx)^4\left(-id(f+gx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},-id(f+gx)^n\right)}{2g^4n} \\
&-\frac{3ibe^{ic}f(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)}{2g^4n} \\
&+\frac{3ibe^{-ic}f(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},id(f+gx)^n\right)}{2g^4n} \\
&-\frac{ibe^{-ic}(f+gx)^4\left(id(f+gx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},id(f+gx)^n\right)}{2g^4n}
\end{aligned}$$

```
[In] Int[x^3*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^4)/4 - ((I/2)*b*E^(I*c)*f^3*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n
])/(g^4*n*((-I)*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*f^3*(f + g*x)*Gamma[n^(-1
), I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^n^(-1)) + (((3*I)/2)*
b*E^(I*c)*f^2*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^4*n*((-I)*d*(f
 + g*x)^n)^(2/n)) - (((3*I)/2)*b*f^2*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n
])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^(2/n)) - (((3*I)/2)*b*E^(I*c)*f*(f + g*
x)^3*Gamma[3/n, (-I)*d*(f + g*x)^n])/(g^4*n*((-I)*d*(f + g*x)^n)^(3/n)) + (
((3*I)/2)*b*f*(f + g*x)^3*Gamma[3/n, I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*
(f + g*x)^n)^(3/n)) + ((I/2)*b*E^(I*c)*(f + g*x)^4*Gamma[4/n, (-I)*d*(f + g
```

```
*x)^n])/(g^4*n*((-I)*d*(f + g*x)^n)^(4/n)) - ((I/2)*b*(f + g*x)^4*Gamma[4/n
, I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^(4/n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \left(ax^3 + bx^3 \sin\left(c + d(f + gx)^n\right)\right) \, dx$$

$$= \frac{ax^4}{4} + b\int x^3 \sin\left(c + d(f+gx)^n\right)\, dx$$

$$= \frac{ax^4}{4} + \frac{b\text{Subst}\left(\int \left(-f^3 \sin(c+dx^n) + 3f^2 x \sin(c+dx^n) - 3fx^2 \sin(c+dx^n) + x^3 \sin(c+dx^n)\right)\, dx, x,\right.}{g^4}$$

$$= \frac{ax^4}{4} + \frac{b\text{Subst}\left(\int x^3 \sin(c+dx^n)\, dx, x, f+gx\right)}{g^4} - \frac{(3bf)\text{Subst}\left(\int x^2 \sin(c+dx^n)\, dx, x, f+gx\right)}{g^4} + \frac{(3bf^2)\,\text{Subst}\left(\int x \sin(c+dx^n)\, dx, x, f+gx\right)}{g^4} - \frac{(bf^3)\,\text{Subst}\left(\int \sin(c+dx^n)\, dx, x, f+gx\right)}{g^4}$$

$$= \frac{ax^4}{4} + \frac{(ib)\text{Subst}\left(\int e^{-ic-idx^n} x^3\, dx, x, f+gx\right)}{2g^4} - \frac{(ib)\text{Subst}\left(\int e^{ic+idx^n} x^3\, dx, x, f+gx\right)}{2g^4} - \frac{(3ibf)\text{Subst}\left(\int e^{-ic-idx^n} x^2\, dx, x, f+gx\right)}{2g^4} + \frac{(3ibf)\text{Subst}\left(\int e^{ic+idx^n} x^2\, dx, x, f+gx\right)}{2g^4} + \frac{(3ibf^2)\,\text{Subst}\left(\int e^{-ic-idx^n} x\, dx, x, f+gx\right)}{2g^4} - \frac{(3ibf^2)\,\text{Subst}\left(\int e^{ic+idx^n} x\, dx, x, f+gx\right)}{2g^4} - \frac{(ibf^3)\,\text{Subst}\left(\int e^{-ic-idx^n}\, dx, x, f+gx\right)}{2g^4} + \frac{(ibf^3)\,\text{Subst}\left(\int e^{ic+idx^n}\, dx, x, f+gx\right)}{2g^4}$$

$$= \frac{ax^4}{4} - \frac{ibe^{ic} f^3 (f+gx) \left(-id(f+gx)^n\right)^{-1/n} \Gamma\left(\frac{1}{n}, -id(f+gx)^n\right)}{2g^4 n} + \frac{ibe^{-ic} f^3 (f+gx) \left(id(f+gx)^n\right)^{-1/n} \Gamma\left(\frac{1}{n}, id(f+gx)^n\right)}{2g^4 n} + \frac{3ibe^{ic} f^2 (f+gx)^2 \left(-id(f+gx)^n\right)^{-2/n} \Gamma\left(\frac{2}{n}, -id(f+gx)^n\right)}{2g^4 n} - \frac{3ibe^{-ic} f^2 (f+gx)^2 \left(id(f+gx)^n\right)^{-2/n} \Gamma\left(\frac{2}{n}, id(f+gx)^n\right)}{2g^4 n} - \frac{3ibe^{ic} f (f+gx)^3 \left(-id(f+gx)^n\right)^{-3/n} \Gamma\left(\frac{3}{n}, -id(f+gx)^n\right)}{2g^4 n} + \frac{3ibe^{-ic} f (f+gx)^3 \left(id(f+gx)^n\right)^{-3/n} \Gamma\left(\frac{3}{n}, id(f+gx)^n\right)}{2g^4 n} + \frac{ibe^{ic} (f+gx)^4 \left(-id(f+gx)^n\right)^{-4/n} \Gamma\left(\frac{4}{n}, -id(f+gx)^n\right)}{2g^4 n} - \frac{ibe^{-ic} (f+gx)^4 \left(id(f+gx)^n\right)^{-4/n} \Gamma\left(\frac{4}{n}, id(f+gx)^n\right)}{2g^4 n}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.79

$$\int x^3(a+b\sin(c+d(f+gx)^n))\,dx=\frac{ax^4}{4}$$
$$-\frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-4/n}\left(f^3\left(-id(f+gx)^n\right)^{3/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)-(f+gx)\left(3f^2(-id(f+\right.\right.}{}$$
$$+\frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-4/n}\left(f^3\left(id(f+gx)^n\right)^{3/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)-(f+gx)\left(3f^2(id(f+gx)^n)\right.\right.}{2}$$

```
[In] Integrate[x^3*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^4)/4 - ((I/2)*b*E^(I*c)*(f + g*x)*(f^3*((-I)*d*(f + g*x)^n)^(3/n)*Gamm
a[n^(-1), (-I)*d*(f + g*x)^n] - (f + g*x)*(3*f^2*((-I)*d*(f + g*x)^n)^(2/n)
*Gamma[2/n, (-I)*d*(f + g*x)^n] - (f + g*x)*(3*f*((-I)*d*(f + g*x)^n)^n^(-1
)*Gamma[3/n, (-I)*d*(f + g*x)^n] - (f + g*x)*Gamma[4/n, (-I)*d*(f + g*x)^n]
))))/(g^4*n*((-I)*d*(f + g*x)^n)^(4/n)) + ((I/2)*b*(f + g*x)*(f^3*(I*d*(f +
 g*x)^n)^(3/n)*Gamma[n^(-1), I*d*(f + g*x)^n] - (f + g*x)*(3*f^2*(I*d*(f +
g*x)^n)^(2/n)*Gamma[2/n, I*d*(f + g*x)^n] - (f + g*x)*(3*f*(I*d*(f + g*x)^n
)^n^(-1)*Gamma[3/n, I*d*(f + g*x)^n] - (f + g*x)*Gamma[4/n, I*d*(f + g*x)^n
]))))/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^(4/n))
```

Maple [F]

$$\int x^3(a+b\sin(c+d(gx+f)^n))\,dx$$

```
[In] int(x^3*(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(x^3*(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [F]

$$\int x^3(a+b\sin(c+d(f+gx)^n))\,dx=\int(b\sin((gx+f)^nd+c)+a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x^3*sin((g*x + f)^n*d + c) + a*x^3, x)
```

Sympy [F]

$$\int x^3(a + b\sin(c + d(f + gx)^n))\,dx = \int x^3(a + b\sin(c + d(f + gx)^n))\,dx$$

```
[In] integrate(x**3*(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Integral(x**3*(a + b*sin(c + d*(f + g*x)**n)), x)
```

Maxima [F]

$$\int x^3(a + b\sin(c + d(f + gx)^n))\,dx = \int (b\sin((gx + f)^n d + c) + a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
[Out] 1/4*a*x^4 + b*integrate(x^3*sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int x^3(a + b\sin(c + d(f + gx)^n))\,dx = \int (b\sin((gx + f)^n d + c) + a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a + b\sin(c + d(f + gx)^n))\,dx = \int x^3\,(a + b\,\sin(c + d\,(f + g\,x)^n))\,dx$$

```
[In] int(x^3*(a + b*sin(c + d*(f + g*x)^n)),x)
[Out] int(x^3*(a + b*sin(c + d*(f + g*x)^n)), x)
```

3.267 $\int x^2(a+b\sin(c+d(f+gx)^n))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 383

$$\begin{aligned}\int x^2(a+b\sin(c+d(f+gx)^n))\,dx &= \frac{ax^3}{3}+\frac{ibe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^3n}\\ &\quad-\frac{ibe^{-ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2g^3n}\\ &\quad-\frac{ibe^{ic}f(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{g^3n}\\ &\quad+\frac{ibe^{-ic}f(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{g^3n}\\ &\quad+\frac{ibe^{ic}(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)}{2g^3n}\\ &\quad-\frac{ibe^{-ic}(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},id(f+gx)^n\right)}{2g^3n}\end{aligned}$$

```
[Out] 1/3*a*x^3+1/2*I*b*exp(I*c)*f^2*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^3/n/((-I
*d*(g*x+f)^n)^(1/n))-1/2*I*b*f^2*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/
g^3/n/((I*d*(g*x+f)^n)^(1/n))-I*b*exp(I*c)*f*(g*x+f)^2*GAMMA(2/n,-I*d*(g*x+
f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(2/n))+I*b*f*(g*x+f)^2*GAMMA(2/n,I*d*(g*x+f)^
n)/exp(I*c)/g^3/n/((I*d*(g*x+f)^n)^(2/n))+1/2*I*b*exp(I*c)*(g*x+f)^3*GAMMA(
3/n,-I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(3/n))-1/2*I*b*(g*x+f)^3*GAMMA(
3/n,I*d*(g*x+f)^n)/exp(I*c)/g^3/n/((I*d*(g*x+f)^n)^(3/n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {14, 3514, 3446, 2239, 3504, 2250}

$$\int x^2(a+b\sin(c+d(f+gx)^n))\,dx$$
$$=\frac{ax^3}{3}+\frac{ibe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^3n}-\frac{ibe^{-ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2g^3n}+\frac{ibe^{ic}(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)}{2g^3n}-\frac{ibe^{ic}f(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{g^3n}+\frac{ibe^{-ic}f(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{g^3n}-\frac{ibe^{-ic}(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},id(f+gx)^n\right)}{2g^3n}$$

```
[In] Int[x^2*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^3)/3 + ((I/2)*b*E^(I*c)*f^2*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n
])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) - ((I/2)*b*f^2*(f + g*x)*Gamma[n^(-1
), I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^n^(-1)) - (I*b*E^(I*c
)*f*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)
^(2/n)) + (I*b*f*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I
*d*(f + g*x)^n)^(2/n)) + ((I/2)*b*E^(I*c)*(f + g*x)^3*Gamma[3/n, (-I)*d*(f
+ g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(3/n)) - ((I/2)*b*(f + g*x)^3*Gamma[
3/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^(3/n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^2 + bx^2 \sin\left(c + d(f + gx)^n\right)\right) \, dx \\
&= \frac{ax^3}{3} + b \int x^2 \sin\left(c + d(f + gx)^n\right) \, dx \\
&= \frac{ax^3}{3} + \frac{b\text{Subst}\left(\int \left(f^2 \sin\left(c + dx^n\right) - 2fx \sin\left(c + dx^n\right) + x^2 \sin\left(c + dx^n\right)\right) \, dx, x, f + gx\right)}{g^3} \\
&= \frac{ax^3}{3} + \frac{b\text{Subst}\left(\int x^2 \sin\left(c + dx^n\right) \, dx, x, f + gx\right)}{g^3} \\
&\quad - \frac{(2bf)\text{Subst}\left(\int x \sin\left(c + dx^n\right) \, dx, x, f + gx\right)}{g^3} \\
&\quad + \frac{\left(bf^2\right) \text{Subst}\left(\int \sin\left(c + dx^n\right) \, dx, x, f + gx\right)}{g^3}
\end{aligned}$$

$$= \frac{ax^3}{3} + \frac{(ib)\text{Subst}\left(\int e^{-ic-idx^n}x^2\,dx, x, f+gx\right)}{2g^3} - \frac{(ib)\text{Subst}\left(\int e^{ic+idx^n}x^2\,dx, x, f+gx\right)}{2g^3} - \frac{(ibf)\text{Subst}\left(\int e^{-ic-idx^n}x\,dx, x, f+gx\right)}{g^3} + \frac{(ibf)\text{Subst}\left(\int e^{ic+idx^n}x\,dx, x, f+gx\right)}{g^3} + \frac{(ibf^2)\,\text{Subst}\left(\int e^{-ic-idx^n}\,dx, x, f+gx\right)}{2g^3} - \frac{(ibf^2)\,\text{Subst}\left(\int e^{ic+idx^n}\,dx, x, f+gx\right)}{2g^3}$$

$$= \frac{ax^3}{3} + \frac{ibe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, -id(f+gx)^n\right)}{2g^3n} - \frac{ibe^{-ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, id(f+gx)^n\right)}{2g^3n} - \frac{ibe^{ic}f(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n}, -id(f+gx)^n\right)}{g^3n} + \frac{ibe^{-ic}f(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n}, id(f+gx)^n\right)}{g^3n} + \frac{ibe^{ic}(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n}, -id(f+gx)^n\right)}{2g^3n} - \frac{ibe^{-ic}(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n}, id(f+gx)^n\right)}{2g^3n}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.82

$$\int x^2(a+b\sin\left(c+d(f+gx)^n\right))\,dx = \frac{ax^3}{3} + \frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-3/n}\left(f^2\left(-id(f+gx)^n\right)^{2/n}\Gamma\left(\frac{1}{n}, -id(f+gx)^n\right) - (f+gx)\left(2f(-id(f+gx\right.\right.}{2g^3n} - \frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-3/n}\left(f^2\left(id(f+gx)^n\right)^{2/n}\Gamma\left(\frac{1}{n}, id(f+gx)^n\right) - (f+gx)\left(2f\left(id(f+gx)^n\right)^{\frac{1}{n}}\right.\right.}{2g^3n}$$

```
[In] Integrate[x^2*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^3)/3 + ((I/2)*b*E^(I*c)*(f + g*x)*(f^2*((-I)*d*(f + g*x)^n)^(2/n)*Gamm
a[n^(-1), (-I)*d*(f + g*x)^n] - (f + g*x)*(2*f*((-I)*d*(f + g*x)^n)^n^(-1)*
Gamma[2/n, (-I)*d*(f + g*x)^n] - (f + g*x)*Gamma[3/n, (-I)*d*(f + g*x)^n]))
)/(g^3*n*((-I)*d*(f + g*x)^n)^(3/n)) - ((I/2)*b*(f + g*x)*(f^2*(I*d*(f + g*
x)^n)^(2/n)*Gamma[n^(-1), I*d*(f + g*x)^n] - (f + g*x)*(2*f*(I*d*(f + g*x)^
n)^n^(-1)*Gamma[2/n, I*d*(f + g*x)^n] - (f + g*x)*Gamma[3/n, I*d*(f + g*x)^
n])))/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^(3/n))
```

Maple [F]

$$\int x^2(a+b\sin(c+d(gx+f)^n))\,dx$$

```
[In] int(x^2*(a+b*sin(c+d*(g*x+f)^n)),x)
[Out] int(x^2*(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))\,dx=\int(b\sin((gx+f)^nd+c)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
[Out] integral(b*x^2*sin((g*x + f)^n*d + c) + a*x^2, x)
```

Sympy [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))\,dx=\int x^2(a+b\sin(c+d(f+gx)^n))\,dx$$

```
[In] integrate(x**2*(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Integral(x**2*(a + b*sin(c + d*(f + g*x)**n)), x)
```

Maxima [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))\,dx=\int(b\sin((gx+f)^nd+c)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
[Out] 1/3*a*x^3 + b*integrate(x^2*sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))\,dx=\int (b\sin((gx+f)^nd+c)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\sin(c+d(f+gx)^n))\,dx=\int x^2\,(a+b\,\sin(c+d\,(f+g\,x)^n))\,dx$$

```
[In] int(x^2*(a + b*sin(c + d*(f + g*x)^n)),x)
```

```
[Out] int(x^2*(a + b*sin(c + d*(f + g*x)^n)), x)
```

3.268 $\int x(a+b\sin(c+d(f+gx)^n))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 255

$$\begin{aligned}
&\int x(a+b\sin(c+d(f+gx)^n))\,dx \\
&= \frac{ax^2}{2}-\frac{ibe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^2n} \\
&\quad+\frac{ibe^{-ic}f(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2g^2n} \\
&\quad+\frac{ibe^{ic}(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^2n} \\
&\quad-\frac{ibe^{-ic}(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{2g^2n}
\end{aligned}$$

```
[Out] 1/2*a*x^2-1/2*I*b*exp(I*c)*f*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^2/n/((-I*d
*(g*x+f)^n)^(1/n))+1/2*I*b*f*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g^2/
n/((I*d*(g*x+f)^n)^(1/n))+1/2*I*b*exp(I*c)*(g*x+f)^2*GAMMA(2/n,-I*d*(g*x+f)
^n)/g^2/n/((-I*d*(g*x+f)^n)^(2/n))-1/2*I*b*(g*x+f)^2*GAMMA(2/n,I*d*(g*x+f)^
n)/exp(I*c)/g^2/n/((I*d*(g*x+f)^n)^(2/n))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {14, 3514, 3446, 2239, 3504, 2250}

$$\begin{aligned}&\int x(a+b\sin(c+d(f+gx)^n))\,dx\\&=\frac{ax^2}{2}+\frac{ibe^{ic}(f+gx)^2(-id(f+gx)^n)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^2n}\\&-\frac{ibe^{ic}f(f+gx)(-id(f+gx)^n)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^2n}\\&+\frac{ibe^{-ic}f(f+gx)(id(f+gx)^n)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2g^2n}\\&-\frac{ibe^{-ic}(f+gx)^2(id(f+gx)^n)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{2g^2n}\end{aligned}$$

```
[In] Int[x*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^2)/2 - ((I/2)*b*E^(I*c)*f*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])
/(g^2*n*((-I)*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*f*(f + g*x)*Gamma[n^(-1), I
*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*E^(I*c
)*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^(
2/n)) - ((I/2)*b*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I
*d*(f + g*x)^n)^(2/n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (ax + bx\sin(c + d(f+gx)^n))\,dx \\
&= \frac{ax^2}{2} + b\int x\sin(c + d(f+gx)^n)\,dx \\
&= \frac{ax^2}{2} + \frac{b\text{Subst}\left(\int (-f\sin(c+dx^n) + x\sin(c+dx^n))\,dx, x, f+gx\right)}{g^2} \\
&= \frac{ax^2}{2} + \frac{b\text{Subst}\left(\int x\sin(c+dx^n)\,dx, x, f+gx\right)}{g^2} - \frac{(bf)\text{Subst}\left(\int \sin(c+dx^n)\,dx, x, f+gx\right)}{g^2} \\
&= \frac{ax^2}{2} + \frac{(ib)\text{Subst}\left(\int e^{-ic-idx^n}x\,dx, x, f+gx\right)}{2g^2} - \frac{(ib)\text{Subst}\left(\int e^{ic+idx^n}x\,dx, x, f+gx\right)}{2g^2} \\
&\quad - \frac{(ibf)\text{Subst}\left(\int e^{-ic-idx^n}\,dx, x, f+gx\right)}{2g^2} + \frac{(ibf)\text{Subst}\left(\int e^{ic+idx^n}\,dx, x, f+gx\right)}{2g^2} \\
&= \frac{ax^2}{2} - \frac{ibe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, -id(f+gx)^n\right)}{2g^2n} \\
&\quad + \frac{ibe^{-ic}f(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, id(f+gx)^n\right)}{2g^2n} \\
&\quad + \frac{ibe^{ic}(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n}, -id(f+gx)^n\right)}{2g^2n} \\
&\quad - \frac{ibe^{-ic}(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n}, id(f+gx)^n\right)}{2g^2n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.84

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\frac{ax^2}{2}+\frac{b(f+gx)\left(-id(f+gx)^n\right)^{-2/n}\left(f(-id(f+gx)^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)-(f+gx)\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)\right)}{2g^2n}+\frac{b(f+gx)\left(id(f+gx)^n\right)^{-2/n}\left(f(id(f+gx)^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)-(f+gx)\Gamma\left(\frac{2}{n},id(f+gx)^n\right)\right)(i\cos(c}{2g^2n}$$

```
[In] Integrate[x*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^2)/2 + (b*(f + g*x)*(f*((-I)*d*(f + g*x)^n)^n^(-1)*Gamma[n^(-1), (-I)*
d*(f + g*x)^n] - (f + g*x)*Gamma[2/n, (-I)*d*(f + g*x)^n])*((-I)*Cos[c] + S
in[c]))/(2*g^2*n*((-I)*d*(f + g*x)^n)^(2/n)) + (b*(f + g*x)*(f*(I*d*(f + g*
x)^n)^n^(-1)*Gamma[n^(-1), I*d*(f + g*x)^n] - (f + g*x)*Gamma[2/n, I*d*(f +
 g*x)^n])*(I*Cos[c] + Sin[c]))/(2*g^2*n*(I*d*(f + g*x)^n)^(2/n))
```

Maple [F]

$$\int x(a+b\sin(c+d(gx+f)^n))\,dx$$

```
[In] int(x*(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(x*(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [F]

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\int(b\sin((gx+f)^nd+c)+a)x\,dx$$

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x*sin((g*x + f)^n*d + c) + a*x, x)
```

Sympy [F]

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\int x(a+b\sin(c+d(f+gx)^n))\,dx$$

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Integral(x*(a + b*sin(c + d*(f + g*x)**n)), x)
```

Maxima [F]

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\int (b\sin((gx+f)^n d+c)+a)x\,dx$$

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + b*integrate(x*sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\int (b\sin((gx+f)^n d+c)+a)x\,dx$$

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\int x\,(a+b\,\sin(c+d\,(f+g\,x)^n))\,dx$$

```
[In] int(x*(a + b*sin(c + d*(f + g*x)^n)),x)
[Out] int(x*(a + b*sin(c + d*(f + g*x)^n)), x)
```

3.269 $\int (a+b\sin(c+d(f+gx)^n))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 122

$$\int (a+b\sin(c+d(f+gx)^n))\,dx=ax+\frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2gn}-\frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2gn}$$

```
[Out] a*x+1/2*I*b*exp(I*c)*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g/n/((-I*d*(g*x+f)^n
)^(1/n))-1/2*I*b*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g/n/((I*d*(g*x+f
)^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3446, 2239}

$$\int (a+b\sin(c+d(f+gx)^n))\,dx=ax+\frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2gn}-\frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2gn}$$

```
[In] Int[a + b*Sin[c + d*(f + g*x)^n],x]
```

```
[Out] a*x + ((I/2)*b*E^(I*c)*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])/(g*n*((
-I)*d*(f + g*x)^n)^n^(-1)) - ((I/2)*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)
^n])/(E^(I*c)*g*n*(I*d*(f + g*x)^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \sin\left(c + d(f+gx)^n\right)\,dx \\
&= ax + \frac{1}{2}(ib)\int e^{-ic-id(f+gx)^n}\,dx - \frac{1}{2}(ib)\int e^{ic+id(f+gx)^n}\,dx \\
&= ax + \frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, -id(f+gx)^n\right)}{2gn} \\
&\quad - \frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, id(f+gx)^n\right)}{2gn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\begin{aligned}
&\int \left(a + b\sin\left(c + d(f+gx)^n\right)\right)\,dx \\
&= ax - \frac{ib(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, id(f+gx)^n\right)(\cos(c) - i\sin(c))}{2gn} \\
&\quad + \frac{ib(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, -id(f+gx)^n\right)(\cos(c) + i\sin(c))}{2gn}
\end{aligned}$$

```
[In] Integrate[a + b*Sin[c + d*(f + g*x)^n],x]
```

```
[Out] a*x - ((I/2)*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n]*(Cos[c] - I*Sin[c])
)/(g*n*(I*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*(f + g*x)*Gamma[n^(-1), (-I)*d*
(f + g*x)^n]*(Cos[c] + I*Sin[c]))/(g*n*((-I)*d*(f + g*x)^n)^n^(-1))
```

Maple [F]

$$\int (a + b\sin(c + d(gx + f)^n))\,dx$$

```
[In] int(a+b*sin(c+d*(g*x+f)^n),x)
[Out] int(a+b*sin(c+d*(g*x+f)^n),x)
```

Fricas [F]

$$\int (a + b\sin(c + d(f + gx)^n))\ dx = \int b\sin((gx + f)^n d + c) + a\,dx$$

```
[In] integrate(a+b*sin(c+d*(g*x+f)^n),x, algorithm="fricas")
[Out] integral(b*sin((g*x + f)^n*d + c) + a, x)
```

Sympy [F]

$$\int (a + b\sin(c + d(f + gx)^n))\ dx = \int (a + b\sin(c + d(f + gx)^n))\ dx$$

```
[In] integrate(a+b*sin(c+d*(g*x+f)**n),x)
[Out] Integral(a + b*sin(c + d*(f + g*x)**n), x)
```

Maxima [F]

$$\int (a + b\sin(c + d(f + gx)^n))\ dx = \int b\sin((gx + f)^n d + c) + a\,dx$$

```
[In] integrate(a+b*sin(c+d*(g*x+f)^n),x, algorithm="maxima")
[Out] a*x + b*integrate(sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int (a + b\sin(c + d(f + gx)^n))\,dx = \int b\sin((gx + f)^n d + c) + a\,dx$$

```
[In] integrate(a+b*sin(c+d*(g*x+f)^n),x, algorithm="giac")
[Out] integrate(b*sin((g*x + f)^n*d + c) + a, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\sin(c + d(f + gx)^n))\,dx = \int a + b\,\sin\left(c + d\,(f + g\,x)^n\right)\,dx$$

```
[In] int(a + b*sin(c + d*(f + g*x)^n),x)
[Out] int(a + b*sin(c + d*(f + g*x)^n), x)
```

3.270 $\int \frac{a+b\sin(c+d(f+gx)^n)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx = a\log(x)+b\text{Int}\left(\frac{\sin\left(c+d(f+gx)^n\right)}{x},x\right)$$

```
[Out] a*ln(x)+b*Unintegrable(sin(c+d*(g*x+f)^n)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx = \int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx$$

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])/x,x]
[Out] a*Log[x] + b*Defer[Int][Sin[c + d*(f + g*x)^n]/x, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a}{x}+\frac{b\sin\left(c+d(f+gx)^n\right)}{x}\right)dx\\ &= a\log(x)+b\int \frac{\sin\left(c+d(f+gx)^n\right)}{x}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx=\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx$$

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x,x]
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sin\left(c+d(gx+f)^n\right)}{x}dx$$

```
[In] int((a+b*sin(c+d*(g*x+f)^n))/x,x)
[Out] int((a+b*sin(c+d*(g*x+f)^n))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx=\int \frac{b\sin\left((gx+f)^n d+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x,x, algorithm="fricas")
[Out] integral((b*sin((g*x + f)^n*d + c) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx=\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))/x,x)
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\sin(c+d(f+gx)^n)}{x}\,dx=\int \frac{b\sin\left(\left(gx+f\right)^n d+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x,x, algorithm="maxima")
[Out] b*integrate(sin((g*x + f)^n*d + c)/x, x) + a*log(x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin(c+d(f+gx)^n)}{x}\,dx=\int \frac{b\sin\left(\left(gx+f\right)^n d+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x,x, algorithm="giac")
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 6.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin(c+d(f+gx)^n)}{x}\,dx=\int \frac{a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)}{x}\,dx$$

```
[In] int((a + b*sin(c + d*(f + g*x)^n))/x,x)
[Out] int((a + b*sin(c + d*(f + g*x)^n))/x, x)
```

3.271 $\int \frac{a+b\sin(c+d(f+gx)^n)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx=-\frac{a}{x}+b\text{Int}\left(\frac{\sin\left(c+d(f+gx)^n\right)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(sin(c+d*(g*x+f)^n)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx=\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx$$

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])/x^2,x]
```

```
[Out] -(a/x) + b*Defer[Int][Sin[c + d*(f + g*x)^n]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^2}+\frac{b\sin\left(c+d(f+gx)^n\right)}{x^2}\right)\,dx\\&=-\frac{a}{x}+b\int \frac{\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx=\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x^2,x]
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sin\left(c+d(gx+f)^n\right)}{x^2}dx$$

```
[In] int((a+b*sin(c+d*(g*x+f)^n))/x^2,x)
[Out] int((a+b*sin(c+d*(g*x+f)^n))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx=\int \frac{b\sin\left((gx+f)^n d+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x^2,x, algorithm="fricas")
[Out] integral((b*sin((g*x + f)^n*d + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 19.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx=\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))/x**2,x)
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{a + b\sin\left(c + d(f + gx)^n\right)}{x^2}\,dx = \int \frac{b\sin\left((gx + f)^n d + c\right) + a}{x^2}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x^2,x, algorithm="maxima")
[Out] b*integrate(sin((g*x + f)^n*d + c)/x^2, x) - a/x
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a + b\sin\left(c + d(f + gx)^n\right)}{x^2}\,dx = \int \frac{b\sin\left((gx + f)^n d + c\right) + a}{x^2}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x^2,x, algorithm="giac")
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a + b\sin\left(c + d(f + gx)^n\right)}{x^2}\,dx = \int \frac{a + b\,\sin\left(c + d\,{\left(f + g\,x\right)}^n\right)}{x^2}\,dx$$

```
[In] int((a + b*sin(c + d*(f + g*x)^n))/x^2,x)
[Out] int((a + b*sin(c + d*(f + g*x)^n))/x^2, x)
```

3.272 $\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 856

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx$$
$$= \frac{(2a^2+b^2)\,f^2x}{2g^2} - \frac{(2a^2+b^2)\,f(f+gx)^2}{2g^3} + \frac{(2a^2+b^2)\,(f+gx)^3}{6g^3}$$
$$+ \frac{iabe^{ic}f^2(f+gx)\,(-id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{g^3n}$$
$$- \frac{iabe^{-ic}f^2(f+gx)\,(id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{g^3n}$$
$$+ \frac{2^{-2-\frac{1}{n}}b^2e^{2ic}f^2(f+gx)\,(-id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{g^3n}$$
$$+ \frac{2^{-2-\frac{1}{n}}b^2e^{-2ic}f^2(f+gx)\,(id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{g^3n}$$
$$- \frac{2iabe^{ic}f(f+gx)^2\,(-id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{g^3n}$$
$$+ \frac{2iabe^{-ic}f(f+gx)^2\,(id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{g^3n}$$
$$- \frac{2^{-1-\frac{2}{n}}b^2e^{2ic}f(f+gx)^2\,(-id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-2id(f+gx)^n\right)}{g^3n}$$
$$- \frac{2^{-1-\frac{2}{n}}b^2e^{-2ic}f(f+gx)^2\,(id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},2id(f+gx)^n\right)}{g^3n}$$
$$+ \frac{iabe^{ic}(f+gx)^3\,(-id(f+gx)^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)}{g^3n}$$
$$- \frac{iabe^{-ic}(f+gx)^3\,(id(f+gx)^n)^{-3/n}\,\Gamma\left(\frac{3}{n},id(f+gx)^n\right)}{g^3n}$$
$$+ \frac{2^{-2-\frac{3}{n}}b^2e^{2ic}(f+gx)^3\,(-id(f+gx)^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-2id(f+gx)^n\right)}{g^3n}$$
$$+ \frac{2^{-2-\frac{3}{n}}b^2e^{-2ic}(f+gx)^3\,(id(f+gx)^n)^{-3/n}\,\Gamma\left(\frac{3}{n},2id(f+gx)^n\right)}{g^3n}$$

```
[Out] 1/2*(2*a^2+b^2)*f^2*x/g^2-1/2*(2*a^2+b^2)*f*(g*x+f)^2/g^3+1/6*(2*a^2+b^2)*(
g*x+f)^3/g^3+I*a*b*exp(I*c)*f^2*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^3/n/((-
I*d*(g*x+f)^n)^(1/n))-I*a*b*f^2*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g
^3/n/((I*d*(g*x+f)^n)^(1/n))+2^(-2-1/n)*b^2*exp(2*I*c)*f^2*(g*x+f)*GAMMA(1/
n,-2*I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(1/n))+2^(-2-1/n)*b^2*f^2*(g*x+
f)*GAMMA(1/n,2*I*d*(g*x+f)^n)/exp(2*I*c)/g^3/n/((I*d*(g*x+f)^n)^(1/n))-2*I*
a*b*exp(I*c)*f*(g*x+f)^2*GAMMA(2/n,-I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^
(2/n))+2*I*a*b*f*(g*x+f)^2*GAMMA(2/n,I*d*(g*x+f)^n)/exp(I*c)/g^3/n/((I*d*(g
```

```
*x+f)^n)^(2/n))-2^(-1-2/n)*b^2*exp(2*I*c)*f*(g*x+f)^2*GAMMA(2/n,-2*I*d*(g*x
+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(2/n))-2^(-1-2/n)*b^2*f*(g*x+f)^2*GAMMA(2/n,
2*I*d*(g*x+f)^n)/exp(2*I*c)/g^3/n/((I*d*(g*x+f)^n)^(2/n))+I*a*b*exp(I*c)*(g
*x+f)^3*GAMMA(3/n,-I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(3/n))-I*a*b*(g*x
+f)^3*GAMMA(3/n,I*d*(g*x+f)^n)/exp(I*c)/g^3/n/((I*d*(g*x+f)^n)^(3/n))+2^(-2
-3/n)*b^2*exp(2*I*c)*(g*x+f)^3*GAMMA(3/n,-2*I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*
x+f)^n)^(3/n))+2^(-2-3/n)*b^2*(g*x+f)^3*GAMMA(3/n,2*I*d*(g*x+f)^n)/exp(2*I*
c)/g^3/n/((I*d*(g*x+f)^n)^(3/n))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules

used = {3514, 3448, 3447, 2239, 3446, 3506, 6, 3505, 2250, 3504}

$$\begin{aligned}
\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx
&= \frac{iabe^{ic}(f+gx)^3\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)(-id(f+gx)^n)^{-3/n}}{g^3n} \\
&+ \frac{2^{-2-\frac{3}{n}}b^2e^{2ic}(f+gx)^3\Gamma\left(\frac{3}{n},-2id(f+gx)^n\right)(-id(f+gx)^n)^{-3/n}}{g^3n} \\
&- \frac{2iabe^{ic}f(f+gx)^2\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)(-id(f+gx)^n)^{-2/n}}{g^3n} \\
&- \frac{2^{-1-\frac{2}{n}}b^2e^{2ic}f(f+gx)^2\Gamma\left(\frac{2}{n},-2id(f+gx)^n\right)(-id(f+gx)^n)^{-2/n}}{g^3n} \\
&+ \frac{iabe^{ic}f^2(f+gx)\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)(-id(f+gx)^n)^{-1/n}}{g^3n} \\
&+ \frac{2^{-2-\frac{1}{n}}b^2e^{2ic}f^2(f+gx)\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)(-id(f+gx)^n)^{-1/n}}{g^3n} \\
&+ \frac{(2a^2+b^2)(f+gx)^3}{6g^3} - \frac{(2a^2+b^2)f(f+gx)^2}{2g^3} + \frac{(2a^2+b^2)f^2x}{2g^2} \\
&- \frac{iabe^{-ic}f^2(f+gx)(id(f+gx)^n)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2^{-2-\frac{1}{n}}b^2e^{-2ic}f^2(f+gx)(id(f+gx)^n)^{-1/n}\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2iabe^{-ic}f(f+gx)^2(id(f+gx)^n)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{g^3n} \\
&- \frac{2^{-1-\frac{2}{n}}b^2e^{-2ic}f(f+gx)^2(id(f+gx)^n)^{-2/n}\Gamma\left(\frac{2}{n},2id(f+gx)^n\right)}{g^3n} \\
&- \frac{iabe^{-ic}(f+gx)^3(id(f+gx)^n)^{-3/n}\Gamma\left(\frac{3}{n},id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2^{-2-\frac{3}{n}}b^2e^{-2ic}(f+gx)^3(id(f+gx)^n)^{-3/n}\Gamma\left(\frac{3}{n},2id(f+gx)^n\right)}{g^3n}
\end{aligned}$$

```
[In] Int[x^2*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] ((2*a^2 + b^2)*f^2*x)/(2*g^2) - ((2*a^2 + b^2)*f*(f + g*x)^2)/(2*g^3) + ((2
*a^2 + b^2)*(f + g*x)^3)/(6*g^3) + (I*a*b*E^(I*c)*f^2*(f + g*x)*Gamma[n^(-1
), (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) - (I*a*b*f^2*(f
 + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^n^
(-1)) + (2^(-2 - n^(-1))*b^2*E^((2*I)*c)*f^2*(f + g*x)*Gamma[n^(-1), (-2*I)
*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*b^2
*f^2*(f + g*x)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^3*n*(I*d*
(f + g*x)^n)^n^(-1)) - ((2*I)*a*b*E^(I*c)*f*(f + g*x)^2*Gamma[2/n, (-I)*d*(
```

```
f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(2/n)) + ((2*I)*a*b*f*(f + g*x)^2*
Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^(2/n)) - (2^(
-1 - 2/n)*b^2*E^((2*I)*c)*f*(f + g*x)^2*Gamma[2/n, (-2*I)*d*(f + g*x)^n])/(
g^3*n*((-I)*d*(f + g*x)^n)^(2/n)) - (2^(-1 - 2/n)*b^2*f*(f + g*x)^2*Gamma[2
/n, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^3*n*(I*d*(f + g*x)^n)^(2/n)) + (I*
a*b*E^(I*c)*(f + g*x)^3*Gamma[3/n, (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f +
 g*x)^n)^(3/n)) - (I*a*b*(f + g*x)^3*Gamma[3/n, I*d*(f + g*x)^n])/(E^(I*c)*
g^3*n*(I*d*(f + g*x)^n)^(3/n)) + (2^(-2 - 3/n)*b^2*E^((2*I)*c)*(f + g*x)^3*
Gamma[3/n, (-2*I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(3/n)) + (2^(
-2 - 3/n)*b^2*(f + g*x)^3*Gamma[3/n, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^3
*n*(I*d*(f + g*x)^n)^(3/n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3448

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

integral

$$=\frac{\text{Subst}\left(\int \left(f^2(a+b\sin(c+dx^n))^2-2fx(a+b\sin(c+dx^n))^2+x^2(a+b\sin(c+dx^n))^2\right)\,dx,x,f+gx\right)}{g^3}$$

$$=\frac{\text{Subst}\left(\int x^2(a+b\sin(c+dx^n))^2\,dx,x,f+gx\right)}{g^3}-\frac{(2f)\text{Subst}\left(\int x(a+b\sin(c+dx^n))^2\,dx,x,f+gx\right)}{g^3}+\frac{f^2\text{Subst}\left(\int (a+b\sin(c+dx^n))^2\,dx,x,f+gx\right)}{g^3}$$

$$=\frac{\text{Subst}\left(\int \left(a^2x^2+\frac{b^2x^2}{2}-\frac{1}{2}b^2x^2\cos(2c+2dx^n)+2abx^2\sin(c+dx^n)\right)\,dx,x,f+gx\right)}{g^3}-\frac{(2f)\text{Subst}\left(\int \left(a^2x+\frac{b^2x}{2}-\frac{1}{2}b^2x\cos(2c+2dx^n)+2abx\sin(c+dx^n)\right)\,dx,x,f+gx\right)}{g^3}+\frac{f^2\text{Subst}\left(\int \left(a^2+\frac{b^2}{2}-\frac{1}{2}b^2\cos(2c+2dx^n)+2ab\sin(c+dx^n)\right)\,dx,x,f+gx\right)}{g^3}$$

$$= \frac{(2a^2+b^2) f^2 x}{2g^2} + \frac{\text{Subst}\left(\int \left(\left(a^2+\frac{b^2}{2}\right) x^2 - \frac{1}{2}b^2x^2\cos(2c+2dx^n) + 2abx^2\sin(c+dx^n)\right) dx, x, f+gx\right)}{g^3} - \frac{(2f)\text{Subst}\left(\int \left(\left(a^2+\frac{b^2}{2}\right) x - \frac{1}{2}b^2x\cos(2c+2dx^n) + 2abx\sin(c+dx^n)\right) dx, x, f+gx\right)}{g^3} + \frac{(2abf^2)\,\text{Subst}\left(\int \sin(c+dx^n)\, dx, x, f+gx\right)}{g^3} - \frac{(b^2f^2)\,\text{Subst}\left(\int \cos(2c+2dx^n)\, dx, x, f+gx\right)}{2g^3}$$

$$= \frac{(2a^2+b^2) f^2 x}{2g^2} - \frac{(2a^2+b^2) f(f+gx)^2}{2g^3} + \frac{(2a^2+b^2)(f+gx)^3}{6g^3} + \frac{(2ab)\text{Subst}\left(\int x^2\sin(c+dx^n)\, dx, x, f+gx\right)}{g^3} - \frac{b^2\text{Subst}\left(\int x^2\cos(2c+2dx^n)\, dx, x, f+gx\right)}{2g^3} - \frac{(4abf)\text{Subst}\left(\int x\sin(c+dx^n)\, dx, x, f+gx\right)}{g^3} + \frac{(b^2f)\,\text{Subst}\left(\int x\cos(2c+2dx^n)\, dx, x, f+gx\right)}{g^3} + \frac{(iabf^2)\,\text{Subst}\left(\int e^{-ic-idx^n}\, dx, x, f+gx\right)}{g^3} - \frac{(iabf^2)\,\text{Subst}\left(\int e^{ic+idx^n}\, dx, x, f+gx\right)}{g^3} - \frac{(b^2f^2)\,\text{Subst}\left(\int e^{-2ic-2idx^n}\, dx, x, f+gx\right)}{4g^3} - \frac{(b^2f^2)\,\text{Subst}\left(\int e^{2ic+2idx^n}\, dx, x, f+gx\right)}{4g^3}$$

$$
\begin{aligned}
&= \frac{(2a^2+b^2)\,f^2x}{2g^2} - \frac{(2a^2+b^2)\,f(f+gx)^2}{2g^3} + \frac{(2a^2+b^2)\,(f+gx)^3}{6g^3} \\
&+ \frac{iabe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{g^3n} \\
&- \frac{iabe^{-ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2^{-2-\frac{1}{n}}b^2e^{2ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2^{-2-\frac{1}{n}}b^2e^{-2ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{g^3n} \\
&+ \frac{(iab)\mathrm{Subst}\left(\int e^{-ic-idx^n}x^2\,dx,x,f+gx\right)}{g^3} - \frac{(iab)\mathrm{Subst}\left(\int e^{ic+idx^n}x^2\,dx,x,f+gx\right)}{g^3} \\
&- \frac{b^2\mathrm{Subst}\left(\int e^{-2ic-2idx^n}x^2\,dx,x,f+gx\right)}{4g^3} - \frac{b^2\mathrm{Subst}\left(\int e^{2ic+2idx^n}x^2\,dx,x,f+gx\right)}{4g^3} \\
&- \frac{(2iabf)\mathrm{Subst}\left(\int e^{-ic-idx^n}x\,dx,x,f+gx\right)}{g^3} \\
&+ \frac{(2iabf)\mathrm{Subst}\left(\int e^{ic+idx^n}x\,dx,x,f+gx\right)}{g^3} \\
&+ \frac{(b^2f)\,\mathrm{Subst}\left(\int e^{-2ic-2idx^n}x\,dx,x,f+gx\right)}{2g^3} \\
&+ \frac{(b^2f)\,\mathrm{Subst}\left(\int e^{2ic+2idx^n}x\,dx,x,f+gx\right)}{2g^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(2a^2+b^2)\,f^2x}{2g^2} - \frac{(2a^2+b^2)\,f(f+gx)^2}{2g^3} + \frac{(2a^2+b^2)\,(f+gx)^3}{6g^3} \\
&+ \frac{iabe^{ic}f^2(f+gx)\,(-id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{g^3n} \\
&- \frac{iabe^{-ic}f^2(f+gx)\,(id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2^{-2-\frac{1}{n}}b^2e^{2ic}f^2(f+gx)\,(-id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2^{-2-\frac{1}{n}}b^2e^{-2ic}f^2(f+gx)\,(id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{g^3n} \\
&- \frac{2iabe^{ic}f(f+gx)^2\,(-id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2iabe^{-ic}f(f+gx)^2\,(id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{g^3n} \\
&- \frac{2^{-1-\frac{2}{n}}b^2e^{2ic}f(f+gx)^2\,(-id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-2id(f+gx)^n\right)}{g^3n} \\
&- \frac{2^{-1-\frac{2}{n}}b^2e^{-2ic}f(f+gx)^2\,(id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},2id(f+gx)^n\right)}{g^3n} \\
&+ \frac{iabe^{ic}(f+gx)^3\,(-id(f+gx)^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)}{g^3n} \\
&- \frac{iabe^{-ic}(f+gx)^3\,(id(f+gx)^n)^{-3/n}\,\Gamma\left(\frac{3}{n},id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2^{-2-\frac{3}{n}}b^2e^{2ic}(f+gx)^3\,(-id(f+gx)^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-2id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2^{-2-\frac{3}{n}}b^2e^{-2ic}(f+gx)^3\,(id(f+gx)^n)^{-3/n}\,\Gamma\left(\frac{3}{n},2id(f+gx)^n\right)}{g^3n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 648, normalized size of antiderivative = 0.76

$$
\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx
$$

$$
= \frac{(2a^2+b^2)\,g^3nx^3+6iabe^{ic}(f+gx)\,(-id(f+gx)^n)^{-3/n}\left(f^2(-id(f+gx)^n)^{2/n}\,\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)-(f+
$$

```
[In] Integrate[x^2*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] ((2*a^2 + b^2)*g^3*n*x^3 + ((6*I)*a*b*E^(I*c)*(f + g*x)*(f^2*((-I)*d*(f + g
*x)^n)^(2/n)*Gamma[n^(-1), (-I)*d*(f + g*x)^n] - (f + g*x)*(2*f*((-I)*d*(f
```

```
+ g*x)^n)^n^(-1)*Gamma[2/n, (-I)*d*(f + g*x)^n] - (f + g*x)*Gamma[3/n, (-I)
*d*(f + g*x)^n])))/((-I)*d*(f + g*x)^n)^(3/n) - ((6*I)*a*b*(f + g*x)*(f^2*(
I*d*(f + g*x)^n)^(2/n)*Gamma[n^(-1), I*d*(f + g*x)^n] - (f + g*x)*(2*f*(I*d
*(f + g*x)^n)^n^(-1)*Gamma[2/n, I*d*(f + g*x)^n] - (f + g*x)*Gamma[3/n, I*d
*(f + g*x)^n])))/(E^(I*c)*(I*d*(f + g*x)^n)^(3/n)) + (3*b^2*E^((2*I)*c)*(f
+ g*x)*(4^n^(-1)*f^2*((-I)*d*(f + g*x)^n)^(2/n)*Gamma[n^(-1), (-2*I)*d*(f +
 g*x)^n] - (f + g*x)*(2^(1 + n^(-1))*f*((-I)*d*(f + g*x)^n)^n^(-1)*Gamma[2/
n, (-2*I)*d*(f + g*x)^n] - (f + g*x)*Gamma[3/n, (-2*I)*d*(f + g*x)^n])))/(2
^((3 + n)/n)*((-I)*d*(f + g*x)^n)^(3/n)) + (3*b^2*(f + g*x)*(4^n^(-1)*f^2*(
I*d*(f + g*x)^n)^(2/n)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n] - (f + g*x)*(2^(1
 + n^(-1))*f*(I*d*(f + g*x)^n)^n^(-1)*Gamma[2/n, (2*I)*d*(f + g*x)^n] - (f
+ g*x)*Gamma[3/n, (2*I)*d*(f + g*x)^n])))/(2^((3 + n)/n)*E^((2*I)*c)*(I*d*(
f + g*x)^n)^(3/n)))/(6*g^3*n)
```

Maple [F]

$$\int x^2(a+b\sin(c+d(gx+f)^n))^2dx$$

[In] int(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x)

[Out] int(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x)

Fricas [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx=\int(b\sin((gx+f)^nd+c)+a)^2x^2\,dx$$

[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")

[Out] integral(-b^2*x^2*cos((g*x + f)^n*d + c)^2 + 2*a*b*x^2*sin((g*x + f)^n*d + c) + (a^2 + b^2)*x^2, x)

Sympy [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx=\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx$$

[In] integrate(x**2*(a+b*sin(c+d*(g*x+f)**n))**2,x)

[Out] Integral(x**2*(a + b*sin(c + d*(f + g*x)**n))**2, x)

Maxima [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx = \int (b\sin((gx+f)^n d+c)+a)^2 x^2\,dx$$

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
[Out] 1/3*a^2*x^3 + 1/6*b^2*x^3 - 1/2*b^2*integrate(x^2*cos(2*(g*x + f)^n*d + 2*c
), x) + 2*a*b*integrate(x^2*sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx = \int (b\sin((gx+f)^n d+c)+a)^2 x^2\,dx$$

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx = \int x^2\,(a+b\,\sin(c+d\,(f+g\,x)^n))^2\,dx$$

```
[In] int(x^2*(a + b*sin(c + d*(f + g*x)^n))^2,x)
[Out] int(x^2*(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

3.273 $\int x(a+b\sin(c+d(f+gx)^n))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 556

$$\begin{aligned}
&\int x(a+b\sin(c+d(f+gx)^n))^2\,dx \\
&= -\frac{(2a^2+b^2)\,fx}{2g}+\frac{(2a^2+b^2)\,(f+gx)^2}{4g^2} \\
&\quad -\frac{iabe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{g^2n} \\
&\quad +\frac{iabe^{-ic}f(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{g^2n} \\
&\quad -\frac{2^{-2-\frac{1}{n}}b^2e^{2ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{g^2n} \\
&\quad -\frac{2^{-2-\frac{1}{n}}b^2e^{-2ic}f(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{g^2n} \\
&\quad +\frac{iabe^{ic}(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{g^2n} \\
&\quad -\frac{iabe^{-ic}(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{g^2n} \\
&\quad +\frac{4^{-1-\frac{1}{n}}b^2e^{2ic}(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-2id(f+gx)^n\right)}{g^2n} \\
&\quad +\frac{4^{-1-\frac{1}{n}}b^2e^{-2ic}(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},2id(f+gx)^n\right)}{g^2n}
\end{aligned}$$

```
[Out] -1/2*(2*a^2+b^2)*f*x/g+1/4*(2*a^2+b^2)*(g*x+f)^2/g^2-I*a*b*exp(I*c)*f*(g*x+
f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^2/n/((-I*d*(g*x+f)^n)^(1/n))+I*a*b*f*(g*x+f)
*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g^2/n/((I*d*(g*x+f)^n)^(1/n))-2^(-2-1/n)
```

```
*b^2*exp(2*I*c)*f*(g*x+f)*GAMMA(1/n,-2*I*d*(g*x+f)^n)/g^2/n/((-I*d*(g*x+f)^
n)^(1/n))-2^(-2-1/n)*b^2*f*(g*x+f)*GAMMA(1/n,2*I*d*(g*x+f)^n)/exp(2*I*c)/g^
2/n/((I*d*(g*x+f)^n)^(1/n))+I*a*b*exp(I*c)*(g*x+f)^2*GAMMA(2/n,-I*d*(g*x+f)
^n)/g^2/n/((-I*d*(g*x+f)^n)^(2/n))-I*a*b*(g*x+f)^2*GAMMA(2/n,I*d*(g*x+f)^n)
/exp(I*c)/g^2/n/((I*d*(g*x+f)^n)^(2/n))+4^(-1-1/n)*b^2*exp(2*I*c)*(g*x+f)^2
*GAMMA(2/n,-2*I*d*(g*x+f)^n)/g^2/n/((-I*d*(g*x+f)^n)^(2/n))+4^(-1-1/n)*b^2*
(g*x+f)^2*GAMMA(2/n,2*I*d*(g*x+f)^n)/exp(2*I*c)/g^2/n/((I*d*(g*x+f)^n)^(2/n
))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3514, 3448, 3447, 2239, 3446, 3506, 6, 3505, 2250, 3504}

$$\begin{aligned}
&\int x(a+b\sin(c+d(f+gx)^n))^2\,dx \\
&= \frac{(2a^2+b^2)(f+gx)^2}{4g^2} - \frac{fx(2a^2+b^2)}{2g} \\
&+ \frac{iabe^{ic}(f+gx)^2(-id(f+gx)^n)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{g^2n} \\
&- \frac{iabe^{ic}f(f+gx)(-id(f+gx)^n)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{g^2n} \\
&+ \frac{iabe^{-ic}f(f+gx)(id(f+gx)^n)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{g^2n} \\
&- \frac{iabe^{-ic}(f+gx)^2(id(f+gx)^n)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{g^2n} \\
&+ \frac{b^2e^{2ic}4^{-\frac{1}{n}-1}(f+gx)^2(-id(f+gx)^n)^{-2/n}\Gamma\left(\frac{2}{n},-2id(f+gx)^n\right)}{g^2n} \\
&- \frac{b^2e^{2ic}f2^{-\frac{1}{n}-2}(f+gx)(-id(f+gx)^n)^{-1/n}\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{g^2n} \\
&- \frac{b^2e^{-2ic}f2^{-\frac{1}{n}-2}(f+gx)(id(f+gx)^n)^{-1/n}\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{g^2n} \\
&+ \frac{b^2e^{-2ic}4^{-\frac{1}{n}-1}(f+gx)^2(id(f+gx)^n)^{-2/n}\Gamma\left(\frac{2}{n},2id(f+gx)^n\right)}{g^2n}
\end{aligned}$$

```
[In] Int[x*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] -1/2*((2*a^2 + b^2)*f*x)/g + ((2*a^2 + b^2)*(f + g*x)^2)/(4*g^2) - (I*a*b*E
^(I*c)*f*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g
*x)^n)^n^(-1)) + (I*a*b*f*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n])/(E^(I*c
)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) - (2^(-2 - n^(-1))*b^2*E^((2*I)*c)*f*(f +
```

```
g*x)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^n^(-
1)) - (2^(-2 - n^(-1))*b^2*f*(f + g*x)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n])/
(E^((2*I)*c)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) + (I*a*b*E^(I*c)*(f + g*x)^2*G
amma[2/n, (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^(2/n)) - (I*a*b*
(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I*d*(f + g*x)^n)^(
2/n)) + (4^(-1 - n^(-1))*b^2*E^((2*I)*c)*(f + g*x)^2*Gamma[2/n, (-2*I)*d*(f
 + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^(2/n)) + (4^(-1 - n^(-1))*b^2*(f +
g*x)^2*Gamma[2/n, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^2*n*(I*d*(f + g*x)^n
)^(2/n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3448

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(-f(a+b\sin(c+dx^n))^2 + x(a+b\sin(c+dx^n))^2\right)\,dx, x, f+gx\right)}{g^2}$$

$$= \frac{\text{Subst}\left(\int x(a+b\sin(c+dx^n))^2\,dx, x, f+gx\right)}{g^2} - \frac{f\text{Subst}\left(\int (a+b\sin(c+dx^n))^2\,dx, x, f+gx\right)}{g^2}$$

$$= \frac{\text{Subst}\left(\int \left(a^2x + \frac{b^2x}{2} - \frac{1}{2}b^2x\cos(2c+2dx^n) + 2abx\sin(c+dx^n)\right)\,dx, x, f+gx\right)}{g^2} - \frac{f\text{Subst}\left(\int \left(a^2 + \frac{b^2}{2} - \frac{1}{2}b^2\cos(2c+2dx^n) + 2ab\sin(c+dx^n)\right)\,dx, x, f+gx\right)}{g^2}$$

$$= -\frac{(2a^2+b^2)\,fx}{2g} + \frac{\text{Subst}\left(\int \left(\left(a^2+\frac{b^2}{2}\right)x - \frac{1}{2}b^2x\cos(2c+2dx^n) + 2abx\sin(c+dx^n)\right)\,dx, x, f+gx\right)}{g^2} - \frac{(2abf)\text{Subst}\left(\int \sin(c+dx^n)\,dx, x, f+gx\right)}{g^2} + \frac{(b^2f)\,\text{Subst}\left(\int \cos(2c+2dx^n)\,dx, x, f+gx\right)}{2g^2}$$

$$= -\frac{(2a^2+b^2)\,fx}{2g} + \frac{(2a^2+b^2)\,(f+gx)^2}{4g^2} + \frac{(2ab)\text{Subst}\left(\int x\sin(c+dx^n)\,dx, x, f+gx\right)}{g^2} - \frac{b^2\text{Subst}\left(\int x\cos(2c+2dx^n)\,dx, x, f+gx\right)}{2g^2} - \frac{(iabf)\text{Subst}\left(\int e^{-ic-idx^n}\,dx, x, f+gx\right)}{g^2} + \frac{(iabf)\text{Subst}\left(\int e^{ic+idx^n}\,dx, x, f+gx\right)}{g^2} + \frac{(b^2f)\,\text{Subst}\left(\int e^{-2ic-2idx^n}\,dx, x, f+gx\right)}{4g^2} + \frac{(b^2f)\,\text{Subst}\left(\int e^{2ic+2idx^n}\,dx, x, f+gx\right)}{4g^2}$$

$$= -\frac{(2a^2+b^2)\,fx}{2g} + \frac{(2a^2+b^2)\,(f+gx)^2}{4g^2} - \frac{iabe^{ic}f(f+gx)\,(-id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n}, -id(f+gx)^n\right)}{g^2n} + \frac{iabe^{-ic}f(f+gx)\,(id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n}, id(f+gx)^n\right)}{g^2n} - \frac{2^{-2-\frac{1}{n}}b^2e^{2ic}f(f+gx)\,(-id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n}, -2id(f+gx)^n\right)}{g^2n} - \frac{2^{-2-\frac{1}{n}}b^2e^{-2ic}f(f+gx)\,(id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n}, 2id(f+gx)^n\right)}{g^2n} + \frac{(iab)\text{Subst}\left(\int e^{-ic-idx^n}x\,dx, x, f+gx\right)}{g^2} - \frac{(iab)\text{Subst}\left(\int e^{ic+idx^n}x\,dx, x, f+gx\right)}{g^2} - \frac{b^2\text{Subst}\left(\int e^{-2ic-2idx^n}x\,dx, x, f+gx\right)}{4g^2} - \frac{b^2\text{Subst}\left(\int e^{2ic+2idx^n}x\,dx, x, f+gx\right)}{4g^2}$$

$$
\begin{aligned}
&= -\frac{(2a^2+b^2)\,fx}{2g} + \frac{(2a^2+b^2)\,(f+gx)^2}{4g^2} \\
&- \frac{iabe^{ic}f(f+gx)\,(-id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{g^2n} \\
&+ \frac{iabe^{-ic}f(f+gx)\,(id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{g^2n} \\
&- \frac{2^{-2-\frac{1}{n}}b^2e^{2ic}f(f+gx)\,(-id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{g^2n} \\
&- \frac{2^{-2-\frac{1}{n}}b^2e^{-2ic}f(f+gx)\,(id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{g^2n} \\
&+ \frac{iabe^{ic}(f+gx)^2\,(-id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{g^2n} \\
&- \frac{iabe^{-ic}(f+gx)^2\,(id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{g^2n} \\
&+ \frac{4^{-1-\frac{1}{n}}b^2e^{2ic}(f+gx)^2\,(-id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-2id(f+gx)^n\right)}{g^2n} \\
&+ \frac{4^{-1-\frac{1}{n}}b^2e^{-2ic}(f+gx)^2\,(id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},2id(f+gx)^n\right)}{g^2n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.78

$$
\int x(a+b\sin(c+d(f+gx)^n))^2\,dx
$$

$$
= \frac{(2a^2+b^2)\,g^2nx^2 - 4^{-1/n}b^2e^{2ic}(f+gx)\,(-id(f+gx)^n)^{-2/n}\left(2^{\frac{1}{n}}f(-id(f+gx)^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right) - \right.}{}
$$

```
[In] Integrate[x*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] ((2*a^2 + b^2)*g^2*n*x^2 - (b^2*E^((2*I)*c)*(f + g*x)*(2^n^(-1)*f*((-I)*d*(
f + g*x)^n)^n^(-1)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n] - (f + g*x)*Gamma[2/
n, (-2*I)*d*(f + g*x)^n]))/(4^n^(-1)*((-I)*d*(f + g*x)^n)^(2/n)) - (b^2*(f
+ g*x)*(2^n^(-1)*f*(I*d*(f + g*x)^n)^n^(-1)*Gamma[n^(-1), (2*I)*d*(f + g*x)
^n] - (f + g*x)*Gamma[2/n, (2*I)*d*(f + g*x)^n]))/(4^n^(-1)*E^((2*I)*c)*(I*
d*(f + g*x)^n)^(2/n)) + (4*a*b*(f + g*x)*(f*((-I)*d*(f + g*x)^n)^n^(-1)*Gam
ma[n^(-1), (-I)*d*(f + g*x)^n] - (f + g*x)*Gamma[2/n, (-I)*d*(f + g*x)^n])*
((-I)*Cos[c] + Sin[c]))/((-I)*d*(f + g*x)^n)^(2/n) + (4*a*b*(f + g*x)*(f*(I
*d*(f + g*x)^n)^n^(-1)*Gamma[n^(-1), I*d*(f + g*x)^n] - (f + g*x)*Gamma[2/n
, I*d*(f + g*x)^n])*(I*Cos[c] + Sin[c]))/(I*d*(f + g*x)^n)^(2/n))/(4*g^2*n)
```

Maple [F]

$$\int x(a+b\sin\left(c+d(gx+f)^n\right))^2dx$$

```
[In] int(x*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(x*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [F]

$$\int x(a+b\sin\left(c+d(f+gx)^n\right))^2\,dx=\int\left(b\sin\left((gx+f)^nd+c\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-b^2*x*cos((g*x + f)^n*d + c)^2 + 2*a*b*x*sin((g*x + f)^n*d + c) +
 (a^2 + b^2)*x, x)
```

Sympy [F]

$$\int x(a+b\sin\left(c+d(f+gx)^n\right))^2\,dx=\int x(a+b\sin\left(c+d(f+gx)^n\right))^2\,dx$$

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Integral(x*(a + b*sin(c + d*(f + g*x)**n))**2, x)
```

Maxima [F]

$$\int x(a+b\sin\left(c+d(f+gx)^n\right))^2\,dx=\int\left(b\sin\left((gx+f)^nd+c\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/4*b^2*x^2 - 1/2*b^2*integrate(x*cos(2*(g*x + f)^n*d + 2*c),
 x) + 2*a*b*integrate(x*sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int x(a+b\sin(c+d(f+gx)^n))^2\,dx = \int (b\sin((gx+f)^n d+c)+a)^2 x\,dx$$

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\sin(c+d(f+gx)^n))^2\,dx = \int x\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)^2\,dx$$

```
[In] int(x*(a + b*sin(c + d*(f + g*x)^n))^2,x)
```

```
[Out] int(x*(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

3.274 $\int (a+b\sin(c+d(f+gx)^n))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 261

$$\begin{aligned}\int (a+b\sin(c+d(f+gx)^n))^2\,dx &= \frac{1}{2}(2a^2+b^2)\,x+\frac{iabe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{gn}\\ &\quad-\frac{iabe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{gn}\\ &\quad+\frac{2^{-2-\frac{1}{n}}b^2e^{2ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{gn}\\ &\quad+\frac{2^{-2-\frac{1}{n}}b^2e^{-2ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{gn}\end{aligned}$$

```
[Out] 1/2*(2*a^2+b^2)*x+I*a*b*exp(I*c)*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g/n/((-I
*d*(g*x+f)^n)^(1/n))-I*a*b*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g/n/((
I*d*(g*x+f)^n)^(1/n))+2^(-2-1/n)*b^2*exp(2*I*c)*(g*x+f)*GAMMA(1/n,-2*I*d*(g
*x+f)^n)/g/n/((-I*d*(g*x+f)^n)^(1/n))+2^(-2-1/n)*b^2*(g*x+f)*GAMMA(1/n,2*I*
d*(g*x+f)^n)/exp(2*I*c)/g/n/((I*d*(g*x+f)^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3448, 3447, 2239, 3446}

$$\begin{aligned}
&\int (a+b\sin (c+d(f+gx)^n))^2\,dx \\
&=\frac{1}{2}x(2a^2+b^2)+\frac{iabe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{gn} \\
&\quad-\frac{iabe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{gn} \\
&\quad+\frac{b^2e^{2ic}2^{-\frac{1}{n}-2}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{gn} \\
&\quad+\frac{b^2e^{-2ic}2^{-\frac{1}{n}-2}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{gn}
\end{aligned}$$

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 + (I*a*b*E^(I*c)*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*
x)^n])/(g*n*((-I)*d*(f + g*x)^n)^n^(-1)) - (I*a*b*(f + g*x)*Gamma[n^(-1), I
*d*(f + g*x)^n])/(E^(I*c)*g*n*(I*d*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*
b^2*E^((2*I)*c)*(f + g*x)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n])/(g*n*((-I)*d
*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*b^2*(f + g*x)*Gamma[n^(-1), (2*I)*
d*(f + g*x)^n])/(E^((2*I)*c)*g*n*(I*d*(f + g*x)^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3448

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2+\frac{b^2}{2}-\frac{1}{2}b^2\cos\left(2c+2d(f+gx)^n\right)+2ab\sin\left(c+d(f+gx)^n\right)\right)\,dx\\
&= \frac{1}{2}\left(2a^2+b^2\right)x+(2ab)\int \sin\left(c+d(f+gx)^n\right)\,dx-\frac{1}{2}b^2\int \cos\left(2c+2d(f+gx)^n\right)\,dx\\
&= \frac{1}{2}\left(2a^2+b^2\right)x+(iab)\int e^{-ic-id(f+gx)^n}\,dx-(iab)\int e^{ic+id(f+gx)^n}\,dx\\
&\quad -\frac{1}{4}b^2\int e^{-2ic-2id(f+gx)^n}\,dx-\frac{1}{4}b^2\int e^{2ic+2id(f+gx)^n}\,dx\\
&= \frac{1}{2}\left(2a^2+b^2\right)x+\frac{iabe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{gn}\\
&\quad -\frac{iabe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{gn}\\
&\quad +\frac{2^{-2-\frac{1}{n}}b^2e^{2ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{gn}\\
&\quad +\frac{2^{-2-\frac{1}{n}}b^2e^{-2ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{gn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.95

$$\int \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2\,dx$$

$$=\frac{2(2a^2+b^2)\,gnx+2^{-1/n}b^2e^{2ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)+2^{-1/n}b^2e^{-2ic}(f+gx)\,(\ldots}{\ldots}$$

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] (2*(2*a^2 + b^2)*g*n*x + (b^2*E^((2*I)*c)*(f + g*x)*Gamma[n^(-1), (-2*I)*d*
(f + g*x)^n])/(2^n^(-1)*((-I)*d*(f + g*x)^n)^n^(-1)) + (b^2*(f + g*x)*Gamma
[n^(-1), (2*I)*d*(f + g*x)^n])/(2^n^(-1)*E^((2*I)*c)*(I*d*(f + g*x)^n)^n^(-
1)) - ((4*I)*a*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n]*(Cos[c] - I*Sin[c
]))/(I*d*(f + g*x)^n)^n^(-1) + ((4*I)*a*b*(f + g*x)*Gamma[n^(-1), (-I)*d*(f
 + g*x)^n]*(Cos[c] + I*Sin[c]))/((-I)*d*(f + g*x)^n)^n^(-1))/(4*g*n)
```

Maple [F]

$$\int (a+b\sin (c+d(gx+f)^{n}))^{2}dx$$

```
[In] int((a+b*sin(c+d*(g*x+f)^n))^2,x)
[Out] int((a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [F]

$$\int (a+b\sin (c+d(f+gx)^{n}))^{2}\,dx=\int (b\sin ((gx+f)^{n}d+c)+a)^{2}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
[Out] integral(-b^2*cos((g*x + f)^n*d + c)^2 + 2*a*b*sin((g*x + f)^n*d + c) + a^2
 + b^2, x)
```

Sympy [F]

$$\int (a+b\sin (c+d(f+gx)^{n}))^{2}\,dx=\int (a+b\sin (c+d(f+gx)^{n}))^{2}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))**2,x)
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))**2, x)
```

Maxima [F]

$$\int (a+b\sin (c+d(f+gx)^{n}))^{2}\,dx=\int (b\sin ((gx+f)^{n}d+c)+a)^{2}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
[Out] a^2*x + 1/2*b^2*x - 1/2*b^2*integrate(cos(2*(g*x + f)^n*d + 2*c), x) + 2*a*
b*integrate(sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int (a + b\sin(c + d(f + gx)^n))^2 \, dx = \int (b\sin((gx + f)^n d + c) + a)^2 \, dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\sin(c + d(f + gx)^n))^2 \, dx = \int (a + b \sin(c + d(f + g x)^n))^2 \, dx$$

```
[In] int((a + b*sin(c + d*(f + g*x)^n))^2,x)
[Out] int((a + b*sin(c + d*(f + g*x)^n))^2, x)
```

3.275 $\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x} dx = \text{Int}\left(\frac{(a+b\sin(c+d(f+gx)^n))^2}{x}, x\right)$$

```
[Out] Unintegrable((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x} dx = \int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x} dx$$

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^2/x,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*(f + g*x)^n])^2/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 3.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx=\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx$$

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x,x]
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sin(c+d(gx+f)^n))^2}{x}dx$$

```
[In] int((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
[Out] int((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx=\int \frac{(b\sin((gx+f)^nd+c)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x,x, algorithm="fricas")
[Out] integral(-(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) - a^
2 - b^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 18.65 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx=\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))**2/x,x)
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx=\int \frac{(b\sin((gx+f)^nd+c)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x,x, algorithm="maxima")
[Out] -1/2*b^2*integrate(cos(2*(g*x + f)^n*d + 2*c)/x, x) + 2*a*b*integrate(sin((
g*x + f)^n*d + c)/x, x) + a^2*log(x) + 1/2*b^2*log(x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx=\int \frac{(b\sin((gx+f)^nd+c)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x,x, algorithm="giac")
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 6.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx=\int \frac{\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)^2}{x}\,dx$$

[In] `int((a + b*sin(c + d*(f + g*x)^n))^2/x,x)`

[Out] `int((a + b*sin(c + d*(f + g*x)^n))^2/x, x)`

3.276 $\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx = \text{Int}\left(\frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}, x\right)$$

```
[Out] Unintegrable((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx = \int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx$$

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*(f + g*x)^n])^2/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx = \int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2,x]
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sin(c+d(gx+f)^n))^2}{x^2}dx$$

```
[In] int((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x)
[Out] int((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx = \int \frac{(b\sin((gx+f)^n d+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x, algorithm="fricas")
[Out] integral(-(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) - a^
2 - b^2)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 51.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx = \int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))**2/x**2,x)
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx = \int \frac{(b\sin((gx+f)^n d+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x, algorithm="maxima")
[Out] -a^2/x - 1/2*(b^2*x*integrate(cos(2*(g*x + f)^n*d + 2*c)/x^2, x) - 4*a*b*x*
integrate(sin((g*x + f)^n*d + c)/x^2, x) + b^2)/x
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx = \int \frac{(b\sin((gx+f)^n d+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x, algorithm="giac")
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a + b\sin(c + d(f + gx)^n))^2}{x^2} \, dx = \int \frac{\left(a + b\ \sin\left(c + d\left(f + g\,x\right)^n\right)\right)^2}{x^2} \, dx$$

```
[In] int((a + b*sin(c + d*(f + g*x)^n))^2/x^2,x)
[Out] int((a + b*sin(c + d*(f + g*x)^n))^2/x^2, x)
```

3.277 $\int \frac{x^2}{a+b\sin(c+d(f+gx)^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\operatorname{Int}\left(\frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)},x\right)$$

```
[Out] Unintegrable(x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

```
[In] Int[x^2/(a + b*Sin[c + d*(f + g*x)^n]),x]
[Out] Defer[Int][x^2/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

```
[In] Integrate[x^2/(a + b*Sin[c + d*(f + g*x)^n]),x]
[Out] Integrate[x^2/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\sin\left(c+d\left(gx+f\right)^n\right)}dx$$

```
[In] int(x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
[Out] int(x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{x^2}{b\sin\left((gx+f)^n d+c\right)+a}\,dx$$

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
[Out] integral(x^2/(b*sin((g*x + f)^n*d + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\text{Timed out}$$

```
[In] integrate(x**2/(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{a + b\sin(c + d(f + gx)^n)} dx = \int \frac{x^2}{b\sin((gx + f)^n d + c) + a} dx$$

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
[Out] integrate(x^2/(b*sin((g*x + f)^n*d + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{a + b\sin(c + d(f + gx)^n)} dx = \int \frac{x^2}{b\sin((gx + f)^n d + c) + a} dx$$

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
[Out] integrate(x^2/(b*sin((g*x + f)^n*d + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 6.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{a + b\sin(c + d(f + gx)^n)} dx = \int \frac{x^2}{a + b\,\sin\left(c + d\,(f + g\,x)^n\right)} dx$$

```
[In] int(x^2/(a + b*sin(c + d*(f + g*x)^n)),x)
[Out] int(x^2/(a + b*sin(c + d*(f + g*x)^n)), x)
```

3.278 $\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\operatorname{Int}\left(\frac{x}{a+b\sin\left(c+d(f+gx)^n\right)},x\right)$$

```
[Out] Unintegrable(x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

```
[In] Int[x/(a + b*Sin[c + d*(f + g*x)^n]),x]
[Out] Defer[Int][x/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Rubi steps

$$\text{integral}=\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx=\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx$$

```
[In] Integrate[x/(a + b*Sin[c + d*(f + g*x)^n]),x]
[Out] Integrate[x/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\sin\left(c+d\left(gx+f\right)^{n}\right)}dx$$

```
[In] int(x/(a+b*sin(c+d*(g*x+f)^n)),x)
[Out] int(x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx=\int \frac{x}{b\sin\left(\left(gx+f\right)^{n}d+c\right)+a}\,dx$$

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
[Out] integral(x/(b*sin((g*x + f)^n*d + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 103.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx=\int \frac{x}{a+b\sin\left(c+d\left(f+gx\right)^{n}\right)}\,dx$$

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Integral(x/(a + b*sin(c + d*(f + g*x)**n)), x)
```

Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx = \int \frac{x}{b\sin\left((gx+f)^n d+c\right)+a}\,dx$$

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
[Out] integrate(x/(b*sin((g*x + f)^n*d + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx = \int \frac{x}{b\sin\left((gx+f)^n d+c\right)+a}\,dx$$

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
[Out] integrate(x/(b*sin((g*x + f)^n*d + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 5.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx = \int \frac{x}{a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)}\,dx$$

```
[In] int(x/(a + b*sin(c + d*(f + g*x)^n)),x)
[Out] int(x/(a + b*sin(c + d*(f + g*x)^n)), x)
```

3.279 $\int \frac{1}{a+b\sin(c+d(f+gx)^n)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\text{Int}\left(\frac{1}{a+b\sin\left(c+d(f+gx)^n\right)},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^(-1),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*(f + g*x)^n])^(-1), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+b\sin (c+d(f+gx)^n)}\,dx=\int \frac{1}{a+b\sin (c+d(f+gx)^n)}\,dx$$

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-1),x]
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-1), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin \left(c+d\left(gx+f\right)^{n}\right)}dx$$

```
[In] int(1/(a+b*sin(c+d*(g*x+f)^n)),x)
[Out] int(1/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+b\sin (c+d(f+gx)^n)}\,dx=\int \frac{1}{b\sin \left((gx+f)^n d+c\right)+a}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
[Out] integral(1/(b*sin((g*x + f)^n*d + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 41.92 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+b\sin (c+d(f+gx)^n)}\,dx=\int \frac{1}{a+b\sin \left(c+d\left(f+gx\right)^{n}\right)}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Integral(1/(a + b*sin(c + d*(f + g*x)**n)), x)
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{1}{b\sin\left((gx+f)^n d+c\right)+a}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
[Out] integrate(1/(b*sin((g*x + f)^n*d + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{1}{b\sin\left((gx+f)^n d+c\right)+a}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
[Out] integrate(1/(b*sin((g*x + f)^n*d + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 6.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{1}{a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)}\,dx$$

```
[In] int(1/(a + b*sin(c + d*(f + g*x)^n)),x)
[Out] int(1/(a + b*sin(c + d*(f + g*x)^n)), x)
```

3.280 $\int \frac{1}{x(a+b\sin(c+d(f+gx)^n))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*Sin[c + d*(f + g*x)^n])),x]
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*(f + g*x)^n])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])),x]
[Out] Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sin\left(c+d\left(gx+f\right)^{n}\right)\right)}dx$$

```
[In] int(1/x/(a+b*sin(c+d*(g*x+f)^n)),x)
[Out] int(1/x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{\left(b\sin\left(\left(gx+f\right)^{n}d+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
[Out] integral(1/(b*x*sin((g*x + f)^n*d + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 106.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+d\left(f+gx\right)^{n}\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Integral(1/(x*(a + b*sin(c + d*(f + g*x)**n))), x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{\left(b\sin\left((gx+f)^n d+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{\left(b\sin\left((gx+f)^n d+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 5.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{x\,\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)}\,dx$$

```
[In] int(1/(x*(a + b*sin(c + d*(f + g*x)^n))),x)
[Out] int(1/(x*(a + b*sin(c + d*(f + g*x)^n))), x)
```

3.281 $\int \frac{1}{x^2(a+b\sin(c+d(f+gx)^n))} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2 (a+b\sin (c+d(f+gx)^n))} dx = \text{Int}\left(\frac{1}{x^2 (a+b\sin (c+d(f+gx)^n))},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 (a+b\sin (c+d(f+gx)^n))} dx = \int \frac{1}{x^2 (a+b\sin (c+d(f+gx)^n))} dx$$

```
[In] Int[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])),x]
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 (a+b\sin (c+d(f+gx)^n))} dx$$

Mathematica [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (a + b \sin (c + d(f + gx)^n))} dx = \int \frac{1}{x^2 (a + b \sin (c + d(f + gx)^n))} dx$$

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])),x]
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a + b \sin (c + d (gx + f)^n))} dx$$

```
[In] int(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
[Out] int(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^2 (a + b \sin (c + d(f + gx)^n))} dx = \int \frac{1}{(b \sin ((gx + f)^n d + c) + a)x^2} dx$$

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
[Out] integral(1/(b*x^2*sin((g*x + f)^n*d + c) + a*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + b \sin (c + d(f + gx)^n))} dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (a + b \sin (c + d(f + gx)^n))} \, dx = \int \frac{1}{(b \sin ((gx + f)^n d + c) + a)x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (a + b \sin (c + d(f + gx)^n))} \, dx = \int \frac{1}{(b \sin ((gx + f)^n d + c) + a)x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 5.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (a + b \sin (c + d(f + gx)^n))} \, dx = \int \frac{1}{x^2 \left(a + b \sin \left(c + d \left(f + g x\right)^n\right)\right)} \, dx$$

```
[In] int(1/(x^2*(a + b*sin(c + d*(f + g*x)^n))),x)
[Out] int(1/(x^2*(a + b*sin(c + d*(f + g*x)^n))), x)
```

3.282 $\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2} dx = \operatorname{Int}\left(\frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}, x\right)$$

```
[Out] Unintegrable(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2} dx = \int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2} dx$$

```
[In] Int[x^2/(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] Defer[Int][x^2/(a + b*Sin[c + d*(f + g*x)^n])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2} dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \$\text{Aborted}$$

```
[In] Integrate[x^2/(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+b\sin(c+d(gx+f)^n))^2}dx$$

```
[In] int(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{x^2}{(b\sin((gx+f)^n d+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-x^2/(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c)
- a^2 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 1509, normalized size of antiderivative = 68.59

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{x^2}{(b\sin((gx+f)^nd+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] (2*(a*b*g*x^3 + a*b*f*x^2)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c
) + 2*(a*b*g*x^3 + a*b*f*x^2)*cos((g*x + f)^n*d + c) - ((a^2*b^2 - b^4)*(g*
x + f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n
*d*g*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((
g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*
d*g*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*si
n((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^
n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x +
f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(
2*(g*x + f)^n*d + 2*c))*integrate(-2*(2*(g*x + f)^n*a^2*d*g*n*x^2*cos((g*x
+ f)^n*d + c)^2 + 2*(g*x + f)^n*a^2*d*g*n*x^2*sin((g*x + f)^n*d + c)^2 + (g
*x + f)^n*a*b*d*g*n*x^2*sin((g*x + f)^n*d + c) - ((g*x + f)^n*a*b*d*g*n*x^2
*sin((g*x + f)^n*d + c) + (2*a*b*f*x - (a*b*g*n - 3*a*b*g)*x^2)*cos((g*x +
f)^n*d + c))*cos(2*(g*x + f)^n*d + 2*c) - (2*a*b*f*x - (a*b*g*n - 3*a*b*g)*
x^2)*cos((g*x + f)^n*d + c) + ((g*x + f)^n*a*b*d*g*n*x^2*cos((g*x + f)^n*d
+ c) - 2*b^2*f*x + (b^2*g*n - 3*b^2*g)*x^2 - (2*a*b*f*x - (a*b*g*n - 3*a*b*
g)*x^2)*sin((g*x + f)^n*d + c))*sin(2*(g*x + f)^n*d + 2*c))/((a^2*b^2 - b^4
)*(g*x + f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x +
 f)^n*d*g*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*
cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x +
f)^n*d*g*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g
*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x
+ f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g
*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)
*cos(2*(g*x + f)^n*d + 2*c)), x) + 2*(b^2*g*x^3 + b^2*f*x^2 + (a*b*g*x^3 +
a*b*f*x^2)*sin((g*x + f)^n*d + c))*sin(2*(g*x + f)^n*d + 2*c))/((a^2*b^2 -
b^4)*(g*x + f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*
x + f)^n*d*g*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g
*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x
 + f)^n*d*g*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*
d*g*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g
*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)
*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g
*n)*cos(2*(g*x + f)^n*d + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{x^2}{(b\sin((gx+f)^n d+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
[Out] integrate(x^2/(b*sin((g*x + f)^n*d + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 5.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{x^2}{(a+b\,\sin(c+d\,(f+g\,x)^n))^2}\,dx$$

```
[In] int(x^2/(a + b*sin(c + d*(f + g*x)^n))^2,x)
[Out] int(x^2/(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

3.283 $\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\text{Int}\left(\frac{x}{(a+b\sin(c+d(f+gx)^n))^2},x\right)$$

```
[Out] Unintegrable(x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$$

```
[In] Int[x/(a + b*Sin[c + d*(f + g*x)^n])^2,x]
[Out] Defer[Int][x/(a + b*Sin[c + d*(f + g*x)^n])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2} \, dx = \$\text{Aborted}$$

```
[In] Integrate[x/(a + b*Sin[c + d*(f + g*x)^n])^2,x]
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\sin(c+d(gx+f)^n))^2} dx$$

```
[In] int(x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
[Out] int(x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2} \, dx = \int \frac{x}{(b\sin((gx+f)^n d+c)+a)^2} \, dx$$

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
[Out] integral(-x/(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) -
a^2 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2} \, dx = \text{Timed out}$$

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)**n))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 1476, normalized size of antiderivative = 73.80

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{x}{(b\sin((gx+f)^n d+c)+a)^2}\,dx$$

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] (2*(a*b*g*x^2 + a*b*f*x)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c)
+ 2*(a*b*g*x^2 + a*b*f*x)*cos((g*x + f)^n*d + c) - ((a^2*b^2 - b^4)*(g*x +
f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g
*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x
+ f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*
n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g
*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d
+ c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n
*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(g
*x + f)^n*d + 2*c))*integrate(-2*(2*(g*x + f)^n*a^2*d*g*n*x*cos((g*x + f)^n
*d + c)^2 + 2*(g*x + f)^n*a^2*d*g*n*x*sin((g*x + f)^n*d + c)^2 + (g*x + f)^
n*a*b*d*g*n*x*sin((g*x + f)^n*d + c) - ((g*x + f)^n*a*b*d*g*n*x*sin((g*x +
f)^n*d + c) + (a*b*f - (a*b*g*n - 2*a*b*g)*x)*cos((g*x + f)^n*d + c))*cos(2
*(g*x + f)^n*d + 2*c) - (a*b*f - (a*b*g*n - 2*a*b*g)*x)*cos((g*x + f)^n*d +
 c) + ((g*x + f)^n*a*b*d*g*n*x*cos((g*x + f)^n*d + c) - b^2*f + (b^2*g*n -
2*b^2*g)*x - (a*b*f - (a*b*g*n - 2*a*b*g)*x)*sin((g*x + f)^n*d + c))*sin(2*
(g*x + f)^n*d + 2*c))/((a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*cos(2*(g*x + f)^n*
d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)^2 +
 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)
^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*sin(2*(g*x + f)^n*d + 2*c)^
2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b
 - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x +
 f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c)
 + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(g*x + f)^n*d + 2*c)), x) + 2*(
b^2*g*x^2 + b^2*f*x + (a*b*g*x^2 + a*b*f*x)*sin((g*x + f)^n*d + c))*sin(2*(
g*x + f)^n*d + 2*c))/((a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*cos(2*(g*x + f)^n*d
 + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)^2 +
4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^
n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*sin(2*(g*x + f)^n*d + 2*c)^2
 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b
- a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x +
f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c)
+ (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(g*x + f)^n*d + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{x}{(b\sin((gx+f)^n d+c)+a)^2}\,dx$$

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
[Out] integrate(x/(b*sin((g*x + f)^n*d + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{x}{(a+b\,\sin(c+d\,(f+g\,x)^n))^2}\,dx$$

```
[In] int(x/(a + b*sin(c + d*(f + g*x)^n))^2,x)
[Out] int(x/(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

3.284 $\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \operatorname{Int}\left(\frac{1}{(a+b\sin(c+d(f+gx)^n))^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$$

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^(-2),x]
[Out] Defer[Int][(a + b*Sin[c + d*(f + g*x)^n])^(-2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$$

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-2),x]
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sin(c+d\,(gx+f)^n))^2}dx$$

```
[In] int(1/(a+b*sin(c+d*(g*x+f)^n))^2,x)
[Out] int(1/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{1}{(b\sin((gx+f)^n d+c)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) -
a^2 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)**n))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 1403, normalized size of antiderivative = 77.94

$$\int \frac{1}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx = \int \frac{1}{\left(b\sin\left((gx+f)^n d+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
[Out] (2*(a*b*g*x + a*b*f)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c) + 2*
(a*b*g*x + a*b*f)*cos((g*x + f)^n*d + c) - ((a^2*b^2 - b^4)*(g*x + f)^n*d*g
*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*cos((
g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d
 + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*sin(2*
(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g*x + f)^
n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (
a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*s
in((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(g*x + f)^
n*d + 2*c))*integrate(-2*(2*(g*x + f)^n*a^2*d*n*cos((g*x + f)^n*d + c)^2 +
2*(g*x + f)^n*a^2*d*n*sin((g*x + f)^n*d + c)^2 + (g*x + f)^n*a*b*d*n*sin((g
*x + f)^n*d + c) - ((g*x + f)^n*a*b*d*n*sin((g*x + f)^n*d + c) - (a*b*n - a
*b)*cos((g*x + f)^n*d + c))*cos(2*(g*x + f)^n*d + 2*c) + (a*b*n - a*b)*cos(
(g*x + f)^n*d + c) + ((g*x + f)^n*a*b*d*n*cos((g*x + f)^n*d + c) + b^2*n -
b^2 + (a*b*n - a*b)*sin((g*x + f)^n*d + c))*sin(2*(g*x + f)^n*d + 2*c))/((a
^2*b^2 - b^4)*(g*x + f)^n*d*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b
^2)*(g*x + f)^n*d*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^
n*d*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(
g*x + f)^n*d*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n
*d*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*n*sin((g*x
+ f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*n - 2*(2*(a^3*b - a*b^3)*(g*x
 + f)^n*d*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*n)*cos(2
*(g*x + f)^n*d + 2*c)), x) + 2*(b^2*g*x + b^2*f + (a*b*g*x + a*b*f)*sin((g*
x + f)^n*d + c))*sin(2*(g*x + f)^n*d + 2*c))/((a^2*b^2 - b^4)*(g*x + f)^n*d
*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*cos
```

```
((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x + f)^n
*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*sin(
2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g*x + f
)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) +
 (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n
*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(g*x + f
)^n*d + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{1}{(b\sin(\left(gx+f\right)^n d+c)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 6.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{1}{{\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)}^2}\,dx$$

```
[In] int(1/(a + b*sin(c + d*(f + g*x)^n))^2,x)
[Out] int(1/(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

3.285 $\int \frac{1}{x(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

```
[In] Int[1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\$\text{Aborted}$$

```
[In] Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sin\left(c+d\left(gx+f\right)^{n}\right)\right)^{2}}dx$$

```
[In] int(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
[Out] int(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left((gx+f)^{n}d+c\right)+a\right)^{2}x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*x*cos((g*x + f)^n*d + c)^2 - 2*a*b*x*sin((g*x + f)^n*d + c
) - (a^2 + b^2)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)**n))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 24.27 (sec) , antiderivative size = 5041, normalized size of antiderivative = 229.14

$$\int \frac{1}{x(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{1}{(b\sin(((gx+f)^n d+c)+a)^2 x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] (2*(a^3*b*g*x + a^3*b*f)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c)
- 2*(b^4*g*x*sin(2*c) + b^4*f*sin(2*c))*cos(2*(g*x + f)^n*d) - 2*((a^3*b -
a*b^3)*g*x + (a^3*b - a*b^3)*f + (a*b^3*g*x*cos(2*c) + a*b^3*f*cos(2*c))*co
s(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*g*x*sin(c) + (a^4 - a^2*b^2)*f*sin(
c))*cos((g*x + f)^n*d) - (a*b^3*g*x*sin(2*c) + a*b^3*f*sin(2*c))*sin(2*(g*x
 + f)^n*d) + 2*((a^4 - a^2*b^2)*g*x*cos(c) + (a^4 - a^2*b^2)*f*cos(c))*sin(
(g*x + f)^n*d))*cos((g*x + f)^n*d + c) + 4*((a^3*b - a*b^3)*g*x*cos(c) + (a
^3*b - a*b^3)*f*cos(c))*cos((g*x + f)^n*d) + ((g*x + f)^n*a^4*b^2*d*g*n*x*c
os(2*(g*x + f)^n*d + 2*c)^2 + (g*x + f)^n*a^4*b^2*d*g*n*x*sin(2*(g*x + f)^n
*d + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x*cos(2*(
g*x + f)^n*d)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^
2 + a^2*b^4)*sin(c)^2)*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d)^2 + (b^6*cos(
2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x*sin(2*(g*x + f)^n*d)^2 + 4*(a^
5*b - 2*a^3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*x*cos(c)*sin((g*x + f)^n*d) + 4*
((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^
2)*(g*x + f)^n*d*g*n*x*sin((g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)
*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6
)*(g*x + f)^n*d*g*n*x - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3
- a*b^5)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d) - (a^2*b^4
 - b^6)*(g*x + f)^n*d*g*n*x*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c)
 + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*(g*x + f)^n*d*g*n*x*sin((g*x + f)^n*d
))*cos(2*(g*x + f)^n*d) - 2*((g*x + f)^n*a^2*b^4*d*g*n*x*cos(2*(g*x + f)^n*
d)*cos(2*c) - (g*x + f)^n*a^2*b^4*d*g*n*x*sin(2*(g*x + f)^n*d)*sin(2*c) + 2
*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x*cos(c)*sin((g*x + f)^n*d) + 2*(a^5*b
 - a^3*b^3)*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 - a^2*
b^4)*(g*x + f)^n*d*g*n*x)*cos(2*(g*x + f)^n*d + 2*c) - 2*(2*((a^3*b^3 - a*b
^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*(g*x + f)^n*d*g*n*
x*cos((g*x + f)^n*d) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*
b^5)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n*x*sin((g*x + f)^n*d) + (a^2*b^4 - b
^6)*(g*x + f)^n*d*g*n*x*sin(2*c))*sin(2*(g*x + f)^n*d) - 2*((g*x + f)^n*a^2
*b^4*d*g*n*x*cos(2*c)*sin(2*(g*x + f)^n*d) + (g*x + f)^n*a^2*b^4*d*g*n*x*co
s(2*(g*x + f)^n*d)*sin(2*c) - 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x*cos((
g*x + f)^n*d)*cos(c) + 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x*sin((g*x + f
)^n*d)*sin(c))*sin(2*(g*x + f)^n*d + 2*c))*integrate(-2*((b^4*g*n*x*sin(2*c
) + b^4*f*sin(2*c))*cos(2*(g*x + f)^n*d) + ((g*x + f)^n*a^3*b*d*g*n*x*sin((
```

g*x + f)^n*d + c) - (a^3*b*g*n*x + a^3*b*f)*cos((g*x + f)^n*d + c))*cos(2*(
g*x + f)^n*d + 2*c) + ((a^3*b - a*b^3)*g*n*x + (a^3*b - a*b^3)*f + ((g*x +
f)^n*a*b^3*d*g*n*x*sin(2*c) + a*b^3*g*n*x*cos(2*c) + a*b^3*f*cos(2*c))*cos(
2*(g*x + f)^n*d) - 2*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*cos(c) - (a^4 - a
^2*b^2)*g*n*x*sin(c) - (a^4 - a^2*b^2)*f*sin(c))*cos((g*x + f)^n*d) + ((g*x
 + f)^n*a*b^3*d*g*n*x*cos(2*c) - a*b^3*g*n*x*sin(2*c) - a*b^3*f*sin(2*c))*s
in(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*sin(c) + (a^4
- a^2*b^2)*g*n*x*cos(c) + (a^4 - a^2*b^2)*f*cos(c))*sin((g*x + f)^n*d))*cos
((g*x + f)^n*d + c) - 2*((a^3*b - a*b^3)*g*n*x*cos(c) + (a^3*b - a*b^3)*f*c
os(c))*cos((g*x + f)^n*d) + (b^4*g*n*x*cos(2*c) + b^4*f*cos(2*c))*sin(2*(g*
x + f)^n*d) - ((g*x + f)^n*a^3*b*d*g*n*x*cos((g*x + f)^n*d + c) + a^2*b^2*g
*n*x + a^2*b^2*f + (a^3*b*g*n*x + a^3*b*f)*sin((g*x + f)^n*d + c))*sin(2*(g
*x + f)^n*d + 2*c) - ((a^3*b - a*b^3)*(g*x + f)^n*d*g*n*x + ((g*x + f)^n*a*
b^3*d*g*n*x*cos(2*c) - a*b^3*g*n*x*sin(2*c) - a*b^3*f*sin(2*c))*cos(2*(g*x
+ f)^n*d) + 2*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*sin(c) + (a^4 - a^2*b^2)
*g*n*x*cos(c) + (a^4 - a^2*b^2)*f*cos(c))*cos((g*x + f)^n*d) - ((g*x + f)^n
*a*b^3*d*g*n*x*sin(2*c) + a*b^3*g*n*x*cos(2*c) + a*b^3*f*cos(2*c))*sin(2*(g
*x + f)^n*d) + 2*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*cos(c) - (a^4 - a^2*b
^2)*g*n*x*sin(c) - (a^4 - a^2*b^2)*f*sin(c))*sin((g*x + f)^n*d))*sin((g*x +
 f)^n*d + c) + 2*((a^3*b - a*b^3)*g*n*x*sin(c) + (a^3*b - a*b^3)*f*sin(c))*
sin((g*x + f)^n*d))/((g*x + f)^n*a^4*b^2*d*g*n*x^2*cos(2*(g*x + f)^n*d + 2*
c)^2 + (g*x + f)^n*a^4*b^2*d*g*n*x^2*sin(2*(g*x + f)^n*d + 2*c)^2 + (b^6*co
s(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x^2*cos(2*(g*x + f)^n*d)^2 + 4
*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)
^2)*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d)^2 + (b^6*cos(2*c)^2 + b^6*sin(
2*c)^2)*(g*x + f)^n*d*g*n*x^2*sin(2*(g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3
 + a*b^5)*(g*x + f)^n*d*g*n*x^2*cos(c)*sin((g*x + f)^n*d) + 4*((a^6 - 2*a^4
*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x + f)^
n*d*g*n*x^2*sin((g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*(g*x + f)^
n*d*g*n*x^2*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*(g*x +
f)^n*d*g*n*x^2 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5
)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d) - (a^2*b^4 - b^
6)*(g*x + f)^n*d*g*n*x^2*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) +
(a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^2*sin((g*x + f)^n*d)
)*cos(2*(g*x + f)^n*d) - 2*((g*x + f)^n*a^2*b^4*d*g*n*x^2*cos(2*(g*x + f)^n
*d)*cos(2*c) - (g*x + f)^n*a^2*b^4*d*g*n*x^2*sin(2*(g*x + f)^n*d)*sin(2*c)
+ 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^2*cos(c)*sin((g*x + f)^n*d) + 2*(
a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2
 - a^2*b^4)*(g*x + f)^n*d*g*n*x^2)*cos(2*(g*x + f)^n*d + 2*c) - 2*(2*((a^3*
b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*(g*x + f)
^n*d*g*n*x^2*cos((g*x + f)^n*d) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a
^3*b^3 - a*b^5)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^2*sin((g*x + f)^n*d) +
 (a^2*b^4 - b^6)*(g*x + f)^n*d*g*n*x^2*sin(2*c))*sin(2*(g*x + f)^n*d) - 2*(
(g*x + f)^n*a^2*b^4*d*g*n*x^2*cos(2*c)*sin(2*(g*x + f)^n*d) + (g*x + f)^n*a
^2*b^4*d*g*n*x^2*cos(2*(g*x + f)^n*d)*sin(2*c) - 2*(a^5*b - a^3*b^3)*(g*x +

```
f)^n*d*g*n*x^2*cos((g*x + f)^n*d)*cos(c) + 2*(a^5*b - a^3*b^3)*(g*x + f)^n
*d*g*n*x^2*sin((g*x + f)^n*d)*sin(c))*sin(2*(g*x + f)^n*d + 2*c)), x) - 2*(
b^4*g*x*cos(2*c) + b^4*f*cos(2*c))*sin(2*(g*x + f)^n*d) + 2*(a^2*b^2*g*x +
a^2*b^2*f + (a^3*b*g*x + a^3*b*f)*sin((g*x + f)^n*d + c))*sin(2*(g*x + f)^n
*d + 2*c) - 2*((a*b^3*g*x*sin(2*c) + a*b^3*f*sin(2*c))*cos(2*(g*x + f)^n*d)
 - 2*((a^4 - a^2*b^2)*g*x*cos(c) + (a^4 - a^2*b^2)*f*cos(c))*cos((g*x + f)^
n*d) + (a*b^3*g*x*cos(2*c) + a*b^3*f*cos(2*c))*sin(2*(g*x + f)^n*d) + 2*((a
^4 - a^2*b^2)*g*x*sin(c) + (a^4 - a^2*b^2)*f*sin(c))*sin((g*x + f)^n*d))*si
n((g*x + f)^n*d + c) - 4*((a^3*b - a*b^3)*g*x*sin(c) + (a^3*b - a*b^3)*f*si
n(c))*sin((g*x + f)^n*d))/((g*x + f)^n*a^4*b^2*d*g*n*x*cos(2*(g*x + f)^n*d
 + 2*c)^2 + (g*x + f)^n*a^4*b^2*d*g*n*x*sin(2*(g*x + f)^n*d + 2*c)^2 + (b^6*
cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x*cos(2*(g*x + f)^n*d)^2 + 4
*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)
^2)*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*
c)^2)*(g*x + f)^n*d*g*n*x*sin(2*(g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3 + a
*b^5)*(g*x + f)^n*d*g*n*x*cos(c)*sin((g*x + f)^n*d) + 4*((a^6 - 2*a^4*b^2 +
 a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x + f)^n*d*g*
n*x*sin((g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*
x*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*(g*x + f)^n*d*g*n
*x - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*s
in(c))*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d) - (a^2*b^4 - b^6)*(g*x + f)^n
*d*g*n*x*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5
)*sin(2*c)*sin(c))*(g*x + f)^n*d*g*n*x*sin((g*x + f)^n*d))*cos(2*(g*x + f)^
n*d) - 2*((g*x + f)^n*a^2*b^4*d*g*n*x*cos(2*(g*x + f)^n*d)*cos(2*c) - (g*x
 + f)^n*a^2*b^4*d*g*n*x*sin(2*(g*x + f)^n*d)*sin(2*c) + 2*(a^5*b - a^3*b^3)*
(g*x + f)^n*d*g*n*x*cos(c)*sin((g*x + f)^n*d) + 2*(a^5*b - a^3*b^3)*(g*x +
f)^n*d*g*n*x*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 - a^2*b^4)*(g*x + f)^n*d*
g*n*x)*cos(2*(g*x + f)^n*d + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c)
 + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d
) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c
))*(g*x + f)^n*d*g*n*x*sin((g*x + f)^n*d) + (a^2*b^4 - b^6)*(g*x + f)^n*d*g
*n*x*sin(2*c))*sin(2*(g*x + f)^n*d) - 2*((g*x + f)^n*a^2*b^4*d*g*n*x*cos(2*
c)*sin(2*(g*x + f)^n*d) + (g*x + f)^n*a^2*b^4*d*g*n*x*cos(2*(g*x + f)^n*d)*
sin(2*c) - 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d)*cos(c
) + 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x*sin((g*x + f)^n*d)*sin(c))*sin(
2*(g*x + f)^n*d + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx = \int \frac{1}{\left(b\sin\left((gx+f)^n d+c\right)+a\right)^2 x}\,dx$$

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 5.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx = \int \frac{1}{x\left(a+b\,\sin\left(c+d\left(f+g\,x\right)^n\right)\right)^2}\,dx$$

```
[In] int(1/(x*(a + b*sin(c + d*(f + g*x)^n))^2),x)
[Out] int(1/(x*(a + b*sin(c + d*(f + g*x)^n))^2), x)
```

3.286 $\int \frac{1}{x^2(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d(f+gx)^n\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x^2 \left(a+b \sin \left(c+d(f+gx)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{1}{x^2 \left(a+b \sin \left(c+d(f+gx)^n\right)\right)^2}\,dx$$

```
[In] Int[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2 \left(a+b \sin \left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d(f+gx)^n\right)\right)^2} \, dx = \$Aborted$$

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
```

```
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d \left(gx+f\right)^n\right)\right)^2} dx$$

```
[In] int(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d(f+gx)^n\right)\right)^2} \, dx = \int \frac{1}{\left(b \sin \left((gx+f)^n d+c\right)+a\right)^2 x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*x^2*sin((g*x + f)^n*d
 + c) - (a^2 + b^2)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d(f+gx)^n\right)\right)^2} \, dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 33.20 (sec) , antiderivative size = 5369, normalized size of antiderivative = 244.05

$$\int \frac{1}{x^2 (a + b\sin (c + d(f + gx)^n))^2} \, dx = \int \frac{1}{(b\sin ((gx + f)^n d + c) + a)^2 x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
[Out] (2*(a^3*b*g*x + a^3*b*f)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c)
- 2*(b^4*g*x*sin(2*c) + b^4*f*sin(2*c))*cos(2*(g*x + f)^n*d) - 2*((a^3*b -
a*b^3)*g*x + (a^3*b - a*b^3)*f + (a*b^3*g*x*cos(2*c) + a*b^3*f*cos(2*c))*co
s(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*g*x*sin(c) + (a^4 - a^2*b^2)*f*sin(
c))*cos((g*x + f)^n*d) - (a*b^3*g*x*sin(2*c) + a*b^3*f*sin(2*c))*sin(2*(g*x
 + f)^n*d) + 2*((a^4 - a^2*b^2)*g*x*cos(c) + (a^4 - a^2*b^2)*f*cos(c))*sin(
(g*x + f)^n*d))*cos((g*x + f)^n*d + c) + 4*((a^3*b - a*b^3)*g*x*cos(c) + (a
^3*b - a*b^3)*f*cos(c))*cos((g*x + f)^n*d) + ((g*x + f)^n*a^4*b^2*d*g*n*x^2
*cos(2*(g*x + f)^n*d + 2*c)^2 + (g*x + f)^n*a^4*b^2*d*g*n*x^2*sin(2*(g*x +
f)^n*d + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x^2*c
os(2*(g*x + f)^n*d)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*
a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d)^2 + (
b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x^2*sin(2*(g*x + f)^n*d)
^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*x^2*cos(c)*sin((g*x +
f)^n*d) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*
b^4)*sin(c)^2)*(g*x + f)^n*d*g*n*x^2*sin((g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^
3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 -
 2*a^2*b^4 + b^6)*(g*x + f)^n*d*g*n*x^2 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*si
n(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^2*cos((g*x
+ f)^n*d) - (a^2*b^4 - b^6)*(g*x + f)^n*d*g*n*x^2*cos(2*c) - 2*((a^3*b^3 -
a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*(g*x + f)^n*d*g
*n*x^2*sin((g*x + f)^n*d))*cos(2*(g*x + f)^n*d) - 2*((g*x + f)^n*a^2*b^4*d*
g*n*x^2*cos(2*(g*x + f)^n*d)*cos(2*c) - (g*x + f)^n*a^2*b^4*d*g*n*x^2*sin(2
*(g*x + f)^n*d)*sin(2*c) + 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^2*cos(c)
*sin((g*x + f)^n*d) + 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^2*cos((g*x +
f)^n*d)*sin(c) + (a^4*b^2 - a^2*b^4)*(g*x + f)^n*d*g*n*x^2)*cos(2*(g*x + f)
^n*d + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*s
in(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d) + 2*((a^3*b^3 - a*
b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n
*x^2*sin((g*x + f)^n*d) + (a^2*b^4 - b^6)*(g*x + f)^n*d*g*n*x^2*sin(2*c))*s
in(2*(g*x + f)^n*d) - 2*((g*x + f)^n*a^2*b^4*d*g*n*x^2*cos(2*c)*sin(2*(g*x
+ f)^n*d) + (g*x + f)^n*a^2*b^4*d*g*n*x^2*cos(2*(g*x + f)^n*d)*sin(2*c) - 2
*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d)*cos(c) + 2*(a^5
*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^2*sin((g*x + f)^n*d)*sin(c))*sin(2*(g*x +
 f)^n*d + 2*c))*integrate(-2*((2*b^4*f*sin(2*c) + (b^4*g*n*sin(2*c) + b^4*g
```

```
*sin(2*c))*x)*cos(2*(g*x + f)^n*d) + ((g*x + f)^n*a^3*b*d*g*n*x*sin((g*x +
f)^n*d + c) - (2*a^3*b*f + (a^3*b*g*n + a^3*b*g)*x)*cos((g*x + f)^n*d + c))
*cos(2*(g*x + f)^n*d + 2*c) + (2*(a^3*b - a*b^3)*f + ((a^3*b - a*b^3)*g*n +
 (a^3*b - a*b^3)*g)*x + ((g*x + f)^n*a*b^3*d*g*n*x*sin(2*c) + 2*a*b^3*f*cos
(2*c) + (a*b^3*g*n*cos(2*c) + a*b^3*g*cos(2*c))*x)*cos(2*(g*x + f)^n*d) - 2
*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*cos(c) - 2*(a^4 - a^2*b^2)*f*sin(c) -
 ((a^4 - a^2*b^2)*g*n*sin(c) + (a^4 - a^2*b^2)*g*sin(c))*x)*cos((g*x + f)^n
*d) + ((g*x + f)^n*a*b^3*d*g*n*x*cos(2*c) - 2*a*b^3*f*sin(2*c) - (a*b^3*g*n
*sin(2*c) + a*b^3*g*sin(2*c))*x)*sin(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*
(g*x + f)^n*d*g*n*x*sin(c) + 2*(a^4 - a^2*b^2)*f*cos(c) + ((a^4 - a^2*b^2)*
g*n*cos(c) + (a^4 - a^2*b^2)*g*cos(c))*x)*sin((g*x + f)^n*d))*cos((g*x + f)
^n*d + c) - 2*(2*(a^3*b - a*b^3)*f*cos(c) + ((a^3*b - a*b^3)*g*n*cos(c) + (
a^3*b - a*b^3)*g*cos(c))*x)*cos((g*x + f)^n*d) + (2*b^4*f*cos(2*c) + (b^4*g
*n*cos(2*c) + b^4*g*cos(2*c))*x)*sin(2*(g*x + f)^n*d) - ((g*x + f)^n*a^3*b*
d*g*n*x*cos((g*x + f)^n*d + c) + 2*a^2*b^2*f + (a^2*b^2*g*n + a^2*b^2*g)*x
+ (2*a^3*b*f + (a^3*b*g*n + a^3*b*g)*x)*sin((g*x + f)^n*d + c))*sin(2*(g*x
+ f)^n*d + 2*c) - ((a^3*b - a*b^3)*(g*x + f)^n*d*g*n*x + ((g*x + f)^n*a*b^3
*d*g*n*x*cos(2*c) - 2*a*b^3*f*sin(2*c) - (a*b^3*g*n*sin(2*c) + a*b^3*g*sin(
2*c))*x)*cos(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*sin(
c) + 2*(a^4 - a^2*b^2)*f*cos(c) + ((a^4 - a^2*b^2)*g*n*cos(c) + (a^4 - a^2*
b^2)*g*cos(c))*x)*cos((g*x + f)^n*d) - ((g*x + f)^n*a*b^3*d*g*n*x*sin(2*c)
+ 2*a*b^3*f*cos(2*c) + (a*b^3*g*n*cos(2*c) + a*b^3*g*cos(2*c))*x)*sin(2*(g*
x + f)^n*d) + 2*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*cos(c) - 2*(a^4 - a^2*
b^2)*f*sin(c) - ((a^4 - a^2*b^2)*g*n*sin(c) + (a^4 - a^2*b^2)*g*sin(c))*x)*
sin((g*x + f)^n*d))*sin((g*x + f)^n*d + c) + 2*(2*(a^3*b - a*b^3)*f*sin(c)
+ ((a^3*b - a*b^3)*g*n*sin(c) + (a^3*b - a*b^3)*g*sin(c))*x)*sin((g*x + f)^
n*d))/((g*x + f)^n*a^4*b^2*d*g*n*x^3*cos(2*(g*x + f)^n*d + 2*c)^2 + (g*x +
f)^n*a^4*b^2*d*g*n*x^3*sin(2*(g*x + f)^n*d + 2*c)^2 + (b^6*cos(2*c)^2 + b^6
*sin(2*c)^2)*(g*x + f)^n*d*g*n*x^3*cos(2*(g*x + f)^n*d)^2 + 4*((a^6 - 2*a^4
*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x + f)^
n*d*g*n*x^3*cos((g*x + f)^n*d)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x +
 f)^n*d*g*n*x^3*sin(2*(g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*(g*x
 + f)^n*d*g*n*x^3*cos(c)*sin((g*x + f)^n*d) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4
)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x + f)^n*d*g*n*x^3*si
n((g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*x^3*co
s((g*x + f)^n*d)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*(g*x + f)^n*d*g*n*x^3
 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin
(c))*(g*x + f)^n*d*g*n*x^3*cos((g*x + f)^n*d) - (a^2*b^4 - b^6)*(g*x + f)^n
*d*g*n*x^3*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b
^5)*sin(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^3*sin((g*x + f)^n*d))*cos(2*(g*x +
 f)^n*d) - 2*((g*x + f)^n*a^2*b^4*d*g*n*x^3*cos(2*(g*x + f)^n*d)*cos(2*c) -
 (g*x + f)^n*a^2*b^4*d*g*n*x^3*sin(2*(g*x + f)^n*d)*sin(2*c) + 2*(a^5*b - a
^3*b^3)*(g*x + f)^n*d*g*n*x^3*cos(c)*sin((g*x + f)^n*d) + 2*(a^5*b - a^3*b^
3)*(g*x + f)^n*d*g*n*x^3*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 - a^2*b^4)*(g
*x + f)^n*d*g*n*x^3)*cos(2*(g*x + f)^n*d + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*c
```

os(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^3*c
os((g*x + f)^n*d) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5
)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^3*sin((g*x + f)^n*d) + (a^2*b^4 - b^
6)*(g*x + f)^n*d*g*n*x^3*sin(2*c))*sin(2*(g*x + f)^n*d) - 2*((g*x + f)^n*a^
2*b^4*d*g*n*x^3*cos(2*c)*sin(2*(g*x + f)^n*d) + (g*x + f)^n*a^2*b^4*d*g*n*x
^3*cos(2*(g*x + f)^n*d)*sin(2*c) - 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^
3*cos((g*x + f)^n*d)*cos(c) + 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^3*sin
((g*x + f)^n*d)*sin(c))*sin(2*(g*x + f)^n*d + 2*c)), x) - 2*(b^4*g*x*cos(2*
c) + b^4*f*cos(2*c))*sin(2*(g*x + f)^n*d) + 2*(a^2*b^2*g*x + a^2*b^2*f + (a
^3*b*g*x + a^3*b*f)*sin((g*x + f)^n*d + c))*sin(2*(g*x + f)^n*d + 2*c) - 2*
((a*b^3*g*x*sin(2*c) + a*b^3*f*sin(2*c))*cos(2*(g*x + f)^n*d) - 2*((a^4 - a
^2*b^2)*g*x*cos(c) + (a^4 - a^2*b^2)*f*cos(c))*cos((g*x + f)^n*d) + (a*b^3*
g*x*cos(2*c) + a*b^3*f*cos(2*c))*sin(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*
g*x*sin(c) + (a^4 - a^2*b^2)*f*sin(c))*sin((g*x + f)^n*d))*sin((g*x + f)^n*
d + c) - 4*((a^3*b - a*b^3)*g*x*sin(c) + (a^3*b - a*b^3)*f*sin(c))*sin((g*x
 + f)^n*d))/((g*x + f)^n*a^4*b^2*d*g*n*x^2*cos(2*(g*x + f)^n*d + 2*c)^2 + (
g*x + f)^n*a^4*b^2*d*g*n*x^2*sin(2*(g*x + f)^n*d + 2*c)^2 + (b^6*cos(2*c)^2
 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x^2*cos(2*(g*x + f)^n*d)^2 + 4*((a^6 -
 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x
 + f)^n*d*g*n*x^2*cos((g*x + f)^n*d)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*
(g*x + f)^n*d*g*n*x^2*sin(2*(g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5
)*(g*x + f)^n*d*g*n*x^2*cos(c)*sin((g*x + f)^n*d) + 4*((a^6 - 2*a^4*b^2 + a
^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x + f)^n*d*g*n*
x^2*sin((g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*
x^2*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*(g*x + f)^n*d*g
*n*x^2 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*
c)*sin(c))*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d) - (a^2*b^4 - b^6)*(g*x
+ f)^n*d*g*n*x^2*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3
 - a*b^5)*sin(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^2*sin((g*x + f)^n*d))*cos(2*
(g*x + f)^n*d) - 2*((g*x + f)^n*a^2*b^4*d*g*n*x^2*cos(2*(g*x + f)^n*d)*cos(
2*c) - (g*x + f)^n*a^2*b^4*d*g*n*x^2*sin(2*(g*x + f)^n*d)*sin(2*c) + 2*(a^5
*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^2*cos(c)*sin((g*x + f)^n*d) + 2*(a^5*b -
a^3*b^3)*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 - a^2*b
^4)*(g*x + f)^n*d*g*n*x^2)*cos(2*(g*x + f)^n*d + 2*c) - 2*(2*((a^3*b^3 - a*
b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*(g*x + f)^n*d*g*n
*x^2*cos((g*x + f)^n*d) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 -
 a*b^5)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^2*sin((g*x + f)^n*d) + (a^2*b^
4 - b^6)*(g*x + f)^n*d*g*n*x^2*sin(2*c))*sin(2*(g*x + f)^n*d) - 2*((g*x + f
)^n*a^2*b^4*d*g*n*x^2*cos(2*c)*sin(2*(g*x + f)^n*d) + (g*x + f)^n*a^2*b^4*d
*g*n*x^2*cos(2*(g*x + f)^n*d)*sin(2*c) - 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*
g*n*x^2*cos((g*x + f)^n*d)*cos(c) + 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x
^2*sin((g*x + f)^n*d)*sin(c))*sin(2*(g*x + f)^n*d + 2*c))

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (a + b\sin(c + d(f + gx)^n))^2} dx = \int \frac{1}{(b\sin((gx + f)^n d + c) + a)^2 x^2} dx$$

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 5.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (a + b\sin(c + d(f + gx)^n))^2} dx = \int \frac{1}{x^2 (a + b\ \sin(c + d (f + g\,x)^n))^2} dx$$

```
[In] int(1/(x^2*(a + b*sin(c + d*(f + g*x)^n))^2),x)
[Out] int(1/(x^2*(a + b*sin(c + d*(f + g*x)^n))^2), x)
```

3.287 $\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx = \text{Int}\left((ex)^m (a+b\sin(c+d(f+gx)^n))^p, x\right)$$

```
[Out] Unintegrable((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx = \int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx$$

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p,x]
[Out] Defer[Int][(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p, x]
```

Rubi steps

$$\text{integral} = \int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx = \int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx$$

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (ex)^m (a+b\sin(c+d(gx+f)^n))^p dx$$

```
[In] int((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x)
[Out] int((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx = \int (ex)^m (b\sin((gx+f)^n d+c)+a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*sin((g*x + f)^n*d + c) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx = \text{Timed out}$$

```
[In] integrate((e*x)**m*(a+b*sin(c+d*(g*x+f)**n))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^m (a + b\sin(c + d(f + gx)^n))^p \, dx = \int (ex)^m (b\sin((gx + f)^n d + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*sin((g*x + f)^n*d + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 171.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^m (a + b\sin(c + d(f + gx)^n))^p \, dx = \int (ex)^m (b\sin((gx + f)^n d + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*sin((g*x + f)^n*d + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 5.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^m (a + b\sin(c + d(f + gx)^n))^p \, dx = \int (e\,x)^m\,\left(a + b\,\sin\left(c + d\,{\left(f + g\,x\right)}^n\right)\right)^p \,dx$$

```
[In] int((e*x)^m*(a + b*sin(c + d*(f + g*x)^n))^p,x)
[Out] int((e*x)^m*(a + b*sin(c + d*(f + g*x)^n))^p, x)
```

3.288 $\int (e+fx)^2 \left(a+b\sin\left(c+\frac{d}{x}\right)\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 224

$$\begin{aligned}\int (e+fx)^2 \left(a+b\sin\left(c+\frac{d}{x}\right)\right) dx &= ae^2x+aefx^2+\frac{1}{3}af^2x^3 \\ &\quad + bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\frac{d}{x}\right) \\ &\quad - bde^2\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) \\ &\quad + \frac{1}{6}bd^3f^2\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) \\ &\quad + bd^2ef\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c)+be^2x\sin\left(c+\frac{d}{x}\right) \\ &\quad - \frac{1}{6}bd^2f^2x\sin\left(c+\frac{d}{x}\right)+befx^2\sin\left(c+\frac{d}{x}\right) \\ &\quad + \frac{1}{3}bf^2x^3\sin\left(c+\frac{d}{x}\right)+bd^2ef\cos(c)\text{Si}\left(\frac{d}{x}\right) \\ &\quad + bde^2\sin(c)\text{Si}\left(\frac{d}{x}\right)-\frac{1}{6}bd^3f^2\sin(c)\text{Si}\left(\frac{d}{x}\right)\end{aligned}$$

```
[Out] a*e^2*x+a*e*f*x^2+1/3*a*f^2*x^3-b*d*e^2*Ci(d/x)*cos(c)+1/6*b*d^3*f^2*Ci(d/x
)*cos(c)+b*d*e*f*x*cos(c+d/x)+1/6*b*d*f^2*x^2*cos(c+d/x)+b*d^2*e*f*cos(c)*S
i(d/x)+b*d^2*e*f*Ci(d/x)*sin(c)+b*d*e^2*Si(d/x)*sin(c)-1/6*b*d^3*f^2*Si(d/x
)*sin(c)+b*e^2*x*sin(c+d/x)-1/6*b*d^2*f^2*x*sin(c+d/x)+b*e*f*x^2*sin(c+d/x)
+1/3*b*f^2*x^3*sin(c+d/x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3512, 14, 3378, 3384, 3380, 3383}

$$\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx = ae^2x+aefx^2+\frac{1}{3}af^2x^3 +\frac{1}{6}bd^3f^2\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) +bd^2ef\sin(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) -bde^2\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) -\frac{1}{6}bd^3f^2\sin(c)\text{Si}\left(\frac{d}{x}\right)+bd^2ef\cos(c)\text{Si}\left(\frac{d}{x}\right) -\frac{1}{6}bd^2f^2x\sin\left(c+\frac{d}{x}\right)+bde^2\sin(c)\text{Si}\left(\frac{d}{x}\right) +be^2x\sin\left(c+\frac{d}{x}\right)+befx^2\sin\left(c+\frac{d}{x}\right) +bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{3}bf^2x^3\sin\left(c+\frac{d}{x}\right) +\frac{1}{6}bdf^2x^2\cos\left(c+\frac{d}{x}\right)$$

```
[In] Int[(e + f*x)^2*(a + b*Sin[c + d/x]),x]
```

```
[Out] a*e^2*x + a*e*f*x^2 + (a*f^2*x^3)/3 + b*d*e*f*x*Cos[c + d/x] + (b*d*f^2*x^2
*Cos[c + d/x])/6 - b*d*e^2*Cos[c]*CosIntegral[d/x] + (b*d^3*f^2*Cos[c]*CosI
ntegral[d/x])/6 + b*d^2*e*f*CosIntegral[d/x]*Sin[c] + b*e^2*x*Sin[c + d/x]
- (b*d^2*f^2*x*Sin[c + d/x])/6 + b*e*f*x^2*Sin[c + d/x] + (b*f^2*x^3*Sin[c
+ d/x])/3 + b*d^2*e*f*Cos[c]*SinIntegral[d/x] + b*d*e^2*Sin[c]*SinIntegral[
d/x] - (b*d^3*f^2*Sin[c]*SinIntegral[d/x])/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \left(\frac{f^2(a+b\sin(c+dx))}{x^4}+\frac{2ef(a+b\sin(c+dx))}{x^3}+\frac{e^2(a+b\sin(c+dx))}{x^2}\right)dx, x, \frac{1}{x}\right)\\
&= -\left(e^2\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)\right)\\
&\quad-(2ef)\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{x^3}\,dx, x, \frac{1}{x}\right)\\
&\quad-f^2\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{x^4}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= -\left(e^2\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{b\sin(c+dx)}{x^2}\right)\,dx,x,\frac{1}{x}\right)\right)$$
$$-(2ef)\text{Subst}\left(\int \left(\frac{a}{x^3}+\frac{b\sin(c+dx)}{x^3}\right)\,dx,x,\frac{1}{x}\right)$$
$$-f^2\text{Subst}\left(\int \left(\frac{a}{x^4}+\frac{b\sin(c+dx)}{x^4}\right)\,dx,x,\frac{1}{x}\right)$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3-\left(be^2\right)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,\frac{1}{x}\right)$$
$$-(2bef)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^3}\,dx,x,\frac{1}{x}\right)-\left(bf^2\right)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^4}\,dx,x,\frac{1}{x}\right)$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+be^2x\sin\left(c+\frac{d}{x}\right)+befx^2\sin\left(c+\frac{d}{x}\right)+\frac{1}{3}bf^2x^3\sin\left(c+\frac{d}{x}\right)$$
$$-\left(bde^2\right)\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx,x,\frac{1}{x}\right)-(bdef)\text{Subst}\left(\int \frac{\cos(c+dx)}{x^2}\,dx,x,\frac{1}{x}\right)$$
$$-\frac{1}{3}\left(bdf^2\right)\text{Subst}\left(\int \frac{\cos(c+dx)}{x^3}\,dx,x,\frac{1}{x}\right)$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+bdefx\cos\left(c+\frac{d}{x}\right)$$
$$+\frac{1}{6}bdf^2x^2\cos\left(c+\frac{d}{x}\right)+be^2x\sin\left(c+\frac{d}{x}\right)+befx^2\sin\left(c+\frac{d}{x}\right)$$
$$+\frac{1}{3}bf^2x^3\sin\left(c+\frac{d}{x}\right)+\left(bd^2ef\right)\text{Subst}\left(\int \frac{\sin(c+dx)}{x}\,dx,x,\frac{1}{x}\right)$$
$$+\frac{1}{6}\left(bd^2f^2\right)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,\frac{1}{x}\right)$$
$$-\left(bde^2\cos(c)\right)\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx,x,\frac{1}{x}\right)$$
$$+\left(bde^2\sin(c)\right)\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx,x,\frac{1}{x}\right)$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\frac{d}{x}\right)$$
$$-bde^2\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)+be^2x\sin\left(c+\frac{d}{x}\right)$$
$$-\frac{1}{6}bd^2f^2x\sin\left(c+\frac{d}{x}\right)+befx^2\sin\left(c+\frac{d}{x}\right)+\frac{1}{3}bf^2x^3\sin\left(c+\frac{d}{x}\right)$$
$$+bde^2\sin(c)\text{Si}\left(\frac{d}{x}\right)+\frac{1}{6}\left(bd^3f^2\right)\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx,x,\frac{1}{x}\right)$$
$$+\left(bd^2ef\cos(c)\right)\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx,x,\frac{1}{x}\right)$$
$$+\left(bd^2ef\sin(c)\right)\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx,x,\frac{1}{x}\right)$$

$$= ae^2x + aefx^2 + \frac{1}{3}af^2x^3 + bdefx\cos\left(c + \frac{d}{x}\right) + \frac{1}{6}bdf^2x^2\cos\left(c + \frac{d}{x}\right) - bde^2\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) + bd^2ef\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c) + be^2x\sin\left(c + \frac{d}{x}\right) - \frac{1}{6}bd^2f^2x\sin\left(c + \frac{d}{x}\right) + befx^2\sin\left(c + \frac{d}{x}\right) + \frac{1}{3}bf^2x^3\sin\left(c + \frac{d}{x}\right) + bd^2ef\cos(c)\text{Si}\left(\frac{d}{x}\right) + bde^2\sin(c)\text{Si}\left(\frac{d}{x}\right) + \frac{1}{6}\left(bd^3f^2\cos(c)\right)\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx, x, \frac{1}{x}\right) - \frac{1}{6}\left(bd^3f^2\sin(c)\right)\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= ae^2x + aefx^2 + \frac{1}{3}af^2x^3 + bdefx\cos\left(c + \frac{d}{x}\right) + \frac{1}{6}bdf^2x^2\cos\left(c + \frac{d}{x}\right) - bde^2\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) + \frac{1}{6}bd^3f^2\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) + bd^2ef\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c) + be^2x\sin\left(c + \frac{d}{x}\right) - \frac{1}{6}bd^2f^2x\sin\left(c + \frac{d}{x}\right) + befx^2\sin\left(c + \frac{d}{x}\right) + \frac{1}{3}bf^2x^3\sin\left(c + \frac{d}{x}\right) + bd^2ef\cos(c)\text{Si}\left(\frac{d}{x}\right) + bde^2\sin(c)\text{Si}\left(\frac{d}{x}\right) - \frac{1}{6}bd^3f^2\sin(c)\text{Si}\left(\frac{d}{x}\right)$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

$$\int (e+fx)^2\left(a + b\sin\left(c + \frac{d}{x}\right)\right)\,dx = \frac{1}{6}\left(bd\,\text{CosIntegral}\left(\frac{d}{x}\right)\left(\left(-6e^2 + d^2f^2\right)\cos(c) + 6def\sin(c)\right) + x\left(2a\left(3e^2 + 3efx + f^2x^2\right) + bdf(6e + fx)\cos\left(c + \frac{d}{x}\right) + b\left(6e^2 + 6efx - f^2\left(d^2 - 2x^2\right)\right)\sin\left(c + \frac{d}{x}\right)\right) - bd\left(-6def\cos(c) + \left(-6e^2 + d^2f^2\right)\sin(c)\right)\text{Si}\left(\frac{d}{x}\right)\right)$$

```
[In] Integrate[(e + f*x)^2*(a + b*Sin[c + d/x]),x]
```

```
[Out] (b*d*CosIntegral[d/x]*((-6*e^2 + d^2*f^2)*Cos[c] + 6*d*e*f*Sin[c]) + x*(2*a
*(3*e^2 + 3*e*f*x + f^2*x^2) + b*d*f*(6*e + f*x)*Cos[c + d/x] + b*(6*e^2 +
```

```
6*e*f*x - f^2*(d^2 - 2*x^2))*Sin[c + d/x]) - b*d*(-6*d*e*f*Cos[c] + (-6*e^2
 + d^2*f^2)*Sin[c])*SinIntegral[d/x])/6
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.84

method	result
parts	$\frac{a(fx+e)^3}{3f} - bd\left(e^2\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d} - \text{Si}\left(\frac{d}{x}\right)\sin(c) + \text{Ci}\left(\frac{d}{x}\right)\cos(c)\right) + 2def\left(-\frac{\sin\left(c+\frac{d}{x}\right)x^2}{2d^2} - \right.\right.$
derivativedivides	$-d\left(-\frac{a f^2 x^3}{3d} - \frac{aef x^2}{d} - \frac{a e^2 x}{d} + b d^2 f^2\left(-\frac{\sin\left(c+\frac{d}{x}\right)x^3}{3d^3} - \frac{\cos\left(c+\frac{d}{x}\right)x^2}{6d^2} + \frac{\sin\left(c+\frac{d}{x}\right)x}{6d} + \frac{\text{Si}\left(\frac{d}{x}\right)\sin(c}{6}\right.\right.$
default	$-d\left(-\frac{a f^2 x^3}{3d} - \frac{aef x^2}{d} - \frac{a e^2 x}{d} + b d^2 f^2\left(-\frac{\sin\left(c+\frac{d}{x}\right)x^3}{3d^3} - \frac{\cos\left(c+\frac{d}{x}\right)x^2}{6d^2} + \frac{\sin\left(c+\frac{d}{x}\right)x}{6d} + \frac{\text{Si}\left(\frac{d}{x}\right)\sin(c}{6}\right.\right.$
risch	$a e^2 x + \frac{a f^2 x^3}{3} + aef x^2 + \frac{bd e^2 e^{-ic} \text{Ei}_1\left(\frac{id}{x}\right)}{2} - \frac{b d^3 f^2 e^{-ic} \text{Ei}_1\left(\frac{id}{x}\right)}{12} - \frac{ib d^2 ef e^{-ic} \text{Ei}_1\left(\frac{id}{x}\right)}{2} + \frac{bd e^2 e^{ic} \text{Ei}}{2}$

```
[In] int((f*x+e)^2*(a+b*sin(c+d/x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*(f*x+e)^3/f-b*d*(e^2*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*cos(c))+
2*d*e*f*(-1/2*sin(c+d/x)/d^2*x^2-1/2*cos(c+d/x)/d*x-1/2*Si(d/x)*cos(c)-1/2*
Ci(d/x)*sin(c))+d^2*f^2*(-1/3*sin(c+d/x)/d^3*x^3-1/6*cos(c+d/x)/d^2*x^2+1/6
*sin(c+d/x)/d*x+1/6*Si(d/x)*sin(c)-1/6*Ci(d/x)*cos(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.83

$$\begin{aligned}\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx &= \frac{1}{3}af^2x^3 + aefx^2 + ae^2x + \frac{1}{6}\left(6\,bd^2ef\,\text{Si}\left(\frac{d}{x}\right) + \left(bd^3f^2 - 6\,bde^2\right)\text{Ci}\left(\frac{d}{x}\right)\right)\cos(c) \\ &\quad + \frac{1}{6}\left(bdf^2x^2 + 6\,bdefx\right)\cos\left(\frac{cx+d}{x}\right) \\ &\quad + \frac{1}{6}\left(6\,bd^2ef\,\text{Ci}\left(\frac{d}{x}\right) - \left(bd^3f^2 - 6\,bde^2\right)\text{Si}\left(\frac{d}{x}\right)\right)\sin(c) \\ &\quad + \frac{1}{6}\left(2\,bf^2x^3 + 6\,befx^2 - \left(bd^2f^2 - 6\,be^2\right)x\right)\sin\left(\frac{cx+d}{x}\right)\end{aligned}$$

```
[In] integrate((f*x+e)^2*(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] 1/3*a*f^2*x^3 + a*e*f*x^2 + a*e^2*x + 1/6*(6*b*d^2*e*f*sin_integral(d/x) +
(b*d^3*f^2 - 6*b*d*e^2)*cos_integral(d/x))*cos(c) + 1/6*(b*d*f^2*x^2 + 6*b*
d*e*f*x)*cos((c*x + d)/x) + 1/6*(6*b*d^2*e*f*cos_integral(d/x) - (b*d^3*f^2
 - 6*b*d*e^2)*sin_integral(d/x))*sin(c) + 1/6*(2*b*f^2*x^3 + 6*b*e*f*x^2 -
(b*d^2*f^2 - 6*b*e^2)*x)*sin((c*x + d)/x)
```

Sympy [F]

$$\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)(e+fx)^2\,dx$$

```
[In] integrate((f*x+e)**2*(a+b*sin(c+d/x)),x)
```

```
[Out] Integral((a + b*sin(c + d/x))*(e + f*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.15

$$\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\frac{1}{3}af^2x^3+aefx^2$$
$$-\frac{1}{2}\left(\left(\left(\mathrm{Ei}\left(\frac{id}{x}\right)+\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\cos(c)-\left(-i\,\mathrm{Ei}\left(\frac{id}{x}\right)+i\,\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d-2x\sin\left(\frac{cx+d}{x}\right)\right)be$$
$$+\frac{1}{2}\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{id}{x}\right)+i\,\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\cos(c)+\left(\mathrm{Ei}\left(\frac{id}{x}\right)+\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d^2+2dx\cos\left(\frac{cx+d}{x}\right)\right.$$
$$+\frac{1}{12}\left(\left(\left(\mathrm{Ei}\left(\frac{id}{x}\right)+\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\cos(c)+\left(i\,\mathrm{Ei}\left(\frac{id}{x}\right)-i\,\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d^3+2dx^2\cos\left(\frac{cx+d}{x}\right)\right.$$
$$+ae^2x$$

```
[In] integrate((f*x+e)^2*(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*f^2*x^3 + a*e*f*x^2 - 1/2*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(
I*d/x) + I*Ei(-I*d/x))*sin(c))*d - 2*x*sin((c*x + d)/x))*b*e^2 + 1/2*(((-I*
Ei(I*d/x) + I*Ei(-I*d/x))*cos(c) + (Ei(I*d/x) + Ei(-I*d/x))*sin(c))*d^2 + 2
*d*x*cos((c*x + d)/x) + 2*x^2*sin((c*x + d)/x))*b*e*f + 1/12*(((Ei(I*d/x) +
 Ei(-I*d/x))*cos(c) + (I*Ei(I*d/x) - I*Ei(-I*d/x))*sin(c))*d^3 + 2*d*x^2*co
s((c*x + d)/x) - 2*(d^2*x - 2*x^3)*sin((c*x + d)/x))*b*f^2 + a*e^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. 2(212) = 424.

Time = 0.38 (sec) , antiderivative size = 1263, normalized size of antiderivative = 5.64

$$\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] 1/6*(b*c^3*d^4*f^2*cos(c)*cos_integral(-c + (c*x + d)/x) + b*c^3*d^4*f^2*si
n(c)*sin_integral(c - (c*x + d)/x) - 3*(c*x + d)*b*c^2*d^4*f^2*cos(c)*cos_i
ntegral(-c + (c*x + d)/x)/x + 6*b*c^3*d^3*e*f*cos_integral(-c + (c*x + d)/x
)*sin(c) - 6*b*c^3*d^3*e*f*cos(c)*sin_integral(c - (c*x + d)/x) - 3*(c*x +
d)*b*c^2*d^4*f^2*sin(c)*sin_integral(c - (c*x + d)/x)/x - 6*b*c^3*d^2*e^2*c
os(c)*cos_integral(-c + (c*x + d)/x) + 3*(c*x + d)^2*b*c*d^4*f^2*cos(c)*cos
_integral(-c + (c*x + d)/x)/x^2 - 18*(c*x + d)*b*c^2*d^3*e*f*cos_integral(-
c + (c*x + d)/x)*sin(c)/x + b*c^2*d^4*f^2*sin((c*x + d)/x) + 18*(c*x + d)*b
*c^2*d^3*e*f*cos(c)*sin_integral(c - (c*x + d)/x)/x - 6*b*c^3*d^2*e^2*sin(c
)*sin_integral(c - (c*x + d)/x) + 3*(c*x + d)^2*b*c*d^4*f^2*sin(c)*sin_inte
gral(c - (c*x + d)/x)/x^2 - 6*b*c^2*d^3*e*f*cos((c*x + d)/x) + b*c*d^4*f^2*
cos((c*x + d)/x) - (c*x + d)^3*b*d^4*f^2*cos(c)*cos_integral(-c + (c*x + d)
/x)/x^3 + 18*(c*x + d)*b*c^2*d^2*e^2*cos(c)*cos_integral(-c + (c*x + d)/x)/
x + 18*(c*x + d)^2*b*c*d^3*e*f*cos_integral(-c + (c*x + d)/x)*sin(c)/x^2 -
2*(c*x + d)*b*c*d^4*f^2*sin((c*x + d)/x)/x - 18*(c*x + d)^2*b*c*d^3*e*f*cos
(c)*sin_integral(c - (c*x + d)/x)/x^2 - (c*x + d)^3*b*d^4*f^2*sin(c)*sin_in
tegral(c - (c*x + d)/x)/x^3 + 18*(c*x + d)*b*c^2*d^2*e^2*sin(c)*sin_integra
l(c - (c*x + d)/x)/x + 12*(c*x + d)*b*c*d^3*e*f*cos((c*x + d)/x)/x - (c*x +
 d)*b*d^4*f^2*cos((c*x + d)/x)/x - 18*(c*x + d)^2*b*c*d^2*e^2*cos(c)*cos_in
tegral(-c + (c*x + d)/x)/x^2 - 6*(c*x + d)^3*b*d^3*e*f*cos_integral(-c + (c
*x + d)/x)*sin(c)/x^3 - 6*b*c^2*d^2*e^2*sin((c*x + d)/x) + 6*b*c*d^3*e*f*si
n((c*x + d)/x) - 2*b*d^4*f^2*sin((c*x + d)/x) + (c*x + d)^2*b*d^4*f^2*sin((
c*x + d)/x)/x^2 + 6*(c*x + d)^3*b*d^3*e*f*cos(c)*sin_integral(c - (c*x + d)
/x)/x^3 - 18*(c*x + d)^2*b*c*d^2*e^2*sin(c)*sin_integral(c - (c*x + d)/x)/x
^2 - 6*a*c^2*d^2*e^2 + 6*a*c*d^3*e*f - 2*a*d^4*f^2 - 6*(c*x + d)^2*b*d^3*e*
f*cos((c*x + d)/x)/x^2 + 6*(c*x + d)^3*b*d^2*e^2*cos(c)*cos_integral(-c + (
c*x + d)/x)/x^3 + 12*(c*x + d)*b*c*d^2*e^2*sin((c*x + d)/x)/x - 6*(c*x + d)
*b*d^3*e*f*sin((c*x + d)/x)/x + 6*(c*x + d)^3*b*d^2*e^2*sin(c)*sin_integral
(c - (c*x + d)/x)/x^3 + 12*(c*x + d)*a*c*d^2*e^2/x - 6*(c*x + d)*a*d^3*e*f/
x - 6*(c*x + d)^2*b*d^2*e^2*sin((c*x + d)/x)/x^2 - 6*(c*x + d)^2*a*d^2*e^2/
x^2)/((c^3 - 3*(c*x + d)*c^2/x + 3*(c*x + d)^2*c/x^2 - (c*x + d)^3/x^3)*d)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\int (e+f\,x)^2\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

```
[In] int((e + f*x)^2*(a + b*sin(c + d/x)),x)
[Out] int((e + f*x)^2*(a + b*sin(c + d/x)), x)
```

3.289 $\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 118

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx = aex+\frac{1}{2}afx^2+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right) - bde\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) + \frac{1}{2}bd^2f\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c) + bex\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bfx^2\sin\left(c+\frac{d}{x}\right) + \frac{1}{2}bd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+bde\sin(c)\text{Si}\left(\frac{d}{x}\right)$$

```
[Out] a*e*x+1/2*a*f*x^2-b*d*e*Ci(d/x)*cos(c)+1/2*b*d*f*x*cos(c+d/x)+1/2*b*d^2*f*c
os(c)*Si(d/x)+1/2*b*d^2*f*Ci(d/x)*sin(c)+b*d*e*Si(d/x)*sin(c)+b*e*x*sin(c+d
/x)+1/2*b*f*x^2*sin(c+d/x)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{3512, 14, 3378, 3384, 3380, 3383\}$

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)dx = aex+\frac{1}{2}afx^2+\frac{1}{2}bd^2f\sin(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) - bde\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)+\frac{1}{2}bd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right) + bde\sin(c)\text{Si}\left(\frac{d}{x}\right)+bex\sin\left(c+\frac{d}{x}\right) + \frac{1}{2}bfx^2\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right)$$

```
[In] Int[(e + f*x)*(a + b*Sin[c + d/x]),x]
```

```
[Out] a*e*x + (a*f*x^2)/2 + (b*d*f*x*Cos[c + d/x])/2 - b*d*e*Cos[c]*CosIntegral[d
/x] + (b*d^2*f*CosIntegral[d/x]*Sin[c])/2 + b*e*x*Sin[c + d/x] + (b*f*x^2*S
in[c + d/x])/2 + (b*d^2*f*Cos[c]*SinIntegral[d/x])/2 + b*d*e*Sin[c]*SinInte
gral[d/x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \left(\frac{f(a+b\sin(c+dx))}{x^3}+\frac{e(a+b\sin(c+dx))}{x^2}\right)\,dx, x, \frac{1}{x}\right)\\
&= -\left(e\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)\right)-f\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= -\left(e\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{b\sin(c+dx)}{x^2}\right)\,dx, x, \frac{1}{x}\right)\right)\\
&\quad -f\text{Subst}\left(\int \left(\frac{a}{x^3}+\frac{b\sin(c+dx)}{x^3}\right)\,dx, x, \frac{1}{x}\right)\\
&= aex+\frac{1}{2}afx^2-(be)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)-(bf)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= aex+\frac{1}{2}afx^2+bex\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bfx^2\sin\left(c+\frac{d}{x}\right)\\
&\quad -(bde)\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx, x, \frac{1}{x}\right)-\frac{1}{2}(bdf)\text{Subst}\left(\int \frac{\cos(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= aex+\frac{1}{2}afx^2+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right)+bex\sin\left(c+\frac{d}{x}\right)\\
&\quad +\frac{1}{2}bfx^2\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}\left(bd^2f\right)\text{Subst}\left(\int \frac{\sin(c+dx)}{x}\,dx, x, \frac{1}{x}\right)\\
&\quad -(bde\cos(c))\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx, x, \frac{1}{x}\right)\\
&\quad +(bde\sin(c))\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= aex + \frac{1}{2}afx^2 + \frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right) - bde\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) + bex\sin\left(c+\frac{d}{x}\right) + \frac{1}{2}bfx^2\sin\left(c+\frac{d}{x}\right) + bde\sin(c)\text{Si}\left(\frac{d}{x}\right) + \frac{1}{2}\left(bd^2f\cos(c)\right)\text{Subst}\left(\int\frac{\sin(dx)}{x}\,dx, x, \frac{1}{x}\right) + \frac{1}{2}\left(bd^2f\sin(c)\right)\text{Subst}\left(\int\frac{\cos(dx)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= aex + \frac{1}{2}afx^2 + \frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right) - bde\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) + \frac{1}{2}bd^2f\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c) + bex\sin\left(c+\frac{d}{x}\right) + \frac{1}{2}bfx^2\sin\left(c+\frac{d}{x}\right) + \frac{1}{2}bd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right) + bde\sin(c)\text{Si}\left(\frac{d}{x}\right)$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.67

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx = \frac{1}{2}\left(bdfx\cos\left(c+\frac{d}{x}\right) + bd\,\text{CosIntegral}\left(\frac{d}{x}\right)(-2e\cos(c)+df\sin(c)) + x(2e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right) + bd(df\cos(c)+2e\sin(c))\text{Si}\left(\frac{d}{x}\right)\right)$$

```
[In] Integrate[(e + f*x)*(a + b*Sin[c + d/x]),x]
```

```
[Out] (b*d*f*x*Cos[c + d/x] + b*d*CosIntegral[d/x]*(-2*e*Cos[c] + d*f*Sin[c]) + x
*(2*e + f*x)*(a + b*Sin[c + d/x]) + b*d*(d*f*Cos[c] + 2*e*Sin[c])*SinIntegr
al[d/x])/2
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

method	result
parts	$a\left(\frac{1}{2}f\,x^2+ex\right)-bd\left(e\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\mathrm{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\mathrm{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\right)+df\left(-\frac{\sin\left(c+\frac{d}{x}\right)x^2}{2d^2}\right.\right.$
derivativedivides	$-d\left(-\frac{af\,x^2}{2d}-\frac{aex}{d}+bdf\left(-\frac{\sin\left(c+\frac{d}{x}\right)x^2}{2d^2}-\frac{\cos\left(c+\frac{d}{x}\right)x}{2d}-\frac{\mathrm{Si}\left(\frac{d}{x}\right)\cos(c)}{2}-\frac{\mathrm{Ci}\left(\frac{d}{x}\right)\sin(c)}{2}\right)+be\left(-\right.\right.$
default	$-d\left(-\frac{af\,x^2}{2d}-\frac{aex}{d}+bdf\left(-\frac{\sin\left(c+\frac{d}{x}\right)x^2}{2d^2}-\frac{\cos\left(c+\frac{d}{x}\right)x}{2d}-\frac{\mathrm{Si}\left(\frac{d}{x}\right)\cos(c)}{2}-\frac{\mathrm{Ci}\left(\frac{d}{x}\right)\sin(c)}{2}\right)+be\left(-\right.\right.$
risch	$aex+\frac{af\,x^2}{2}+\frac{bde\,\mathrm{e}^{-ic}\,\mathrm{Ei}_1\left(\frac{id}{x}\right)}{2}-\frac{ib\,d^2f\,\mathrm{e}^{-ic}\,\mathrm{Ei}_1\left(\frac{id}{x}\right)}{4}+\frac{bde\,\mathrm{e}^{ic}\,\mathrm{Ei}_1\left(-\frac{id}{x}\right)}{2}+\frac{ib\,d^2f\,\mathrm{e}^{ic}\,\mathrm{Ei}_1\left(-\frac{id}{x}\right)}{4}+$

```
[In] int((f*x+e)*(a+b*sin(c+d/x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*f*x^2+e*x)-b*d*(e*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*cos(c))+d*
f*(-1/2*sin(c+d/x)/d^2*x^2-1/2*cos(c+d/x)/d*x-1/2*Si(d/x)*cos(c)-1/2*Ci(d/x
)*sin(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int(e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\frac{1}{2}\,bdfx\cos\left(\frac{cx+d}{x}\right)+\frac{1}{2}\,afx^2+aex+\frac{1}{2}\left(bd^2f\,\mathrm{Si}\left(\frac{d}{x}\right)-2\,bde\,\mathrm{Ci}\left(\frac{d}{x}\right)\right)\cos\left(c\right)+\frac{1}{2}\left(bd^2f\,\mathrm{Ci}\left(\frac{d}{x}\right)+2\,bde\,\mathrm{Si}\left(\frac{d}{x}\right)\right)\sin\left(c\right)+\frac{1}{2}\left(bfx^2+2\,bex\right)\sin\left(\frac{cx+d}{x}\right)$$

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] 1/2*b*d*f*x*cos((c*x + d)/x) + 1/2*a*f*x^2 + a*e*x + 1/2*(b*d^2*f*sin_integ
ral(d/x) - 2*b*d*e*cos_integral(d/x))*cos(c) + 1/2*(b*d^2*f*cos_integral(d/
x) + 2*b*d*e*sin_integral(d/x))*sin(c) + 1/2*(b*f*x^2 + 2*b*e*x)*sin((c*x +
 d)/x)
```

Sympy [F]

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)(e+fx)\,dx$$

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x)),x)
[Out] Integral((a + b*sin(c + d/x))*(e + f*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.30

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\frac{1}{2}afx^2 -\frac{1}{2}\left(\left(\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos(c)-\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin(c)\right)d-2\,x\sin\left(\frac{cx+d}{x}\right)\right)be +\frac{1}{4}\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos(c)+\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin(c)\right)d^2+2\,dx\cos\left(\frac{cx+d}{x}\right) \right. + aex$$

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x)),x, algorithm="maxima")
[Out] 1/2*a*f*x^2 - 1/2*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(
-I*d/x))*sin(c))*d - 2*x*sin((c*x + d)/x))*b*e + 1/4*(((-I*Ei(I*d/x) + I*Ei
(-I*d/x))*cos(c) + (Ei(I*d/x) + Ei(-I*d/x))*sin(c))*d^2 + 2*d*x*cos((c*x +
d)/x) + 2*x^2*sin((c*x + d)/x))*b*f + a*e*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(108) = 216.

Time = 0.32 (sec) , antiderivative size = 520, normalized size of antiderivative = 4.41

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx =\frac{bc^2d^3f\,\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)\sin(c)-bc^2d^3f\cos(c)\,\mathrm{Si}\left(c-\frac{cx+d}{x}\right)-2\,bc^2d^2e\cos(c)\,\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)-\frac{2\,(cx+d)bcd^3f\,\mathrm{C}\cdots}{\cdots}}{\cdots}$$

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] 1/2*(b*c^2*d^3*f*cos_integral(-c + (c*x + d)/x)*sin(c) - b*c^2*d^3*f*cos(c)
*sin_integral(c - (c*x + d)/x) - 2*b*c^2*d^2*e*cos(c)*cos_integral(-c + (c*
x + d)/x) - 2*(c*x + d)*b*c*d^3*f*cos_integral(-c + (c*x + d)/x)*sin(c)/x +
 2*(c*x + d)*b*c*d^3*f*cos(c)*sin_integral(c - (c*x + d)/x)/x - 2*b*c^2*d^2
*e*sin(c)*sin_integral(c - (c*x + d)/x) - b*c*d^3*f*cos((c*x + d)/x) + 4*(c
*x + d)*b*c*d^2*e*cos(c)*cos_integral(-c + (c*x + d)/x)/x + (c*x + d)^2*b*d
^3*f*cos_integral(-c + (c*x + d)/x)*sin(c)/x^2 - (c*x + d)^2*b*d^3*f*cos(c)
*sin_integral(c - (c*x + d)/x)/x^2 + 4*(c*x + d)*b*c*d^2*e*sin(c)*sin_integ
ral(c - (c*x + d)/x)/x + (c*x + d)*b*d^3*f*cos((c*x + d)/x)/x - 2*(c*x + d)
^2*b*d^2*e*cos(c)*cos_integral(-c + (c*x + d)/x)/x^2 - 2*b*c*d^2*e*sin((c*x
 + d)/x) + b*d^3*f*sin((c*x + d)/x) - 2*(c*x + d)^2*b*d^2*e*sin(c)*sin_inte
gral(c - (c*x + d)/x)/x^2 - 2*a*c*d^2*e + a*d^3*f + 2*(c*x + d)*b*d^2*e*sin
((c*x + d)/x)/x + 2*(c*x + d)*a*d^2*e/x)/((c^2 - 2*(c*x + d)*c/x + (c*x + d
)^2/x^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\int (e+f\,x)\,\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

```
[In] int((e + f*x)*(a + b*sin(c + d/x)),x)
```

```
[Out] int((e + f*x)*(a + b*sin(c + d/x)), x)
```

3.290 $\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx = ax-bd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)+bx\sin\left(c+\frac{d}{x}\right)+bd\sin(c)\text{Si}\left(\frac{d}{x}\right)$$

```
[Out] a*x-b*d*Ci(d/x)*cos(c)+b*d*Si(d/x)*sin(c)+b*x*sin(c+d/x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3442, 3378, 3384, 3380, 3383}

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx = ax-bd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)+bd\sin(c)\text{Si}\left(\frac{d}{x}\right)+bx\sin\left(c+\frac{d}{x}\right)$$

```
[In] Int[a + b*Sin[c + d/x],x]
```

```
[Out] a*x - b*d*Cos[c]*CosIntegral[d/x] + b*x*Sin[c + d/x] + b*d*Sin[c]*SinIntegr
al[d/x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \sin\left(c+\frac{d}{x}\right)\,dx \\
&= ax - b\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= ax + bx\sin\left(c+\frac{d}{x}\right) - (bd)\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx, x, \frac{1}{x}\right) \\
&= ax + bx\sin\left(c+\frac{d}{x}\right) - (bd\cos(c))\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx, x, \frac{1}{x}\right) \\
&\quad + (bd\sin(c))\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx, x, \frac{1}{x}\right) \\
&= ax - bd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) + bx\sin\left(c+\frac{d}{x}\right) + bd\sin(c)\text{Si}\left(\frac{d}{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=ax+bx\cos\left(\frac{d}{x}\right)\sin(c)+bx\cos(c)\sin\left(\frac{d}{x}\right)-bd\left(\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)-\sin(c)\text{Si}\left(\frac{d}{x}\right)\right)$$

```
[In] Integrate[a + b*Sin[c + d/x],x]
[Out] a*x + b*x*Cos[d/x]*Sin[c] + b*x*Cos[c]*Sin[d/x] - b*d*(Cos[c]*CosIntegral[d
/x] - Sin[c]*SinIntegral[d/x])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

method	result	siz
default	$ax-bd\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\text{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\text{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\right)$	43
parts	$ax-bd\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\text{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\text{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\right)$	43
derivativedivides	$-d\left(-\frac{ax}{d}+b\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\text{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\text{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\right)\right)$	48
risch	$ax-\frac{i\pi\,\text{csgn}\left(\frac{d}{x}\right)\mathrm{e}^{-ic}bd}{2}+i\,\text{Si}\left(\frac{d}{x}\right)\mathrm{e}^{-ic}bd+\frac{\mathrm{e}^{-ic}\,\text{Ei}_1\left(-\frac{id}{x}\right)bd}{2}+\frac{\mathrm{e}^{ic}\,\text{Ei}_1\left(-\frac{id}{x}\right)bd}{2}+bx\sin\left(\frac{cx+d}{x}\right)$	87

```
[In] int(a+b*sin(c+d/x),x,method=_RETURNVERBOSE)
[Out] a*x-b*d*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*cos(c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=-bd\cos\left(c\right)\text{Ci}\left(\frac{d}{x}\right)+bd\sin\left(c\right)\text{Si}\left(\frac{d}{x}\right)+bx\sin\left(\frac{cx+d}{x}\right)+ax$$

```
[In] integrate(a+b*sin(c+d/x),x, algorithm="fricas")
[Out] -b*d*cos(c)*cos_integral(d/x) + b*d*sin(c)*sin_integral(d/x) + b*x*sin((c*x
 + d)/x) + a*x
```

Sympy [F]

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

```
[In] integrate(a+b*sin(c+d/x),x)
[Out] Integral(a + b*sin(c + d/x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx = -\frac{1}{2}\left(\left(\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos\left(c\right)-\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin\left(c\right)\right)d-2\,x\sin\left(\frac{cx+d}{x}\right)\right) + ax$$

```
[In] integrate(a+b*sin(c+d/x),x, algorithm="maxima")
[Out] -1/2*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(-I*d/x))*sin(
c))*d - 2*x*sin((c*x + d)/x))*b + a*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(38) = 76.

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.61

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx = ax - \frac{\left(cd^2\cos\left(c\right)\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)+cd^2\sin\left(c\right)\mathrm{Si}\left(c-\frac{cx+d}{x}\right)-\frac{(cx+d)d^2\cos(c)\,\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)}{x}-\frac{(cx+d)d^2\sin(c)\,\mathrm{Si}\left(c-\frac{cx+d}{x}\right)}{x}\right.}{\left(c-\frac{cx+d}{x}\right)d}$$

```
[In] integrate(a+b*sin(c+d/x),x, algorithm="giac")
[Out] a*x - (c*d^2*cos(c)*cos_integral(-c + (c*x + d)/x) + c*d^2*sin(c)*sin_integ
ral(c - (c*x + d)/x) - (c*x + d)*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)/
x - (c*x + d)*d^2*sin(c)*sin_integral(c - (c*x + d)/x)/x + d^2*sin((c*x + d
)/x))*b/((c - (c*x + d)/x)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\int a+b\,\sin\left(c+\frac{d}{x}\right)\,dx$$

```
[In] int(a + b*sin(c + d/x),x)
[Out] int(a + b*sin(c + d/x), x)
```

3.291 $\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 103

$$\begin{aligned}\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx &= \frac{a\log\left(f+\frac{e}{x}\right)}{f}+\frac{a\log(x)}{f}-\frac{b\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c)}{f}\\&\quad+\frac{b\,\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(c-\frac{df}{e}\right)}{f}\\&\quad+\frac{b\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f}-\frac{b\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f}\end{aligned}$$

```
[Out] a*ln(f+e/x)/f+a*ln(x)/f+b*cos(c-d*f/e)*Si(d*(f/e+1/x))/f-b*cos(c)*Si(d/x)/f
-b*Ci(d/x)*sin(c)/f+b*Ci(d*(f/e+1/x))*sin(c-d*f/e)/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3512, 14, 3384, 3380, 3383, 3398}

$$\begin{aligned}\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx &= \frac{a\log\left(\frac{e}{x}+f\right)}{f}+\frac{a\log(x)}{f}+\frac{b\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f}\\&\quad-\frac{b\sin(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)}{f}\\&\quad+\frac{b\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f}-\frac{b\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f}\end{aligned}$$

```
[In] Int[(a + b*Sin[c + d/x])/(e + f*x),x]
```

```
[Out] (a*Log[f + e/x])/f + (a*Log[x])/f - (b*CosIntegral[d/x]*Sin[c])/f + (b*CosI
ntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/f + (b*Cos[c - (d*f)/e]*SinInte
gral[d*(f/e + x^(-1))])/f - (b*Cos[c]*SinIntegral[d/x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \left(\frac{a+b\sin(c+dx)}{fx} - \frac{e(a+b\sin(c+dx))}{f(f+ex)}\right)\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{x}\,dx, x, \frac{1}{x}\right)}{f} + \frac{e\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a}{x} + \frac{b\sin(c+dx)}{x}\right)\,dx, x, \frac{1}{x}\right)}{f} + \frac{e\text{Subst}\left(\int \left(\frac{a}{f+ex} + \frac{b\sin(c+dx)}{f+ex}\right)\,dx, x, \frac{1}{x}\right)}{f}$$

$$= \frac{a\log\left(f+\frac{e}{x}\right)}{f} + \frac{a\log(x)}{f} - \frac{b\text{Subst}\left(\int \frac{\sin(c+dx)}{x}\,dx, x, \frac{1}{x}\right)}{f} + \frac{(be)\text{Subst}\left(\int \frac{\sin(c+dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{f}$$

$$= \frac{a\log\left(f+\frac{e}{x}\right)}{f} + \frac{a\log(x)}{f} - \frac{(b\cos(c))\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx, x, \frac{1}{x}\right)}{f}$$
$$+ \frac{\left(be\cos\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{f}$$
$$- \frac{(b\sin(c))\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx, x, \frac{1}{x}\right)}{f}$$
$$+ \frac{\left(be\sin\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{f}$$

$$= \frac{a\log\left(f+\frac{e}{x}\right)}{f} + \frac{a\log(x)}{f} - \frac{b\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c)}{f}$$
$$+ \frac{b\,\text{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(c-\frac{df}{e}\right)}{f} + \frac{b\cos\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{f} - \frac{b\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx$$
$$= \frac{a\log(e+fx) - b\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c) + b\,\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(c-\frac{df}{e}\right) + b\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\right.\right.}{f}$$

```
[In] Integrate[(a + b*Sin[c + d/x])/(e + f*x),x]
```

```
[Out] (a*Log[e + f*x] - b*CosIntegral[d/x]*Sin[c] + b*CosIntegral[d*(f/e + x^(-1)
)]*Sin[c - (d*f)/e] + b*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] - b*
Cos[c]*SinIntegral[d/x])/f
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.25

method	result
parts	$\frac{a\ln(fx+e)}{f}-bd\left(-\frac{e\left(\frac{\mathrm{Si}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+df}{e}\right)}{e}-\frac{\mathrm{Ci}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}\right)}{df}+\frac{\mathrm{Si}\left(\frac{d}{x}\right)\cos(c)+\mathrm{Ci}\left(\frac{d}{x}\right)\cdots}{df}\cdots\right.$
risch	$-\frac{ib\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{Ei}_1\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{2f}+\frac{ib\,\mathrm{Ei}_1\left(\frac{id}{x}\right)\mathrm{e}^{-ic}}{2f}+\frac{ib\,\mathrm{e}^{\frac{i(ce-df)}{e}}\,\mathrm{Ei}_1\left(-\frac{id}{x}-ic-\frac{-ice+ifd}{e}\right)}{2f}-\frac{ib\,\mathrm{Ei}_1\left(-\frac{id}{x}\right)\mathrm{e}\cdots}{2f}$
derivativedivides	$-d\left(\frac{a\ln\left(\frac{d}{x}\right)}{fd}-\frac{a\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{fd}+\frac{b\left(\mathrm{Si}\left(\frac{d}{x}\right)\cos(c)+\mathrm{Ci}\left(\frac{d}{x}\right)\sin(c)\right)}{fd}-\frac{be\left(-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+\cdots}{e}\right)}{e}\cdots\right.}{\cdots}\right.$
default	$-d\left(\frac{a\ln\left(\frac{d}{x}\right)}{fd}-\frac{a\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{fd}+\frac{b\left(\mathrm{Si}\left(\frac{d}{x}\right)\cos(c)+\mathrm{Ci}\left(\frac{d}{x}\right)\sin(c)\right)}{fd}-\frac{be\left(-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+\cdots}{e}\right)}{e}\cdots\right.}{\cdots}\right.$

```
[In] int((a+b*sin(c+d/x))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] a/f*ln(f*x+e)-b*d*(-e/d/f*(Si(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e-Ci(d/
x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e)+1/d/f*(Si(d/x)*cos(c)+Ci(d/x)*sin(c)
))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx = -\frac{b\,\mathrm{Ci}\left(\frac{d}{x}\right)\sin\left(c\right)+b\,\mathrm{Ci}\left(\frac{dfx+de}{ex}\right)\sin\left(-\frac{ce-df}{e}\right)+b\cos\left(c\right)\mathrm{Si}\left(\frac{d}{x}\right)-b\cos\left(-\frac{ce-df}{e}\right)\mathrm{Si}\left(\frac{dfx+de}{ex}\right)-a\log\left(fx+e\right)\cdots}{f}$$

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e),x, algorithm="fricas")
```

```
[Out] -(b*cos_integral(d/x)*sin(c) + b*cos_integral((d*f*x + d*e)/(e*x))*sin(-(c*
e - d*f)/e) + b*cos(c)*sin_integral(d/x) - b*cos(-(c*e - d*f)/e)*sin_integr
al((d*f*x + d*e)/(e*x)) - a*log(f*x + e))/f
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx=\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx$$

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e),x)
```

```
[Out] Integral((a + b*sin(c + d/x))/(e + f*x), x)
```

Maxima [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx=\int \frac{b\sin\left(c+\frac{d}{x}\right)+a}{fx+e}\,dx$$

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e),x, algorithm="maxima")
```

```
[Out] b*(integrate(1/2*sin((c*x + d)/x)/((f*x + e)*cos((c*x + d)/x)^2 + (f*x + e)
*sin((c*x + d)/x)^2), x) + integrate(1/2*sin((c*x + d)/x)/(f*x + e), x)) +
a*log(f*x + e)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.64

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx=-\frac{bd\,\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)\sin(c)-bd\,\mathrm{Ci}\left(-\frac{ce-df-\frac{(cx+d)e}{x}}{e}\right)\sin\left(\frac{ce-df}{e}\right)-bd\cos(c)\,\mathrm{Si}\left(c-\frac{cx+d}{x}\right)+bd\cos\left(\frac{ce-df}{e}\right)}{df}$$

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e),x, algorithm="giac")
```

```
[Out] -(b*d*cos_integral(-c + (c*x + d)/x)*sin(c) - b*d*cos_integral(-(c*e - d*f
- (c*x + d)*e/x)/e)*sin((c*e - d*f)/e) - b*d*cos(c)*sin_integral(c - (c*x +
 d)/x) + b*d*cos((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e)
 - a*d*log(c*e - d*f - (c*x + d)*e/x) + a*d*log(c - (c*x + d)/x))/(d*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx = \int \frac{a+b\,\sin\left(c+\frac{d}{x}\right)}{e+f\,x}\,dx$$

```
[In] int((a + b*sin(c + d/x))/(e + f*x),x)
[Out] int((a + b*sin(c + d/x))/(e + f*x), x)
```

3.292 $\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 94

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx = \frac{a}{e\left(f+\frac{e}{x}\right)} - \frac{bd\cos\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{bd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}$$

```
[Out] a/e/(f+e/x)-b*d*Ci(d*(f/e+1/x))*cos(c-d*f/e)/e^2+b*d*Si(d*(f/e+1/x))*sin(c-
d*f/e)/e^2+b*sin(c+d/x)/e/(f+e/x)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3512, 3398, 3378, 3384, 3380, 3383}

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx = \frac{a}{e\left(\frac{e}{x}+f\right)} - \frac{bd\cos\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2} + \frac{bd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)}$$

```
[In] Int[(a + b*Sin[c + d/x])/(e + f*x)^2,x]
```

```
[Out] a/(e*(f + e/x)) - (b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))])/e^2
+ (b*Sin[c + d/x])/(e*(f + e/x)) + (b*d*Sin[c - (d*f)/e]*SinIntegral[d*(f/e
 + x^(-1))])/e^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int \left(\frac{a}{(f+ex)^2}+\frac{b\sin(c+dx)}{(f+ex)^2}\right)\,dx, x, \frac{1}{x}\right)\end{aligned}$$

$$= \frac{a}{e\left(f+\frac{e}{x}\right)} - b\text{Subst}\left(\int \frac{\sin(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a}{e\left(f+\frac{e}{x}\right)} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(c+dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{a}{e\left(f+\frac{e}{x}\right)} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} - \frac{\left(bd\cos\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e} + \frac{\left(bd\sin\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{a}{e\left(f+\frac{e}{x}\right)} - \frac{bd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{bd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx$$
$$= \frac{-bd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right) + \frac{e\left(-ae+bfx\sin\left(c+\frac{d}{x}\right)\right)}{f(e+fx)} + bd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}$$

```
[In] Integrate[(a + b*Sin[c + d/x])/(e + f*x)^2,x]
[Out] (-(b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))]) + (e*(-(a*e) + b*f*x
*Sin[c + d/x]))/(f*(e + f*x)) + b*d*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x
^(-1))])/e^2
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

method	result
parts	$-\frac{a}{f(fx+e)}-bd\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{\frac{\mathrm{Si}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\frac{\mathrm{Ci}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+df}{e}\right)}{e}}{e}\right)$
derivativedivides	$-d\left(-\frac{a}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+b\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\frac{\mathrm{Ci}\left(\frac{d}{x}+c+\frac{-ce+}{e}\right.}{}}{e}\right.\right.$
default	$-d\left(-\frac{a}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+b\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\frac{\mathrm{Ci}\left(\frac{d}{x}+c+\frac{-ce+}{e}\right.}{}}{e}\right.\right.$
risch	$-\frac{a}{f(fx+e)}+\frac{bd\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\mathrm{Ei}_1\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{2e^2}+\frac{bd\,\mathrm{e}^{\frac{i(ce-df)}{e}}\mathrm{Ei}_1\left(-\frac{id}{x}-ic-\frac{-ice+ifd}{e}\right)}{2e^2}+\frac{ibd\sin\left(\frac{cx+d}{x}\right)}{e\left(-ice+ifd+e\left(ic+\right.\right.}$

```
[In] int((a+b*sin(c+d/x))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/f/(f*x+e)-b*d*(-sin(c+d/x)/(-c*e+d*f+e*(c+d/x))/e+(Si(d/x+c+(-c*e+d*f)/e
)*sin((-c*e+d*f)/e)/e+Ci(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e)/e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx$$
$$=\frac{befx\sin\left(\frac{cx+d}{x}\right)-ae^2-(bdf^2x+bdef)\cos\left(-\frac{ce-df}{e}\right)\mathrm{Ci}\left(\frac{dfx+de}{ex}\right)-(bdf^2x+bdef)\sin\left(-\frac{ce-df}{e}\right)\mathrm{Si}\left(\frac{dfx+de}{ex}\right)}{e^2f^2x+e^3f}$$

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] (b*e*f*x*sin((c*x + d)/x) - a*e^2 - (b*d*f^2*x + b*d*e*f)*cos(-(c*e - d*f)/
e)*cos_integral((d*f*x + d*e)/(e*x)) - (b*d*f^2*x + b*d*e*f)*sin(-(c*e - d*
f)/e)*sin_integral((d*f*x + d*e)/(e*x)))/(e^2*f^2*x + e^3*f)
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx=\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{\left(e+fx\right)^2}\,dx$$

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sin(c + d/x))/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx = \int \frac{b\sin\left(c+\frac{d}{x}\right)+a}{(fx+e)^2}\,dx$$

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] b*(integrate(1/2*sin((c*x + d)/x)/(f^2*x^2 + 2*e*f*x + e^2), x) + integrate
(1/2*sin((c*x + d)/x)/((f^2*x^2 + 2*e*f*x + e^2)*cos((c*x + d)/x)^2 + (f^2*
x^2 + 2*e*f*x + e^2)*sin((c*x + d)/x)^2), x)) - a/(f^2*x + e*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(94) = 188$.

Time = 0.31 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.61

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx =$$

$$-\frac{bcd^2e\cos\left(\frac{ce-df}{e}\right)\operatorname{Ci}\left(-\frac{ce-df-\frac{(cx+d)e}{x}}{e}\right)-bd^3f\cos\left(\frac{ce-df}{e}\right)\operatorname{Ci}\left(-\frac{ce-df-\frac{(cx+d)e}{x}}{e}\right)+bcd^2e\sin\left(\frac{ce-df}{e}\right)\operatorname{Si}\left(\frac{ce-\cdots}{\cdots}\right)\cdots}{\cdots}$$

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -(b*c*d^2*e*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)
 - b*d^3*f*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)
+ b*c*d^2*e*sin((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e)
- b*d^3*f*sin((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) -
(c*x + d)*b*d^2*e*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e
/x)/e)/x - (c*x + d)*b*d^2*e*sin((c*e - d*f)/e)*sin_integral((c*e - d*f - (
c*x + d)*e/x)/e)/x + b*d^2*e*sin((c*x + d)/x) + a*d^2*e)/((c*e^3 - d*e^2*f
- (c*x + d)*e^3/x)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx = \int \frac{a+b\,\sin\left(c+\frac{d}{x}\right)}{{\left(e+f\,x\right)}^2}\,dx$$

```
[In] int((a + b*sin(c + d/x))/(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(c + d/x))/(e + f*x)^2, x)
```

3.293 $\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 233

$$\begin{aligned}\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx = &-\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(f+\frac{e}{x}\right)} \\ &-\frac{bd\cos\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3} \\ &-\frac{bd^2f\operatorname{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(c-\frac{df}{e}\right)}{2e^4} \\ &-\frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)} \\ &-\frac{bd^2f\cos\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2e^4}+\frac{bd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}\end{aligned}$$

```
[Out] -1/2*a*f/e^2/(f+e/x)^2+a/e^2/(f+e/x)-b*d*Ci(d*(f/e+1/x))*cos(c-d*f/e)/e^3-1
/2*b*d*f*cos(c+d/x)/e^3/(f+e/x)-1/2*b*d^2*f*cos(c-d*f/e)*Si(d*(f/e+1/x))/e^
4-1/2*b*d^2*f*Ci(d*(f/e+1/x))*sin(c-d*f/e)/e^4+b*d*Si(d*(f/e+1/x))*sin(c-d*
f/e)/e^3-1/2*b*f*sin(c+d/x)/e^2/(f+e/x)^2+b*sin(c+d/x)/e^2/(f+e/x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3512, 3398, 3378, 3384, 3380, 3383}

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx = \frac{a}{e^2\left(\frac{e}{x}+f\right)}-\frac{af}{2e^2\left(\frac{e}{x}+f\right)^2} - \frac{bd^2f\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2e^4} - \frac{bd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3} - \frac{bd^2f\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2e^4} + \frac{bd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3} - \frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(\frac{e}{x}+f\right)} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(\frac{e}{x}+f\right)} - \frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(\frac{e}{x}+f\right)^2}$$

```
[In] Int[(a + b*Sin[c + d/x])/(e + f*x)^3,x]
```

```
[Out] -1/2*(a*f)/(e^2*(f + e/x)^2) + a/(e^2*(f + e/x)) - (b*d*f*Cos[c + d/x])/(2*
e^3*(f + e/x)) - (b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))])/e^3 -
 (b*d^2*f*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/(2*e^4) - (b*f*Si
n[c + d/x])/(2*e^2*(f + e/x)^2) + (b*Sin[c + d/x])/(e^2*(f + e/x)) - (b*d^2
*f*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/(2*e^4) + (b*d*Sin[c - (
d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/e^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \left(-\frac{f(a+b\sin(c+dx))}{e(f+ex)^3}+\frac{a+b\sin(c+dx)}{e(f+ex)^2}\right)\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{(f+ex)^2}\,dx,x,\frac{1}{x}\right)}{e}+\frac{f\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{(f+ex)^3}\,dx,x,\frac{1}{x}\right)}{e}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a}{(f+ex)^2}+\frac{b\sin(c+dx)}{(f+ex)^2}\right)\,dx,x,\frac{1}{x}\right)}{e}+\frac{f\text{Subst}\left(\int \left(\frac{a}{(f+ex)^3}+\frac{b\sin(c+dx)}{(f+ex)^3}\right)\,dx,x,\frac{1}{x}\right)}{e}\\
&= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{b\text{Subst}\left(\int \frac{\sin(c+dx)}{(f+ex)^2}\,dx,x,\frac{1}{x}\right)}{e}+\frac{(bf)\text{Subst}\left(\int \frac{\sin(c+dx)}{(f+ex)^3}\,dx,x,\frac{1}{x}\right)}{e}\\
&= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}\\
&\quad-\frac{(bd)\text{Subst}\left(\int \frac{\cos(c+dx)}{f+ex}\,dx,x,\frac{1}{x}\right)}{e^2}+\frac{(bdf)\text{Subst}\left(\int \frac{\cos(c+dx)}{(f+ex)^2}\,dx,x,\frac{1}{x}\right)}{2e^2}
\end{aligned}$$

$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2} + \frac{a}{e^2\left(f+\frac{e}{x}\right)} - \frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(f+\frac{e}{x}\right)} - \frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)} - \frac{(bd^2f)\operatorname{Subst}\left(\int \frac{\sin(c+dx)}{f+ex}\,dx,x,\frac{1}{x}\right)}{2e^3} - \frac{\left(bd\cos\left(c-\frac{df}{e}\right)\right)\operatorname{Subst}\left(\int \frac{\cos\left(\frac{df}{e}+dx\right)}{f+ex}\,dx,x,\frac{1}{x}\right)}{e^2} + \frac{\left(bd\sin\left(c-\frac{df}{e}\right)\right)\operatorname{Subst}\left(\int \frac{\sin\left(\frac{df}{e}+dx\right)}{f+ex}\,dx,x,\frac{1}{x}\right)}{e^2}$$

$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2} + \frac{a}{e^2\left(f+\frac{e}{x}\right)} - \frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(f+\frac{e}{x}\right)} - \frac{bd\cos\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3} - \frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)} + \frac{bd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3} - \frac{\left(bd^2f\cos\left(c-\frac{df}{e}\right)\right)\operatorname{Subst}\left(\int \frac{\sin\left(\frac{df}{e}+dx\right)}{f+ex}\,dx,x,\frac{1}{x}\right)}{2e^3} - \frac{\left(bd^2f\sin\left(c-\frac{df}{e}\right)\right)\operatorname{Subst}\left(\int \frac{\cos\left(\frac{df}{e}+dx\right)}{f+ex}\,dx,x,\frac{1}{x}\right)}{2e^3}$$

$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2} + \frac{a}{e^2\left(f+\frac{e}{x}\right)} - \frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(f+\frac{e}{x}\right)} - \frac{bd\cos\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3} - \frac{bd^2f\operatorname{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(c-\frac{df}{e}\right)}{2e^4} - \frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)} - \frac{bd^2f\cos\left(c-\frac{df}{e}\right)\operatorname{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{2e^4} + \frac{bd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3}$$

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.65

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx =$$

$$-\frac{bd\,\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\left(2e\cos\left(c-\frac{df}{e}\right)+df\sin\left(c-\frac{df}{e}\right)\right)+\frac{e\left(ae^3+bdf^2x(e+fx)\cos\left(c+\frac{d}{x}\right)-befx(2e+fx)\sin\left(c+\frac{d}{x}\right)\right)}{f(e+fx)^2}}{2e^4}$$

```
[In] Integrate[(a + b*Sin[c + d/x])/(e + f*x)^3,x]
```

```
[Out] -1/2*(b*d*CosIntegral[d*(f/e + x^(-1))]*(2*e*Cos[c - (d*f)/e] + d*f*Sin[c -
 (d*f)/e]) + (e*(a*e^3 + b*d*f^2*x*(e + f*x)*Cos[c + d/x] - b*e*f*x*(2*e +
f*x)*Sin[c + d/x]))/(f*(e + f*x)^2) + b*d*(d*f*Cos[c - (d*f)/e] - 2*e*Sin[c
 - (d*f)/e])*SinIntegral[d*(f/e + x^(-1))])/e^4
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{a}{2f(fx+e)^{2}}+\frac{ib\,d^{2}\mathrm{e}^{-\frac{i(ce-df)}{e}}\operatorname{Ei}_{1}\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)f}{4e^{4}}+\frac{bd\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\operatorname{Ei}_{1}\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{2e^{3}}-\frac{ib\,d^{2}\mathrm{e}^{\frac{i(ce-df)}{e}}\cdots}{\cdots}$
parts	$-\frac{a}{2f(fx+e)^{2}}-bd\left(\frac{-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{\frac{\operatorname{Si}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\frac{\operatorname{Ci}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+df}{e}\right)}{e}}{e}}{e}+\cdots\right.$
derivativedivides	$-d\left(-\frac{a}{e^{2}\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}-\frac{(ce-df)a}{2e^{2}\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)^{2}}+\frac{b\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\operatorname{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\cdots}{e}\cdots}{\cdots}\right.}{e}\cdots\right.$
default	$-d\left(-\frac{a}{e^{2}\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}-\frac{(ce-df)a}{2e^{2}\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)^{2}}+\frac{b\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\operatorname{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\cdots}{e}\cdots}{\cdots}\right.}{e}\cdots\right.$

```
[In] int((a+b*sin(c+d/x))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/f/(f*x+e)^2+1/4*I*b*d^2/e^4*exp(-I*(c*e-d*f)/e)*Ei(1,I*d/x+I*c-I*(c*
e-d*f)/e)*f+1/2*b*d/e^3*exp(-I*(c*e-d*f)/e)*Ei(1,I*d/x+I*c-I*(c*e-d*f)/e)-1
/4*I*b*d^2*exp(I*(c*e-d*f)/e)*Ei(1,-I*d/x-I*c-(-I*c*e+I*f*d)/e)/e^4*f+1/2*b
*d*exp(I*(c*e-d*f)/e)*Ei(1,-I*d/x-I*c-(-I*c*e+I*f*d)/e)/e^3+1/4*I*b/e^3*x*(
2*I*d^3*f^4*x^3+2*I*d^3*e^3*f+6*I*d^3*e*f^3*x^2+6*I*d^3*e^2*f^2*x)/(f*x+e)^
2/(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2)*cos((c*x+d)/x)-1/4*b/e^2*x*(-2*d^2*f^3*
x^3-8*d^2*e*f^2*x^2-10*d^2*e^2*f*x-4*d^2*e^3)/(f*x+e)^2/(d^2*f^2*x^2+2*d^2*
e*f*x+d^2*e^2)*sin((c*x+d)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.41

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx =$$
$$-\frac{ae^4+\left(2\left(bdef^3x^2+2\,bde^2f^2x+bde^3f\right)\operatorname{Ci}\left(\frac{dfx+de}{ex}\right)+\left(bd^2f^4x^2+2\,bd^2ef^3x+bd^2e^2f^2\right)\operatorname{Si}\left(\frac{dfx+de}{ex}\right)\right)\cos\left(\right.}{}$$

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*e^4 + (2*(b*d*e*f^3*x^2 + 2*b*d*e^2*f^2*x + b*d*e^3*f)*cos_integral
((d*f*x + d*e)/(e*x)) + (b*d^2*f^4*x^2 + 2*b*d^2*e*f^3*x + b*d^2*e^2*f^2)*s
in_integral((d*f*x + d*e)/(e*x)))*cos(-(c*e - d*f)/e) + (b*d*e*f^3*x^2 + b*
d*e^2*f^2*x)*cos((c*x + d)/x) - ((b*d^2*f^4*x^2 + 2*b*d^2*e*f^3*x + b*d^2*e
^2*f^2)*cos_integral((d*f*x + d*e)/(e*x)) - 2*(b*d*e*f^3*x^2 + 2*b*d*e^2*f^
2*x + b*d*e^3*f)*sin_integral((d*f*x + d*e)/(e*x)))*sin(-(c*e - d*f)/e) - (
b*e^2*f^2*x^2 + 2*b*e^3*f*x)*sin((c*x + d)/x))/(e^4*f^3*x^2 + 2*e^5*f^2*x +
 e^6*f)
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx = \int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{\left(e+fx\right)^3}\,dx$$

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)**3,x)
```

```
[Out] Integral((a + b*sin(c + d/x))/(e + f*x)**3, x)
```

Maxima [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx = \int \frac{b\sin\left(c+\frac{d}{x}\right)+a}{\left(fx+e\right)^3}\,dx$$

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] b*(integrate(1/2*sin((c*x + d)/x)/(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)
, x) + integrate(1/2*sin((c*x + d)/x)/((f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x +
 e^3)*cos((c*x + d)/x)^2 + (f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)*sin((c
*x + d)/x)^2), x)) - 1/2*a/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. 2(223) = 446.

Time = 0.38 (sec) , antiderivative size = 1501, normalized size of antiderivative = 6.44

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2*d^3*e^2*f*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)*sin((c*e
 - d*f)/e) - 2*b*c*d^4*e*f^2*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)*s
in((c*e - d*f)/e) + b*d^5*f^3*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)*
sin((c*e - d*f)/e) - b*c^2*d^3*e^2*f*cos((c*e - d*f)/e)*sin_integral((c*e -
 d*f - (c*x + d)*e/x)/e) + 2*b*c*d^4*e*f^2*cos((c*e - d*f)/e)*sin_integral(
(c*e - d*f - (c*x + d)*e/x)/e) - b*d^5*f^3*cos((c*e - d*f)/e)*sin_integral(
(c*e - d*f - (c*x + d)*e/x)/e) + 2*b*c^2*d^2*e^3*cos((c*e - d*f)/e)*cos_int
egral(-(c*e - d*f - (c*x + d)*e/x)/e) - 4*b*c*d^3*e^2*f*cos((c*e - d*f)/e)*
cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e) + 2*b*d^4*e*f^2*cos((c*e - d*f
)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e) - 2*(c*x + d)*b*c*d^3*e^2
*f*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)*sin((c*e - d*f)/e)/x + 2*(c
*x + d)*b*d^4*e*f^2*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)*sin((c*e -
 d*f)/e)/x + 2*(c*x + d)*b*c*d^3*e^2*f*cos((c*e - d*f)/e)*sin_integral((c*e
 - d*f - (c*x + d)*e/x)/e)/x - 2*(c*x + d)*b*d^4*e*f^2*cos((c*e - d*f)/e)*s
in_integral((c*e - d*f - (c*x + d)*e/x)/e)/x + 2*b*c^2*d^2*e^3*sin((c*e - d
*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) - 4*b*c*d^3*e^2*f*sin((c
*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) + 2*b*d^4*e*f^2*si
n((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) - b*c*d^3*e^2*
f*cos((c*x + d)/x) + b*d^4*e*f^2*cos((c*x + d)/x) - 4*(c*x + d)*b*c*d^2*e^3
*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)/x + 4*(c*x
 + d)*b*d^3*e^2*f*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e
/x)/e)/x + (c*x + d)^2*b*d^3*e^2*f*cos_integral(-(c*e - d*f - (c*x + d)*e/x
)/e)*sin((c*e - d*f)/e)/x^2 - (c*x + d)^2*b*d^3*e^2*f*cos((c*e - d*f)/e)*si
n_integral((c*e - d*f - (c*x + d)*e/x)/e)/x^2 - 4*(c*x + d)*b*c*d^2*e^3*sin
((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e)/x + 4*(c*x + d)
*b*d^3*e^2*f*sin((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e)
/x + (c*x + d)*b*d^3*e^2*f*cos((c*x + d)/x)/x + 2*(c*x + d)^2*b*d^2*e^3*cos
((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)/x^2 + 2*b*c*d^
2*e^3*sin((c*x + d)/x) - b*d^3*e^2*f*sin((c*x + d)/x) + 2*(c*x + d)^2*b*d^2
*e^3*sin((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e)/x^2 + 2
*a*c*d^2*e^3 - a*d^3*e^2*f - 2*(c*x + d)*b*d^2*e^3*sin((c*x + d)/x)/x - 2*(
c*x + d)*a*d^2*e^3/x)/((c^2*e^6 - 2*c*d*e^5*f + d^2*e^4*f^2 - 2*(c*x + d)*c
*e^6/x + 2*(c*x + d)*d*e^5*f/x + (c*x + d)^2*e^6/x^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx = \int \frac{a+b\,\sin\left(c+\frac{d}{x}\right)}{\left(e+f\,x\right)^3}\,dx$$

```
[In] int((a + b*sin(c + d/x))/(e + f*x)^3,x)
[Out] int((a + b*sin(c + d/x))/(e + f*x)^3, x)
```

3.294 $\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 254

$$\begin{aligned}\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx &= a^2ex+\frac{1}{2}a^2fx^2+abdfx\cos\left(c+\frac{d}{x}\right)\\ &\quad-2abde\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)\\ &\quad-b^2d^2f\cos(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right)\\ &\quad+abd^2f\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c)\\ &\quad-b^2de\,\text{CosIntegral}\left(\frac{2d}{x}\right)\sin(2c)\\ &\quad+2abex\sin\left(c+\frac{d}{x}\right)+abfx^2\sin\left(c+\frac{d}{x}\right)\\ &\quad+b^2dfx\cos\left(c+\frac{d}{x}\right)\sin\left(c+\frac{d}{x}\right)\\ &\quad+b^2ex\sin^2\left(c+\frac{d}{x}\right)+\frac{1}{2}b^2fx^2\sin^2\left(c+\frac{d}{x}\right)\\ &\quad+abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+2abde\sin(c)\text{Si}\left(\frac{d}{x}\right)\\ &\quad-b^2de\cos(2c)\text{Si}\left(\frac{2d}{x}\right)+b^2d^2f\sin(2c)\text{Si}\left(\frac{2d}{x}\right)\end{aligned}$$

```
[Out] a^2*e*x+1/2*a^2*f*x^2-2*a*b*d*e*Ci(d/x)*cos(c)-b^2*d^2*f*Ci(2*d/x)*cos(2*c)
+a*b*d*f*x*cos(c+d/x)+a*b*d^2*f*cos(c)*Si(d/x)-b^2*d*e*cos(2*c)*Si(2*d/x)+a
*b*d^2*f*Ci(d/x)*sin(c)+2*a*b*d*e*Si(d/x)*sin(c)-b^2*d*e*Ci(2*d/x)*sin(2*c)
+b^2*d^2*f*Si(2*d/x)*sin(2*c)+2*a*b*e*x*sin(c+d/x)+a*b*f*x^2*sin(c+d/x)+b^2
```

```
*d*f*x*cos(c+d/x)*sin(c+d/x)+b^2*e*x*sin(c+d/x)^2+1/2*b^2*f*x^2*sin(c+d/x)^
2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {3512, 3398, 3378, 3384, 3380, 3383, 3395, 29, 3393, 3394, 12}

$$\begin{aligned}\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx = {} & a^2ex+\frac{1}{2}a^2fx^2+abd^2f\sin(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)\\ & -2abde\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)\\ & +abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)\\ & +2abde\sin(c)\text{Si}\left(\frac{d}{x}\right)+2abex\sin\left(c+\frac{d}{x}\right)\\ & +abfx^2\sin\left(c+\frac{d}{x}\right)+abdfx\cos\left(c+\frac{d}{x}\right)\\ & -b^2d^2f\cos(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right)\\ & -b^2de\sin(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right)\\ & +b^2d^2f\sin(2c)\text{Si}\left(\frac{2d}{x}\right)-b^2de\cos(2c)\text{Si}\left(\frac{2d}{x}\right)\\ & +b^2ex\sin^2\left(c+\frac{d}{x}\right)+\frac{1}{2}b^2fx^2\sin^2\left(c+\frac{d}{x}\right)\\ & +b^2dfx\sin\left(c+\frac{d}{x}\right)\cos\left(c+\frac{d}{x}\right)\end{aligned}$$

```
[In] Int[(e + f*x)*(a + b*Sin[c + d/x])^2,x]
```

```
[Out] a^2*e*x + (a^2*f*x^2)/2 + a*b*d*f*x*Cos[c + d/x] - 2*a*b*d*e*Cos[c]*CosInte
gral[d/x] - b^2*d^2*f*Cos[2*c]*CosIntegral[(2*d)/x] + a*b*d^2*f*CosIntegral
[d/x]*Sin[c] - b^2*d*e*CosIntegral[(2*d)/x]*Sin[2*c] + 2*a*b*e*x*Sin[c + d/
x] + a*b*f*x^2*Sin[c + d/x] + b^2*d*f*x*Cos[c + d/x]*Sin[c + d/x] + b^2*e*x
*Sin[c + d/x]^2 + (b^2*f*x^2*Sin[c + d/x]^2)/2 + a*b*d^2*f*Cos[c]*SinIntegr
al[d/x] + 2*a*b*d*e*Sin[c]*SinIntegral[d/x] - b^2*d*e*Cos[2*c]*SinIntegral[
(2*d)/x] + b^2*d^2*f*Sin[2*c]*SinIntegral[(2*d)/x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
```

```
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int\left(\frac{f(a+b\sin(c+dx))^2}{x^3}+\frac{e(a+b\sin(c+dx))^2}{x^2}\right)dx,x,\frac{1}{x}\right)\\
&= -\left(e\text{Subst}\left(\int\frac{(a+b\sin(c+dx))^2}{x^2}\,dx,x,\frac{1}{x}\right)\right)-f\text{Subst}\left(\int\frac{(a+b\sin(c+dx))^2}{x^3}\,dx,x,\frac{1}{x}\right)\\
&= -\left(e\text{Subst}\left(\int\left(\frac{a^2}{x^2}+\frac{2ab\sin(c+dx)}{x^2}+\frac{b^2\sin^2(c+dx)}{x^2}\right)dx,x,\frac{1}{x}\right)\right)\\
&\quad -f\text{Subst}\left(\int\left(\frac{a^2}{x^3}+\frac{2ab\sin(c+dx)}{x^3}+\frac{b^2\sin^2(c+dx)}{x^3}\right)dx,x,\frac{1}{x}\right)\\
&= a^2ex+\frac{1}{2}a^2fx^2-(2abe)\text{Subst}\left(\int\frac{\sin(c+dx)}{x^2}\,dx,x,\frac{1}{x}\right)\\
&\quad -\left(b^2e\right)\text{Subst}\left(\int\frac{\sin^2(c+dx)}{x^2}\,dx,x,\frac{1}{x}\right)-(2abf)\text{Subst}\left(\int\frac{\sin(c+dx)}{x^3}\,dx,x,\frac{1}{x}\right)\\
&\quad -\left(b^2f\right)\text{Subst}\left(\int\frac{\sin^2(c+dx)}{x^3}\,dx,x,\frac{1}{x}\right)
\end{aligned}$$

$$= a^2ex + \frac{1}{2}a^2fx^2 + 2abex\sin\left(c+\frac{d}{x}\right) + abfx^2\sin\left(c+\frac{d}{x}\right) + b^2dfx\cos\left(c+\frac{d}{x}\right)\sin\left(c+\frac{d}{x}\right) + b^2ex\sin^2\left(c+\frac{d}{x}\right) + \frac{1}{2}b^2fx^2\sin^2\left(c+\frac{d}{x}\right) - (2abde)\text{Subst}\left(\int\frac{\cos(c+dx)}{x}\,dx,x,\frac{1}{x}\right) - (2b^2de)\,\text{Subst}\left(\int\frac{\sin(2c+2dx)}{2x}\,dx,x,\frac{1}{x}\right) - (abdf)\text{Subst}\left(\int\frac{\cos(c+dx)}{x^2}\,dx,x,\frac{1}{x}\right) - \left(b^2d^2f\right)\text{Subst}\left(\int\frac{1}{x}\,dx,x,\frac{1}{x}\right) + \left(2b^2d^2f\right)\text{Subst}\left(\int\frac{\sin^2(c+dx)}{x}\,dx,x,\frac{1}{x}\right)$$

$$= a^2ex + \frac{1}{2}a^2fx^2 + abdfx\cos\left(c+\frac{d}{x}\right) + b^2d^2f\log(x) + 2abex\sin\left(c+\frac{d}{x}\right) + abfx^2\sin\left(c+\frac{d}{x}\right) + b^2dfx\cos\left(c+\frac{d}{x}\right)\sin\left(c+\frac{d}{x}\right) + b^2ex\sin^2\left(c+\frac{d}{x}\right) + \frac{1}{2}b^2fx^2\sin^2\left(c+\frac{d}{x}\right) - \left(b^2de\right)\text{Subst}\left(\int\frac{\sin(2c+2dx)}{x}\,dx,x,\frac{1}{x}\right) + \left(abd^2f\right)\text{Subst}\left(\int\frac{\sin(c+dx)}{x}\,dx,x,\frac{1}{x}\right) + \left(2b^2d^2f\right)\text{Subst}\left(\int\left(\frac{1}{2x}-\frac{\cos(2c+2dx)}{2x}\right)dx,x,\frac{1}{x}\right) - (2abde\cos(c))\text{Subst}\left(\int\frac{\cos(dx)}{x}\,dx,x,\frac{1}{x}\right) + (2abde\sin(c))\text{Subst}\left(\int\frac{\sin(dx)}{x}\,dx,x,\frac{1}{x}\right)$$

$$= a^2ex + \frac{1}{2}a^2fx^2 + abdfx\cos\left(c+\frac{d}{x}\right) - 2abde\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) + 2abex\sin\left(c+\frac{d}{x}\right) + abfx^2\sin\left(c+\frac{d}{x}\right) + b^2dfx\cos\left(c+\frac{d}{x}\right)\sin\left(c+\frac{d}{x}\right) + b^2ex\sin^2\left(c+\frac{d}{x}\right) + \frac{1}{2}b^2fx^2\sin^2\left(c+\frac{d}{x}\right) + 2abde\sin(c)\text{Si}\left(\frac{d}{x}\right) - \left(b^2d^2f\right)\text{Subst}\left(\int\frac{\cos(2c+2dx)}{x}\,dx,x,\frac{1}{x}\right) + \left(abd^2f\cos(c)\right)\text{Subst}\left(\int\frac{\sin(dx)}{x}\,dx,x,\frac{1}{x}\right) - \left(b^2de\cos(2c)\right)\text{Subst}\left(\int\frac{\sin(2dx)}{x}\,dx,x,\frac{1}{x}\right) + \left(abd^2f\sin(c)\right)\text{Subst}\left(\int\frac{\cos(dx)}{x}\,dx,x,\frac{1}{x}\right) - \left(b^2de\sin(2c)\right)\text{Subst}\left(\int\frac{\cos(2dx)}{x}\,dx,x,\frac{1}{x}\right)$$

$$= a^2ex + \frac{1}{2}a^2fx^2 + abdfx\cos\left(c+\frac{d}{x}\right) - 2abde\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) + abd^2f\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c) - b^2de\,\text{CosIntegral}\left(\frac{2d}{x}\right)\sin(2c) + 2abex\sin\left(c+\frac{d}{x}\right) + abfx^2\sin\left(c+\frac{d}{x}\right) + b^2dfx\cos\left(c+\frac{d}{x}\right)\sin\left(c+\frac{d}{x}\right) + b^2ex\sin^2\left(c+\frac{d}{x}\right) + \frac{1}{2}b^2fx^2\sin^2\left(c+\frac{d}{x}\right) + abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right) + 2abde\sin(c)\text{Si}\left(\frac{d}{x}\right) - b^2de\cos(2c)\text{Si}\left(\frac{2d}{x}\right) - \left(b^2d^2f\cos(2c)\right)\text{Subst}\left(\int\frac{\cos(2dx)}{x}\,dx, x, \frac{1}{x}\right) + \left(b^2d^2f\sin(2c)\right)\text{Subst}\left(\int\frac{\sin(2dx)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= a^2ex + \frac{1}{2}a^2fx^2 + abdfx\cos\left(c+\frac{d}{x}\right) - 2abde\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) - b^2d^2f\cos(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right) + abd^2f\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c) - b^2de\,\text{CosIntegral}\left(\frac{2d}{x}\right)\sin(2c) + 2abex\sin\left(c+\frac{d}{x}\right) + abfx^2\sin\left(c+\frac{d}{x}\right) + b^2dfx\cos\left(c+\frac{d}{x}\right)\sin\left(c+\frac{d}{x}\right) + b^2ex\sin^2\left(c+\frac{d}{x}\right) + \frac{1}{2}b^2fx^2\sin^2\left(c+\frac{d}{x}\right) + abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right) + 2abde\sin(c)\text{Si}\left(\frac{d}{x}\right) - b^2de\cos(2c)\text{Si}\left(\frac{2d}{x}\right) + b^2d^2f\sin(2c)\text{Si}\left(\frac{2d}{x}\right)$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.99

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = \frac{1}{4}\left(4a^2ex+2b^2ex+2a^2fx^2+b^2fx^2 + 4abdfx\cos\left(c+\frac{d}{x}\right)-2b^2ex\cos\left(2\left(c+\frac{d}{x}\right)\right) - b^2fx^2\cos\left(2\left(c+\frac{d}{x}\right)\right) + 4abd\,\text{CosIntegral}\left(\frac{d}{x}\right)(-2e\cos(c)+df\sin(c)) - 4b^2d\,\text{CosIntegral}\left(\frac{2d}{x}\right)(df\cos(2c)+e\sin(2c)) + 8abex\sin\left(c+\frac{d}{x}\right)+4abfx^2\sin\left(c+\frac{d}{x}\right) + 2b^2dfx\sin\left(2\left(c+\frac{d}{x}\right)\right)+4abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right) + 8abde\sin(c)\text{Si}\left(\frac{d}{x}\right)-4b^2de\cos(2c)\text{Si}\left(\frac{2d}{x}\right) + 4b^2d^2f\sin(2c)\text{Si}\left(\frac{2d}{x}\right)\right)$$

```
[In] Integrate[(e + f*x)*(a + b*Sin[c + d/x])^2,x]
```

```
[Out] (4*a^2*e*x + 2*b^2*e*x + 2*a^2*f*x^2 + b^2*f*x^2 + 4*a*b*d*f*x*Cos[c + d/x]
 - 2*b^2*e*x*Cos[2*(c + d/x)] - b^2*f*x^2*Cos[2*(c + d/x)] + 4*a*b*d*CosInt
egral[d/x]*(-2*e*Cos[c] + d*f*Sin[c]) - 4*b^2*d*CosIntegral[(2*d)/x]*(d*f*C
os[2*c] + e*Sin[2*c]) + 8*a*b*e*x*Sin[c + d/x] + 4*a*b*f*x^2*Sin[c + d/x] +
 2*b^2*d*f*x*Sin[2*(c + d/x)] + 4*a*b*d^2*f*Cos[c]*SinIntegral[d/x] + 8*a*b
*d*e*Sin[c]*SinIntegral[d/x] - 4*b^2*d*e*Cos[2*c]*SinIntegral[(2*d)/x] + 4*
b^2*d^2*f*Sin[2*c]*SinIntegral[(2*d)/x])/4
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.98

method	result
parts	$a^2\left(\frac{1}{2}f\,x^2+ex\right)-b^2d\left(-\frac{ex}{2d}-\frac{e\left(-\frac{2\cos\left(2c+\frac{2d}{x}\right)x}{d}-4\,\mathrm{Si}\left(\frac{2d}{x}\right)\cos(2c)-4\,\mathrm{Ci}\left(\frac{2d}{x}\right)\sin(2c)\right)}{4}-\frac{f\,x^2}{4d}-\frac{df\left(-\right.}{}\right.$
derivativedivides	$-d\left(-\frac{a^2f\,x^2}{2d}-\frac{a^2ex}{d}+2abfd\left(-\frac{\sin\left(c+\frac{d}{x}\right)x^2}{2d^2}-\frac{\cos\left(c+\frac{d}{x}\right)x}{2d}-\frac{\mathrm{Si}\left(\frac{d}{x}\right)\cos(c)}{2}-\frac{\mathrm{Ci}\left(\frac{d}{x}\right)\sin(c)}{2}\right)+2ab\right.$
default	$-d\left(-\frac{a^2f\,x^2}{2d}-\frac{a^2ex}{d}+2abfd\left(-\frac{\sin\left(c+\frac{d}{x}\right)x^2}{2d^2}-\frac{\cos\left(c+\frac{d}{x}\right)x}{2d}-\frac{\mathrm{Si}\left(\frac{d}{x}\right)\cos(c)}{2}-\frac{\mathrm{Ci}\left(\frac{d}{x}\right)\sin(c)}{2}\right)+2ab\right.$
risch	$a^2ex+\frac{a^2f\,x^2}{2}+abde\,\mathrm{e}^{-ic}\,\mathrm{Ei}_1\left(\frac{id}{x}\right)-\frac{iab\,d^2f\,\mathrm{e}^{-ic}\,\mathrm{Ei}_1\left(\frac{id}{x}\right)}{2}+\frac{b^2ex}{2}+\frac{b^2f\,x^2}{4}+\frac{\mathrm{e}^{-2ic}\,\mathrm{Ei}_1\left(\frac{2id}{x}\right)b^2d^2f}{2}-$

```
[In] int((f*x+e)*(a+b*sin(c+d/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/2*f*x^2+e*x)-b^2*d*(-1/2*e/d*x-1/4*e*(-2*cos(2*c+2*d/x)/d*x-4*Si(2*d
/x)*cos(2*c)-4*Ci(2*d/x)*sin(2*c))-1/4/d*f*x^2-1/4*d*f*(-cos(2*c+2*d/x)/d^2
*x^2+2*sin(2*c+2*d/x)/d*x+4*Si(2*d/x)*sin(2*c)-4*Ci(2*d/x)*cos(2*c)))-2*a*b
*d*(e*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*cos(c))+d*f*(-1/2*sin(c+d/x)/
d^2*x^2-1/2*cos(c+d/x)/d*x-1/2*Si(d/x)*cos(c)-1/2*Ci(d/x)*sin(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx \\
&= abdfx\cos\left(\frac{cx+d}{x}\right)+\frac{1}{2}\left(a^2+b^2\right)fx^2+\left(a^2+b^2\right)ex \\
&\quad -\frac{1}{2}\left(b^2fx^2+2\,b^2ex\right)\cos\left(\frac{cx+d}{x}\right)^2-\left(b^2d^2f\,\mathrm{Ci}\left(\frac{2\,d}{x}\right)+b^2de\,\mathrm{Si}\left(\frac{2\,d}{x}\right)\right)\cos\left(2\,c\right) \\
&\quad +\left(abd^2f\,\mathrm{Si}\left(\frac{d}{x}\right)-2\,abde\,\mathrm{Ci}\left(\frac{d}{x}\right)\right)\cos\left(c\right) \\
&\quad +\left(b^2d^2f\,\mathrm{Si}\left(\frac{2\,d}{x}\right)-b^2de\,\mathrm{Ci}\left(\frac{2\,d}{x}\right)\right)\sin\left(2\,c\right) \\
&\quad +\left(abd^2f\,\mathrm{Ci}\left(\frac{d}{x}\right)+2\,abde\,\mathrm{Si}\left(\frac{d}{x}\right)\right)\sin\left(c\right) \\
&\quad +\left(b^2dfx\cos\left(\frac{cx+d}{x}\right)+abfx^2+2\,abex\right)\sin\left(\frac{cx+d}{x}\right)
\end{aligned}$$

[In] integrate((f*x+e)*(a+b*sin(c+d/x))^2,x, algorithm="fricas")

```
[Out] a*b*d*f*x*cos((c*x + d)/x) + 1/2*(a^2 + b^2)*f*x^2 + (a^2 + b^2)*e*x - 1/2*
(b^2*f*x^2 + 2*b^2*e*x)*cos((c*x + d)/x)^2 - (b^2*d^2*f*cos_integral(2*d/x)
 + b^2*d*e*sin_integral(2*d/x))*cos(2*c) + (a*b*d^2*f*sin_integral(d/x) - 2
*a*b*d*e*cos_integral(d/x))*cos(c) + (b^2*d^2*f*sin_integral(2*d/x) - b^2*d
*e*cos_integral(2*d/x))*sin(2*c) + (a*b*d^2*f*cos_integral(d/x) + 2*a*b*d*e
*sin_integral(d/x))*sin(c) + (b^2*d*f*x*cos((c*x + d)/x) + a*b*f*x^2 + 2*a*
b*e*x)*sin((c*x + d)/x)
```

Sympy [F]

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx=\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2(e+fx)\,dx$$

[In] integrate((f*x+e)*(a+b*sin(c+d/x))**2,x)

[Out] Integral((a + b*sin(c + d/x))**2*(e + f*x), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.26

$$\begin{aligned}\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx=&\ \frac{1}{2}\,a^2fx^2\\ &-\left(\left(\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos\left(c\right)-\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin\left(c\right)\right)d-2\,x\sin\left(\frac{cx+d}{x}\right)\right)ab\\ &-\frac{1}{2}\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{2i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{2i\,d}{x}\right)\right)\cos\left(2\,c\right)+\left(\mathrm{Ei}\left(\frac{2i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{2i\,d}{x}\right)\right)\sin\left(2\,c\right)\right)d+x\cos\left(\frac{2\,(c\right.\right.\\ &+\frac{1}{2}\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos\left(c\right)+\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin\left(c\right)\right)d^2+2\,dx\cos\left(\frac{cx+d}{x}\right)\right.\\ &-\frac{1}{4}\left(2\left(\left(\mathrm{Ei}\left(\frac{2i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{2i\,d}{x}\right)\right)\cos\left(2\,c\right)+\left(i\,\mathrm{Ei}\left(\frac{2i\,d}{x}\right)-i\,\mathrm{Ei}\left(-\frac{2i\,d}{x}\right)\right)\sin\left(2\,c\right)\right)d^2+x^2\cos\left(\frac{2\,(\right.\right.\\ &+a^2ex\end{aligned}$$

[In] integrate((f*x+e)*(a+b*sin(c+d/x))^2,x, algorithm="maxima")

```
[Out] 1/2*a^2*f*x^2 - (((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(-I
*d/x))*sin(c))*d - 2*x*sin((c*x + d)/x))*a*b*e - 1/2*(((-I*Ei(2*I*d/x) + I*
Ei(-2*I*d/x))*cos(2*c) + (Ei(2*I*d/x) + Ei(-2*I*d/x))*sin(2*c))*d + x*cos(2
*(c*x + d)/x) - x)*b^2*e + 1/2*(((-I*Ei(I*d/x) + I*Ei(-I*d/x))*cos(c) + (Ei
```

```
(I*d/x) + Ei(-I*d/x))*sin(c))*d^2 + 2*d*x*cos((c*x + d)/x) + 2*x^2*sin((c*x
 + d)/x))*a*b*f - 1/4*(2*((Ei(2*I*d/x) + Ei(-2*I*d/x))*cos(2*c) + (I*Ei(2*I
*d/x) - I*Ei(-2*I*d/x))*sin(2*c))*d^2 + x^2*cos(2*(c*x + d)/x) - 2*d*x*sin(
2*(c*x + d)/x) - x^2)*b^2*f + a^2*e*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1125 vs. $2(250) = 500$.

Time = 0.32 (sec) , antiderivative size = 1125, normalized size of antiderivative = 4.43

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] -1/4*(4*b^2*c^2*d^3*f*cos(2*c)*cos_integral(-2*c + 2*(c*x + d)/x) - 4*a*b*c
^2*d^3*f*cos_integral(-c + (c*x + d)/x)*sin(c) + 4*b^2*c^2*d^3*f*sin(2*c)*s
in_integral(2*c - 2*(c*x + d)/x) + 4*a*b*c^2*d^3*f*cos(c)*sin_integral(c -
(c*x + d)/x) + 8*a*b*c^2*d^2*e*cos(c)*cos_integral(-c + (c*x + d)/x) - 8*(c
*x + d)*b^2*c*d^3*f*cos(2*c)*cos_integral(-2*c + 2*(c*x + d)/x)/x + 4*b^2*c
^2*d^2*e*cos_integral(-2*c + 2*(c*x + d)/x)*sin(2*c) + 8*(c*x + d)*a*b*c*d^
3*f*cos_integral(-c + (c*x + d)/x)*sin(c)/x - 4*b^2*c^2*d^2*e*cos(2*c)*sin_
integral(2*c - 2*(c*x + d)/x) - 8*(c*x + d)*b^2*c*d^3*f*sin(2*c)*sin_integr
al(2*c - 2*(c*x + d)/x)/x - 8*(c*x + d)*a*b*c*d^3*f*cos(c)*sin_integral(c -
 (c*x + d)/x)/x + 8*a*b*c^2*d^2*e*sin(c)*sin_integral(c - (c*x + d)/x) + 4*
a*b*c*d^3*f*cos((c*x + d)/x) - 16*(c*x + d)*a*b*c*d^2*e*cos(c)*cos_integral
(-c + (c*x + d)/x)/x + 4*(c*x + d)^2*b^2*d^3*f*cos(2*c)*cos_integral(-2*c +
 2*(c*x + d)/x)/x^2 - 8*(c*x + d)*b^2*c*d^2*e*cos_integral(-2*c + 2*(c*x +
d)/x)*sin(2*c)/x - 4*(c*x + d)^2*a*b*d^3*f*cos_integral(-c + (c*x + d)/x)*s
in(c)/x^2 + 2*b^2*c*d^3*f*sin(2*(c*x + d)/x) + 8*(c*x + d)*b^2*c*d^2*e*cos(
2*c)*sin_integral(2*c - 2*(c*x + d)/x)/x + 4*(c*x + d)^2*b^2*d^3*f*sin(2*c)
*sin_integral(2*c - 2*(c*x + d)/x)/x^2 + 4*(c*x + d)^2*a*b*d^3*f*cos(c)*sin
_integral(c - (c*x + d)/x)/x^2 - 16*(c*x + d)*a*b*c*d^2*e*sin(c)*sin_integr
al(c - (c*x + d)/x)/x - 2*b^2*c*d^2*e*cos(2*(c*x + d)/x) + b^2*d^3*f*cos(2*
(c*x + d)/x) - 4*(c*x + d)*a*b*d^3*f*cos((c*x + d)/x)/x + 8*(c*x + d)^2*a*b
*d^2*e*cos(c)*cos_integral(-c + (c*x + d)/x)/x^2 + 4*(c*x + d)^2*b^2*d^2*e*
cos_integral(-2*c + 2*(c*x + d)/x)*sin(2*c)/x^2 - 2*(c*x + d)*b^2*d^3*f*sin
(2*(c*x + d)/x)/x + 8*a*b*c*d^2*e*sin((c*x + d)/x) - 4*a*b*d^3*f*sin((c*x +
 d)/x) - 4*(c*x + d)^2*b^2*d^2*e*cos(2*c)*sin_integral(2*c - 2*(c*x + d)/x)
/x^2 + 8*(c*x + d)^2*a*b*d^2*e*sin(c)*sin_integral(c - (c*x + d)/x)/x^2 + 4
*a^2*c*d^2*e + 2*b^2*c*d^2*e - 2*a^2*d^3*f - b^2*d^3*f + 2*(c*x + d)*b^2*d^
2*e*cos(2*(c*x + d)/x)/x - 8*(c*x + d)*a*b*d^2*e*sin((c*x + d)/x)/x - 4*(c*
x + d)*a^2*d^2*e/x - 2*(c*x + d)*b^2*d^2*e/x)/((c^2 - 2*(c*x + d)*c/x + (c*
x + d)^2/x^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx=\int (e+f\,x)\,\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2\,dx$$

```
[In] int((e + f*x)*(a + b*sin(c + d/x))^2,x)
[Out] int((e + f*x)*(a + b*sin(c + d/x))^2, x)
```

3.295 $\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx$

Optimal result

Integrand size = 14, antiderivative size = 94

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = a^2x-2abd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)-b^2d\,\text{CosIntegral}\left(\frac{2d}{x}\right)\sin(2c)+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)+2abd\sin(c)\text{Si}\left(\frac{d}{x}\right)-b^2d\cos(2c)\text{Si}\left(\frac{2d}{x}\right)$$

```
[Out] a^2*x-2*a*b*d*Ci(d/x)*cos(c)-b^2*d*cos(2*c)*Si(2*d/x)+2*a*b*d*Si(d/x)*sin(c
)-b^2*d*Ci(2*d/x)*sin(2*c)+2*a*b*x*sin(c+d/x)+b^2*x*sin(c+d/x)^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3442, 3398, 3378, 3384, 3380, 3383, 3394, 12}

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = a^2x-2abd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)+2abd\sin(c)\text{Si}\left(\frac{d}{x}\right)+2abx\sin\left(c+\frac{d}{x}\right)-b^2d\sin(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right)-b^2d\cos(2c)\text{Si}\left(\frac{2d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)$$

```
[In] Int[(a + b*Sin[c + d/x])^2,x]
```

```
[Out] a^2*x - 2*a*b*d*Cos[c]*CosIntegral[d/x] - b^2*d*CosIntegral[(2*d)/x]*Sin[2*
c] + 2*a*b*x*Sin[c + d/x] + b^2*x*Sin[c + d/x]^2 + 2*a*b*d*Sin[c]*SinIntegr
al[d/x] - b^2*d*Cos[2*c]*SinIntegral[(2*d)/x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
```

```
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a^2}{x^2}+\frac{2ab\sin(c+dx)}{x^2}+\frac{b^2\sin^2(c+dx)}{x^2}\right)dx, x, \frac{1}{x}\right)\\
&= a^2x-(2ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)-b^2\text{Subst}\left(\int \frac{\sin^2(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= a^2x+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)-(2abd)\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx, x, \frac{1}{x}\right)\\
&\quad-\left(2b^2d\right)\text{Subst}\left(\int \frac{\sin(2c+2dx)}{2x}\,dx, x, \frac{1}{x}\right)\\
&= a^2x+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)-\left(b^2d\right)\text{Subst}\left(\int \frac{\sin(2c+2dx)}{x}\,dx, x, \frac{1}{x}\right)\\
&\quad-(2abd\cos(c))\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx, x, \frac{1}{x}\right)+(2abd\sin(c))\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= a^2x-2abd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)\\
&\quad+2abd\sin(c)\text{Si}\left(\frac{d}{x}\right)-\left(b^2d\cos(2c)\right)\text{Subst}\left(\int \frac{\sin(2dx)}{x}\,dx, x, \frac{1}{x}\right)\\
&\quad-\left(b^2d\sin(2c)\right)\text{Subst}\left(\int \frac{\cos(2dx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= a^2x-2abd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)-b^2d\,\text{CosIntegral}\left(\frac{2d}{x}\right)\sin(2c)\\
&\quad+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)+2abd\sin(c)\text{Si}\left(\frac{d}{x}\right)-b^2d\cos(2c)\text{Si}\left(\frac{2d}{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = \frac{1}{2}\left(2a^2x+b^2x-b^2x\cos\left(2\left(c+\frac{d}{x}\right)\right) - 4abd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) - 2b^2d\,\text{CosIntegral}\left(\frac{2d}{x}\right)\sin(2c)+4abx\sin\left(c+\frac{d}{x}\right) + 4abd\sin(c)\text{Si}\left(\frac{d}{x}\right)-2b^2d\cos(2c)\text{Si}\left(\frac{2d}{x}\right)\right)$$

```
[In] Integrate[(a + b*Sin[c + d/x])^2,x]
```

```
[Out] (2*a^2*x + b^2*x - b^2*x*Cos[2*(c + d/x)] - 4*a*b*d*Cos[c]*CosIntegral[d/x]
 - 2*b^2*d*CosIntegral[(2*d)/x]*Sin[2*c] + 4*a*b*x*Sin[c + d/x] + 4*a*b*d*S
in[c]*SinIntegral[d/x] - 2*b^2*d*Cos[2*c]*SinIntegral[(2*d)/x])/2
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

method	result
parts	$a^2x-b^2d\left(-\frac{x}{2d}+\frac{\cos\left(2c+\frac{2d}{x}\right)x}{2d}+\text{Si}\left(\frac{2d}{x}\right)\cos\left(2c\right)+\text{Ci}\left(\frac{2d}{x}\right)\sin\left(2c\right)\right)-2abd\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}\right.$
derivativedivides	$-d\left(-\frac{a^2x}{d}+2ab\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\text{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\text{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\right)-\frac{b^2x}{2d}-\frac{b^2\left(-\frac{2\cos\left(2c+\frac{2d}{x}\right)x}{d}\right.}{}\right.$
default	$-d\left(-\frac{a^2x}{d}+2ab\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\text{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\text{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\right)-\frac{b^2x}{2d}-\frac{b^2\left(-\frac{2\cos\left(2c+\frac{2d}{x}\right)x}{d}\right.}{}\right.$
risch	$\frac{e^{-2ic}\pi\,\text{csgn}\left(\frac{d}{x}\right)b^2d}{2}-e^{-2ic}\,\text{Si}\left(\frac{2d}{x}\right)b^2d+\frac{i\,\text{Ei}_1\left(-\frac{2id}{x}\right)e^{-2ic}b^2d}{2}-\frac{idb^2\,\text{Ei}_1\left(-\frac{2id}{x}\right)e^{2ic}}{2}+abd\,\text{Ei}_1\left(-\frac{id}{x}\right.$

```
[In] int((a+b*sin(c+d/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x-b^2*d*(-1/2*x/d+1/2*cos(2*c+2*d/x)/d*x+Si(2*d/x)*cos(2*c)+Ci(2*d/x)*s
in(2*c))-2*a*b*d*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*cos(c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx = -b^2x\cos\left(\frac{cx+d}{x}\right)^2 - 2\,abd\cos(c)\,\mathrm{Ci}\left(\frac{d}{x}\right) - b^2d\,\mathrm{Ci}\left(\frac{2\,d}{x}\right)\sin(2\,c) - b^2d\cos(2\,c)\,\mathrm{Si}\left(\frac{2\,d}{x}\right) + 2\,abd\sin(c)\,\mathrm{Si}\left(\frac{d}{x}\right) + 2\,abx\sin\left(\frac{cx+d}{x}\right) + \left(a^2+b^2\right)x$$

```
[In] integrate((a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] -b^2*x*cos((c*x + d)/x)^2 - 2*a*b*d*cos(c)*cos_integral(d/x) - b^2*d*cos_in
tegral(2*d/x)*sin(2*c) - b^2*d*cos(2*c)*sin_integral(2*d/x) + 2*a*b*d*sin(c
)*sin_integral(d/x) + 2*a*b*x*sin((c*x + d)/x) + (a^2 + b^2)*x
```

Sympy [F]

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx = \int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx$$

```
[In] integrate((a+b*sin(c+d/x))**2,x)
```

```
[Out] Integral((a + b*sin(c + d/x))**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.46

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx = -\left(\left(\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos(c) - \left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin(c)\right)d - 2\,x\sin\left(\frac{cx+d}{x}\right)\right)ab - \frac{1}{2}\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{2i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{2i\,d}{x}\right)\right)\cos(2\,c) + \left(\mathrm{Ei}\left(\frac{2i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{2i\,d}{x}\right)\right)\sin(2\,c)\right)d + x\cos\left(\frac{2\,(cx}{x}\right.\right.\right. + a^2x$$

```
[In] integrate((a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(-I*d/x))*sin(c))*
d - 2*x*sin((c*x + d)/x))*a*b - 1/2*(((-I*Ei(2*I*d/x) + I*Ei(-2*I*d/x))*cos
(2*c) + (Ei(2*I*d/x) + Ei(-2*I*d/x))*sin(2*c))*d + x*cos(2*(c*x + d)/x) - x
)*b^2 + a^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(94) = 188$.

Time = 0.38 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.24

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx =$$

$$-\frac{4\,abcd^2\cos(c)\,\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)+2\,b^2cd^2\,\mathrm{Ci}\left(-2\,c+\frac{2\,(cx+d)}{x}\right)\sin(2\,c)-2\,b^2cd^2\cos(2\,c)\,\mathrm{Si}\left(2\,c-\frac{2\,(cx+d)}{x}\right)}{}$$

```
[In] integrate((a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b*c*d^2*cos(c)*cos_integral(-c + (c*x + d)/x) + 2*b^2*c*d^2*cos_i
ntegral(-2*c + 2*(c*x + d)/x)*sin(2*c) - 2*b^2*c*d^2*cos(2*c)*sin_integral(
2*c - 2*(c*x + d)/x) + 4*a*b*c*d^2*sin(c)*sin_integral(c - (c*x + d)/x) - 4
*(c*x + d)*a*b*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)/x - 2*(c*x + d)*b^
2*d^2*cos_integral(-2*c + 2*(c*x + d)/x)*sin(2*c)/x + 2*(c*x + d)*b^2*d^2*c
os(2*c)*sin_integral(2*c - 2*(c*x + d)/x)/x - 4*(c*x + d)*a*b*d^2*sin(c)*si
n_integral(c - (c*x + d)/x)/x - b^2*d^2*cos(2*(c*x + d)/x) + 4*a*b*d^2*sin(
(c*x + d)/x) + 2*a^2*d^2 + b^2*d^2)/((c - (c*x + d)/x)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = \int \left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2 dx$$

```
[In] int((a + b*sin(c + d/x))^2,x)
```

```
[Out] int((a + b*sin(c + d/x))^2, x)
```

3.296 $\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 255

$$\begin{aligned}\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx &= -\frac{b^2\cos\left(2c-\frac{2df}{e}\right)\operatorname{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2f} \\ &\quad+\frac{b^2\cos(2c)\operatorname{CosIntegral}\left(\frac{2d}{x}\right)}{2f}+\frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{b^2\log\left(f+\frac{e}{x}\right)}{2f} \\ &\quad+\frac{a^2\log(x)}{f}+\frac{b^2\log(x)}{2f}-\frac{2ab\operatorname{CosIntegral}\left(\frac{d}{x}\right)\sin(c)}{f} \\ &\quad+\frac{2ab\operatorname{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(c-\frac{df}{e}\right)}{f} \\ &\quad+\frac{2ab\cos\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f} \\ &\quad+\frac{b^2\sin\left(2c-\frac{2df}{e}\right)\operatorname{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2f} \\ &\quad-\frac{2ab\cos(c)\operatorname{Si}\left(\frac{d}{x}\right)}{f}-\frac{b^2\sin(2c)\operatorname{Si}\left(\frac{2d}{x}\right)}{2f}\end{aligned}$$

```
[Out] 1/2*b^2*Ci(2*d/x)*cos(2*c)/f-1/2*b^2*Ci(2*d*(f/e+1/x))*cos(2*c-2*d*f/e)/f+a
^2*ln(f+e/x)/f+1/2*b^2*ln(f+e/x)/f+a^2*ln(x)/f+1/2*b^2*ln(x)/f+2*a*b*cos(c-
d*f/e)*Si(d*(f/e+1/x))/f-2*a*b*cos(c)*Si(d/x)/f-2*a*b*Ci(d/x)*sin(c)/f-1/2*
b^2*Si(2*d/x)*sin(2*c)/f+1/2*b^2*Si(2*d*(f/e+1/x))*sin(2*c-2*d*f/e)/f+2*a*b
*Ci(d*(f/e+1/x))*sin(c-d*f/e)/f
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3512, 3398, 3384, 3380, 3383, 3393}

$$\begin{aligned}
\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx &= \frac{a^2\log\left(\frac{e}{x}+f\right)}{f}+\frac{a^2\log(x)}{f} \\
&+\frac{2ab\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f} \\
&-\frac{2ab\sin(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)}{f}+\frac{2ab\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f} \\
&-\frac{2ab\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f}-\frac{b^2\cos\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2f} \\
&+\frac{b^2\cos(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right)}{2f}+\frac{b^2\sin\left(2c-\frac{2df}{e}\right)\text{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2f} \\
&-\frac{b^2\sin(2c)\text{Si}\left(\frac{2d}{x}\right)}{2f}+\frac{b^2\log\left(\frac{e}{x}+f\right)}{2f}+\frac{b^2\log(x)}{2f}
\end{aligned}$$

```
[In] Int[(a + b*Sin[c + d/x])^2/(e + f*x),x]
```

```
[Out] -1/2*(b^2*Cos[2*c - (2*d*f)/e]*CosIntegral[2*d*(f/e + x^(-1))])/f + (b^2*Co
s[2*c]*CosIntegral[(2*d)/x])/(2*f) + (a^2*Log[f + e/x])/f + (b^2*Log[f + e/
x])/(2*f) + (a^2*Log[x])/f + (b^2*Log[x])/(2*f) - (2*a*b*CosIntegral[d/x]*S
in[c])/f + (2*a*b*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/f + (2*a*
b*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/f + (b^2*Sin[2*c - (2*d*f
)/e]*SinIntegral[2*d*(f/e + x^(-1))])/(2*f) - (2*a*b*Cos[c]*SinIntegral[d/x
])/f - (b^2*Sin[2*c]*SinIntegral[(2*d)/x])/(2*f)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \left(\frac{(a+b\sin(c+dx))^2}{fx} - \frac{e(a+b\sin(c+dx))^2}{f(f+ex)}\right) dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{x} dx, x, \frac{1}{x}\right)}{f} + \frac{e\text{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{f+ex} dx, x, \frac{1}{x}\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^2}{x} + \frac{2ab\sin(c+dx)}{x} + \frac{b^2\sin^2(c+dx)}{x}\right) dx, x, \frac{1}{x}\right)}{f} \\
&\quad + \frac{e\text{Subst}\left(\int \left(\frac{a^2}{f+ex} + \frac{2ab\sin(c+dx)}{f+ex} + \frac{b^2\sin^2(c+dx)}{f+ex}\right) dx, x, \frac{1}{x}\right)}{f} \\
&= \frac{a^2\log\left(f+\frac{e}{x}\right)}{f} + \frac{a^2\log(x)}{f} - \frac{(2ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{x} dx, x, \frac{1}{x}\right)}{f} \\
&\quad - \frac{b^2\text{Subst}\left(\int \frac{\sin^2(c+dx)}{x} dx, x, \frac{1}{x}\right)}{f} + \frac{(2abe)\text{Subst}\left(\int \frac{\sin(c+dx)}{f+ex} dx, x, \frac{1}{x}\right)}{f} \\
&\quad + \frac{(b^2e)\,\text{Subst}\left(\int \frac{\sin^2(c+dx)}{f+ex} dx, x, \frac{1}{x}\right)}{f}
\end{aligned}
$$

$$= \frac{a^2\log\left(f+\frac{e}{x}\right)}{f} + \frac{a^2\log(x)}{f} - \frac{b^2\text{Subst}\left(\int\left(\frac{1}{2x} - \frac{\cos(2c+2dx)}{2x}\right)\,dx, x, \frac{1}{x}\right)}{f}$$
$$+ \frac{(b^2e)\,\text{Subst}\left(\int\left(\frac{1}{2(f+ex)} - \frac{\cos(2c+2dx)}{2(f+ex)}\right)\,dx, x, \frac{1}{x}\right)}{f}$$
$$- \frac{(2ab\cos(c))\text{Subst}\left(\int\frac{\sin(dx)}{x}\,dx, x, \frac{1}{x}\right)}{f}$$
$$+ \frac{\left(2abe\cos\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{f}$$
$$- \frac{(2ab\sin(c))\text{Subst}\left(\int\frac{\cos(dx)}{x}\,dx, x, \frac{1}{x}\right)}{f}$$
$$+ \frac{\left(2abe\sin\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{f}$$

$$= \frac{a^2\log\left(f+\frac{e}{x}\right)}{f} + \frac{b^2\log\left(f+\frac{e}{x}\right)}{2f} + \frac{a^2\log(x)}{f} + \frac{b^2\log(x)}{2f}$$
$$- \frac{2ab\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c)}{f} + \frac{2ab\,\text{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(c-\frac{df}{e}\right)}{f}$$
$$+ \frac{2ab\cos\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{f} - \frac{2ab\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f}$$
$$+ \frac{b^2\text{Subst}\left(\int\frac{\cos(2c+2dx)}{x}\,dx, x, \frac{1}{x}\right)}{2f} - \frac{(b^2e)\,\text{Subst}\left(\int\frac{\cos(2c+2dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{2f}$$

$$= \frac{a^2\log\left(f+\frac{e}{x}\right)}{f} + \frac{b^2\log\left(f+\frac{e}{x}\right)}{2f} + \frac{a^2\log(x)}{f} + \frac{b^2\log(x)}{2f} - \frac{2ab\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c)}{f} + \frac{2ab\,\text{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(c-\frac{df}{e}\right)}{f} + \frac{2ab\cos\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{f} - \frac{2ab\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f} + \frac{(b^2\cos(2c))\,\text{Subst}\left(\int\frac{\cos(2dx)}{x}\,dx,x,\frac{1}{x}\right)}{2f} - \frac{\left(b^2e\cos\left(2c-\frac{2df}{e}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{2df}{e}+2dx\right)}{f+ex}\,dx,x,\frac{1}{x}\right)}{2f} - \frac{(b^2\sin(2c))\,\text{Subst}\left(\int\frac{\sin(2dx)}{x}\,dx,x,\frac{1}{x}\right)}{2f} + \frac{\left(b^2e\sin\left(2c-\frac{2df}{e}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{2df}{e}+2dx\right)}{f+ex}\,dx,x,\frac{1}{x}\right)}{2f}$$

$$= -\frac{b^2\cos\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{2f} + \frac{b^2\cos(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right)}{2f} + \frac{a^2\log\left(f+\frac{e}{x}\right)}{f} + \frac{b^2\log\left(f+\frac{e}{x}\right)}{2f} + \frac{a^2\log(x)}{f} + \frac{b^2\log(x)}{2f} - \frac{2ab\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c)}{f} + \frac{2ab\,\text{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(c-\frac{df}{e}\right)}{f} + \frac{2ab\cos\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{f} + \frac{b^2\sin\left(2c-\frac{2df}{e}\right)\text{Si}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{2f} - \frac{2ab\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f} - \frac{b^2\sin(2c)\text{Si}\left(\frac{2d}{x}\right)}{2f}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.76

$$\int\frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx = \frac{-b^2\cos\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)+b^2\cos(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right)+2a^2\log(e+fx)+b^2\log(e+fx}{}$$

```
[In] Integrate[(a + b*Sin[c + d/x])^2/(e + f*x),x]
```

```
[Out] (-(b^2*Cos[2*c - (2*d*f)/e]*CosIntegral[2*d*(f/e + x^(-1))]) + b^2*Cos[2*c]
*CosIntegral[(2*d)/x] + 2*a^2*Log[e + f*x] + b^2*Log[e + f*x] - 4*a*b*CosIn
```

```
tegral[d/x]*Sin[c] + 4*a*b*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e] +
 4*a*b*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + b^2*Sin[2*c - (2*d*
f)/e]*SinIntegral[2*d*(f/e + x^(-1))] - 4*a*b*Cos[c]*SinIntegral[d/x] - b^2
*Sin[2*c]*SinIntegral[(2*d)/x])/(2*f)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.18

method	result
parts	$\frac{\ln(fx+e)a^2}{f} - \frac{b^2\ln\left(\frac{d}{x}\right)}{2f} + \frac{b^2\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{2f} - \frac{b^2\,\mathrm{Si}\left(\frac{2d}{x}+2c+\frac{-2ce+2df}{e}\right)\sin\left(\frac{-2ce+2df}{e}\right)}{2f} - \frac{b^2\,\mathrm{Ci}\left(\frac{2d}{x}+2\right.}{}$
risch	$-\frac{iab\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{Ei}_1\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{f} + \frac{iab\,\mathrm{Ei}_1\left(\frac{id}{x}\right)\mathrm{e}^{-ic}}{f} + \frac{\ln(fx+e)a^2}{f} + \frac{\ln(fx+e)b^2}{2f} - \frac{b^2\,\mathrm{Ei}_1\left(\frac{2id}{x}\right)\mathrm{e}^{-2ic}}{4f}$
derivativedivides	$-d\left(\frac{a^2\ln\left(\frac{d}{x}\right)}{fd} - \frac{a^2\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{fd} - \frac{2abe\left(-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+df}{e}\right)}{e} - \frac{\mathrm{Ci}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin\left(\right.}{e}\right.}{fd}\right.$
default	$-d\left(\frac{a^2\ln\left(\frac{d}{x}\right)}{fd} - \frac{a^2\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{fd} - \frac{2abe\left(-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+df}{e}\right)}{e} - \frac{\mathrm{Ci}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin\left(\right.}{e}\right.}{fd}\right.$

```
[In] int((a+b*sin(c+d/x))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] ln(f*x+e)/f*a^2-1/2*b^2/f*ln(d/x)+1/2*b^2/f*ln(-c*e+d*f+e*(c+d/x))-1/2*b^2/
f*Si(2*d/x+2*c+2*(-c*e+d*f)/e)*sin(2*(-c*e+d*f)/e)-1/2*b^2/f*Ci(2*d/x+2*c+2
*(-c*e+d*f)/e)*cos(2*(-c*e+d*f)/e)-1/2*b^2*Si(2*d/x)*sin(2*c)/f+1/2*b^2*Ci(
2*d/x)*cos(2*c)/f-2*a*b*d*(-e/d/f*(Si(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f)/e)
/e-Ci(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e)+1/d/f*(Si(d/x)*cos(c)+Ci(d/x
)*sin(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx$$
$$= \frac{b^2\cos\left(2\,c\right)\mathrm{Ci}\left(\frac{2\,d}{x}\right) - b^2\cos\left(-\frac{2\,(ce-df)}{e}\right)\mathrm{Ci}\left(\frac{2\,(dfx+de)}{ex}\right) - 4\,ab\,\mathrm{Ci}\left(\frac{d}{x}\right)\sin\left(c\right) - 4\,ab\,\mathrm{Ci}\left(\frac{dfx+de}{ex}\right)\sin\left(-\frac{ce-df}{e}\right)}{}$$

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cos(2*c)*cos_integral(2*d/x) - b^2*cos(-2*(c*e - d*f)/e)*cos_integ
ral(2*(d*f*x + d*e)/(e*x)) - 4*a*b*cos_integral(d/x)*sin(c) - 4*a*b*cos_int
egral((d*f*x + d*e)/(e*x))*sin(-(c*e - d*f)/e) - b^2*sin(2*c)*sin_integral(
2*d/x) - 4*a*b*cos(c)*sin_integral(d/x) - b^2*sin(-2*(c*e - d*f)/e)*sin_int
egral(2*(d*f*x + d*e)/(e*x)) + 4*a*b*cos(-(c*e - d*f)/e)*sin_integral((d*f*
x + d*e)/(e*x)) + (2*a^2 + b^2)*log(f*x + e))/f
```

Sympy [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx = \int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx$$

```
[In] integrate((a+b*sin(c+d/x))**2/(f*x+e),x)
```

```
[Out] Integral((a + b*sin(c + d/x))**2/(e + f*x), x)
```

Maxima [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx = \int \frac{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}{fx+e}\,dx$$

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^2*log(f*x + e)/f - 1/2*(2*b^2*f*integrate(1/4*cos(2*(c*x + d)/x)/((f*x +
e)*cos(2*(c*x + d)/x)^2 + (f*x + e)*sin(2*(c*x + d)/x)^2), x) + 2*b^2*f*int
egrate(1/4*cos(2*(c*x + d)/x)/(f*x + e), x) - 2*a*b*f*integrate(sin((c*x +
d)/x)/((f*x + e)*cos((c*x + d)/x)^2 + (f*x + e)*sin((c*x + d)/x)^2), x) - 2
*a*b*f*integrate(sin((c*x + d)/x)/(f*x + e), x) - b^2*log(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.42

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx$$
$$= \frac{b^2d\cos(2c)\,\mathrm{Ci}\left(-2c+\frac{2(cx+d)}{x}\right) - b^2d\cos\left(\frac{2(ce-df)}{e}\right)\mathrm{Ci}\left(-\frac{2\left(ce-df-\frac{(cx+d)e}{x}\right)}{e}\right) - 4abd\,\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)\sin(c)}{}$$

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*cos(2*c)*cos_integral(-2*c + 2*(c*x + d)/x) - b^2*d*cos(2*(c*e -
 d*f)/e)*cos_integral(-2*(c*e - d*f - (c*x + d)*e/x)/e) - 4*a*b*d*cos_integ
ral(-c + (c*x + d)/x)*sin(c) + 4*a*b*d*cos_integral(-(c*e - d*f - (c*x + d)
*e/x)/e)*sin((c*e - d*f)/e) + b^2*d*sin(2*c)*sin_integral(2*c - 2*(c*x + d)
/x) + 4*a*b*d*cos(c)*sin_integral(c - (c*x + d)/x) - b^2*d*sin(2*(c*e - d*f
)/e)*sin_integral(2*(c*e - d*f - (c*x + d)*e/x)/e) - 4*a*b*d*cos((c*e - d*f
)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) + 2*a^2*d*log(c*e - d*f -
(c*x + d)*e/x) + b^2*d*log(c*e - d*f - (c*x + d)*e/x) - 2*a^2*d*log(c - (c*
x + d)/x) - b^2*d*log(c - (c*x + d)/x))/(d*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx=\int \frac{{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)}^2}{e+f\,x} \,d x$$

```
[In] int((a + b*sin(c + d/x))^2/(e + f*x),x)
```

```
[Out] int((a + b*sin(c + d/x))^2/(e + f*x), x)
```

3.297 $\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 195

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx = \frac{a^2}{e\left(f+\frac{e}{x}\right)} - \frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2} - \frac{b^2d\,\text{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(2c-\frac{2df}{e}\right)}{e^2} + \frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{2abd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2} - \frac{b^2d\cos\left(2c-\frac{2df}{e}\right)\text{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}$$

```
[Out] a^2/e/(f+e/x)-2*a*b*d*Ci(d*(f/e+1/x))*cos(c-d*f/e)/e^2-b^2*d*cos(2*c-2*d*f/
e)*Si(2*d*(f/e+1/x))/e^2-b^2*d*Ci(2*d*(f/e+1/x))*sin(2*c-2*d*f/e)/e^2+2*a*b
*d*Si(d*(f/e+1/x))*sin(c-d*f/e)/e^2+2*a*b*sin(c+d/x)/e/(f+e/x)+b^2*sin(c+d/
x)^2/e/(f+e/x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

= {3512, 3398, 3378, 3384, 3380, 3383, 3394, 12}

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx = \frac{a^2}{e\left(\frac{e}{x}+f\right)} - \frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2} + \frac{2abd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2} + \frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)} - \frac{b^2d\sin\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2} - \frac{b^2d\cos\left(2c-\frac{2df}{e}\right)\text{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)}$$

```
[In] Int[(a + b*Sin[c + d/x])^2/(e + f*x)^2,x]
```

```
[Out] a^2/(e*(f + e/x)) - (2*a*b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))]
)/e^2 - (b^2*d*CosIntegral[2*d*(f/e + x^(-1))]*Sin[2*c - (2*d*f)/e])/e^2 +
(2*a*b*Sin[c + d/x])/(e*(f + e/x)) + (b^2*Sin[c + d/x]^2)/(e*(f + e/x)) + (
2*a*b*d*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/e^2 - (b^2*d*Cos[2*
c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))])/e^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a^2}{(f+ex)^2}+\frac{2ab\sin(c+dx)}{(f+ex)^2}+\frac{b^2\sin^2(c+dx)}{(f+ex)^2}\right)\,dx, x, \frac{1}{x}\right)\\
&= \frac{a^2}{e\left(f+\frac{e}{x}\right)}-(2ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)-b^2\text{Subst}\left(\int \frac{\sin^2(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a^2}{e\left(f+\frac{e}{x}\right)}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)}+\frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)}\\
&\quad -\frac{(2abd)\text{Subst}\left(\int \frac{\cos(c+dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e}-\frac{(2b^2d)\,\text{Subst}\left(\int \frac{\sin(2c+2dx)}{2(f+ex)}\,dx, x, \frac{1}{x}\right)}{e}
\end{aligned}$$

$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} + \frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} - \frac{(b^2d)\,\text{Subst}\left(\int \frac{\sin(2c+2dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e}$$
$$- \frac{\left(2abd\cos\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e}$$
$$+ \frac{\left(2abd\sin\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e}$$
$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} - \frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2}$$
$$+ \frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{2abd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2}$$
$$- \frac{\left(b^2d\cos\left(2c-\frac{2df}{e}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2df}{e}+2dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e}$$
$$- \frac{\left(b^2d\sin\left(2c-\frac{2df}{e}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2df}{e}+2dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e}$$
$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} - \frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2}$$
$$- \frac{b^2d\,\text{CosIntegral}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(2c-\frac{2df}{e}\right)}{e^2} + \frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)}$$
$$+ \frac{2abd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2} - \frac{b^2d\cos\left(2c-\frac{2df}{e}\right)\text{Si}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx =$$
$$-\frac{2a^2e^2+b^2e^2+b^2efx\cos\left(2\left(c+\frac{d}{x}\right)\right)+4abdf(e+fx)\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)+2b^2df(e+}{}$$

```
[In] Integrate[(a + b*Sin[c + d/x])^2/(e + f*x)^2,x]
```

```
[Out] -1/2*(2*a^2*e^2 + b^2*e^2 + b^2*e*f*x*Cos[2*(c + d/x)] + 4*a*b*d*f*(e + f*x
)*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))] + 2*b^2*d*f*(e + f*x)*CosI
```

```
ntegral[2*d*(f/e + x^(-1))]*Sin[2*c - (2*d*f)/e] - 4*a*b*e*f*x*Sin[c + d/x]
 - 4*a*b*d*e*f*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] - 4*a*b*d*f^2
*x*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + 2*b^2*d*e*f*Cos[2*c - (
2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 2*b^2*d*f^2*x*Cos[2*c - (2*d*f)
/e]*SinIntegral[2*d*(f/e + x^(-1))])/(e^2*f*(e + f*x))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.50

method	result
parts	$-\frac{a^2}{f(fx+e)}-b^2d\left(-\frac{1}{2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{\cos\left(2c+\frac{2d}{x}\right)}{2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{2\,\mathrm{Si}\left(\frac{2d}{x}+2c+\frac{-2ce+2df}{e}\right)\cos\left(\frac{-2ce+2df}{e}\right)}{e}\right.$
derivativedivides	$-d\left(-\frac{a^2}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+2ab\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\frac{\mathrm{Ci}\left(\frac{d}{x}+c+\right.}{}}{e}\right.\right.$
default	$-d\left(-\frac{a^2}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+2ab\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\frac{\mathrm{Ci}\left(\frac{d}{x}+c+\right.}{}}{e}\right.\right.$
risch	$\frac{abd\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{Ei}_1\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{e^2}-\frac{a^2}{f(fx+e)}-\frac{b^2}{2f(fx+e)}+\frac{id\,b^2\mathrm{e}^{-\frac{2i(ce-df)}{e}}\,\mathrm{Ei}_1\left(\frac{2id}{x}+2ic-\frac{2i(ce-df)}{e}\right)}{2e^2}-\frac{id}{}$

```
[In] int((a+b*sin(c+d/x))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/(f*x+e)*a^2-b^2*d*(-1/2/(-c*e+d*f+e*(c+d/x))/e+1/2*cos(2*c+2*d/x)/(-c*
e+d*f+e*(c+d/x))/e+1/2*(2*Si(2*d/x+2*c+2*(-c*e+d*f)/e)*cos(2*(-c*e+d*f)/e)/
e-2*Ci(2*d/x+2*c+2*(-c*e+d*f)/e)*sin(2*(-c*e+d*f)/e)/e)/e)-2*a*b*d*(-sin(c+
d/x)/(-c*e+d*f+e*(c+d/x))/e+(Si(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e+Ci(
d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e)/e)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx =$$
$$-\frac{2\,b^2efx\cos\left(\frac{cx+d}{x}\right)^2-4\,abefx\sin\left(\frac{cx+d}{x}\right)-b^2efx+(2\,a^2+b^2)e^2+4\,(abdf^2x+abdef)\cos\left(-\frac{ce-df}{e}\right)\mathrm{Ci}}{}$$

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*e*f*x*cos((c*x + d)/x)^2 - 4*a*b*e*f*x*sin((c*x + d)/x) - b^2*e
*f*x + (2*a^2 + b^2)*e^2 + 4*(a*b*d*f^2*x + a*b*d*e*f)*cos(-(c*e - d*f)/e)*
cos_integral((d*f*x + d*e)/(e*x)) - 2*(b^2*d*f^2*x + b^2*d*e*f)*cos_integra
l(2*(d*f*x + d*e)/(e*x))*sin(-2*(c*e - d*f)/e) + 2*(b^2*d*f^2*x + b^2*d*e*f
)*cos(-2*(c*e - d*f)/e)*sin_integral(2*(d*f*x + d*e)/(e*x)) + 4*(a*b*d*f^2*
x + a*b*d*e*f)*sin(-(c*e - d*f)/e)*sin_integral((d*f*x + d*e)/(e*x)))/(e^2*
f^2*x + e^3*f)
```

Sympy [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx = \int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{\left(e+fx\right)^2}\,dx$$

```
[In] integrate((a+b*sin(c+d/x))**2/(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sin(c + d/x))**2/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx = \int \frac{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -a^2/(f^2*x + e*f) - 1/2*(b^2 + 2*(b^2*f^2*x + b^2*e*f)*integrate(1/4*cos(2
*(c*x + d)/x)/(f^2*x^2 + 2*e*f*x + e^2), x) + 2*(b^2*f^2*x + b^2*e*f)*integ
rate(1/4*cos(2*(c*x + d)/x)/((f^2*x^2 + 2*e*f*x + e^2)*cos(2*(c*x + d)/x)^2
 + (f^2*x^2 + 2*e*f*x + e^2)*sin(2*(c*x + d)/x)^2), x) - 2*(a*b*f^2*x + a*b
*e*f)*integrate(sin((c*x + d)/x)/(f^2*x^2 + 2*e*f*x + e^2), x) - 2*(a*b*f^2
*x + a*b*e*f)*integrate(sin((c*x + d)/x)/((f^2*x^2 + 2*e*f*x + e^2)*cos((c*
x + d)/x)^2 + (f^2*x^2 + 2*e*f*x + e^2)*sin((c*x + d)/x)^2), x))/(f^2*x + e
*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. 2(195) = 390.

Time = 0.31 (sec) , antiderivative size = 686, normalized size of antiderivative = 3.52

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx =$$

$$-\frac{4\,abcd^2e\cos\left(\frac{ce-df}{e}\right)\operatorname{Ci}\left(-\frac{ce-df-\frac{(cx+d)e}{x}}{e}\right)-4\,abd^3f\cos\left(\frac{ce-df}{e}\right)\operatorname{Ci}\left(-\frac{ce-df-\frac{(cx+d)e}{x}}{e}\right)+2\,b^2cd^2e\operatorname{Ci}\left(-\frac{2\left(ce-\right.}{}\right.}{}$$

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b*c*d^2*e*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)
*e/x)/e) - 4*a*b*d^3*f*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x +
 d)*e/x)/e) + 2*b^2*c*d^2*e*cos_integral(-2*(c*e - d*f - (c*x + d)*e/x)/e)*
sin(2*(c*e - d*f)/e) - 2*b^2*d^3*f*cos_integral(-2*(c*e - d*f - (c*x + d)*e
/x)/e)*sin(2*(c*e - d*f)/e) - 2*b^2*c*d^2*e*cos(2*(c*e - d*f)/e)*sin_integr
al(2*(c*e - d*f - (c*x + d)*e/x)/e) + 2*b^2*d^3*f*cos(2*(c*e - d*f)/e)*sin_
integral(2*(c*e - d*f - (c*x + d)*e/x)/e) + 4*a*b*c*d^2*e*sin((c*e - d*f)/e
)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) - 4*a*b*d^3*f*sin((c*e - d*f)
/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) - 4*(c*x + d)*a*b*d^2*e*cos
((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)/x - 2*(c*x + d
)*b^2*d^2*e*cos_integral(-2*(c*e - d*f - (c*x + d)*e/x)/e)*sin(2*(c*e - d*f
)/e)/x + 2*(c*x + d)*b^2*d^2*e*cos(2*(c*e - d*f)/e)*sin_integral(2*(c*e - d
*f - (c*x + d)*e/x)/e)/x - 4*(c*x + d)*a*b*d^2*e*sin((c*e - d*f)/e)*sin_int
egral((c*e - d*f - (c*x + d)*e/x)/e)/x - b^2*d^2*e*cos(2*(c*x + d)/x) + 4*a
*b*d^2*e*sin((c*x + d)/x) + 2*a^2*d^2*e + b^2*d^2*e)/((c*e^3 - d*e^2*f - (c
*x + d)*e^3/x)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx = \int \frac{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}{\left(e+f\,x\right)^2}\,dx$$

```
[In] int((a + b*sin(c + d/x))^2/(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(c + d/x))^2/(e + f*x)^2, x)
```

3.298 $\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 470

$$\begin{aligned}
\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx =& -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)} \\
&-\frac{2abd\cos\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3} \\
&+\frac{b^2d^2f\cos\left(2c-\frac{2df}{e}\right)\operatorname{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4} \\
&-\frac{b^2d\operatorname{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(2c-\frac{2df}{e}\right)}{e^3} \\
&-\frac{abd^2f\operatorname{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(c-\frac{df}{e}\right)}{e^4} \\
&-\frac{abf\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)^2}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)} \\
&-\frac{b^2df\cos\left(c+\frac{d}{x}\right)\sin\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}-\frac{b^2f\sin^2\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2} \\
&+\frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}-\frac{abd^2f\cos\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4} \\
&+\frac{2abd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3} \\
&-\frac{b^2d\cos\left(2c-\frac{2df}{e}\right)\operatorname{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3} \\
&-\frac{b^2d^2f\sin\left(2c-\frac{2df}{e}\right)\operatorname{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4}
\end{aligned}$$

```
[Out] -1/2*a^2*f/e^2/(f+e/x)^2+a^2/e^2/(f+e/x)+b^2*d^2*f*Ci(2*d*(f/e+1/x))*cos(2*
c-2*d*f/e)/e^4-2*a*b*d*Ci(d*(f/e+1/x))*cos(c-d*f/e)/e^3-a*b*d*f*cos(c+d/x)/
e^3/(f+e/x)-a*b*d^2*f*cos(c-d*f/e)*Si(d*(f/e+1/x))/e^4-b^2*d*cos(2*c-2*d*f/
e)*Si(2*d*(f/e+1/x))/e^3-b^2*d*Ci(2*d*(f/e+1/x))*sin(2*c-2*d*f/e)/e^3-b^2*d
^2*f*Si(2*d*(f/e+1/x))*sin(2*c-2*d*f/e)/e^4-a*b*d^2*f*Ci(d*(f/e+1/x))*sin(c
-d*f/e)/e^4+2*a*b*d*Si(d*(f/e+1/x))*sin(c-d*f/e)/e^3-a*b*f*sin(c+d/x)/e^2/(
f+e/x)^2+2*a*b*sin(c+d/x)/e^2/(f+e/x)-b^2*d*f*cos(c+d/x)*sin(c+d/x)/e^3/(f+
e/x)-1/2*b^2*f*sin(c+d/x)^2/e^2/(f+e/x)^2+b^2*sin(c+d/x)^2/e^2/(f+e/x)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3512, 3398, 3378, 3384, 3380, 3383, 3395, 31, 3393, 3394, 12}

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx = \frac{a^2}{e^2\left(\frac{e}{x}+f\right)}-\frac{a^2 f}{2e^2\left(\frac{e}{x}+f\right)^2} -\frac{abd^2 f\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4} -\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3} -\frac{abd^2 f\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4} +\frac{2abd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(\frac{e}{x}+f\right)} +\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e^2\left(\frac{e}{x}+f\right)}-\frac{abf\sin\left(c+\frac{d}{x}\right)}{e^2\left(\frac{e}{x}+f\right)^2} +\frac{b^2d^2f\cos\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4} -\frac{b^2d\sin\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3} -\frac{b^2d^2f\sin\left(2c-\frac{2df}{e}\right)\text{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4} -\frac{b^2d\cos\left(2c-\frac{2df}{e}\right)\text{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3} -\frac{b^2df\sin\left(c+\frac{d}{x}\right)\cos\left(c+\frac{d}{x}\right)}{e^3\left(\frac{e}{x}+f\right)} +\frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e^2\left(\frac{e}{x}+f\right)}-\frac{b^2f\sin^2\left(c+\frac{d}{x}\right)}{2e^2\left(\frac{e}{x}+f\right)^2}$$

```
[In] Int[(a + b*Sin[c + d/x])^2/(e + f*x)^3,x]
```

```
[Out] -1/2*(a^2*f)/(e^2*(f + e/x)^2) + a^2/(e^2*(f + e/x)) - (a*b*d*f*Cos[c + d/x
])/(e^3*(f + e/x)) - (2*a*b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))
])/e^3 + (b^2*d^2*f*Cos[2*c - (2*d*f)/e]*CosIntegral[2*d*(f/e + x^(-1))])/e
^4 - (b^2*d*CosIntegral[2*d*(f/e + x^(-1))]*Sin[2*c - (2*d*f)/e])/e^3 - (a*
b*d^2*f*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/e^4 - (a*b*f*Sin[c
+ d/x])/(e^2*(f + e/x)^2) + (2*a*b*Sin[c + d/x])/(e^2*(f + e/x)) - (b^2*d*f
*Cos[c + d/x]*Sin[c + d/x])/(e^3*(f + e/x)) - (b^2*f*Sin[c + d/x]^2)/(2*e^2
*(f + e/x)^2) + (b^2*Sin[c + d/x]^2)/(e^2*(f + e/x)) - (a*b*d^2*f*Cos[c - (
d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/e^4 + (2*a*b*d*Sin[c - (d*f)/e]*SinI
ntegral[d*(f/e + x^(-1))])/e^3 - (b^2*d*Cos[2*c - (2*d*f)/e]*SinIntegral[2*
d*(f/e + x^(-1))])/e^3 - (b^2*d^2*f*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f
/e + x^(-1))])/e^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \left(-\frac{f(a+b\sin(c+dx))^2}{e(f+ex)^3} + \frac{(a+b\sin(c+dx))^2}{e(f+ex)^2}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{(f+ex)^2} dx, x, \frac{1}{x}\right)}{e} + \frac{f\text{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{(f+ex)^3} dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a^2}{(f+ex)^2}+\frac{2ab\sin(c+dx)}{(f+ex)^2}+\frac{b^2\sin^2(c+dx)}{(f+ex)^2}\right)\,dx,x,\frac{1}{x}\right)}{e} + \frac{f\text{Subst}\left(\int \left(\frac{a^2}{(f+ex)^3}+\frac{2ab\sin(c+dx)}{(f+ex)^3}+\frac{b^2\sin^2(c+dx)}{(f+ex)^3}\right)\,dx,x,\frac{1}{x}\right)}{e}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{(2ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{(f+ex)^2}\,dx,x,\frac{1}{x}\right)}{e} - \frac{b^2\text{Subst}\left(\int \frac{\sin^2(c+dx)}{(f+ex)^2}\,dx,x,\frac{1}{x}\right)}{e}+\frac{(2abf)\text{Subst}\left(\int \frac{\sin(c+dx)}{(f+ex)^3}\,dx,x,\frac{1}{x}\right)}{e} + \frac{(b^2f)\,\text{Subst}\left(\int \frac{\sin^2(c+dx)}{(f+ex)^3}\,dx,x,\frac{1}{x}\right)}{e}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abf\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)^2}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)} - \frac{b^2df\cos\left(c+\frac{d}{x}\right)\sin\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}-\frac{b^2f\sin^2\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}-\frac{(2abd)\text{Subst}\left(\int \frac{\cos(c+dx)}{f+ex}\,dx,x,\frac{1}{x}\right)}{e^2} - \frac{(2b^2d)\,\text{Subst}\left(\int \frac{\sin(2c+2dx)}{2(f+ex)}\,dx,x,\frac{1}{x}\right)}{e^2}+\frac{(b^2d^2f)\,\text{Subst}\left(\int \frac{1}{f+ex}\,dx,x,\frac{1}{x}\right)}{e^3} - \frac{(2b^2d^2f)\,\text{Subst}\left(\int \frac{\sin^2(c+dx)}{f+ex}\,dx,x,\frac{1}{x}\right)}{e^3}+\frac{(abdf)\text{Subst}\left(\int \frac{\cos(c+dx)}{(f+ex)^2}\,dx,x,\frac{1}{x}\right)}{e^2}$$

$$= -\frac{a^2 f}{2e^2 \left(f+\frac{e}{x}\right)^2} + \frac{a^2}{e^2 \left(f+\frac{e}{x}\right)} - \frac{abdf \cos\left(c+\frac{d}{x}\right)}{e^3 \left(f+\frac{e}{x}\right)} + \frac{b^2 d^2 f \log\left(f+\frac{e}{x}\right)}{e^4} - \frac{abf \sin\left(c+\frac{d}{x}\right)}{e^2 \left(f+\frac{e}{x}\right)^2} + \frac{2ab \sin\left(c+\frac{d}{x}\right)}{e^2 \left(f+\frac{e}{x}\right)} - \frac{b^2 df \cos\left(c+\frac{d}{x}\right) \sin\left(c+\frac{d}{x}\right)}{e^3 \left(f+\frac{e}{x}\right)} - \frac{b^2 f \sin^2\left(c+\frac{d}{x}\right)}{2e^2 \left(f+\frac{e}{x}\right)^2} + \frac{b^2 \sin^2\left(c+\frac{d}{x}\right)}{e^2 \left(f+\frac{e}{x}\right)} - \frac{(b^2 d)\,\text{Subst}\left(\int \frac{\sin(2c+2dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e^2} - \frac{(abd^2 f)\,\text{Subst}\left(\int \frac{\sin(c+dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e^3} - \frac{(2b^2 d^2 f)\,\text{Subst}\left(\int \left(\frac{1}{2(f+ex)} - \frac{\cos(2c+2dx)}{2(f+ex)}\right)\,dx, x, \frac{1}{x}\right)}{e^3} - \frac{\left(2abd \cos\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{\left(2abd \sin\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e^2}$$

$$= -\frac{a^2 f}{2e^2 \left(f+\frac{e}{x}\right)^2} + \frac{a^2}{e^2 \left(f+\frac{e}{x}\right)} - \frac{abdf \cos\left(c+\frac{d}{x}\right)}{e^3 \left(f+\frac{e}{x}\right)} - \frac{2abd \cos\left(c-\frac{df}{e}\right) \text{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3} - \frac{abf \sin\left(c+\frac{d}{x}\right)}{e^2 \left(f+\frac{e}{x}\right)^2} + \frac{2ab \sin\left(c+\frac{d}{x}\right)}{e^2 \left(f+\frac{e}{x}\right)} - \frac{b^2 df \cos\left(c+\frac{d}{x}\right) \sin\left(c+\frac{d}{x}\right)}{e^3 \left(f+\frac{e}{x}\right)} - \frac{b^2 f \sin^2\left(c+\frac{d}{x}\right)}{2e^2 \left(f+\frac{e}{x}\right)^2} + \frac{b^2 \sin^2\left(c+\frac{d}{x}\right)}{e^2 \left(f+\frac{e}{x}\right)} + \frac{2abd \sin\left(c-\frac{df}{e}\right) \text{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3} + \frac{(b^2 d^2 f)\,\text{Subst}\left(\int \frac{\cos(2c+2dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e^3} - \frac{\left(b^2 d \cos\left(2c-\frac{2df}{e}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2df}{e}+2dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e^2} - \frac{\left(abd^2 f \cos\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e^3} - \frac{\left(b^2 d \sin\left(2c-\frac{2df}{e}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2df}{e}+2dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e^2} - \frac{\left(abd^2 f \sin\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e^3}$$

$$
\begin{aligned}
&= -\frac{a^2 f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)} \\
&\quad -\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3} \\
&\quad -\frac{b^2 d\,\text{CosIntegral}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(2c-\frac{2df}{e}\right)}{e^3} \\
&\quad -\frac{abd^2 f\,\text{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(c-\frac{df}{e}\right)}{e^4}-\frac{abf\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)^2} \\
&\quad +\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}-\frac{b^2 df\cos\left(c+\frac{d}{x}\right)\sin\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}-\frac{b^2 f\sin^2\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2} \\
&\quad +\frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}-\frac{abd^2 f\cos\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^4} \\
&\quad +\frac{2abd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3}-\frac{b^2 d\cos\left(2c-\frac{2df}{e}\right)\text{Si}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3} \\
&\quad +\frac{\left(b^2 d^2 f\cos\left(2c-\frac{2df}{e}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{2df}{e}+2dx\right)}{f+ex}\,dx,x,\frac{1}{x}\right)}{e^3} \\
&\quad -\frac{\left(b^2 d^2 f\sin\left(2c-\frac{2df}{e}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{2df}{e}+2dx\right)}{f+ex}\,dx,x,\frac{1}{x}\right)}{e^3}
\end{aligned}
$$

$$= -\frac{a^2 f}{2e^2\left(f+\frac{e}{x}\right)^2} + \frac{a^2}{e^2\left(f+\frac{e}{x}\right)} - \frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)} - \frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3} + \frac{b^2d^2f\cos\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{e^4} - \frac{b^2d\,\text{CosIntegral}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(2c-\frac{2df}{e}\right)}{e^3} - \frac{abd^2f\,\text{CosIntegral}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(c-\frac{df}{e}\right)}{e^4} - \frac{abf\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)^2} + \frac{2ab\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)} - \frac{b^2df\cos\left(c+\frac{d}{x}\right)\sin\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)} - \frac{b^2f\sin^2\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)} - \frac{abd^2f\cos\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^4} + \frac{2abd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3} - \frac{b^2d\cos\left(2c-\frac{2df}{e}\right)\text{Si}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3} - \frac{b^2d^2f\sin\left(2c-\frac{2df}{e}\right)\text{Si}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{e^4}$$

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.57

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx = -\frac{2a^2e^4+b^2e^4+4abde^2f^2x\cos\left(c+\frac{d}{x}\right)+4abdef^3x^2\cos\left(c+\frac{d}{x}\right)+2b^2e^3fx\cos\left(2\left(c+\frac{d}{x}\right)\right)+b^2e^2f^2x^2\cos\left(\right.}{}$$

```
[In] Integrate[(a + b*Sin[c + d/x])^2/(e + f*x)^3,x]
```

```
[Out] -1/4*(2*a^2*e^4 + b^2*e^4 + 4*a*b*d*e^2*f^2*x*Cos[c + d/x] + 4*a*b*d*e*f^3*
x^2*Cos[c + d/x] + 2*b^2*e^3*f*x*Cos[2*(c + d/x)] + b^2*e^2*f^2*x^2*Cos[2*(
c + d/x)] - 4*b^2*d*f*(e + f*x)^2*CosIntegral[2*d*(f/e + x^(-1))]*(d*f*Cos[
2*c - (2*d*f)/e] - e*Sin[2*c - (2*d*f)/e]) + 4*a*b*d*f*(e + f*x)^2*CosInteg
ral[d*(f/e + x^(-1))]*(2*e*Cos[c - (d*f)/e] + d*f*Sin[c - (d*f)/e]) - 8*a*b
*e^3*f*x*Sin[c + d/x] - 4*a*b*e^2*f^2*x^2*Sin[c + d/x] + 2*b^2*d*e^2*f^2*x*
Sin[2*(c + d/x)] + 2*b^2*d*e*f^3*x^2*Sin[2*(c + d/x)] + 4*a*b*d^2*e^2*f^2*C
os[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + 8*a*b*d^2*e*f^3*x*Cos[c - (
d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + 4*a*b*d^2*f^4*x^2*Cos[c - (d*f)/e]*
SinIntegral[d*(f/e + x^(-1))] - 8*a*b*d*e^3*f*Sin[c - (d*f)/e]*SinIntegral[
d*(f/e + x^(-1))] - 16*a*b*d*e^2*f^2*x*Sin[c - (d*f)/e]*SinIntegral[d*(f/e
```

```
+ x^(-1))] - 8*a*b*d*e*f^3*x^2*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1)
)] + 4*b^2*d*e^3*f*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 8
*b^2*d*e^2*f^2*x*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 4*b
^2*d*e*f^3*x^2*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 4*b^2
*d^2*e^2*f^2*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 8*b^2*d
^2*e*f^3*x*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 4*b^2*d^2
*f^4*x^2*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))])/(e^4*f*(e +
f*x)^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.84

method	result
risch	$\frac{iab\,d^2 e^{-\frac{i(ce-df)}{e}} \operatorname{Ei}_1\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right) f}{2e^4}+\frac{abd\,e^{-\frac{i(ce-df)}{e}} \operatorname{Ei}_1\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{e^3}-\frac{a^2}{2f(fx+e)^2}-\frac{b^2}{4f(fx+e)^2}-$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((a+b*sin(c+d/x))^2/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*a*b*d^2/e^4*exp(-I*(c*e-d*f)/e)*Ei(1,I*d/x+I*c-I*(c*e-d*f)/e)*f+a*b*d
/e^3*exp(-I*(c*e-d*f)/e)*Ei(1,I*d/x+I*c-I*(c*e-d*f)/e)-1/2/f/(f*x+e)^2*a^2-
1/4/f*b^2/(f*x+e)^2-1/2*d^2*b^2/e^4*exp(-2*I*(c*e-d*f)/e)*Ei(1,2*I*d/x+2*I*
c-2*I*(c*e-d*f)/e)*f+1/2*I*d*b^2/e^3*exp(-2*I*(c*e-d*f)/e)*Ei(1,2*I*d/x+2*I
*c-2*I*(c*e-d*f)/e)-1/2*d^2*b^2*exp(2*I*(c*e-d*f)/e)*Ei(1,-2*I*d/x-2*I*c-2*
(-I*c*e+I*f*d)/e)/e^4*f-1/2*I*d*b^2*exp(2*I*(c*e-d*f)/e)*Ei(1,-2*I*d/x-2*I*
c-2*(-I*c*e+I*f*d)/e)/e^3-1/2*I*a*b*d^2*exp(I*(c*e-d*f)/e)*Ei(1,-I*d/x-I*c-
(-I*c*e+I*f*d)/e)/e^4*f+a*b*d*exp(I*(c*e-d*f)/e)*Ei(1,-I*d/x-I*c-(-I*c*e+I*
f*d)/e)/e^3+1/2*I*a*b/e^3*x*(2*I*d^3*f^4*x^3+2*I*d^3*e^3*f+6*I*d^3*e*f^3*x^
2+6*I*d^3*e^2*f^2*x)/(f*x+e)^2/(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2)*cos((c*x+d
)/x)-1/2*a*b/e^2*x*(-2*d^2*f^3*x^3-8*d^2*e*f^2*x^2-10*d^2*e^2*f*x-4*d^2*e^3
)/(f*x+e)^2/(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2)*sin((c*x+d)/x)+1/8*b^2*x/e^2*
(-2*d^2*f^3*x^3-8*d^2*e*f^2*x^2-10*d^2*e^2*f*x-4*d^2*e^3)/(f*x+e)^2/(d^2*f^
2*x^2+2*d^2*e*f*x+d^2*e^2)*cos(2*(c*x+d)/x)+1/8*I*b^2*x/e^3*(4*I*d^3*f^4*x^
3+4*I*d^3*e^3*f+12*I*d^3*e*f^3*x^2+12*I*d^3*e^2*f^2*x)/(f*x+e)^2/(d^2*f^2*x
^2+2*d^2*e*f*x+d^2*e^2)*sin(2*(c*x+d)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 710, normalized size of antiderivative = 1.51

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx$$

$$= \frac{b^2e^2f^2x^2+2\,b^2e^3fx-(2\,a^2+b^2)e^4-2\,(b^2e^2f^2x^2+2\,b^2e^3fx)\cos\left(\frac{cx+d}{x}\right)^2-4\left(2\,(abdef^3x^2+2\,abde^2f^2x+\right.}{}$$

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*e^2*f^2*x^2 + 2*b^2*e^3*f*x - (2*a^2 + b^2)*e^4 - 2*(b^2*e^2*f^2*x
^2 + 2*b^2*e^3*f*x)*cos((c*x + d)/x)^2 - 4*(2*(a*b*d*e*f^3*x^2 + 2*a*b*d*e^
2*f^2*x + a*b*d*e^3*f)*cos_integral((d*f*x + d*e)/(e*x)) + (a*b*d^2*f^4*x^2
 + 2*a*b*d^2*e*f^3*x + a*b*d^2*e^2*f^2)*sin_integral((d*f*x + d*e)/(e*x)))*
cos(-(c*e - d*f)/e) + 4*((b^2*d^2*f^4*x^2 + 2*b^2*d^2*e*f^3*x + b^2*d^2*e^2
*f^2)*cos_integral(2*(d*f*x + d*e)/(e*x)) - (b^2*d*e*f^3*x^2 + 2*b^2*d*e^2*
f^2*x + b^2*d*e^3*f)*sin_integral(2*(d*f*x + d*e)/(e*x)))*cos(-2*(c*e - d*f
)/e) - 4*(a*b*d*e*f^3*x^2 + a*b*d*e^2*f^2*x)*cos((c*x + d)/x) + 4*((a*b*d^2
*f^4*x^2 + 2*a*b*d^2*e*f^3*x + a*b*d^2*e^2*f^2)*cos_integral((d*f*x + d*e)/
(e*x)) - 2*(a*b*d*e*f^3*x^2 + 2*a*b*d*e^2*f^2*x + a*b*d*e^3*f)*sin_integral
((d*f*x + d*e)/(e*x)))*sin(-(c*e - d*f)/e) + 4*((b^2*d*e*f^3*x^2 + 2*b^2*d*
e^2*f^2*x + b^2*d*e^3*f)*cos_integral(2*(d*f*x + d*e)/(e*x)) + (b^2*d^2*f^4
*x^2 + 2*b^2*d^2*e*f^3*x + b^2*d^2*e^2*f^2)*sin_integral(2*(d*f*x + d*e)/(e
*x)))*sin(-2*(c*e - d*f)/e) + 4*(a*b*e^2*f^2*x^2 + 2*a*b*e^3*f*x - (b^2*d*e
*f^3*x^2 + b^2*d*e^2*f^2*x)*cos((c*x + d)/x))*sin((c*x + d)/x))/(e^4*f^3*x^
2 + 2*e^5*f^2*x + e^6*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sin(c+d/x))**2/(f*x+e)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx=\int \frac{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}{(fx+e)^3}\,dx$$

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a^2/(f^3*x^2 + 2*e*f^2*x + e^2*f) - 1/4*(b^2 + 4*(b^2*f^3*x^2 + 2*b^2*
e*f^2*x + b^2*e^2*f)*integrate(1/4*cos(2*(c*x + d)/x)/(f^3*x^3 + 3*e*f^2*x^
2 + 3*e^2*f*x + e^3), x) + 4*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*inte
grate(1/4*cos(2*(c*x + d)/x)/((f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)*cos
(2*(c*x + d)/x)^2 + (f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)*sin(2*(c*x +
d)/x)^2), x) - 4*(a*b*f^3*x^2 + 2*a*b*e*f^2*x + a*b*e^2*f)*integrate(sin((c
*x + d)/x)/(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3), x) - 4*(a*b*f^3*x^2 +
 2*a*b*e*f^2*x + a*b*e^2*f)*integrate(sin((c*x + d)/x)/((f^3*x^3 + 3*e*f^2*
x^2 + 3*e^2*f*x + e^3)*cos((c*x + d)/x)^2 + (f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*
f*x + e^3)*sin((c*x + d)/x)^2), x))/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3078 vs. $2(466) = 932$.

Time = 0.59 (sec) , antiderivative size = 3078, normalized size of antiderivative = 6.55

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*b^2*c^2*d^3*e^2*f*cos(2*(c*e - d*f)/e)*cos_integral(-2*(c*e - d*f -
(c*x + d)*e/x)/e) - 8*b^2*c*d^4*e*f^2*cos(2*(c*e - d*f)/e)*cos_integral(-2*
(c*e - d*f - (c*x + d)*e/x)/e) + 4*b^2*d^5*f^3*cos(2*(c*e - d*f)/e)*cos_int
egral(-2*(c*e - d*f - (c*x + d)*e/x)/e) - 4*a*b*c^2*d^3*e^2*f*cos_integral(
-(c*e - d*f - (c*x + d)*e/x)/e)*sin((c*e - d*f)/e) + 8*a*b*c*d^4*e*f^2*cos_
integral(-(c*e - d*f - (c*x + d)*e/x)/e)*sin((c*e - d*f)/e) - 4*a*b*d^5*f^3
*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)*sin((c*e - d*f)/e) + 4*b^2*c^
2*d^3*e^2*f*sin(2*(c*e - d*f)/e)*sin_integral(2*(c*e - d*f - (c*x + d)*e/x)
/e) - 8*b^2*c*d^4*e*f^2*sin(2*(c*e - d*f)/e)*sin_integral(2*(c*e - d*f - (c
*x + d)*e/x)/e) + 4*b^2*d^5*f^3*sin(2*(c*e - d*f)/e)*sin_integral(2*(c*e -
d*f - (c*x + d)*e/x)/e) + 4*a*b*c^2*d^3*e^2*f*cos((c*e - d*f)/e)*sin_integr
al((c*e - d*f - (c*x + d)*e/x)/e) - 8*a*b*c*d^4*e*f^2*cos((c*e - d*f)/e)*si
n_integral((c*e - d*f - (c*x + d)*e/x)/e) + 4*a*b*d^5*f^3*cos((c*e - d*f)/e
)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) - 8*a*b*c^2*d^2*e^3*cos((c*e
- d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e) + 16*a*b*c*d^3*e^2*f
```

```
*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e) - 8*a*b*d^
4*e*f^2*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e) - 8
*(c*x + d)*b^2*c*d^3*e^2*f*cos(2*(c*e - d*f)/e)*cos_integral(-2*(c*e - d*f
- (c*x + d)*e/x)/e)/x + 8*(c*x + d)*b^2*d^4*e*f^2*cos(2*(c*e - d*f)/e)*cos_
integral(-2*(c*e - d*f - (c*x + d)*e/x)/e)/x - 4*b^2*c^2*d^2*e^3*cos_integr
al(-2*(c*e - d*f - (c*x + d)*e/x)/e)*sin(2*(c*e - d*f)/e) + 8*b^2*c*d^3*e^2
*f*cos_integral(-2*(c*e - d*f - (c*x + d)*e/x)/e)*sin(2*(c*e - d*f)/e) - 4*
b^2*d^4*e*f^2*cos_integral(-2*(c*e - d*f - (c*x + d)*e/x)/e)*sin(2*(c*e - d
*f)/e) + 8*(c*x + d)*a*b*c*d^3*e^2*f*cos_integral(-(c*e - d*f - (c*x + d)*e
/x)/e)*sin((c*e - d*f)/e)/x - 8*(c*x + d)*a*b*d^4*e*f^2*cos_integral(-(c*e
- d*f - (c*x + d)*e/x)/e)*sin((c*e - d*f)/e)/x + 4*b^2*c^2*d^2*e^3*cos(2*(c
*e - d*f)/e)*sin_integral(2*(c*e - d*f - (c*x + d)*e/x)/e) - 8*b^2*c*d^3*e^
2*f*cos(2*(c*e - d*f)/e)*sin_integral(2*(c*e - d*f - (c*x + d)*e/x)/e) + 4*
b^2*d^4*e*f^2*cos(2*(c*e - d*f)/e)*sin_integral(2*(c*e - d*f - (c*x + d)*e/
x)/e) - 8*(c*x + d)*b^2*c*d^3*e^2*f*sin(2*(c*e - d*f)/e)*sin_integral(2*(c*
e - d*f - (c*x + d)*e/x)/e)/x + 8*(c*x + d)*b^2*d^4*e*f^2*sin(2*(c*e - d*f)
/e)*sin_integral(2*(c*e - d*f - (c*x + d)*e/x)/e)/x - 8*(c*x + d)*a*b*c*d^3
*e^2*f*cos((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e)/x + 8
*(c*x + d)*a*b*d^4*e*f^2*cos((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x
+ d)*e/x)/e)/x - 8*a*b*c^2*d^2*e^3*sin((c*e - d*f)/e)*sin_integral((c*e - d
*f - (c*x + d)*e/x)/e) + 16*a*b*c*d^3*e^2*f*sin((c*e - d*f)/e)*sin_integral
((c*e - d*f - (c*x + d)*e/x)/e) - 8*a*b*d^4*e*f^2*sin((c*e - d*f)/e)*sin_in
tegral((c*e - d*f - (c*x + d)*e/x)/e) + 4*a*b*c*d^3*e^2*f*cos((c*x + d)/x)
- 4*a*b*d^4*e*f^2*cos((c*x + d)/x) + 16*(c*x + d)*a*b*c*d^2*e^3*cos((c*e -
d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)/x - 16*(c*x + d)*a*b*d
^3*e^2*f*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)/x
+ 4*(c*x + d)^2*b^2*d^3*e^2*f*cos(2*(c*e - d*f)/e)*cos_integral(-2*(c*e - d
*f - (c*x + d)*e/x)/e)/x^2 + 8*(c*x + d)*b^2*c*d^2*e^3*cos_integral(-2*(c*e
 - d*f - (c*x + d)*e/x)/e)*sin(2*(c*e - d*f)/e)/x - 8*(c*x + d)*b^2*d^3*e^2
*f*cos_integral(-2*(c*e - d*f - (c*x + d)*e/x)/e)*sin(2*(c*e - d*f)/e)/x -
4*(c*x + d)^2*a*b*d^3*e^2*f*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)*si
n((c*e - d*f)/e)/x^2 + 2*b^2*c*d^3*e^2*f*sin(2*(c*x + d)/x) - 2*b^2*d^4*e*f
^2*sin(2*(c*x + d)/x) - 8*(c*x + d)*b^2*c*d^2*e^3*cos(2*(c*e - d*f)/e)*sin_
integral(2*(c*e - d*f - (c*x + d)*e/x)/e)/x + 8*(c*x + d)*b^2*d^3*e^2*f*cos
(2*(c*e - d*f)/e)*sin_integral(2*(c*e - d*f - (c*x + d)*e/x)/e)/x + 4*(c*x
+ d)^2*b^2*d^3*e^2*f*sin(2*(c*e - d*f)/e)*sin_integral(2*(c*e - d*f - (c*x
+ d)*e/x)/e)/x^2 + 4*(c*x + d)^2*a*b*d^3*e^2*f*cos((c*e - d*f)/e)*sin_integ
ral((c*e - d*f - (c*x + d)*e/x)/e)/x^2 + 16*(c*x + d)*a*b*c*d^2*e^3*sin((c*
e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e)/x - 16*(c*x + d)*a*
b*d^3*e^2*f*sin((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e)/
x + 2*b^2*c*d^2*e^3*cos(2*(c*x + d)/x) - b^2*d^3*e^2*f*cos(2*(c*x + d)/x) -
 4*(c*x + d)*a*b*d^3*e^2*f*cos((c*x + d)/x)/x - 8*(c*x + d)^2*a*b*d^2*e^3*c
os((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)/x^2 - 4*(c*x
 + d)^2*b^2*d^2*e^3*cos_integral(-2*(c*e - d*f - (c*x + d)*e/x)/e)*sin(2*(c
*e - d*f)/e)/x^2 - 2*(c*x + d)*b^2*d^3*e^2*f*sin(2*(c*x + d)/x)/x - 8*a*b*c
```

```
*d^2*e^3*sin((c*x + d)/x) + 4*a*b*d^3*e^2*f*sin((c*x + d)/x) + 4*(c*x + d)^
2*b^2*d^2*e^3*cos(2*(c*e - d*f)/e)*sin_integral(2*(c*e - d*f - (c*x + d)*e/
x)/e)/x^2 - 8*(c*x + d)^2*a*b*d^2*e^3*sin((c*e - d*f)/e)*sin_integral((c*e
- d*f - (c*x + d)*e/x)/e)/x^2 - 4*a^2*c*d^2*e^3 - 2*b^2*c*d^2*e^3 + 2*a^2*d
^3*e^2*f + b^2*d^3*e^2*f - 2*(c*x + d)*b^2*d^2*e^3*cos(2*(c*x + d)/x)/x + 8
*(c*x + d)*a*b*d^2*e^3*sin((c*x + d)/x)/x + 4*(c*x + d)*a^2*d^2*e^3/x + 2*(
c*x + d)*b^2*d^2*e^3/x)/((c^2*e^6 - 2*c*d*e^5*f + d^2*e^4*f^2 - 2*(c*x + d)
*c*e^6/x + 2*(c*x + d)*d*e^5*f/x + (c*x + d)^2*e^6/x^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx=\int \frac{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}{{\left(e+f\,x\right)}^3}\,dx$$

```
[In] int((a + b*sin(c + d/x))^2/(e + f*x)^3,x)
[Out] int((a + b*sin(c + d/x))^2/(e + f*x)^3, x)
```

3.299 $\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\text{Int}\left(\frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] Int[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
[Out] Defer[Int][(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Rubi steps

$$\text{integral}=\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
[Out] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^2}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

```
[In] int((f*x+e)^2/(a+b*sin(c+d/x)),x)
[Out] int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="fricas")
[Out] integral((f^2*x^2 + 2*e*f*x + e^2)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)**2/(a+b*sin(c+d/x)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="maxima")
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Giac [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="giac")
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 6.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(e+f\,x)^2}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] int((e + f*x)^2/(a + b*sin(c + d/x)),x)
[Out] int((e + f*x)^2/(a + b*sin(c + d/x)), x)
```

3.300 $\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\text{Int}\left(\frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable((f*x+e)/(a+b*sin(c+d/x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] Int[(e + f*x)/(a + b*Sin[c + d/x]),x]
```

```
[Out] Defer[Int][(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Rubi steps

$$\text{integral}=\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] Integrate[(e + f*x)/(a + b*Sin[c + d/x]),x]
[Out] Integrate[(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{fx+e}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

```
[In] int((f*x+e)/(a+b*sin(c+d/x)),x)
[Out] int((f*x+e)/(a+b*sin(c+d/x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="fricas")
[Out] integral((f*x + e)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="maxima")
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Giac [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="giac")
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 6.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+f\,x}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] int((e + f*x)/(a + b*sin(c + d/x)),x)
[Out] int((e + f*x)/(a + b*sin(c + d/x)), x)
```

3.301 $\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\text{Int}\left(\frac{1}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(c+d/x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] Int[(a + b*Sin[c + d/x])^(-1),x]
[Out] Defer[Int][(a + b*Sin[c + d/x])^(-1), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] Integrate[(a + b*Sin[c + d/x])^(-1),x]
[Out] Integrate[(a + b*Sin[c + d/x])^(-1), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

```
[In] int(1/(a+b*sin(c+d/x)),x)
[Out] int(1/(a+b*sin(c+d/x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d/x)),x, algorithm="fricas")
[Out] integral(1/(b*sin((c*x + d)/x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 27.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d/x)),x)
[Out] Integral(1/(a + b*sin(c + d/x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d/x)),x, algorithm="maxima")
[Out] integrate(1/(b*sin(c + d/x) + a), x)
```

Giac [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d/x)),x, algorithm="giac")
[Out] integrate(1/(b*sin(c + d/x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 5.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] int(1/(a + b*sin(c + d/x)),x)
[Out] int(1/(a + b*sin(c + d/x)), x)
```

3.302 $\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \operatorname{Int}\left(\frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable((f*x+e)/(a+b*sin(c+d/x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] Int[(e + f*x)/(a + b*Sin[c + d/x]),x]
[Out] Defer[Int][(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] Integrate[(e + f*x)/(a + b*Sin[c + d/x]),x]
[Out] Integrate[(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{fx+e}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

```
[In] int((f*x+e)/(a+b*sin(c+d/x)),x)
[Out] int((f*x+e)/(a+b*sin(c+d/x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="fricas")
[Out] integral((f*x + e)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="maxima")
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Giac [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="giac")
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+f\,x}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] int((e + f*x)/(a + b*sin(c + d/x)),x)
[Out] int((e + f*x)/(a + b*sin(c + d/x)), x)
```

3.303 $\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\operatorname{Int}\left(\frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] Int[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
[Out] Defer[Int][(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Rubi steps

$$\text{integral}=\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
[Out] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^2}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

```
[In] int((f*x+e)^2/(a+b*sin(c+d/x)),x)
[Out] int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="fricas")
[Out] integral((f^2*x^2 + 2*e*f*x + e^2)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**2/(a+b*sin(c+d/x)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="maxima")
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Giac [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="giac")
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \int \frac{(e+f\,x)^2}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] int((e + f*x)^2/(a + b*sin(c + d/x)),x)
[Out] int((e + f*x)^2/(a + b*sin(c + d/x)), x)
```

3.304 $\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} dx = \text{Int}\left(\frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}, x\right)$$

```
[Out] Unintegrable((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} dx = \int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} dx$$

```
[In] Int[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
[Out] Defer[Int][(e + f*x)^2/(a + b*Sin[c + d/x])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \$\text{Aborted}$$

```
[In] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

```
[In] int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
[Out] int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
[Out] integral(-(f^2*x^2 + 2*e*f*x + e^2)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*
x + d)/x) - a^2 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**2/(a+b*sin(c+d/x))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 24.08 (sec) , antiderivative size = 1281, normalized size of antiderivative = 58.23

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b*f^2*x^4 + 2*a*b*e*f*x^3 + a*b*e^2*x^2)*cos(2*(c*x + d)/x)*cos((c*x
 + d)/x) + 2*(a*b*f^2*x^4 + 2*a*b*e*f*x^3 + a*b*e^2*x^2)*cos((c*x + d)/x) +
 ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d
)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2
 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4
*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b
^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))*integrate(-
2*(2*(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2)*cos((c*x + d)/x)^2 + 2*(a^
2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2)*sin((c*x + d)/x)^2 + (2*(2*a*b*f^2
*x^3 + 3*a*b*e*f*x^2 + a*b*e^2*x)*cos((c*x + d)/x) - (a*b*d*f^2*x^2 + 2*a*b
*d*e*f*x + a*b*d*e^2)*sin((c*x + d)/x))*cos(2*(c*x + d)/x) + 2*(2*a*b*f^2*x
^3 + 3*a*b*e*f*x^2 + a*b*e^2*x)*cos((c*x + d)/x) + (4*b^2*f^2*x^3 + 6*b^2*e
*f*x^2 + 2*b^2*e^2*x + (a*b*d*f^2*x^2 + 2*a*b*d*e*f*x + a*b*d*e^2)*cos((c*x
 + d)/x) + 2*(2*a*b*f^2*x^3 + 3*a*b*e*f*x^2 + a*b*e^2*x)*sin((c*x + d)/x))*
sin(2*(c*x + d)/x) + (a*b*d*f^2*x^2 + 2*a*b*d*e*f*x + a*b*d*e^2)*sin((c*x +
 d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((
c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (
a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x
)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*
b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x)
+ 2*(b^2*f^2*x^4 + 2*b^2*e*f*x^3 + b^2*e^2*x^2 + (a*b*f^2*x^4 + 2*a*b*e*f*x
^3 + a*b*e^2*x^2)*sin((c*x + d)/x))*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*
cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b -
a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x
 + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*s
in((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/
x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))
```

Giac [N/A]

Not integrable

Time = 7.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="giac")
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(e+f\,x)^2}{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

```
[In] int((e + f*x)^2/(a + b*sin(c + d/x))^2,x)
[Out] int((e + f*x)^2/(a + b*sin(c + d/x))^2, x)
```

3.305 $\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\mathrm{Int}\left(\frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

```
[In] Int[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
```

```
[Out] Defer[Int][(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 16.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

```
[In] Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
[Out] Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{fx+e}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

```
[In] int((f*x+e)/(a+b*sin(c+d/x))^2,x)
[Out] int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
[Out] integral(-(f*x + e)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*x + d)/x) - a^2
- b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 1103, normalized size of antiderivative = 55.15

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b*f*x^3 + a*b*e*x^2)*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*(a*b*f*
x^3 + a*b*e*x^2)*cos((c*x + d)/x) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2
 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x +
d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4
- a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a
^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)
*d)*cos(2*(c*x + d)/x))*integrate(-2*(2*(a^2*d*f*x + a^2*d*e)*cos((c*x + d)
/x)^2 + 2*(a^2*d*f*x + a^2*d*e)*sin((c*x + d)/x)^2 + ((3*a*b*f*x^2 + 2*a*b*
e*x)*cos((c*x + d)/x) - (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x))*cos(2*(c*x
+ d)/x) + (3*a*b*f*x^2 + 2*a*b*e*x)*cos((c*x + d)/x) + (3*b^2*f*x^2 + 2*b^2
*e*x + (a*b*d*f*x + a*b*d*e)*cos((c*x + d)/x) + (3*a*b*f*x^2 + 2*a*b*e*x)*s
in((c*x + d)/x))*sin(2*(c*x + d)/x) + (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x
))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x +
 d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b
^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 +
 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a
*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x) + 2*(
b^2*f*x^3 + b^2*e*x^2 + (a*b*f*x^3 + a*b*e*x^2)*sin((c*x + d)/x))*sin(2*(c*
x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*co
s((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x)
+ (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d
)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a
^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))
```

Giac [N/A]

Not integrable

Time = 5.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="giac")
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{e+f\,x}{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

```
[In] int((e + f*x)/(a + b*sin(c + d/x))^2,x)
[Out] int((e + f*x)/(a + b*sin(c + d/x))^2, x)
```

3.306 $\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} dx = \text{Int}\left(\frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}, x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(c+d/x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} dx = \int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} dx$$

```
[In] Int[(a + b*Sin[c + d/x])^(-2),x]
[Out] Defer[Int][(a + b*Sin[c + d/x])^(-2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

```
[In] Integrate[(a + b*Sin[c + d/x])^(-2),x]
[Out] Integrate[(a + b*Sin[c + d/x])^(-2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

```
[In] int(1/(a+b*sin(c+d/x))^2,x)
[Out] int(1/(a+b*sin(c+d/x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.36

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*x + d)/x) - a^2 - b^2),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sin(c+d/x))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 974, normalized size of antiderivative = 69.57

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \int \frac{1}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(2*a*b*x^2*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*a*b*x^2*cos((c*x + d)/x
) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x
+ d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*
b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2
+ 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b -
a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))*integrat
e(-2*(2*a^2*d*cos((c*x + d)/x)^2 + 2*a^2*d*sin((c*x + d)/x)^2 + 2*a*b*x*cos
((c*x + d)/x) + a*b*d*sin((c*x + d)/x) + (2*a*b*x*cos((c*x + d)/x) - a*b*d*
sin((c*x + d)/x))*cos(2*(c*x + d)/x) + (a*b*d*cos((c*x + d)/x) + 2*a*b*x*si
n((c*x + d)/x) + 2*b^2*x)*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x
 + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*c
os((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2
 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x +
d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*
b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x) + 2*(a*b*x^2*sin((c*x + d)/x) + b^2*x
^2)*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 -
a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*
(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d
*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4
)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c
*x + d)/x))
```

Giac [N/A]

Not integrable

Time = 3.64 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*sin(c+d/x))^2,x, algorithm="giac")
[Out] integrate((b*sin(c + d/x) + a)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 6.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

```
[In] int(1/(a + b*sin(c + d/x))^2,x)
[Out] int(1/(a + b*sin(c + d/x))^2, x)
```

3.307 $\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

```
[In] Int[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
[Out] Defer[Int][(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

```
[In] Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
[Out] Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{fx+e}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

```
[In] int((f*x+e)/(a+b*sin(c+d/x))^2,x)
[Out] int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
[Out] integral(-(f*x + e)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*x + d)/x) - a^2
- b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 1103, normalized size of antiderivative = 55.15

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
[Out] -(2*(a*b*f*x^3 + a*b*e*x^2)*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*(a*b*f*
x^3 + a*b*e*x^2)*cos((c*x + d)/x) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2
 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x +
d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4
- a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a
^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)
*d)*cos(2*(c*x + d)/x))*integrate(-2*(2*(a^2*d*f*x + a^2*d*e)*cos((c*x + d)
/x)^2 + 2*(a^2*d*f*x + a^2*d*e)*sin((c*x + d)/x)^2 + ((3*a*b*f*x^2 + 2*a*b*
e*x)*cos((c*x + d)/x) - (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x))*cos(2*(c*x
+ d)/x) + (3*a*b*f*x^2 + 2*a*b*e*x)*cos((c*x + d)/x) + (3*b^2*f*x^2 + 2*b^2
*e*x + (a*b*d*f*x + a*b*d*e)*cos((c*x + d)/x) + (3*a*b*f*x^2 + 2*a*b*e*x)*s
in((c*x + d)/x))*sin(2*(c*x + d)/x) + (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x
))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x +
 d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b
^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 +
 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a
*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x) + 2*(
b^2*f*x^3 + b^2*e*x^2 + (a*b*f*x^3 + a*b*e*x^2)*sin((c*x + d)/x))*sin(2*(c*
x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*co
s((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x)
+ (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d
)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a
^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))
```

Giac [N/A]

Not integrable

Time = 4.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="giac")
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{e+f\,x}{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

```
[In] int((e + f*x)/(a + b*sin(c + d/x))^2,x)
[Out] int((e + f*x)/(a + b*sin(c + d/x))^2, x)
```

3.308 $\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

```
[In] Int[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
[Out] Defer[Int][(e + f*x)^2/(a + b*Sin[c + d/x])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \$\text{Aborted}$$

```
[In] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

```
[In] int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
[Out] int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
[Out] integral(-(f^2*x^2 + 2*e*f*x + e^2)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*
x + d)/x) - a^2 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**2/(a+b*sin(c+d/x))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 24.28 (sec) , antiderivative size = 1281, normalized size of antiderivative = 58.23

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b*f^2*x^4 + 2*a*b*e*f*x^3 + a*b*e^2*x^2)*cos(2*(c*x + d)/x)*cos((c*x
 + d)/x) + 2*(a*b*f^2*x^4 + 2*a*b*e*f*x^3 + a*b*e^2*x^2)*cos((c*x + d)/x) +
 ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d
)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2
 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4
*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b
^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))*integrate(-
2*(2*(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2)*cos((c*x + d)/x)^2 + 2*(a^
2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2)*sin((c*x + d)/x)^2 + (2*(2*a*b*f^2
*x^3 + 3*a*b*e*f*x^2 + a*b*e^2*x)*cos((c*x + d)/x) - (a*b*d*f^2*x^2 + 2*a*b
*d*e*f*x + a*b*d*e^2)*sin((c*x + d)/x))*cos(2*(c*x + d)/x) + 2*(2*a*b*f^2*x
^3 + 3*a*b*e*f*x^2 + a*b*e^2*x)*cos((c*x + d)/x) + (4*b^2*f^2*x^3 + 6*b^2*e
*f*x^2 + 2*b^2*e^2*x + (a*b*d*f^2*x^2 + 2*a*b*d*e*f*x + a*b*d*e^2)*cos((c*x
 + d)/x) + 2*(2*a*b*f^2*x^3 + 3*a*b*e*f*x^2 + a*b*e^2*x)*sin((c*x + d)/x))*
sin(2*(c*x + d)/x) + (a*b*d*f^2*x^2 + 2*a*b*d*e*f*x + a*b*d*e^2)*sin((c*x +
 d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((
c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (
a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x
)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*
b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x)
+ 2*(b^2*f^2*x^4 + 2*b^2*e*f*x^3 + b^2*e^2*x^2 + (a*b*f^2*x^4 + 2*a*b*e*f*x
^3 + a*b*e^2*x^2)*sin((c*x + d)/x))*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*
cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b -
a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x
 + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*s
in((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/
x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))
```

Giac [N/A]

Not integrable

Time = 7.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="giac")
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(e+f\,x)^2}{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

```
[In] int((e + f*x)^2/(a + b*sin(c + d/x))^2,x)
[Out] int((e + f*x)^2/(a + b*sin(c + d/x))^2, x)
```

3.309 $\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx=\text{Int}\left((e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p,x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*sin(c+d/x))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx=\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$$

```
[In] Int[(e + f*x)^m*(a + b*Sin[c + d/x])^p,x]
```

```
[Out] Defer[Int][(e + f*x)^m*(a + b*Sin[c + d/x])^p, x]
```

Rubi steps

$$\text{integral}=\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p \, dx = \int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p \, dx$$

```
[In] Integrate[(e + f*x)^m*(a + b*Sin[c + d/x])^p,x]
[Out] Integrate[(e + f*x)^m*(a + b*Sin[c + d/x])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (fx+e)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p dx$$

```
[In] int((f*x+e)^m*(a+b*sin(c+d/x))^p,x)
[Out] int((f*x+e)^m*(a+b*sin(c+d/x))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p \, dx = \int (fx+e)^m \left(b\sin\left(c+\frac{d}{x}\right)+a\right)^p dx$$

```
[In] integrate((f*x+e)^m*(a+b*sin(c+d/x))^p,x, algorithm="fricas")
[Out] integral((f*x + e)^m*(b*sin((c*x + d)/x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p \, dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**m*(a+b*sin(c+d/x))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx=\int (fx+e)^m\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^p\,dx$$

```
[In] integrate((f*x+e)^m*(a+b*sin(c+d/x))^p,x, algorithm="maxima")
[Out] integrate((f*x + e)^m*(b*sin(c + d/x) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx=\int (fx+e)^m\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^p\,dx$$

```
[In] integrate((f*x+e)^m*(a+b*sin(c+d/x))^p,x, algorithm="giac")
[Out] integrate((f*x + e)^m*(b*sin(c + d/x) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 6.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx=\int (e+f\,x)^m\,\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$$

```
[In] int((e + f*x)^m*(a + b*sin(c + d/x))^p,x)
[Out] int((e + f*x)^m*(a + b*sin(c + d/x))^p, x)
```

3.310 $\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 115

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{e^{ia}x^m(-ibx)^{-m}\csc(a+bx)\Gamma(1+m,-ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b} - \frac{e^{-ia}x^m(ibx)^{-m}\csc(a+bx)\Gamma(1+m,ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}$$

```
[Out] -1/2*exp(I*a)*x^m*csc(b*x+a)*GAMMA(1+m,-I*b*x)*(c*sin(b*x+a)^3)^(1/3)/b/((-
I*b*x)^m)-1/2*x^m*csc(b*x+a)*GAMMA(1+m,I*b*x)*(c*sin(b*x+a)^3)^(1/3)/b/exp(
I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6852, 3389, 2212}

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{e^{ia}x^m(-ibx)^{-m}\csc(a+bx)\Gamma(m+1,-ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b} - \frac{e^{-ia}x^m(ibx)^{-m}\csc(a+bx)\Gamma(m+1,ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}$$

```
[In] Int[x^m*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] -1/2*(E^(I*a)*x^m*Csc[a + b*x]*Gamma[1 + m, (-I)*b*x]*(c*Sin[a + b*x]^3)^(1
/3))/(b*((-I)*b*x)^m) - (x^m*Csc[a + b*x]*Gamma[1 + m, I*b*x]*(c*Sin[a + b*
x]^3)^(1/3))/(2*b*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x^m\sin(a+bx)\,dx\\
&= \frac{1}{2}\left(i\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int e^{-i(a+bx)}x^m\,dx\\
&\quad -\frac{1}{2}\left(i\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int e^{i(a+bx)}x^m\,dx\\
&= -\frac{e^{ia}x^m(-ibx)^{-m}\csc(a+bx)\Gamma(1+m,-ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}\\
&\quad -\frac{e^{-ia}x^m(ibx)^{-m}\csc(a+bx)\Gamma(1+m,ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{e^{-ia}x^m(b^2x^2)^{-m}\csc(a+bx)\left(e^{2ia}(ibx)^m\Gamma(1+m,-ibx)+(-ibx)^m\Gamma(1+m,ibx)\right)\sqrt[3]{c\sin^3(a+bx)}}{2b}$$

```
[In] Integrate[x^m*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] -1/2*(x^m*Csc[a + b*x]*(E^((2*I)*a)*(I*b*x)^m*Gamma[1 + m, (-I)*b*x] + ((-I
)*b*x)^m*Gamma[1 + m, I*b*x])*(c*Sin[a + b*x]^3)^(1/3))/(b*E^(I*a)*(b^2*x^2
)^m)
```

Maple [F]

$$\int x^m\left(c\left(\sin^3(bx+a)\right)\right)^{\frac{1}{3}}dx$$

```
[In] int(x^m*(c*sin(b*x+a)^3)^(1/3),x)
```

```
[Out] int(x^m*(c*sin(b*x+a)^3)^(1/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{\left(e^{(-m\log(ib)-ia)}\Gamma(m+1,ibx)+e^{(-m\log(-ib)+ia)}\Gamma(m+1,-ibx)\right)\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{1}{3}}}{2b\sin(bx+a)}$$

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-m*log(I*b) - I*a)*gamma(m + 1, I*b*x) + e^(-m*log(-I*b) + I*a)*ga
mma(m + 1, -I*b*x))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)/(b*sin(b*x
 + a))
```

Sympy [F]

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = \int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx$$

```
[In] integrate(x**m*(c*sin(b*x+a)**3)**(1/3),x)
[Out] Integral(x**m*(c*sin(a + b*x)**3)**(1/3), x)
```

Maxima [F]

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x^m, x)
```

Giac [F]

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = \int x^m \left(c\sin\left(a+b\,x\right)^3\right)^{1/3}\,dx$$

```
[In] int(x^m*(c*sin(a + b*x)^3)^(1/3),x)
[Out] int(x^m*(c*sin(a + b*x)^3)^(1/3), x)
```

3.311 $\int x^3 \sqrt[3]{c \sin^3(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 96

$$\int x^3 \sqrt[3]{c \sin^3(a+bx)}\,dx = -\frac{6 \sqrt[3]{c \sin^3(a+bx)}}{b^4} + \frac{3x^2 \sqrt[3]{c \sin^3(a+bx)}}{b^2} + \frac{6x \cot(a+bx) \sqrt[3]{c \sin^3(a+bx)}}{b^3} - \frac{x^3 \cot(a+bx) \sqrt[3]{c \sin^3(a+bx)}}{b}$$

```
[Out] -6*(c*sin(b*x+a)^3)^(1/3)/b^4+3*x^2*(c*sin(b*x+a)^3)^(1/3)/b^2+6*x*cot(b*x+
a)*(c*sin(b*x+a)^3)^(1/3)/b^3-x^3*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6852, 3377, 2717}

$$\int x^3 \sqrt[3]{c \sin^3(a+bx)}\,dx = -\frac{6 \sqrt[3]{c \sin^3(a+bx)}}{b^4} + \frac{6x \cot(a+bx) \sqrt[3]{c \sin^3(a+bx)}}{b^3} + \frac{3x^2 \sqrt[3]{c \sin^3(a+bx)}}{b^2} - \frac{x^3 \cot(a+bx) \sqrt[3]{c \sin^3(a+bx)}}{b}$$

```
[In] Int[x^3*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] (-6*(c*Sin[a + b*x]^3)^(1/3))/b^4 + (3*x^2*(c*Sin[a + b*x]^3)^(1/3))/b^2 +
(6*x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b^3 - (x^3*Cot[a + b*x]*(c*Sin[
a + b*x]^3)^(1/3))/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x^3\sin(a+bx)\,dx\\
&= -\frac{x^3\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}+\frac{\left(3\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x^2\cos(a+bx)\,dx}{b}\\
&= \frac{3x^2\sqrt[3]{c\sin^3(a+bx)}}{b^2}-\frac{x^3\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}\\
&\quad-\frac{\left(6\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x\sin(a+bx)\,dx}{b^2}\\
&= \frac{3x^2\sqrt[3]{c\sin^3(a+bx)}}{b^2}+\frac{6x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3}\\
&\quad-\frac{x^3\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}\\
&\quad-\frac{\left(6\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int\cos(a+bx)\,dx}{b^3}
\end{aligned}
$$

$$= -\frac{6\sqrt[3]{c\sin^3(a+bx)}}{b^4} + \frac{3x^2\sqrt[3]{c\sin^3(a+bx)}}{b^2} + \frac{6x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3} - \frac{x^3\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int x^3\sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{(6-3b^2x^2+bx(-6+b^2x^2)\cot(a+bx))\sqrt[3]{c\sin^3(a+bx)}}{b^4}$$

```
[In] Integrate[x^3*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] -(((6 - 3*b^2*x^2 + b*x*(-6 + b^2*x^2)*Cot[a + b*x])*(c*Sin[a + b*x]^3)^(1/
3))/b^4)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.57

method	result
risch	$-\frac{i(b^3x^3+3ix^2b^2-6bx-6i)\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}\mathrm{e}^{2i(bx+a)}}{2b^4\left(\mathrm{e}^{2i(bx+a)}-1\right)} - \frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}(b^3x^3-3ix^2b^2-6bx+6i)}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)b^4}$

```
[In] int(x^3*(c*sin(b*x+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^4*(b^3*x^3+3*I*x^2*b^2-6*b*x-6*I)/(exp(2*I*(b*x+a))-1)*(I*c*exp(-3
*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(1/3)*exp(2*I*(b*x+a))-1/2*I*(I*c*exp(-
3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(1/3)/(exp(2*I*(b*x+a))-1)*(b^3*x^3-3*
I*x^2*b^2-6*b*x+6*I)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int x^3 \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{\left((b^3x^3 - 6\,bx)\cos\left(bx+a\right) - 3\left(b^2x^2 - 2\right)\sin\left(bx+a\right)\right)\left(-\left(c\cos\left(bx+a\right)^2 - c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}}{b^4\sin\left(bx+a\right)}$$

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
[Out] -((b^3*x^3 - 6*b*x)*cos(b*x + a) - 3*(b^2*x^2 - 2)*sin(b*x + a))*(-(c*cos(b
*x + a)^2 - c)*sin(b*x + a))^(1/3)/(b^4*sin(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.34

$$\int x^3 \sqrt[3]{c\sin^3(a+bx)}\,dx = \begin{cases} \frac{x^4\sqrt[3]{c\sin^3\left(a\right)}}{4} \\ 0 \\ -\frac{x^3\sqrt[3]{c\sin^3\left(a+bx\right)}\cos\left(a+bx\right)}{b\sin\left(a+bx\right)} + \frac{3x^2\sqrt[3]{c\sin^3\left(a+bx\right)}}{b^2} + \frac{6x\sqrt[3]{c\sin^3\left(a+bx\right)}\cos\left(a+bx\right)}{b^3\sin\left(a+bx\right)} - \frac{6\sqrt[3]{c\sin^3\left(a+bx\right)}}{b^4} \end{cases}$$

```
[In] integrate(x**3*(c*sin(b*x+a)**3)**(1/3),x)
[Out] Piecewise((x**4*(c*sin(a)**3)**(1/3)/4, Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a,
-b*x + pi)), (-x**3*(c*sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*x)
) + 3*x**2*(c*sin(a + b*x)**3)**(1/3)/b**2 + 6*x*(c*sin(a + b*x)**3)**(1/3)
*cos(a + b*x)/(b**3*sin(a + b*x)) - 6*(c*sin(a + b*x)**3)**(1/3)/b**4, True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.52

$$\int x^3 \sqrt[3]{c\sin^3(a+bx)}\,dx = \frac{3\left((bx+a)\cos\left(bx+a\right) - \sin\left(bx+a\right)\right)a^2c^{\frac{1}{3}} - 3\left(\left((bx+a)^2 - 2\right)\cos\left(bx+a\right) - 2\left(bx+a\right)\sin\left(bx+a\right)\right)ac}{2\,b^4}$$

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*(3*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a^2*c^(1/3) - 3*(((b*x + a)^
2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*a*c^(1/3) + 4*a^3*c^(1/3)/(
sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1) + (((b*x + a)^3 - 6*b*x - 6*a)*cos
(b*x + a) - 3*((b*x + a)^2 - 2)*sin(b*x + a))*c^(1/3))/b^4
```

Giac [F]

$$\int x^3 \sqrt[3]{c \sin^3(a+bx)}\, dx = \int \left(c \sin\left(bx+a\right)^3\right)^{\frac{1}{3}} x^3\, dx$$

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14

$$\int x^3 \sqrt[3]{c \sin^3(a+bx)}\, dx = \frac{2^{1/3} \left(c \left(3 \sin\left(a+b\,x\right)-\sin\left(3\,a+3\,b\,x\right)\right)\right)^{1/3} \left(3\,b^2\,x^2-12 \sin\left(a+b\,x\right)^2+6\,b\,x\,\sin\left(2\,a+2\,b\,x\right)+3\,b^2\,x^2\,(2\,\mathrm{s}\right.}{4\,b^4 \sin\left(a+b\,x\right)^2}$$

```
[In] int(x^3*(c*sin(a + b*x)^3)^(1/3),x)
```

```
[Out] (2^(1/3)*(c*(3*sin(a + b*x) - sin(3*a + 3*b*x)))^(1/3)*(3*b^2*x^2 - 12*sin(
a + b*x)^2 + 6*b*x*sin(2*a + 2*b*x) + 3*b^2*x^2*(2*sin(a + b*x)^2 - 1) - b^
3*x^3*sin(2*a + 2*b*x)))/(4*b^4*sin(a + b*x)^2)
```

3.312 $\int x^2 \sqrt[3]{c \sin^3(a+bx)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 74

$$\int x^2 \sqrt[3]{c \sin^3(a+bx)} \, dx = \frac{2x\sqrt[3]{c \sin^3(a+bx)}}{b^2} + \frac{2\cot(a+bx)\sqrt[3]{c \sin^3(a+bx)}}{b^3} - \frac{x^2\cot(a+bx)\sqrt[3]{c \sin^3(a+bx)}}{b}$$

```
[Out] 2*x*(c*sin(b*x+a)^3)^(1/3)/b^2+2*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b^3-x^2*
cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6852, 3377, 2718}

$$\int x^2 \sqrt[3]{c \sin^3(a+bx)} \, dx = \frac{2\cot(a+bx)\sqrt[3]{c \sin^3(a+bx)}}{b^3} + \frac{2x\sqrt[3]{c \sin^3(a+bx)}}{b^2} - \frac{x^2\cot(a+bx)\sqrt[3]{c \sin^3(a+bx)}}{b}$$

```
[In] Int[x^2*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] (2*x*(c*Sin[a + b*x]^3)^(1/3))/b^2 + (2*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/
3))/b^3 - (x^2*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x^2\sin(a+bx)\,dx \\
&= -\frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}+\frac{\left(2\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x\cos(a+bx)\,dx}{b} \\
&= \frac{2x\sqrt[3]{c\sin^3(a+bx)}}{b^2}-\frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b} \\
&\quad -\frac{\left(2\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int\sin(a+bx)\,dx}{b^2} \\
&= \frac{2x\sqrt[3]{c\sin^3(a+bx)}}{b^2}+\frac{2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3}-\frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$
\int x^2\sqrt[3]{c\sin^3(a+bx)}\,dx=\frac{(2bx+(2-b^2x^2)\cot(a+bx))\sqrt[3]{c\sin^3(a+bx)}}{b^3}
$$

```
[In] Integrate[x^2*(c*Sin[a + b*x]^3)^(1/3),x]
[Out] ((2*b*x + (2 - b^2*x^2)*Cot[a + b*x])*(c*Sin[a + b*x]^3)^(1/3))/b^3
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.80

method	result	size
risch	$-\frac{i(x^2b^2+2ibx-2)\left(ic\,e^{-3i(bx+a)}\left(e^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}e^{2i(bx+a)}}{2b^3\left(e^{2i(bx+a)}-1\right)}-\frac{i\left(ic\,e^{-3i(bx+a)}\left(e^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}(x^2b^2-2ibx-2)}{2\left(e^{2i(bx+a)}-1\right)b^3}$	133

```
[In] int(x^2*(c*sin(b*x+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^3*(x^2*b^2+2*I*b*x-2)/(exp(2*I*(b*x+a))-1)*(I*c*exp(-3*I*(b*x+a))*
(exp(2*I*(b*x+a))-1)^3)^(1/3)*exp(2*I*(b*x+a))-1/2*I*(I*c*exp(-3*I*(b*x+a))
*(exp(2*I*(b*x+a))-1)^3)^(1/3)/(exp(2*I*(b*x+a))-1)*(x^2*b^2-2*I*b*x-2)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int x^2 \sqrt[3]{c\sin^3(a+bx)}\,dx = \frac{(2\,bx\sin(bx+a)-(b^2x^2-2)\cos(bx+a))\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{1}{3}}}{b^3\sin(bx+a)}$$

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] (2*b*x*sin(b*x + a) - (b^2*x^2 - 2)*cos(b*x + a))*(-(c*cos(b*x + a)^2 - c)*
sin(b*x + a))^(1/3)/(b^3*sin(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int x^2 \sqrt[3]{c\sin^3(a+bx)}\,dx = \begin{cases} \frac{x^3\sqrt[3]{c\sin^3(a)}}{3} & \text{for}\ b=0 \\ 0 & \text{for}\ a=-bx\vee a=- \\ -\frac{x^2\sqrt[3]{c\sin^3(a+bx)}\cos(a+bx)}{b\sin(a+bx)}+\frac{2x\sqrt[3]{c\sin^3(a+bx)}}{b^2}+\frac{2\sqrt[3]{c\sin^3(a+bx)}\cos(a+bx)}{b^3\sin(a+bx)} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(c*sin(b*x+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x**3*(c*sin(a)**3)**(1/3)/3, Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a,
-b*x + pi)), (-x**2*(c*sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*x)
) + 2*x*(c*sin(a + b*x)**3)**(1/3)/b**2 + 2*(c*sin(a + b*x)**3)**(1/3)*cos(
a + b*x)/(b**3*sin(a + b*x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.34

$$\int x^2 \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{2\left((bx+a)\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)ac^{\frac{1}{3}} - \left(\left((bx+a)^2-2\right)\cos\left(bx+a\right)-2\left(bx+a\right)\sin\left(bx+a\right)\right)c^{\frac{1}{3}} +}{2\,b^3}$$

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/2*(2*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a*c^(1/3) - (((b*x + a)^2 -
 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*c^(1/3) + 4*a^2*c^(1/3)/(sin(b
*x + a)^2/(cos(b*x + a) + 1)^2 + 1))/b^3
```

Giac [F]

$$\int x^2 \sqrt[3]{c\sin^3(a+bx)}\,dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}} x^2\,dx$$

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x^2, x)
```

Mupad [B] (verification not implemented)

Time = 6.82 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int x^2 \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{\left(2\,c\,\left(3\,\sin\left(a+b\,x\right)-\sin\left(3\,a+3\,b\,x\right)\right)\right)^{1/3}\,\left(\sin\left(2\,a+2\,b\,x\right)+b\,x-\frac{b^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{2}-b\,x\,\cos\left(2\,a+2\,b\,x\right)\right)}{b^3\,\left(\cos\left(2\,a+2\,b\,x\right)-1\right)}$$

```
[In] int(x^2*(c*sin(a + b*x)^3)^(1/3),x)
```

```
[Out] -((2*c*(3*sin(a + b*x) - sin(3*a + 3*b*x)))^(1/3)*(sin(2*a + 2*b*x) + b*x -
 (b^2*x^2*sin(2*a + 2*b*x))/2 - b*x*cos(2*a + 2*b*x)))/(b^3*(cos(2*a + 2*b*
x) - 1))
```

3.313 $\int x \sqrt[3]{c \sin^3(a+bx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int x \sqrt[3]{c \sin^3(a+bx)}\,dx = \frac{\sqrt[3]{c \sin^3(a+bx)}}{b^2} - \frac{x \cot(a+bx) \sqrt[3]{c \sin^3(a+bx)}}{b}$$

```
[Out] (c*sin(b*x+a)^3)^(1/3)/b^2-x*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6852, 3377, 2717}

$$\int x \sqrt[3]{c \sin^3(a+bx)}\,dx = \frac{\sqrt[3]{c \sin^3(a+bx)}}{b^2} - \frac{x \cot(a+bx) \sqrt[3]{c \sin^3(a+bx)}}{b}$$

```
[In] Int[x*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] (c*Sin[a + b*x]^3)^(1/3)/b^2 - (x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x\sin(a+bx)\,dx\\ &= -\frac{x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}+\frac{\left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int\cos(a+bx)\,dx}{b}\\ &= \frac{\sqrt[3]{c\sin^3(a+bx)}}{b^2}-\frac{x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx=\frac{(1-bx\cot(a+bx))\sqrt[3]{c\sin^3(a+bx)}}{b^2}$$

```
[In] Integrate[x*(c*Sin[a + b*x]^3)^(1/3),x]
[Out] ((1 - b*x*Cot[a + b*x])*(c*Sin[a + b*x]^3)^(1/3))/b^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.60

method	result	size
risch	$-\frac{i(bx+i)\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}\mathrm{e}^{2i(bx+a)}}{2b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)}-\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}(bx-i)}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)b^2}$	117

```
[In] int(x*(c*sin(b*x+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^2*(b*x+I)/(exp(2*I*(b*x+a))-1)*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*
x+a))-1)^3)^(1/3)*exp(2*I*(b*x+a))-1/2*I*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b
*x+a))-1)^3)^(1/3)/(exp(2*I*(b*x+a))-1)*(b*x-I)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{(bx\cos(bx+a)-\sin(bx+a))\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{1}{3}}}{b^2\sin(bx+a)}$$

```
[In] integrate(x*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(b*x*cos(b*x + a) - sin(b*x + a))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(
1/3)/(b^2*sin(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx = \begin{cases} \frac{x^2\sqrt[3]{c\sin^3(a)}}{2} & \text{for } b=0 \\ 0 & \text{for } a=-bx \vee a=-bx+\pi \\ -\frac{x\sqrt[3]{c\sin^3(a+bx)}\cos(a+bx)}{b\sin(a+bx)}+\frac{\sqrt[3]{c\sin^3(a+bx)}}{b^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(c*sin(b*x+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x**2*(c*sin(a)**3)**(1/3)/2, Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a,
-b*x + pi)), (-x*(c*sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*x)) +
 (c*sin(a + b*x)**3)**(1/3)/b**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx = \frac{((bx+a)\cos(bx+a)-\sin(bx+a))c^{\frac{1}{3}} + \frac{4\,ac^{\frac{1}{3}}}{\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1}}{2\,b^2}$$

```
[In] integrate(x*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x + a)*cos(b*x + a) - sin(b*x + a))*c^(1/3) + 4*a*c^(1/3)/(sin(b*x
 + a)^2/(cos(b*x + a) + 1)^2 + 1))/b^2
```

Giac [F]

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx = \int \left(c\sin(bx+a)^3\right)^{\frac{1}{3}} x\,dx$$

```
[In] integrate(x*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x, x)
```

Mupad [B] (verification not implemented)

Time = 6.78 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx = \frac{\left(\frac{\sin(a+b\,x)^2}{2}-\frac{b\,x\,\sin(2\,a+2\,b\,x)}{4}\right)\,\left(2\,c\,(3\,\sin(a+b\,x)-\sin(3\,a+3\,b\,x))\right)^{1/3}}{b^2\sin(a+b\,x)^2}$$

```
[In] int(x*(c*sin(a + b*x)^3)^(1/3),x)
```

```
[Out] ((sin(a + b*x)^2/2 - (b*x*sin(2*a + 2*b*x))/4)*(2*c*(3*sin(a + b*x) - sin(3
*a + 3*b*x)))^(1/3))/(b^2*sin(a + b*x)^2)
```

3.314 $\int \sqrt[3]{c\sin^3(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx=-\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

```
[Out] -cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3286, 2718}

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx=-\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

```
[In] Int[(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int\sin(a+bx)\,dx\\ &= -\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int\sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

```
[In] Integrate[(c*Sin[a + b*x]^3)^(1/3),x]
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 105, normalized size of antiderivative = 4.20

method	result	size
risch	$-\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}\mathrm{e}^{2i(bx+a)}}{2b\left(\mathrm{e}^{2i(bx+a)}-1\right)}-\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)b}$	105

```
[In] int((c*sin(b*x+a)^3)^(1/3),x,method=_RETURNVERBOSE)
[Out] -1/2*I/b/(exp(2*I*(b*x+a))-1)*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3
)^(1/3)*exp(2*I*(b*x+a))-1/2*I*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^
3)^(1/3)/(exp(2*I*(b*x+a))-1)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}\cos\left(bx+a\right)}{b\sin\left(bx+a\right)}$$

```
[In] integrate((c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
[Out] -(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)*cos(b*x + a)/(b*sin(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(22) = 44$.

Time = 0.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx = \begin{cases} x\sqrt[3]{c\sin^3\left(a\right)} & \text{for}\: b=0 \\ 0 & \text{for}\: a=-bx \vee a=-bx+\pi \\ -\frac{\sqrt[3]{c\sin^3\left(a+bx\right)}\cos\left(a+bx\right)}{b\sin\left(a+bx\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate((c*sin(b*x+a)**3)**(1/3),x)
[Out] Piecewise((x*(c*sin(a)**3)**(1/3), Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a, -b*x
+ pi)), (-(c*sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{2\,c^{\frac{1}{3}}}{b\left(\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1\right)}$$

```
[In] integrate((c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
[Out] -2*c^(1/3)/(b*(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1))
```

Giac [F]

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a)^3)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{\sin\left(2\,a+2\,b\,x\right)\,\left(2\,c\,\left(3\,\sin\left(a+b\,x\right)-\sin\left(3\,a+3\,b\,x\right)\right)\right)^{1/3}}{4\,b\sin\left(a+b\,x\right)^2}$$

```
[In] int((c*sin(a + b*x)^3)^(1/3),x)
[Out] -(sin(2*a + 2*b*x)*(2*c*(3*sin(a + b*x) - sin(3*a + 3*b*x)))^(1/3))/(4*b*si
n(a + b*x)^2)
```

3.315 $\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 55

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx = \text{CosIntegral}(bx)\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)} + \cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\text{Si}(bx)$$

```
[Out] cos(a)*csc(b*x+a)*Si(b*x)*(c*sin(b*x+a)^3)^(1/3)+Ci(b*x)*csc(b*x+a)*sin(a)*
(c*sin(b*x+a)^3)^(1/3)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6852, 3384, 3380, 3383}

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx = \sin(a)\,\text{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} + \cos(a)\text{Si}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}$$

```
[In] Int[(c*Sin[a + b*x]^3)^(1/3)/x,x]
```

```
[Out] CosIntegral[b*x]*Csc[a + b*x]*Sin[a]*(c*Sin[a + b*x]^3)^(1/3) + Cos[a]*Csc[
a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*SinIntegral[b*x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int\frac{\sin(a+bx)}{x}\,dx\\
&= \left(\cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int\frac{\sin(bx)}{x}\,dx\\
&\quad+\left(\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)}\right)\int\frac{\cos(bx)}{x}\,dx\\
&= \text{CosIntegral}(bx)\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)}+\cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\,\text{Si}(bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$
\int\frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx=\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}(\text{CosIntegral}(bx)\sin(a)+\cos(a)\text{Si}(bx))
$$

```
[In] Integrate[(c*Sin[a + b*x]^3)^(1/3)/x,x]
```

```
[Out] Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(CosIntegral[b*x]*Sin[a] + Cos[a]*Sin
Integral[b*x])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.93

method	result	size
risch	$-\frac{\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{1}{3}}\left(i\mathrm{e}^{ibx}\pi\ \mathrm{csgn}(bx)-2i\mathrm{e}^{ibx}\ \mathrm{Si}(bx)+\mathrm{Ei}_{1}(-ibx)\mathrm{e}^{i(bx+2a)}-\mathrm{e}^{ibx}\ \mathrm{Ei}_{1}(-ibx)\right)}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)}$	106

```
[In] int((c*sin(b*x+a)^3)^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(1/3)*(I*exp(I*b*x)*Pi*
csgn(b*x)-2*I*exp(I*b*x)*Si(b*x)+Ei(1,-I*b*x)*exp(I*(b*x+2*a))-exp(I*b*x)*E
i(1,-I*b*x))/(exp(2*I*(b*x+a))-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx = \frac{\left(-i\,\mathrm{Ei}(i\,bx)\,e^{(i\,a)}+i\,\mathrm{Ei}(-i\,bx)\,e^{(-i\,a)}\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}}{2\,\sin\left(bx+a\right)}$$

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*Ei(I*b*x)*e^(I*a) + I*Ei(-I*b*x)*e^(-I*a))*(-(c*cos(b*x + a)^2 - c)
*sin(b*x + a))^(1/3)/sin(b*x + a)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx = \int \frac{\sqrt[3]{c\sin^3\left(a+bx\right)}}{x}\,dx$$

```
[In] integrate((c*sin(b*x+a)**3)**(1/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(1/3)/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt[3]{c \sin^3(a+bx)}}{x} \, dx = \frac{1}{4} \left((i E_1(i bx) - i E_1(-i bx)) \cos(a) + (E_1(i bx) + E_1(-i bx)) \sin(a) \right) c^{\frac{1}{3}}$$

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x,x, algorithm="maxima")
[Out] 1/4*((I*exp_integral_e(1, I*b*x) - I*exp_integral_e(1, -I*b*x))*cos(a) + (e
xp_integral_e(1, I*b*x) + exp_integral_e(1, -I*b*x))*sin(a))*c^(1/3)
```

Giac [F]

$$\int \frac{\sqrt[3]{c \sin^3(a+bx)}}{x} \, dx = \int \frac{\left(c \sin(bx+a)^3\right)^{\frac{1}{3}}}{x} \, dx$$

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x,x, algorithm="giac")
[Out] integrate((c*sin(b*x + a)^3)^(1/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c \sin^3(a+bx)}}{x} \, dx = \int \frac{\left(c \sin(a+b x)^3\right)^{1/3}}{x} \, dx$$

```
[In] int((c*sin(a + b*x)^3)^(1/3)/x,x)
[Out] int((c*sin(a + b*x)^3)^(1/3)/x, x)
```

3.316 $\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = -\frac{\sqrt[3]{c\sin^3(a+bx)}}{x} + b\cos(a)\,\text{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} - b\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)}\text{Si}(bx)$$

```
[Out] -(c*sin(b*x+a)^3)^(1/3)/x+b*Ci(b*x)*cos(a)*csc(b*x+a)*(c*sin(b*x+a)^3)^(1/3
)-b*csc(b*x+a)*Si(b*x)*sin(a)*(c*sin(b*x+a)^3)^(1/3)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6852, 3378, 3384, 3380, 3383}

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = b\cos(a)\,\text{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} - b\sin(a)\text{Si}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} - \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}$$

```
[In] Int[(c*Sin[a + b*x]^3)^(1/3)/x^2,x]
```

```
[Out] -((c*Sin[a + b*x]^3)^(1/3)/x) + b*Cos[a]*CosIntegral[b*x]*Csc[a + b*x]*(c*S
in[a + b*x]^3)^(1/3) - b*Csc[a + b*x]*Sin[a]*(c*Sin[a + b*x]^3)^(1/3)*SinIn
tegral[b*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\sin(a+bx)}{x^2}\,dx \\
&= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{x} + \left(b\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\cos(a+bx)}{x}\,dx \\
&= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{x} + \left(b\cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\cos(bx)}{x}\,dx \\
&\quad - \left(b\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\sin(bx)}{x}\,dx
\end{aligned}$$

$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{x} + b\cos(a)\,\text{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} - b\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)}\text{Si}(bx)$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = \frac{\sqrt[3]{c\sin^3(a+bx)}(-1 + bx\cos(a)\,\text{CosIntegral}(bx)\csc(a+bx) - bx\csc(a+bx)\sin(a)\text{Si}(bx))}{x}$$

```
[In] Integrate[(c*Sin[a + b*x]^3)^(1/3)/x^2,x]
```

```
[Out] ((c*Sin[a + b*x]^3)^(1/3)*(-1 + b*x*Cos[a]*CosIntegral[b*x]*Csc[a + b*x] - b*x*Csc[a + b*x]*Sin[a]*SinIntegral[b*x]))/x
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}\left(-\mathrm{e}^{ibx}\,\mathrm{Ei}_1(ibx)bx-\mathrm{Ei}_1(-ibx)\mathrm{e}^{i(bx+2a)}bx+i\mathrm{e}^{2i(bx+a)}-i\right)}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)x}$	102

```
[In] int((c*sin(b*x+a)^3)^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(1/3)*(-exp(I*b*x)*Ei(
1,I*b*x)*b*x-Ei(1,-I*b*x)*exp(I*(b*x+2*a))*b*x+I*exp(2*I*(b*x+a))-I)/(exp(2
*I*(b*x+a))-1)/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = \frac{\left(bx\mathrm{Ei}(i\,bx)\,e^{(i\,a)} + bx\mathrm{Ei}(-i\,bx)\,e^{(-i\,a)} - 2\,\sin\left(bx+a\right)\right)\left(-\left(c\cos\left(bx+a\right)^2 - c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}}{2\,x\sin\left(bx+a\right)}$$

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*Ei(I*b*x)*e^(I*a) + b*x*Ei(-I*b*x)*e^(-I*a) - 2*sin(b*x + a))*(-(c
*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)/(x*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = \int \frac{\sqrt[3]{c\sin^3\left(a+bx\right)}}{x^2}\,dx$$

```
[In] integrate((c*sin(b*x+a)**3)**(1/3)/x**2,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(1/3)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.97

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = \frac{\left(\left(\left(\sqrt{3}-i\right)E_2(i\,bx) + \left(\sqrt{3}+i\right)E_2(-i\,bx)\right)\cos\left(a\right)^3 + \left(\left(\sqrt{3}-i\right)E_2(i\,bx) + \left(\sqrt{3}+i\right)E_2(-i\,bx)\right)\cos\left(a\right)\sin\left(\right.}{}$$

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*(((sqrt(3) - I)*exp_integral_e(2, I*b*x) + (sqrt(3) + I)*exp_integral_e
(2, -I*b*x))*cos(a)^3 + ((sqrt(3) - I)*exp_integral_e(2, I*b*x) + (sqrt(3)
+ I)*exp_integral_e(2, -I*b*x))*cos(a)*sin(a)^2 + ((-I*sqrt(3) - 1)*exp_int
egral_e(2, I*b*x) + (I*sqrt(3) - 1)*exp_integral_e(2, -I*b*x))*sin(a)^3 - (
```

```
(sqrt(3) + I)*exp_integral_e(2, I*b*x) + (sqrt(3) - I)*exp_integral_e(2, -I
*b*x))*cos(a) + (((-I*sqrt(3) - 1)*exp_integral_e(2, I*b*x) + (I*sqrt(3) -
1)*exp_integral_e(2, -I*b*x))*cos(a)^2 + (I*sqrt(3) - 1)*exp_integral_e(2,
I*b*x) + (-I*sqrt(3) - 1)*exp_integral_e(2, -I*b*x))*sin(a))*b*c^(1/3)/(a*c
os(a)^2 + a*sin(a)^2 - (b*x + a)*(cos(a)^2 + sin(a)^2))
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}}}{x^2}\,dx$$

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^2,x, algorithm="giac")
[Out] integrate((c*sin(b*x + a)^3)^(1/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = \int \frac{\left(c\sin\left(a+b\,x\right)^3\right)^{1/3}}{x^2}\,dx$$

```
[In] int((c*sin(a + b*x)^3)^(1/3)/x^2,x)
[Out] int((c*sin(a + b*x)^3)^(1/3)/x^2, x)
```

3.317 $\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 116

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx = -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} - \frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x} - \frac{1}{2}b^2\,\text{CosIntegral}(bx)\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)} - \frac{1}{2}b^2\cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\,\text{Si}(bx)$$

```
[Out] -1/2*(c*sin(b*x+a)^3)^(1/3)/x^2-1/2*b*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/x-1
/2*b^2*cos(a)*csc(b*x+a)*Si(b*x)*(c*sin(b*x+a)^3)^(1/3)-1/2*b^2*Ci(b*x)*csc
(b*x+a)*sin(a)*(c*sin(b*x+a)^3)^(1/3)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6852, 3378, 3384, 3380, 3383}

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx = -\frac{1}{2}b^2\sin(a)\,\text{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} - \frac{1}{2}b^2\cos(a)\text{Si}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} - \frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} - \frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x}$$

```
[In] Int[(c*Sin[a + b*x]^3)^(1/3)/x^3,x]
```

```
[Out] -1/2*(c*Sin[a + b*x]^3)^(1/3)/x^2 - (b*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3
))/(2*x) - (b^2*CosIntegral[b*x]*Csc[a + b*x]*Sin[a]*(c*Sin[a + b*x]^3)^(1/
3))/2 - (b^2*Cos[a]*Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*SinIntegral[b*x])
/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\sin(a+bx)}{x^3}\,dx \\
&= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2}+\frac{1}{2}\left(b\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\cos(a+bx)}{x^2}\,dx
\end{aligned}$$

$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} - \frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x} - \frac{1}{2}\left(b^2\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int\frac{\sin(a+bx)}{x}\,dx$$

$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} - \frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x} - \frac{1}{2}\left(b^2\cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int\frac{\sin(bx)}{x}\,dx - \frac{1}{2}\left(b^2\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)}\right)\int\frac{\cos(bx)}{x}\,dx$$

$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} - \frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x} - \frac{1}{2}b^2\,\text{CosIntegral}(bx)\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)} - \frac{1}{2}b^2\cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\text{Si}(bx)$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

$$\int\frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx = -\frac{\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}(bx\cos(a+bx)+b^2x^2\,\text{CosIntegral}(bx)\sin(a)+\sin(a+bx)+b^2x^2\cos(a)\text{Si}(b}{2x^2}$$

```
[In] Integrate[(c*Sin[a + b*x]^3)^(1/3)/x^3,x]
[Out] -1/2*(Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(b*x*Cos[a + b*x] + b^2*x^2*Cos
Integral[b*x]*Sin[a] + Sin[a + b*x] + b^2*x^2*Cos[a]*SinIntegral[b*x]))/x^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{\left(ic\,e^{-3i(bx+a)}\left(e^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}\left(e^{ibx}\,\text{Ei}_1(ibx)x^2b^2-\text{Ei}_1(-ibx)e^{i(bx+2a)}x^2b^2+ie^{2i(bx+a)}xb+ibx+e^{2i(bx+a)}-1\right)}{4\left(e^{2i(bx+a)}-1\right)x^2}$	123

```
[In] int((c*sin(b*x+a)^3)^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(1/3)*(exp(I*b*x)*Ei(1,
I*b*x)*x^2*b^2-Ei(1,-I*b*x)*exp(I*(b*x+2*a))*x^2*b^2+I*exp(2*I*(b*x+a))*x*b
+I*b*x+exp(2*I*(b*x+a))-1)/(exp(2*I*(b*x+a))-1)/x^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx$$
$$= \frac{\left(i\,b^2x^2\text{Ei}(i\,bx)\,e^{(i\,a)} - i\,b^2x^2\text{Ei}(-i\,bx)\,e^{(-i\,a)} - 2\,bx\cos\left(bx+a\right) - 2\,\sin\left(bx+a\right)\right)\left(-\left(c\cos\left(bx+a\right)^2 - c\right)\text{si}\right.}{4\,x^2\sin\left(bx+a\right)}$$

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(I*b^2*x^2*Ei(I*b*x)*e^(I*a) - I*b^2*x^2*Ei(-I*b*x)*e^(-I*a) - 2*b*x*co
s(b*x + a) - 2*sin(b*x + a))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)/(
x^2*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx = \int \frac{\sqrt[3]{c\sin^3\left(a+bx\right)}}{x^3}\,dx$$

```
[In] integrate((c*sin(b*x+a)**3)**(1/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(1/3)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx =$$
$$-\frac{\left(\left(\left(\sqrt{3}-i\right)E_3(i\,bx) + \left(\sqrt{3}+i\right)E_3(-i\,bx)\right)\cos\left(a\right)^3 + \left(\left(\sqrt{3}-i\right)E_3(i\,bx) + \left(\sqrt{3}+i\right)E_3(-i\,bx)\right)\cos\left(a\right)\text{s}\right.}{}$$

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*(((sqrt(3) - I)*exp_integral_e(3, I*b*x) + (sqrt(3) + I)*exp_integral_
e(3, -I*b*x))*cos(a)^3 + ((sqrt(3) - I)*exp_integral_e(3, I*b*x) + (sqrt(3)
 + I)*exp_integral_e(3, -I*b*x))*cos(a)*sin(a)^2 + ((-I*sqrt(3) - 1)*exp_in
tegral_e(3, I*b*x) + (I*sqrt(3) - 1)*exp_integral_e(3, -I*b*x))*sin(a)^3 -
((sqrt(3) + I)*exp_integral_e(3, I*b*x) + (sqrt(3) - I)*exp_integral_e(3, -
I*b*x))*cos(a) + (((-I*sqrt(3) - 1)*exp_integral_e(3, I*b*x) + (I*sqrt(3) -
 1)*exp_integral_e(3, -I*b*x))*cos(a)^2 + (I*sqrt(3) - 1)*exp_integral_e(3,
 I*b*x) + (-I*sqrt(3) - 1)*exp_integral_e(3, -I*b*x))*sin(a))*b^2*c^(1/3)/(
a^2*cos(a)^2 + a^2*sin(a)^2 + (b*x + a)^2*(cos(a)^2 + sin(a)^2) - 2*(a*cos(
a)^2 + a*sin(a)^2)*(b*x + a))
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx=\int \frac{\left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}}}{x^3}\,dx$$

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx=\int \frac{\left(c\sin\left(a+b\,x\right)^3\right)^{1/3}}{x^3}\,dx$$

```
[In] int((c*sin(a + b*x)^3)^(1/3)/x^3,x)
```

```
[Out] int((c*sin(a + b*x)^3)^(1/3)/x^3, x)
```

3.318 $\int x^m \sqrt[3]{c \sin^3(a+bx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 153

$$\int x^m \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \frac{1}{4} i e^{ia} x^{1+m} \left(-ibx^2\right)^{\frac{1}{2}(-1-m)} \csc\left(a+bx^2\right) \Gamma\left(\frac{1+m}{2}, -ibx^2\right) \sqrt[3]{c \sin^3(a+bx^2)} - \frac{1}{4} i e^{-ia} x^{1+m} \left(ibx^2\right)^{\frac{1}{2}(-1-m)} \csc\left(a+bx^2\right) \Gamma\left(\frac{1+m}{2}, ibx^2\right) \sqrt[3]{c \sin^3(a+bx^2)}$$

```
[Out] 1/4*I*exp(I*a)*x^(1+m)*(-I*b*x^2)^(-1/2-1/2*m)*csc(b*x^2+a)*GAMMA(1/2+1/2*m
,-I*b*x^2)*(c*sin(b*x^2+a)^3)^(1/3)-1/4*I*x^(1+m)*(I*b*x^2)^(-1/2-1/2*m)*cs
c(b*x^2+a)*GAMMA(1/2+1/2*m,I*b*x^2)*(c*sin(b*x^2+a)^3)^(1/3)/exp(I*a)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6852, 3470, 2250}

$$\int x^m \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \frac{1}{4} i e^{ia} x^{m+1} \left(-ibx^2\right)^{\frac{1}{2}(-m-1)} \csc\left(a+bx^2\right) \Gamma\left(\frac{m+1}{2}, -ibx^2\right) \sqrt[3]{c \sin^3(a+bx^2)} - \frac{1}{4} i e^{-ia} x^{m+1} \left(ibx^2\right)^{\frac{1}{2}(-m-1)} \csc\left(a+bx^2\right) \Gamma\left(\frac{m+1}{2}, ibx^2\right) \sqrt[3]{c \sin^3(a+bx^2)}$$

```
[In] Int[x^m*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (I/4)*E^(I*a)*x^(1 + m)*((-I)*b*x^2)^((-1 - m)/2)*Csc[a + b*x^2]*Gamma[(1 +
 m)/2, (-I)*b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3) - ((I/4)*x^(1 + m)*(I*b*x^2)^
((-1 - m)/2)*Csc[a + b*x^2]*Gamma[(1 + m)/2, I*b*x^2]*(c*Sin[a + b*x^2]^3)^
(1/3))/E^(I*a)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int x^m\sin\left(a+bx^2\right)\,dx \\
&= \frac{1}{2}\left(i\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int e^{-ia-ibx^2}x^m\,dx \\
&\quad -\frac{1}{2}\left(i\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int e^{ia+ibx^2}x^m\,dx \\
&= \frac{1}{4}ie^{ia}x^{1+m}\left(-ibx^2\right)^{\frac{1}{2}(-1-m)}\csc\left(a+bx^2\right)\Gamma\left(\frac{1+m}{2},-ibx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)} \\
&\quad -\frac{1}{4}ie^{-ia}x^{1+m}\left(ibx^2\right)^{\frac{1}{2}(-1-m)}\csc\left(a+bx^2\right)\Gamma\left(\frac{1+m}{2},ibx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int x^m \sqrt[3]{c \sin^3\left(a+bx^2\right)}\,dx = \frac{1}{4} i x^{1+m} \left(b^2x^4\right)^{\frac{1}{2}(-1-m)} \csc\left(a+bx^2\right)\left(-\left(-ibx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},ibx^2\right)(\cos(a)-i\sin(a)) + \left(ibx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},-ibx^2\right)(\cos(a)+i\sin(a))\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}$$

```
[In] Integrate[x^m*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (I/4)*x^(1 + m)*(b^2*x^4)^((-1 - m)/2)*Csc[a + b*x^2]*(-(((-I)*b*x^2)^((1 +
 m)/2)*Gamma[(1 + m)/2, I*b*x^2]*(Cos[a] - I*Sin[a])) + (I*b*x^2)^((1 + m)/
2)*Gamma[(1 + m)/2, (-I)*b*x^2]*(Cos[a] + I*Sin[a]))*(c*Sin[a + b*x^2]^3)^(
1/3)
```

Maple [F]

$$\int x^m \left(c\left(\sin^3\left(b\,x^2+a\right)\right)\right)^{\frac{1}{3}}dx$$

```
[In] int(x^m*(c*sin(b*x^2+a)^3)^(1/3),x)
```

```
[Out] int(x^m*(c*sin(b*x^2+a)^3)^(1/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int x^m \sqrt[3]{c \sin^3\left(a+bx^2\right)}\,dx = -\frac{\left(e^{\left(-\frac{1}{2}(m-1)\log(ib)-ia\right)}\Gamma\left(\frac{1}{2}m+\frac{1}{2},i\,bx^2\right)+e^{\left(-\frac{1}{2}(m-1)\log(-ib)+ia\right)}\Gamma\left(\frac{1}{2}m+\frac{1}{2},-i\,bx^2\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2\right.\right.}{4\,b\sin\left(bx^2+a\right)}$$

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(e^(-1/2*(m - 1)*log(I*b) - I*a)*gamma(1/2*m + 1/2, I*b*x^2) + e^(-1/2
*(m - 1)*log(-I*b) + I*a)*gamma(1/2*m + 1/2, -I*b*x^2))*(-(c*cos(b*x^2 + a)
^2 - c)*sin(b*x^2 + a))^(1/3)/(b*sin(b*x^2 + a))
```

Sympy [F]

$$\int x^m \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \int x^m \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$$

```
[In] integrate(x**m*(c*sin(b*x**2+a)**3)**(1/3),x)
[Out] Integral(x**m*(c*sin(a + b*x**2)**3)**(1/3), x)
```

Maxima [F]

$$\int x^m \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \int \left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^m, x)
```

Giac [F]

$$\int x^m \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \int \left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \int x^m \left(c\sin\left(b\,x^2+a\right)^3\right)^{1/3}\,dx$$

```
[In] int(x^m*(c*sin(a + b*x^2)^3)^(1/3),x)
[Out] int(x^m*(c*sin(a + b*x^2)^3)^(1/3), x)
```

3.319 $\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{2b^2} - \frac{x^2 \cot(a+bx^2) \sqrt[3]{c \sin^3(a+bx^2)}}{2b}$$

```
[Out] 1/2*(c*sin(b*x^2+a)^3)^(1/3)/b^2-1/2*x^2*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(1
/3)/b
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 3460, 3377, 2717}

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{2b^2} - \frac{x^2 \cot(a+bx^2) \sqrt[3]{c \sin^3(a+bx^2)}}{2b}$$

```
[In] Int[x^3*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (c*Sin[a + b*x^2]^3)^(1/3)/(2*b^2) - (x^2*Cot[a + b*x^2]*(c*Sin[a + b*x^2]^
3)^(1/3))/(2*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int x^3\sin\left(a+bx^2\right)\,dx\\
&= \frac{1}{2}\left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx,x,x^2\right)\\
&= -\frac{x^2\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b}\\
&\quad+\frac{\left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int\cos(a+bx)\,dx,x,x^2\right)}{2b}\\
&= \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b^2}-\frac{x^2\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

$$\int x^3\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=-\frac{\left(-1+bx^2\cot\left(a+bx^2\right)\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b^2}$$

```
[In] Integrate[x^3*(c*Sin[a + b*x^2]^3)^(1/3),x]
[Out] -1/2*((-1 + b*x^2*Cot[a + b*x^2])*(c*Sin[a + b*x^2]^3)^(1/3))/b^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.33

method	result	size
risch	$-\frac{i(b\,x^2+i)\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{1}{3}}\mathrm{e}^{2i\left(b\,x^2+a\right)}}{4b^2\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)}-\frac{i\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{1}{3}}(b\,x^2-i)}{4\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)b^2}$	135

```
[In] int(x^3*(c*sin(b*x^2+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/b^2*(b*x^2+I)/(exp(2*I*(b*x^2+a))-1)*(I*c*exp(-3*I*(b*x^2+a))*(exp(2
*I*(b*x^2+a))-1)^3)^(1/3)*exp(2*I*(b*x^2+a))-1/4*I*(I*c*exp(-3*I*(b*x^2+a))
*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)/(exp(2*I*(b*x^2+a))-1)*(b*x^2-I)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int x^3\sqrt[3]{c\sin^3(a+bx^2)}\,dx = -\frac{(bx^2\cos(bx^2+a)-\sin(bx^2+a))\left(-\left(c\cos(bx^2+a)^2-c\right)\sin(bx^2+a)\right)^{\frac{1}{3}}}{2\,b^2\sin(bx^2+a)}$$

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^2*cos(b*x^2 + a) - sin(b*x^2 + a))*(-(c*cos(b*x^2 + a)^2 - c)*sin
(b*x^2 + a))^(1/3)/(b^2*sin(b*x^2 + a))
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47

$$\int x^3\sqrt[3]{c\sin^3(a+bx^2)}\,dx = \begin{cases} \frac{x^4\sqrt[3]{c\sin^3(a)}}{4} & \text{for } b=0 \\ 0 & \text{for } a=-bx^2 \vee a=-bx^2+\pi \\ -\frac{x^2\sqrt[3]{c\sin^3(a+bx^2)}\cos(a+bx^2)}{2b\sin(a+bx^2)}+\frac{\sqrt[3]{c\sin^3(a+bx^2)}}{2b^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*(c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x**4*(c*sin(a)**3)**(1/3)/4, Eq(b, 0)), (0, Eq(a, -b*x**2) | Eq(
a, -b*x**2 + pi)), (-x**2*(c*sin(a + b*x**2)**3)**(1/3)*cos(a + b*x**2)/(2*
b*sin(a + b*x**2)) + (c*sin(a + b*x**2)**3)**(1/3)/(2*b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.55

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \frac{(bx^2 \cos(bx^2+a) - \sin(bx^2+a))c^{\frac{1}{3}}}{4\,b^2}$$

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2*cos(b*x^2 + a) - sin(b*x^2 + a))*c^(1/3)/b^2
```

Giac [F]

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \int \left(c \sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}} x^3\,dx$$

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \frac{\left(\frac{\sin(b\,x^2+a)^2}{4} - \frac{b\,x^2\,\sin(2\,b\,x^2+2\,a)}{8}\right)\left(-2\,c\,(\sin(3\,b\,x^2+3\,a) - 3\,\sin(b\,x^2+a))\right)^{1/3}}{b^2\sin(b\,x^2+a)^2}$$

```
[In] int(x^3*(c*sin(a + b*x^2)^3)^(1/3),x)
```

```
[Out] ((sin(a + b*x^2)^2/4 - (b*x^2*sin(2*a + 2*b*x^2))/8)*(-2*c*(sin(3*a + 3*b*x
^2) - 3*sin(a + b*x^2)))^(1/3))/(b^2*sin(a + b*x^2)^2)
```

3.320 $\int x^2 \sqrt[3]{c \sin^3(a+bx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 155

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = -\frac{x\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b} + \frac{\sqrt{\frac{\pi}{2}}\cos(a)\csc(a+bx^2)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sqrt[3]{c\sin^3(a+bx^2)}}{2b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\csc(a+bx^2)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\sqrt[3]{c\sin^3(a+bx^2)}}{2b^{3/2}}$$

```
[Out] -1/2*x*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(1/3)/b+1/4*cos(a)*csc(b*x^2+a)*Fres
nelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*(c*sin(b*x^2+a)^3)^(1/3)*2^(1/2)*Pi^(1/2)/
b^(3/2)-1/4*csc(b*x^2+a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*(c*sin
(b*x^2+a)^3)^(1/3)*2^(1/2)*Pi^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6852, 3466, 3435, 3433, 3432}

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b^{3/2}} - \frac{x\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b}$$

```
[In] Int[x^2*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] -1/2*(x*Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/b + (Sqrt[Pi/2]*Cos[a]*C
sc[a + b*x^2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*(c*Sin[a + b*x^2]^3)^(1/3))/(2
*b^(3/2)) - (Sqrt[Pi/2]*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a
]*(c*Sin[a + b*x^2]^3)^(1/3))/(2*b^(3/2))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int x^2\sin\left(a+bx^2\right)\,dx$$

$$= -\frac{x\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b} + \frac{\left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\cos\left(a+bx^2\right)\,dx}{2b}$$

$$= -\frac{x\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b} + \frac{\left(\cos(a)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}\right)\int\cos(bx^2)\,dx}{2b} - \frac{\left(\csc(a+bx^2)\sin(a)\sqrt[3]{c\sin^3(a+bx^2)}\right)\int\sin(bx^2)\,dx}{2b}$$

$$= -\frac{x\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b} + \frac{\sqrt{\frac{\pi}{2}}\cos(a)\csc(a+bx^2)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sqrt[3]{c\sin^3(a+bx^2)}}{2b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\csc(a+bx^2)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\sqrt[3]{c\sin^3(a+bx^2)}}{2b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

$$\int x^2\sqrt[3]{c\sin^3(a+bx^2)}\,dx = -\frac{\csc(a+bx^2)\left(2\sqrt{b}x\cos(a+bx^2)-\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\right.}{4b^{3/2}}$$

```
[In] Integrate[x^2*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] -1/4*(Csc[a + b*x^2]*(2*Sqrt[b]*x*Cos[a + b*x^2] - Sqrt[2*Pi]*Cos[a]*Fresne
lC[Sqrt[b]*Sqrt[2/Pi]*x] + Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]
)*(c*Sin[a + b*x^2]^3)^(1/3))/b^(3/2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.55

method	result
risch	$\frac{\left(ic\,e^{-3i(b\,x^2+a)}\left(e^{2i(b\,x^2+a)}-1\right)^3\right)^{\frac{1}{3}}\left(-\frac{ix\,e^{2i(b\,x^2+a)}}{2b}+\frac{i\sqrt{\pi}\,\text{erf}(\sqrt{-ib}\,x)\,e^{i(b\,x^2+2a)}}{4b\sqrt{-ib}}\right)}{2\,e^{2i(b\,x^2+a)}-2} - \frac{ix\left(ic\,e^{-3i(b\,x^2+a)}\left(e^{2i(b\,x^2+a)}-1\right)^3\right.}{4b\left(e^{2i(b\,x^2+a)}-1\right)}$

```
[In] int(x^2*(c*sin(b*x^2+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(exp(2*I*(b*x^2+a))-1)*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^
3)^(1/3)*(-1/2*I/b*x*exp(2*I*(b*x^2+a))+1/4*I/b*Pi^(1/2)/(-I*b)^(1/2)*erf((
-I*b)^(1/2)*x)*exp(I*(b*x^2+2*a)))-1/4*I*x/b/(exp(2*I*(b*x^2+a))-1)*(I*c*ex
p(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)+1/8*I*(I*c*exp(-3*I*(b*x^
2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)/b
*Pi^(1/2)/(I*b)^(1/2)*erf((I*b)^(1/2)*x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86

$$\int x^2 \sqrt[3]{c\sin^3(a+bx^2)}\,dx = -\frac{\left(4\,bx\cos\left(bx^2+a\right)-\sqrt{2}\left(\pi e^{(i\,a)}+\pi e^{(-i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)-\sqrt{2}\left(i\,\pi e^{(i\,a)}-i\,\pi e^{(-i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)}{8\,b^2\sin\left(bx^2+a\right)}$$

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cos(b*x^2 + a) - sqrt(2)*(pi*e^(I*a) + pi*e^(-I*a))*sqrt(b/pi)*
fresnel_cos(sqrt(2)*x*sqrt(b/pi)) - sqrt(2)*(I*pi*e^(I*a) - I*pi*e^(-I*a))*
sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)))*(-(c*cos(b*x^2 + a)^2 - c)*si
n(b*x^2 + a))^(1/3)/(b^2*sin(b*x^2 + a))
```

Sympy [F]

$$\int x^2 \sqrt[3]{c\sin^3(a+bx^2)}\,dx = \int x^2 \sqrt[3]{c\sin^3(a+bx^2)}\,dx$$

```
[In] integrate(x**2*(c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Integral(x**2*(c*sin(a + b*x**2)**3)**(1/3), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \frac{8\,b^2 c^{\frac{1}{3}} x \cos(bx^2+a) + \sqrt{2}\sqrt{\pi}\Big(((i-1)\cos(a) + (i+1)\sin(a))\operatorname{erf}\big(\sqrt{i\,b}x\big) + (-(i+1)\cos(a) - (i-1$$

$$32\,b^3$$

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/32*(8*b^2*c^(1/3)*x*cos(b*x^2 + a) + sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) +
(I + 1)*sin(a))*erf(sqrt(I*b)*x) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf(s
qrt(-I*b)*x))*b^(3/2)*c^(1/3))/b^3
```

Giac [F]

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \int \left(c \sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}} x^2\,dx$$

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \int x^2 \left(c \sin\left(b\,x^2+a\right)^3\right)^{1/3}\,dx$$

```
[In] int(x^2*(c*sin(a + b*x^2)^3)^(1/3),x)
```

```
[Out] int(x^2*(c*sin(a + b*x^2)^3)^(1/3), x)
```

3.321 $\int x\sqrt[3]{c\sin^3(a+bx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int x\sqrt[3]{c\sin^3(a+bx^2)}\,dx=-\frac{\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b}$$

```
[Out] -1/2*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(1/3)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6847, 3286, 2718}

$$\int x\sqrt[3]{c\sin^3(a+bx^2)}\,dx=-\frac{\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b}$$

```
[In] Int[x*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] -1/2*(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
```

```
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sqrt[3]{c\sin^3(a+bx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int \sin(a+bx)\,dx, x, x^2\right)\\
&= -\frac{\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$
\int x\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = -\frac{\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b}
$$

```
[In] Integrate[x*(c*Sin[a + b*x^2]^3)^(1/3),x]
[Out] -1/2*(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.84

method	result	size
risch	$-\frac{i\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{1}{3}}\mathrm{e}^{2i\left(b\,x^2+a\right)}}{4b\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)}-\frac{i\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{1}{3}}}{4b\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)}$	119

```
[In] int(x*(c*sin(b*x^2+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/b/(exp(2*I*(b*x^2+a))-1)*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a)
)-1)^3)^(1/3)*exp(2*I*(b*x^2+a))-1/4*I/b/(exp(2*I*(b*x^2+a))-1)*(I*c*exp(-3
*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int x\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=-\frac{\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}\cos\left(bx^2+a\right)}{2\,b\sin\left(bx^2+a\right)}$$

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)*cos(b*x^2 + a)/(b*sin
(b*x^2 + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(27) = 54.

Time = 0.64 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

$$\int x\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=\begin{cases}\frac{x^2\sqrt[3]{c\sin^3\left(a\right)}}{2} & \text{for } b=0\\ 0 & \text{for } a=-bx^2\vee a=-bx^2+\pi\\ -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}\cos\left(a+bx^2\right)}{2b\sin\left(a+bx^2\right)} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x**2*(c*sin(a)**3)**(1/3)/2, Eq(b, 0)), (0, Eq(a, -b*x**2) | Eq(
a, -b*x**2 + pi)), (-(c*sin(a + b*x**2)**3)**(1/3)*cos(a + b*x**2)/(2*b*sin
(a + b*x**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int x\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=\frac{c^{\frac{1}{3}}\cos\left(bx^2+a\right)}{4\,b}$$

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
[Out] 1/4*c^(1/3)*cos(b*x^2 + a)/b
```

Giac [F]

$$\int x\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=\int\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}x\,dx$$

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x, x)
```

Mupad [B] (verification not implemented)

Time = 6.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int x\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=-\frac{\sin\left(2\,b\,x^2+2\,a\right)\,\left(-2\,c\left(\sin\left(3\,b\,x^2+3\,a\right)-3\,\sin\left(b\,x^2+a\right)\right)\right)^{1/3}}{8\,b\sin\left(b\,x^2+a\right)^2}$$

```
[In] int(x*(c*sin(a + b*x^2)^3)^(1/3),x)
[Out] -(sin(2*a + 2*b*x^2)*(-2*c*(sin(3*a + 3*b*x^2) - 3*sin(a + b*x^2)))^(1/3))/
(8*b*sin(a + b*x^2)^2)
```

3.322 $\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 117

$$\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \frac{\sqrt{\frac{\pi}{2}}\cos(a)\csc\left(a+bx^2\right)\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}} + \frac{\sqrt{\frac{\pi}{2}}\csc\left(a+bx^2\right)\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}$$

```
[Out] 1/2*cos(a)*csc(b*x^2+a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*(c*sin(b*x^2+a
)^3)^(1/3)*2^(1/2)*Pi^(1/2)/b^(1/2)+1/2*csc(b*x^2+a)*FresnelC(x*b^(1/2)*2^(
1/2)/Pi^(1/2))*sin(a)*(c*sin(b*x^2+a)^3)^(1/3)*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6852, 3434, 3433, 3432}

$$\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \frac{\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}} + \frac{\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}$$

```
[In] Int[(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (Sqrt[Pi/2]*Cos[a]*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*(c*Sin[a +
 b*x^2]^3)^(1/3))/Sqrt[b] + (Sqrt[Pi/2]*Csc[a + b*x^2]*FresnelC[Sqrt[b]*Sqr
t[2/Pi]*x]*Sin[a]*(c*Sin[a + b*x^2]^3)^(1/3))/Sqrt[b]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\sin\left(a+bx^2\right)\,dx\\
&= \left(\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\sin\left(bx^2\right)\,dx\\
&\quad+\left(\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\cos\left(bx^2\right)\,dx\\
&= \frac{\sqrt{\frac{\pi}{2}}\cos(a)\csc\left(a+bx^2\right)\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}\\
&\quad+\frac{\sqrt{\frac{\pi}{2}}\csc\left(a+bx^2\right)\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \sqrt[3]{c\sin^3(a+bx^2)}\,dx$$
$$=\frac{\sqrt{\frac{\pi}{2}}\csc(a+bx^2)\left(\cos(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\right)\sqrt[3]{c\sin^3(a+bx^2)}}{\sqrt{b}}$$

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(1/3),x]
[Out] (Sqrt[Pi/2]*Csc[a + b*x^2]*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] + Fresnel
C[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])*(c*Sin[a + b*x^2]^3)^(1/3))/Sqrt[b]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34

method	result
risch	$\frac{\operatorname{erf}(\sqrt{-ib}\,x)\sqrt{\pi}\left(ic\,e^{-3i(b\,x^2+a)}\left(e^{2i(b\,x^2+a)}-1\right)^3\right)^{\frac{1}{3}}e^{i(b\,x^2+2a)}}{4\sqrt{-ib}\left(e^{2i(b\,x^2+a)}-1\right)}-\frac{\left(ic\,e^{-3i(b\,x^2+a)}\left(e^{2i(b\,x^2+a)}-1\right)^3\right)^{\frac{1}{3}}e^{ib\,x^2}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{ib}\,x\right)}{4\left(e^{2i(b\,x^2+a)}-1\right)\sqrt{ib}}$

```
[In] int((c*sin(b*x^2+a)^3)^(1/3),x,method=_RETURNVERBOSE)
[Out] 1/4*erf((-I*b)^(1/2)*x)/(-I*b)^(1/2)*Pi^(1/2)/(exp(2*I*(b*x^2+a))-1)*(I*c*e
xp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)*exp(I*(b*x^2+2*a))-1/4*(
I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)/(exp(2*I*(b*x^2+a))
-1)*exp(I*b*x^2)*Pi^(1/2)/(I*b)^(1/2)*erf((I*b)^(1/2)*x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \sqrt[3]{c\sin^3(a+bx^2)}\,dx$$
$$=\frac{\left(\sqrt{2}\left(-i\,\pi e^{(i\,a)}+i\,\pi e^{(-i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{2}\left(\pi e^{(i\,a)}+\pi e^{(-i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\right)\left(-\left(c\cos(bx^2+a)\right.\right.}{4\,b\sin(bx^2+a)}$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

[Out] 1/4*(sqrt(2)*(-I*pi*e^(I*a) + I*pi*e^(-I*a))*sqrt(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi)) + sqrt(2)*(pi*e^(I*a) + pi*e^(-I*a))*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)/(b*sin(b*x^2 + a))

Sympy [F]

$$\int \sqrt[3]{c\sin^3(a+bx^2)}\,dx = \int \sqrt[3]{c\sin^3(a+bx^2)}\,dx$$

[In] integrate((c*sin(b*x**2+a)**3)**(1/3),x)

[Out] Integral((c*sin(a + b*x**2)**3)**(1/3), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \sqrt[3]{c\sin^3(a+bx^2)}\,dx = \frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\cos(a)+(i-1)\sin(a)\right)\operatorname{erf}\left(\sqrt{i\,b}x\right)+\left((i-1)\cos(a)-(i+1)\sin(a)\right)\operatorname{erf}\left(\sqrt{-i\,b}x\right)\right)}{16\sqrt{b}}$$

[In] integrate((c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")

[Out] 1/16*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf(sqrt(I*b)*x) + ((I - 1)*cos(a) - (I + 1)*sin(a))*erf(sqrt(-I*b)*x))*c^(1/3)/sqrt(b)

Giac [F]

$$\int \sqrt[3]{c\sin^3(a+bx^2)}\,dx = \int \left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}\,dx$$

[In] integrate((c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")

[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{c\sin^3(a+bx^2)}\,dx = \int \left(c\sin\left(b\,x^2+a\right)^3\right)^{1/3}\,dx$$

```
[In] int((c*sin(a + b*x^2)^3)^(1/3),x)
[Out] int((c*sin(a + b*x^2)^3)^(1/3), x)
```

3.323 $\int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x} \, dx = \frac{1}{2} \text{CosIntegral}(bx^2) \csc(a+bx^2) \sin(a) \sqrt[3]{c \sin^3(a+bx^2)} + \frac{1}{2} \cos(a) \csc(a+bx^2) \sqrt[3]{c \sin^3(a+bx^2)} \text{Si}(bx^2)$$

```
[Out] 1/2*cos(a)*csc(b*x^2+a)*Si(b*x^2)*(c*sin(b*x^2+a)^3)^(1/3)+1/2*Ci(b*x^2)*cs
c(b*x^2+a)*sin(a)*(c*sin(b*x^2+a)^3)^(1/3)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 3458, 3457, 3456}

$$\int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x} \, dx = \frac{1}{2} \sin(a) \text{CosIntegral}(bx^2) \csc(a+bx^2) \sqrt[3]{c \sin^3(a+bx^2)} + \frac{1}{2} \cos(a) \text{Si}(bx^2) \csc(a+bx^2) \sqrt[3]{c \sin^3(a+bx^2)}$$

```
[In] Int[(c*Sin[a + b*x^2]^3)^(1/3)/x,x]
```

```
[Out] (CosIntegral[b*x^2]*Csc[a + b*x^2]*Sin[a]*(c*Sin[a + b*x^2]^3)^(1/3))/2 + (
Cos[a]*Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*SinIntegral[b*x^2])/2
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\frac{\sin\left(a+bx^2\right)}{x}\,dx\\
&= \left(\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\frac{\sin\left(bx^2\right)}{x}\,dx\\
&\quad+\left(\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\frac{\cos\left(bx^2\right)}{x}\,dx\\
&= \frac{1}{2}\text{CosIntegral}\left(bx^2\right)\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\\
&\quad+\frac{1}{2}\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\text{Si}\left(bx^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}\,dx=\frac{1}{2}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\text{CosIntegral}\left(bx^2\right)\sin(a)+\cos(a)\text{Si}\left(bx^2\right)\right)$$

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(1/3)/x,x]
[Out] (Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*(CosIntegral[b*x^2]*Sin[a] + Cos
[a]*SinIntegral[b*x^2]))/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.75

method	result
risch	$-\frac{\left(i c\,{\mathrm e}^{-3 i \left(b \,x^{2}+a \right)} \left({\mathrm e}^{2 i \left(b \,x^{2}+a \right)}-1\right)^{3}\right)^{\frac{1}{3}} \left(i {\mathrm e}^{i b \,x^{2}} \pi \,\mathrm{csgn}\left(b \,x^{2}\right)-2 i {\mathrm e}^{i b \,x^{2}} \mathrm{Si}\left(b \,x^{2}\right)+\mathrm{Ei}_{1}\left(-i b \,x^{2}\right) {\mathrm e}^{i \left(b \,x^{2}+2 a \right)}-{\mathrm e}^{i b \,x^{2}} \mathrm{Ei}_{1}\left(-i b \,x^{2}\right)\right)}{4 \left({\mathrm e}^{2 i \left(b \,x^{2}+a \right)}-1\right)}$

[In] int((c*sin(b*x^2+a)^3)^(1/3)/x,x,method=_RETURNVERBOSE)

[Out] -1/4*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)*(I*exp(I*b*x^2)*Pi*csgn(b*x^2)-2*I*exp(I*b*x^2)*Si(b*x^2)+Ei(1,-I*b*x^2)*exp(I*(b*x^2+2*a))-exp(I*b*x^2)*Ei(1,-I*b*x^2))/(exp(2*I*(b*x^2+a))-1)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x}\,dx = \frac{\left(-i\,\mathrm{Ei}(i\,bx^2)\,e^{(i\,a)}+i\,\mathrm{Ei}(-i\,bx^2)\,e^{(-i\,a)}\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}}{4\,\sin\left(bx^2+a\right)}$$

[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x,x, algorithm="fricas")

[Out] 1/4*(-I*Ei(I*b*x^2)*e^(I*a) + I*Ei(-I*b*x^2)*e^(-I*a))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)/sin(b*x^2 + a)

Sympy [F]

$$\int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x}\,dx = \int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x}\,dx$$

[In] integrate((c*sin(b*x**2+a)**3)**(1/3)/x,x)

[Out] Integral((c*sin(a + b*x**2)**3)**(1/3)/x, x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x}\,dx = \frac{1}{8}\left(\left(i\operatorname{Ei}(i\,bx^2)-i\operatorname{Ei}(-i\,bx^2)\right)\cos(a)-\left(\operatorname{Ei}(i\,bx^2)+\operatorname{Ei}(-i\,bx^2)\right)\sin(a)\right)c^{\frac{1}{3}}$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x,x, algorithm="maxima")
[Out] 1/8*((I*Ei(I*b*x^2) - I*Ei(-I*b*x^2))*cos(a) - (Ei(I*b*x^2) + Ei(-I*b*x^2))
*sin(a))*c^(1/3)
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x}\,dx = \int \frac{\left(c\sin(bx^2+a)^3\right)^{\frac{1}{3}}}{x}\,dx$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x,x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x}\,dx = \int \frac{\left(c\sin(b\,x^2+a)^3\right)^{1/3}}{x}\,dx$$

```
[In] int((c*sin(a + b*x^2)^3)^(1/3)/x,x)
[Out] int((c*sin(a + b*x^2)^3)^(1/3)/x, x)
```

3.324 $\int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 135

$$\int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x^2} \, dx = -\frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x} + \sqrt{b}\sqrt{2\pi} \cos(a) \csc(a + bx^2) \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) \sqrt[3]{c \sin^3(a+bx^2)} - \sqrt{b}\sqrt{2\pi} \csc(a + bx^2) \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) \sin(a) \sqrt[3]{c \sin^3(a+bx^2)}$$

```
[Out] -(c*sin(b*x^2+a)^3)^(1/3)/x+cos(a)*csc(b*x^2+a)*FresnelC(x*b^(1/2)*2^(1/2)/
Pi^(1/2))*(c*sin(b*x^2+a)^3)^(1/3)*b^(1/2)*2^(1/2)*Pi^(1/2)-csc(b*x^2+a)*Fr
esnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*(c*sin(b*x^2+a)^3)^(1/3)*b^(1/2)*
2^(1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6852, 3468, 3435, 3433, 3432}

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^2}\,dx = \sqrt{2\pi}\sqrt{b}\cos(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3(a+bx^2)} - \sqrt{2\pi}\sqrt{b}\sin(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3(a+bx^2)} - \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x}$$

```
[In] Int[(c*Sin[a + b*x^2]^3)^(1/3)/x^2,x]
```

```
[Out] -((c*Sin[a + b*x^2]^3)^(1/3)/x) + Sqrt[b]*Sqrt[2*Pi]*Cos[a]*Csc[a + b*x^2]*
FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*(c*Sin[a + b*x^2]^3)^(1/3) - Sqrt[b]*Sqrt[2*
Pi]*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]*(c*Sin[a + b*x^2]^
3)^(1/3)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\frac{\sin\left(a+bx^2\right)}{x^2}\,dx\\
&= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}+\left(2b\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\cos\left(a+bx^2\right)\,dx\\
&= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}+\left(2b\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\cos\left(bx^2\right)\,dx\\
&\quad-\left(2b\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\sin\left(bx^2\right)\,dx\\
&= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}+\sqrt{b}\sqrt{2\pi}\cos(a)\csc\left(a+bx^2\right)\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\\
&\quad-\sqrt{b}\sqrt{2\pi}\csc\left(a+bx^2\right)\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx = \frac{\left(-1+\sqrt{b}\sqrt{2\pi}x\cos(a)\csc\left(a+bx^2\right)\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{b}\sqrt{2\pi}x\csc\left(a+bx^2\right)\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin\right.}{x}$$

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(1/3)/x^2,x]
```

```
[Out] ((-1 + Sqrt[b]*Sqrt[2*Pi]*x*Cos[a]*Csc[a + b*x^2]*FresnelC[Sqrt[b]*Sqrt[2/P
i]*x] - Sqrt[b]*Sqrt[2*Pi]*x*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*
Sin[a])*(c*Sin[a + b*x^2]^3)^(1/3))/x
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.72

method	result
risch	$\frac{\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{1}{3}}\left(-\frac{\mathrm{e}^{2i\left(b\,x^2+a\right)}}{x}+\frac{ib\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)\mathrm{e}^{i\left(b\,x^2+2a\right)}}{\sqrt{-ib}}\right)}{2\,\mathrm{e}^{2i\left(b\,x^2+a\right)}-2}+\frac{\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{1}{3}}}{2x\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)}$

[In] int((c*sin(b*x^2+a)^3)^(1/3)/x^2,x,method=_RETURNVERBOSE)

[Out] 1/2/(exp(2*I*(b*x^2+a))-1)*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)*(-1/x*exp(2*I*(b*x^2+a))+I*b*Pi^(1/2)/(-I*b)^(1/2)*erf((-I*b)^(1/2)*x)*exp(I*(b*x^2+2*a)))+1/2/x/(exp(2*I*(b*x^2+a))-1)*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)+1/2*I*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)*b*Pi^(1/2)/(I*b)^(1/2)*erf((I*b)^(1/2)*x)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx$$
$$=\frac{\left(\sqrt{2}\left(\pi xe^{(ia)}+\pi xe^{(-ia)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{2}\left(i\,\pi xe^{(ia)}-i\,\pi xe^{(-ia)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)-2\,\sin\left(bx^2+a\right)\right.}{2\,x\sin\left(bx^2+a\right)}$$

[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^2,x, algorithm="fricas")

[Out] 1/2*(sqrt(2)*(pi*x*e^(I*a) + pi*x*e^(-I*a))*sqrt(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi)) + sqrt(2)*(I*pi*x*e^(I*a) - I*pi*x*e^(-I*a))*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) - 2*sin(b*x^2 + a))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)/(x*sin(b*x^2 + a))

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx = \int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx$$

```
[In] integrate((c*sin(b*x**2+a)**3)**(1/3)/x**2,x)
[Out] Integral((c*sin(a + b*x**2)**3)**(1/3)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx$$
$$= \frac{\sqrt{bx^2}\left(\left(\left(i-1\right)\sqrt{2}\Gamma\left(-\frac{1}{2}, i\,bx^2\right) - \left(i+1\right)\sqrt{2}\Gamma\left(-\frac{1}{2}, -i\,bx^2\right)\right)\cos\left(a\right) + \left(\left(i+1\right)\sqrt{2}\Gamma\left(-\frac{1}{2}, i\,bx^2\right) - \left(i-1\right)\right.}{16\,x}$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^2,x, algorithm="maxima")
[Out] 1/16*sqrt(b*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I + 1)*sqrt(2)*g
amma(-1/2, -I*b*x^2))*cos(a) + ((I + 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I -
 1)*sqrt(2)*gamma(-1/2, -I*b*x^2))*sin(a))*c^(1/3)/x
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}}{x^2}\,dx$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^2,x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^2}\,dx = \int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{1/3}}{x^2}\,dx$$

```
[In] int((c*sin(a + b*x^2)^3)^(1/3)/x^2,x)
[Out] int((c*sin(a + b*x^2)^3)^(1/3)/x^2, x)
```

3.325 $\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^3}\,dx = -\frac{\sqrt[3]{c\sin^3(a+bx^2)}}{2x^2} + \frac{1}{2}b\cos(a)\,\text{CosIntegral}(bx^2)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)} - \frac{1}{2}b\csc(a+bx^2)\sin(a)\sqrt[3]{c\sin^3(a+bx^2)}\,\text{Si}(bx^2)$$

```
[Out] -1/2*(c*sin(b*x^2+a)^3)^(1/3)/x^2+1/2*b*Ci(b*x^2)*cos(a)*csc(b*x^2+a)*(c*si
n(b*x^2+a)^3)^(1/3)-1/2*b*csc(b*x^2+a)*Si(b*x^2)*sin(a)*(c*sin(b*x^2+a)^3)^
(1/3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6852, 3460, 3378, 3384, 3380, 3383}

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^3}\,dx = \frac{1}{2}b\cos(a)\,\text{CosIntegral}(bx^2)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)} - \frac{1}{2}b\sin(a)\text{Si}(bx^2)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)} - \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{2x^2}$$

```
[In] Int[(c*Sin[a + b*x^2]^3)^(1/3)/x^3,x]
```

```
[Out] -1/2*(c*Sin[a + b*x^2]^3)^(1/3)/x^2 + (b*Cos[a]*CosIntegral[b*x^2]*Csc[a +
b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/2 - (b*Csc[a + b*x^2]*Sin[a]*(c*Sin[a +
b*x^2]^3)^(1/3)*SinIntegral[b*x^2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\frac{\sin\left(a+bx^2\right)}{x^3}\,dx\\
&= \frac{1}{2}\left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int\frac{\sin(a+bx)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2x^2}+\frac{1}{2}\left(b\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int\frac{\cos(a+bx)}{x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2x^2}\\
&\quad+\frac{1}{2}\left(b\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int\frac{\cos(bx)}{x}\,dx,x,x^2\right)\\
&\quad-\frac{1}{2}\left(b\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2x^2}+\frac{1}{2}b\cos(a)\,\text{CosIntegral}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\\
&\quad-\frac{1}{2}b\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\text{Si}\left(bx^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$
\int\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx = -\frac{\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(-bx^2\cos(a)\,\text{CosIntegral}\left(bx^2\right)+\sin\left(a+bx^2\right)+bx^2\sin(a)\text{Si}\left(bx^2\right)\right)}{2x^2}
$$

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(1/3)/x^3,x]
```

```
[Out] -1/2*(Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*(-(b*x^2*Cos[a]*CosIntegral
[b*x^2]) + Sin[a + b*x^2] + b*x^2*Sin[a]*SinIntegral[b*x^2]))/x^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.21

method	result	size
risch	$-\frac{\left(i c\,\mathrm{e}^{-3 i \left(b\,x^{2}+a\right)} \left(\mathrm{e}^{2 i \left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{1}{3}} \left(i b\,\mathrm{Ei}_{1}\left(-i b\,x^{2}\right) \mathrm{e}^{i \left(b\,x^{2}+2 a\right)} x^{2}+i \mathrm{e}^{i b\,x^{2}} b\,\mathrm{Ei}_{1}\left(i b\,x^{2}\right) x^{2}+\mathrm{e}^{2 i \left(b\,x^{2}+a\right)}-1\right)}{4 \left(\mathrm{e}^{2 i \left(b\,x^{2}+a\right)}-1\right) x^{2}}$	119

```
[In] int((c*sin(b*x^2+a)^3)^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)*(I*b*Ei(1,-I*
b*x^2)*exp(I*(b*x^2+2*a))*x^2+I*exp(I*b*x^2)*b*Ei(1,I*b*x^2)*x^2+exp(2*I*(b
*x^2+a))-1)/(exp(2*I*(b*x^2+a))-1)/x^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx = \frac{\left(bx^2\mathrm{Ei}\left(i\,bx^2\right)e^{(i\,a)}+bx^2\mathrm{Ei}\left(-i\,bx^2\right)e^{(-i\,a)}-2\,\sin\left(bx^2+a\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}}{4\,x^2\sin\left(bx^2+a\right)}$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b*x^2*Ei(I*b*x^2)*e^(I*a) + b*x^2*Ei(-I*b*x^2)*e^(-I*a) - 2*sin(b*x^2
+ a))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)/(x^2*sin(b*x^2 + a))
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx = \int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx$$

```
[In] integrate((c*sin(b*x**2+a)**3)**(1/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(1/3)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^3}\,dx = -\frac{1}{8}\left(\left(\Gamma\left(-1,i\,bx^2\right)+\Gamma\left(-1,-i\,bx^2\right)\right)\cos(a)-\left(i\,\Gamma\left(-1,i\,bx^2\right)-i\,\Gamma\left(-1,-i\,bx^2\right)\right)\sin(a)\right)bc^{\frac{1}{3}}$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^3,x, algorithm="maxima")
[Out] -1/8*((gamma(-1, I*b*x^2) + gamma(-1, -I*b*x^2))*cos(a) - (I*gamma(-1, I*b*
x^2) - I*gamma(-1, -I*b*x^2))*sin(a))*b*c^(1/3)
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}}{x^3}\,dx$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^3,x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^3}\,dx = \int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{1/3}}{x^3}\,dx$$

```
[In] int((c*sin(a + b*x^2)^3)^(1/3)/x^3,x)
[Out] int((c*sin(a + b*x^2)^3)^(1/3)/x^3, x)
```

3.326 $\int x^m \sqrt[3]{c \sin^3(a+bx^n)}\, dx$

Optimal result

Integrand size = 20, antiderivative size = 157

$$\int x^m \sqrt[3]{c \sin^3(a+bx^n)}\, dx = \frac{ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\csc(a+bx^n)\,\Gamma\left(\frac{1+m}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\csc(a+bx^n)\,\Gamma\left(\frac{1+m}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x^(1+m)*csc(a+b*x^n)*GAMMA((1+m)/n,-I*b*x^n)*(c*sin(a+b*x^n)
^3)^(1/3)/n/((-I*b*x^n)^((1+m)/n))-1/2*I*x^(1+m)*csc(a+b*x^n)*GAMMA((1+m)/n
,I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6852, 3504, 2250}

$$\int x^m \sqrt[3]{c \sin^3(a+bx^n)}\, dx = \frac{ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\csc(a+bx^n)\,\Gamma\left(\frac{m+1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\csc(a+bx^n)\,\Gamma\left(\frac{m+1}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

```
[In] Int[x^m*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x^(1 + m)*Csc[a + b*x^n]*Gamma[(1 + m)/n, (-I)*b*x^n]*(c*Sin
[a + b*x^n]^3)^(1/3))/(n*((-I)*b*x^n)^((1 + m)/n)) - ((I/2)*x^(1 + m)*Csc[a
 + b*x^n]*Gamma[(1 + m)/n, I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*
(I*b*x^n)^((1 + m)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int x^m\sin\left(a+bx^n\right)\,dx\\
&= \frac{1}{2}\left(i\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int e^{-ia-ibx^n}x^m\,dx\\
&\quad-\frac{1}{2}\left(i\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int e^{ia+ibx^n}x^m\,dx\\
&= \frac{ie^{ia}x^{1+m}\left(-ibx^n\right)^{-\frac{1+m}{n}}\csc\left(a+bx^n\right)\Gamma\left(\frac{1+m}{n},-ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}\\
&\quad-\frac{ie^{-ia}x^{1+m}\left(ibx^n\right)^{-\frac{1+m}{n}}\csc\left(a+bx^n\right)\Gamma\left(\frac{1+m}{n},ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

$$\int x^m \sqrt[3]{c \sin^3(a+bx^n)}\,dx$$
$$= \frac{ix^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}} \csc(a+bx^n) \left(-(-ibx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, ibx^n\right)(\cos(a) - i\sin(a)) + (ibx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -ibx^n\right.\right.}{2n}$$

```
[In] Integrate[x^m*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*x^(1 + m)*Csc[a + b*x^n]*(-(((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n
, I*b*x^n]*(Cos[a] - I*Sin[a])) + (I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-
I)*b*x^n]*(Cos[a] + I*Sin[a]))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))
^((1 + m)/n))
```

Maple [F]

$$\int x^m \left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{1}{3}} dx$$

```
[In] int(x^m*(c*sin(a+b*x^n)^3)^(1/3),x)
```

```
[Out] int(x^m*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Fricas [F]

$$\int x^m \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \int \left(c \sin(bx^n+a)^3\right)^{\frac{1}{3}} x^m\,dx$$

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x^m, x)
```

Sympy [F]

$$\int x^m \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \int x^m \sqrt[3]{c \sin^3(a+bx^n)}\,dx$$

```
[In] integrate(x**m*(c*sin(a+b*x**n)**3)**(1/3),x)
```

```
[Out] Integral(x**m*(c*sin(a + b*x**n)**3)**(1/3), x)
```

Maxima [F]

$$\int x^m \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \int \left(c \sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^m, x)
```

Giac [F]

$$\int x^m \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \int \left(c \sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \int x^m \left(c \sin\left(a+b\,x^n\right)^3\right)^{1/3}\,dx$$

```
[In] int(x^m*(c*sin(a + b*x^n)^3)^(1/3),x)
[Out] int(x^m*(c*sin(a + b*x^n)^3)^(1/3), x)
```

3.327 $\int x^3 \sqrt[3]{c \sin^3(a+bx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \frac{ie^{ia}x^4(-ibx^n)^{-4/n}\csc(a+bx^n)\Gamma\left(\frac{4}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^4(ibx^n)^{-4/n}\csc(a+bx^n)\Gamma\left(\frac{4}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x^4*csc(a+b*x^n)*GAMMA(4/n,-I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/3
)/n/((-I*b*x^n)^(4/n))-1/2*I*x^4*csc(a+b*x^n)*GAMMA(4/n,I*b*x^n)*(c*sin(a+b
*x^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^(4/n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6852, 3504, 2250}

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \frac{ie^{ia}x^4(-ibx^n)^{-4/n}\Gamma\left(\frac{4}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^4(ibx^n)^{-4/n}\Gamma\left(\frac{4}{n},ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

```
[In] Int[x^3*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x^4*Csc[a + b*x^n]*Gamma[4/n, (-I)*b*x^n]*(c*Sin[a + b*x^n]^
3)^(1/3))/(n*((-I)*b*x^n)^(4/n)) - ((I/2)*x^4*Csc[a + b*x^n]*Gamma[4/n, I*b
*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^(4/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int x^3\sin\left(a+bx^n\right)\,dx \\
&= \frac{1}{2}\left(i\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int e^{-ia-ibx^n}x^3\,dx \\
&\quad -\frac{1}{2}\left(i\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int e^{ia+ibx^n}x^3\,dx \\
&= \frac{ie^{ia}x^4(-ibx^n)^{-4/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{4}{n},-ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n} \\
&\quad -\frac{ie^{-ia}x^4(ibx^n)^{-4/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{4}{n},ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int x^3\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx$$

$$= \frac{ix^4\left(b^2x^{2n}\right)^{-4/n}\csc\left(a+bx^n\right)\left(-(-ibx^n)^{4/n}\Gamma\left(\frac{4}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{4/n}\Gamma\left(\frac{4}{n},-ibx^n\right)(\cos(a)\right.}{2n}$$

```
[In] Integrate[x^3*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*x^4*Csc[a + b*x^n]*(-(((-I)*b*x^n)^(4/n)*Gamma[4/n, I*b*x^n]*(Cos[a]
 - I*Sin[a])) + (I*b*x^n)^(4/n)*Gamma[4/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))
*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^(4/n))
```

Maple [F]

$$\int x^{3}\left(c\left(\sin^{3}\left(a+b\,x^{n}\right)\right)\right)^{\frac{1}{3}}dx$$

```
[In] int(x^3*(c*sin(a+b*x^n)^3)^(1/3),x)
[Out] int(x^3*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Fricas [F]

$$\int x^{3}\sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}\,dx=\int\left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{1}{3}}x^{3}\,dx$$

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x^3, x)
```

Sympy [F]

$$\int x^{3}\sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}\,dx=\int x^{3}\sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}\,dx$$

```
[In] integrate(x**3*(c*sin(a+b*x**n)**3)**(1/3),x)
[Out] Integral(x**3*(c*sin(a + b*x**n)**3)**(1/3), x)
```

Maxima [F]

$$\int x^{3}\sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}\,dx=\int\left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{1}{3}}x^{3}\,dx$$

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^3, x)
```

Giac [F]

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \int \left(c \sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}} x^3\,dx$$

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \int x^3 \left(c \sin\left(a+b\,x^n\right)^3\right)^{1/3}\,dx$$

```
[In] int(x^3*(c*sin(a + b*x^n)^3)^(1/3),x)
[Out] int(x^3*(c*sin(a + b*x^n)^3)^(1/3), x)
```

3.328 $\int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \frac{ie^{ia}x^3(-ibx^n)^{-3/n}\csc(a+bx^n)\,\Gamma\left(\frac{3}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^3(ibx^n)^{-3/n}\csc(a+bx^n)\,\Gamma\left(\frac{3}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x^3*csc(a+b*x^n)*GAMMA(3/n,-I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/3
)/n/((-I*b*x^n)^(3/n))-1/2*I*x^3*csc(a+b*x^n)*GAMMA(3/n,I*b*x^n)*(c*sin(a+b
*x^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^(3/n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6852, 3504, 2250}

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \frac{ie^{ia}x^3(-ibx^n)^{-3/n}\Gamma\left(\frac{3}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^3(ibx^n)^{-3/n}\Gamma\left(\frac{3}{n},ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

```
[In] Int[x^2*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x^3*Csc[a + b*x^n]*Gamma[3/n, (-I)*b*x^n]*(c*Sin[a + b*x^n]^
3)^(1/3))/(n*((-I)*b*x^n)^(3/n)) - ((I/2)*x^3*Csc[a + b*x^n]*Gamma[3/n, I*b
*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^(3/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int x^2\sin\left(a+bx^n\right)\,dx\\
&= \frac{1}{2}\left(i\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int e^{-ia-ibx^n}x^2\,dx\\
&\quad-\frac{1}{2}\left(i\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int e^{ia+ibx^n}x^2\,dx\\
&= \frac{ie^{ia}x^3(-ibx^n)^{-3/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{3}{n},-ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}\\
&\quad-\frac{ie^{-ia}x^3(ibx^n)^{-3/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{3}{n},ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int x^2\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx$$
$$=\frac{ix^3(b^2x^{2n})^{-3/n}\csc\left(a+bx^n\right)\left(-(-ibx^n)^{3/n}\Gamma\left(\frac{3}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{3/n}\Gamma\left(\frac{3}{n},-ibx^n\right)(\cos(a)+\right.}{2n}$$

```
[In] Integrate[x^2*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*x^3*Csc[a + b*x^n]*(-(((-I)*b*x^n)^(3/n)*Gamma[3/n, I*b*x^n]*(Cos[a]
 - I*Sin[a])) + (I*b*x^n)^(3/n)*Gamma[3/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))
*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^(3/n))
```

Maple [F]

$$\int x^{2}\left(c\left(\sin^{3}\left(a+b\,x^{n}\right)\right)\right)^{\frac{1}{3}}dx$$

```
[In] int(x^2*(c*sin(a+b*x^n)^3)^(1/3),x)
[Out] int(x^2*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Fricas [F]

$$\int x^2 \sqrt[3]{c \sin^3(a + bx^n)}\, dx = \int \left(c \sin\left(bx^n + a\right)^3\right)^{\frac{1}{3}} x^2\, dx$$

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x^2, x)
```

Sympy [F]

$$\int x^2 \sqrt[3]{c \sin^3(a + bx^n)}\, dx = \int x^2 \sqrt[3]{c \sin^3(a + bx^n)}\, dx$$

```
[In] integrate(x**2*(c*sin(a+b*x**n)**3)**(1/3),x)
[Out] Integral(x**2*(c*sin(a + b*x**n)**3)**(1/3), x)
```

Maxima [F]

$$\int x^2 \sqrt[3]{c \sin^3(a + bx^n)}\, dx = \int \left(c \sin\left(bx^n + a\right)^3\right)^{\frac{1}{3}} x^2\, dx$$

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^2, x)
```

Giac [F]

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \int \left(c \sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}} x^2\,dx$$

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \int x^2 \left(c \sin\left(a+b\,x^n\right)^3\right)^{1/3} dx$$

```
[In] int(x^2*(c*sin(a + b*x^n)^3)^(1/3),x)
[Out] int(x^2*(c*sin(a + b*x^n)^3)^(1/3), x)
```

3.329 $\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 143

$$\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx = \frac{ie^{ia}x^2(-ibx^n)^{-2/n}\csc(a+bx^n)\,\Gamma\left(\frac{2}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^2(ibx^n)^{-2/n}\csc(a+bx^n)\,\Gamma\left(\frac{2}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x^2*csc(a+b*x^n)*GAMMA(2/n,-I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/3
)/n/((-I*b*x^n)^(2/n))-1/2*I*x^2*csc(a+b*x^n)*GAMMA(2/n,I*b*x^n)*(c*sin(a+b
*x^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^(2/n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6852, 3504, 2250}

$$\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx = \frac{ie^{ia}x^2(-ibx^n)^{-2/n}\Gamma\left(\frac{2}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^2(ibx^n)^{-2/n}\Gamma\left(\frac{2}{n},ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

```
[In] Int[x*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x^2*Csc[a + b*x^n]*Gamma[2/n, (-I)*b*x^n]*(c*Sin[a + b*x^n]^
3)^(1/3))/(n*((-I)*b*x^n)^(2/n)) - ((I/2)*x^2*Csc[a + b*x^n]*Gamma[2/n, I*b
*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^(2/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int x\sin\left(a+bx^n\right)\,dx \\
&= \frac{1}{2}\left(i\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int e^{-ia-ibx^n}x\,dx \\
&\quad -\frac{1}{2}\left(i\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int e^{ia+ibx^n}x\,dx \\
&= \frac{ie^{ia}x^2(-ibx^n)^{-2/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{2}{n},-ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n} \\
&\quad -\frac{ie^{-ia}x^2(ibx^n)^{-2/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{2}{n},ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int x\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx$$

$$= \frac{ix^2(b^2x^{2n})^{-2/n}\csc\left(a+bx^n\right)\left(-(-ibx^n)^{2/n}\Gamma\left(\frac{2}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{2/n}\Gamma\left(\frac{2}{n},-ibx^n\right)(\cos(a)\right.}{2n}$$

```
[In] Integrate[x*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*x^2*Csc[a + b*x^n]*(-(((-I)*b*x^n)^(2/n)*Gamma[2/n, I*b*x^n]*(Cos[a]
 - I*Sin[a])) + (I*b*x^n)^(2/n)*Gamma[2/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))
*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^(2/n))
```

Maple [F]

$$\int x\left(c\left(\sin^{3}\left(a+b\,x^{n}\right)\right)\right)^{\frac{1}{3}}dx$$

```
[In] int(x*(c*sin(a+b*x^n)^3)^(1/3),x)
[Out] int(x*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Fricas [F]

$$\int x\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx=\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}x\,dx$$

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x, x)
```

Sympy [F]

$$\int x\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx=\int x\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx$$

```
[In] integrate(x*(c*sin(a+b*x**n)**3)**(1/3),x)
[Out] Integral(x*(c*sin(a + b*x**n)**3)**(1/3), x)
```

Maxima [F]

$$\int x\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx=\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}x\,dx$$

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x, x)
```

Giac [F]

$$\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx=\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}x\,dx$$

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx=\int x\left(c\sin\left(a+b\,x^n\right)^3\right)^{1/3}\,dx$$

```
[In] int(x*(c*sin(a + b*x^n)^3)^(1/3),x)
[Out] int(x*(c*sin(a + b*x^n)^3)^(1/3), x)
```

3.330 $\int \sqrt[3]{c \sin^3(a+bx^n)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \frac{ie^{ia}x(-ibx^n)^{-1/n}\csc(a+bx^n)\,\Gamma\left(\frac{1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x(ibx^n)^{-1/n}\csc(a+bx^n)\,\Gamma\left(\frac{1}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x*csc(a+b*x^n)*GAMMA(1/n,-I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/3)/
n/((-I*b*x^n)^(1/n))-1/2*I*x*csc(a+b*x^n)*GAMMA(1/n,I*b*x^n)*(c*sin(a+b*x^n
)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6852, 3446, 2239}

$$\int \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \frac{ie^{ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

```
[In] Int[(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x*Csc[a + b*x^n]*Gamma[n^(-1), (-I)*b*x^n]*(c*Sin[a + b*x^n]
^3)^(1/3))/(n*((-I)*b*x^n)^n^(-1)) - ((I/2)*x*Csc[a + b*x^n]*Gamma[n^(-1),
I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int\sin\left(a+bx^n\right)\,dx\\
&= \frac{1}{2}\left(i\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int e^{-ia-ibx^n}\,dx\\
&\quad-\frac{1}{2}\left(i\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int e^{ia+ibx^n}\,dx\\
&= \frac{ie^{ia}x(-ibx^n)^{-1/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{1}{n},-ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}\\
&\quad-\frac{ie^{-ia}x(ibx^n)^{-1/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{1}{n},ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

$$\int\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx$$

$$=\frac{ix(b^2x^{2n})^{-1/n}\csc\left(a+bx^n\right)\left(-(-ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-ibx^n\right)(\cos(a)+i\,s\right.}{2n}$$

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*x*Csc[a + b*x^n]*(-(((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n]*(Cos[
a] - I*Sin[a])) + (I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x^n]*(Cos[a] + I*Si
n[a]))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^n^(-1))
```

Maple [F]

$$\int \left(c\left(\sin^{3}\left(a+b\,x^{n}\right)\right)\right)^{\frac{1}{3}}dx$$

```
[In] int((c*sin(a+b*x^n)^3)^(1/3),x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}\,dx = \int \left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}\,dx = \int \sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}\,dx$$

```
[In] integrate((c*sin(a+b*x**n)**3)**(1/3),x)
```

```
[Out] Integral((c*sin(a + b*x**n)**3)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}\,dx = \int \left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \int \left(c\sin\left(a+b\,x^n\right)^3\right)^{1/3}\,dx$$

```
[In] int((c*sin(a + b*x^n)^3)^(1/3),x)
[Out] int((c*sin(a + b*x^n)^3)^(1/3), x)
```

3.331 $\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx = \frac{\text{CosIntegral}(bx^n)\csc(a+bx^n)\sin(a)\sqrt[3]{c\sin^3(a+bx^n)}}{n} + \frac{\cos(a)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\text{Si}(bx^n)}{n}$$

```
[Out] cos(a)*csc(a+b*x^n)*Si(b*x^n)*(c*sin(a+b*x^n)^3)^(1/3)/n+Ci(b*x^n)*csc(a+b*
x^n)*sin(a)*(c*sin(a+b*x^n)^3)^(1/3)/n
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 3458, 3457, 3456}

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx = \frac{\sin(a)\,\text{CosIntegral}(bx^n)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{n} + \frac{\cos(a)\text{Si}(bx^n)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{n}$$

```
[In] Int[(c*Sin[a + b*x^n]^3)^(1/3)/x,x]
```

```
[Out] (CosIntegral[b*x^n]*Csc[a + b*x^n]*Sin[a]*(c*Sin[a + b*x^n]^3)^(1/3))/n + (
Cos[a]*Csc[a + b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3)*SinIntegral[b*x^n])/n
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int\frac{\sin\left(a+bx^n\right)}{x}\,dx \\
&= \left(\cos(a)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int\frac{\sin\left(bx^n\right)}{x}\,dx \\
&\quad+\left(\csc\left(a+bx^n\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int\frac{\cos\left(bx^n\right)}{x}\,dx \\
&= \frac{\text{CosIntegral}\left(bx^n\right)\csc\left(a+bx^n\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{n} \\
&\quad+\frac{\cos(a)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,\text{Si}\left(bx^n\right)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx = \frac{\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}(\text{CosIntegral}(bx^n)\sin(a)+\cos(a)\text{Si}(bx^n))}{n}$$

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(1/3)/x,x]
```

```
[Out] (Csc[a + b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3)*(CosIntegral[b*x^n]*Sin[a] + Cos
[a]*SinIntegral[b*x^n]))/n
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.83 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.79

method	result	siz
risch	$-\frac{\left(ic\,\mathrm{e}^{-3i(a+b\,x^n)}\left(\mathrm{e}^{2i(a+b\,x^n)}-1\right)^3\right)^{\frac{1}{3}}\left(i\mathrm{e}^{ib\,x^n}\pi\,\mathrm{csgn}(b\,x^n)-2i\mathrm{e}^{ib\,x^n}\,\mathrm{Si}(b\,x^n)+\mathrm{Ei}_1(-ib\,x^n)\mathrm{e}^{i(b\,x^n+2a)}-\mathrm{e}^{ib\,x^n}\,\mathrm{Ei}_1(-ib\,x^n)\right)}{2n\left(\mathrm{e}^{2i(a+b\,x^n)}-1\right)}$	131

```
[In] int((c*sin(a+b*x^n)^3)^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(I*c*exp(-3*I*(a+b*x^n))*(exp(2*I*(a+b*x^n))-1)^3)^(1/3)*(I*exp(I*b*x^
n)*Pi*csgn(b*x^n)-2*I*exp(I*b*x^n)*Si(b*x^n)+Ei(1,-I*b*x^n)*exp(I*(b*x^n+2*
a))-exp(I*b*x^n)*Ei(1,-I*b*x^n))/n/(exp(2*I*(a+b*x^n))-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx = \frac{\left(-i\,\mathrm{Ei}(i\,bx^n)\,e^{(i\,a)}+i\,\mathrm{Ei}(-i\,bx^n)\,e^{(-i\,a)}\right)\left(-\left(c\cos\left(bx^n+a\right)^2-c\right)\sin\left(bx^n+a\right)\right)^{\frac{1}{3}}}{2\,n\sin\left(bx^n+a\right)}$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*Ei(I*b*x^n)*e^(I*a) + I*Ei(-I*b*x^n)*e^(-I*a))*(-(c*cos(b*x^n + a)^
2 - c)*sin(b*x^n + a))^(1/3)/(n*sin(b*x^n + a))
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx = \int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx$$

```
[In] integrate((c*sin(a+b*x**n)**3)**(1/3)/x,x)
[Out] Integral((c*sin(a + b*x**n)**3)**(1/3)/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx = \frac{\left(\left((\sqrt{3}+i)\text{Ei}(i\,bx^n) - (\sqrt{3}+i)\text{Ei}(-i\,bx^n) - (\sqrt{3}-i)\text{Ei}\left(i\,be^{\left(n\overline{\log(x)}\right)}\right) + (\sqrt{3}-i)\text{Ei}\left(-i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\right.}{}$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x,x, algorithm="maxima")
[Out] 1/8*(((sqrt(3) + I)*Ei(I*b*x^n) - (sqrt(3) + I)*Ei(-I*b*x^n) - (sqrt(3) - I
)*Ei(I*b*e^(n*conjugate(log(x)))) + (sqrt(3) - I)*Ei(-I*b*e^(n*conjugate(lo
g(x)))))*cos(a) - ((-I*sqrt(3) + 1)*Ei(I*b*x^n) + (-I*sqrt(3) + 1)*Ei(-I*b*
x^n) + (I*sqrt(3) + 1)*Ei(I*b*e^(n*conjugate(log(x)))) + (I*sqrt(3) + 1)*Ei
(-I*b*e^(n*conjugate(log(x)))))*sin(a))*c^(1/3)/n
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx = \int \frac{\left(c\sin(bx^n+a)^3\right)^{\frac{1}{3}}}{x}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x,x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c \sin^3(a+bx^n)}}{x}\,dx = \int \frac{\left(c \sin(a+b\,x^n)^3\right)^{1/3}}{x}\,dx$$

```
[In] int((c*sin(a + b*x^n)^3)^(1/3)/x,x)
[Out] int((c*sin(a + b*x^n)^3)^(1/3)/x, x)
```

3.332 $\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 139

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx = \frac{ie^{ia}(-ibx^n)^{\frac{1}{n}}\csc(a+bx^n)\,\Gamma\left(-\frac{1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx} - \frac{ie^{-ia}(ibx^n)^{\frac{1}{n}}\csc(a+bx^n)\,\Gamma\left(-\frac{1}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx}$$

```
[Out] 1/2*I*exp(I*a)*(-I*b*x^n)^(1/n)*csc(a+b*x^n)*GAMMA(-1/n,-I*b*x^n)*(c*sin(a+
b*x^n)^3)^(1/3)/n/x-1/2*I*(I*b*x^n)^(1/n)*csc(a+b*x^n)*GAMMA(-1/n,I*b*x^n)*
(c*sin(a+b*x^n)^3)^(1/3)/exp(I*a)/n/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6852, 3504, 2250}

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx = \frac{ie^{ia}(-ibx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx} - \frac{ie^{-ia}(ibx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx}$$

```
[In] Int[(c*Sin[a + b*x^n]^3)^(1/3)/x^2,x]
```

```
[Out] ((I/2)*E^(I*a)*((-I)*b*x^n)^n^(-1)*Csc[a + b*x^n]*Gamma[-n^(-1), (-I)*b*x^n
]*(c*Sin[a + b*x^n]^3)^(1/3))/(n*x) - ((I/2)*(I*b*x^n)^n^(-1)*Csc[a + b*x^n
]*Gamma[-n^(-1), I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*x)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int\frac{\sin(a+bx^n)}{x^2}\,dx\\
&= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int\frac{e^{-ia-ibx^n}}{x^2}\,dx\\
&\quad-\frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int\frac{e^{ia+ibx^n}}{x^2}\,dx\\
&= \frac{ie^{ia}(-ibx^n)^{\frac{1}{n}}\csc(a+bx^n)\,\Gamma\left(-\frac{1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx}\\
&\quad-\frac{ie^{-ia}(ibx^n)^{\frac{1}{n}}\csc(a+bx^n)\,\Gamma\left(-\frac{1}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

$$\int\frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx$$

$$=\frac{i\csc(a+bx^n)\left(-(ibx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},ibx^n\right)(\cos(a)-i\sin(a))+(-ibx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-ibx^n\right)(\cos(a)+i\sin(a))\right)\sqrt[3]{\ldots}}{2nx}$$

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(1/3)/x^2,x]
```

```
[Out] ((I/2)*Csc[a + b*x^n]*(-((I*b*x^n)^n^(-1)*Gamma[-n^(-1), I*b*x^n]*(Cos[a] -
 I*Sin[a])) + ((-I)*b*x^n)^n^(-1)*Gamma[-n^(-1), (-I)*b*x^n]*(Cos[a] + I*Si
n[a]))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*x)
```

Maple [F]

$$\int \frac{\left(c \left(\sin^{3}\left(a +b \,x^{n}\right)\right)\right)^{\frac{1}{3}}}{x^{2}}d x$$

```
[In] int((c*sin(a+b*x^n)^3)^(1/3)/x^2,x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(1/3)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{c \sin^3(a+bx^n)}}{x^2} \, dx=\int \frac{\left(c \sin\left(b x^{n} + a\right)^{3}\right)^{\frac{1}{3}}}{x^{2}} \, dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c \sin^3(a+bx^n)}}{x^2} \, dx=\int \frac{\sqrt[3]{c \sin^{3}{\left(a + b x^{n} \right)}}}{x^{2}}\, dx$$

```
[In] integrate((c*sin(a+b*x**n)**3)**(1/3)/x**2,x)
```

```
[Out] Integral((c*sin(a + b*x**n)**3)**(1/3)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c \sin^3(a+bx^n)}}{x^2} \, dx=\int \frac{\left(c \sin\left(b x^{n} + a\right)^{3}\right)^{\frac{1}{3}}}{x^{2}} \, dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}}{x^2}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^2,x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx = \int \frac{\left(c\sin\left(a+b\,x^n\right)^3\right)^{1/3}}{x^2}\,dx$$

```
[In] int((c*sin(a + b*x^n)^3)^(1/3)/x^2,x)
[Out] int((c*sin(a + b*x^n)^3)^(1/3)/x^2, x)
```

3.333 $\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = \frac{ie^{ia}(-ibx^n)^{2/n}\csc(a+bx^n)\,\Gamma\left(-\frac{2}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx^2} - \frac{ie^{-ia}(ibx^n)^{2/n}\csc(a+bx^n)\,\Gamma\left(-\frac{2}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx^2}$$

```
[Out] 1/2*I*exp(I*a)*(-I*b*x^n)^(2/n)*csc(a+b*x^n)*GAMMA(-2/n,-I*b*x^n)*(c*sin(a+
b*x^n)^3)^(1/3)/n/x^2-1/2*I*(I*b*x^n)^(2/n)*csc(a+b*x^n)*GAMMA(-2/n,I*b*x^n
)*(c*sin(a+b*x^n)^3)^(1/3)/exp(I*a)/n/x^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6852, 3504, 2250}

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = \frac{ie^{ia}(-ibx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx^2} - \frac{ie^{-ia}(ibx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx^2}$$

```
[In] Int[(c*Sin[a + b*x^n]^3)^(1/3)/x^3,x]
```

```
[Out] ((I/2)*E^(I*a)*((-I)*b*x^n)^(2/n)*Csc[a + b*x^n]*Gamma[-2/n, (-I)*b*x^n]*(c
*Sin[a + b*x^n]^3)^(1/3))/(n*x^2) - ((I/2)*(I*b*x^n)^(2/n)*Csc[a + b*x^n]*G
amma[-2/n, I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*x^2)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int\frac{\sin\left(a+bx^n\right)}{x^3}\,dx\\
&= \frac{1}{2}\left(i\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int\frac{e^{-ia-ibx^n}}{x^3}\,dx\\
&\quad-\frac{1}{2}\left(i\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\right)\int\frac{e^{ia+ibx^n}}{x^3}\,dx\\
&= \frac{ie^{ia}\left(-ibx^n\right)^{2/n}\csc\left(a+bx^n\right)\Gamma\left(-\frac{2}{n},-ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2nx^2}\\
&\quad-\frac{ie^{-ia}\left(ibx^n\right)^{2/n}\csc\left(a+bx^n\right)\Gamma\left(-\frac{2}{n},ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2nx^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$
\int\frac{\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{x^3}\,dx
$$

$$
=\frac{i\csc\left(a+bx^n\right)\left(-\left(ibx^n\right)^{2/n}\Gamma\left(-\frac{2}{n},ibx^n\right)\left(\cos(a)-i\sin(a)\right)+\left(-ibx^n\right)^{2/n}\Gamma\left(-\frac{2}{n},-ibx^n\right)\left(\cos(a)+i\sin(a)\right)\right)}{2nx^2}
$$

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(1/3)/x^3,x]
```

```
[Out] ((I/2)*Csc[a + b*x^n]*(-((I*b*x^n)^(2/n)*Gamma[-2/n, I*b*x^n]*(Cos[a] - I*S
in[a])) + ((-I)*b*x^n)^(2/n)*Gamma[-2/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))*(
c*Sin[a + b*x^n]^3)^(1/3))/(n*x^2)
```

Maple [F]

$$\int \frac{\left(c\left(\sin^{3}\left(a+b\,x^{n}\right)\right)\right)^{\frac{1}{3}}}{x^{3}}dx$$

```
[In] int((c*sin(a+b*x^n)^3)^(1/3)/x^3,x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(1/3)/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}}{x^{3}}\,dx=\int \frac{\left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{1}{3}}}{x^{3}}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}}{x^{3}}\,dx=\int \frac{\sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}}{x^{3}}\,dx$$

```
[In] integrate((c*sin(a+b*x**n)**3)**(1/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x**n)**3)**(1/3)/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}}{x^{3}}\,dx=\int \frac{\left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{1}{3}}}{x^{3}}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}}{x^3}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^3,x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = \int \frac{\left(c\sin\left(a+b\,x^n\right)^3\right)^{1/3}}{x^3}\,dx$$

```
[In] int((c*sin(a + b*x^n)^3)^(1/3)/x^3,x)
[Out] int((c*sin(a + b*x^n)^3)^(1/3)/x^3, x)
```

3.334 $\int x^m (c\sin^3(a+bx))^{2/3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 169

$$\int x^m \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \frac{x^{1+m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(1+m)} + \frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\csc^2(a+bx)\Gamma(1+m,-2ibx)\left(c\sin^3(a+bx)\right)^{2/3}}{b} - \frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\csc^2(a+bx)\Gamma(1+m,2ibx)\left(c\sin^3(a+bx)\right)^{2/3}}{b}$$

```
[Out] 1/2*x^(1+m)*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)/(1+m)+I*2^(-3-m)*exp(2*I*a)
*x^m*csc(b*x+a)^2*GAMMA(1+m,-2*I*b*x)*(c*sin(b*x+a)^3)^(2/3)/b/((-I*b*x)^m)
-I*2^(-3-m)*x^m*csc(b*x+a)^2*GAMMA(1+m,2*I*b*x)*(c*sin(b*x+a)^3)^(2/3)/b/ex
p(2*I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6852, 3393, 3388, 2212}

$$\int x^m \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \frac{x^{m+1}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(m+1)} + \frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\csc^2(a+bx)\Gamma(m+1,-2ibx)\left(c\sin^3(a+bx)\right)^{2/3}}{b} - \frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\csc^2(a+bx)\Gamma(m+1,2ibx)\left(c\sin^3(a+bx)\right)^{2/3}}{b}$$

```
[In] Int[x^m*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (x^(1 + m)*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/(2*(1 + m)) + (I*2^(-3
- m)*E^((2*I)*a)*x^m*Csc[a + b*x]^2*Gamma[1 + m, (-2*I)*b*x]*(c*Sin[a + b*x
]^3)^(2/3))/(b*((-I)*b*x)^m) - (I*2^(-3 - m)*x^m*Csc[a + b*x]^2*Gamma[1 + m
, (2*I)*b*x]*(c*Sin[a + b*x]^3)^(2/3))/(b*E^((2*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^m\sin^2(a+bx)\,dx\\
&= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int\left(\frac{x^m}{2}-\frac{1}{2}x^m\cos(2a+2bx)\right)dx\\
&= \frac{x^{1+m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(1+m)}\\
&\quad-\frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^m\cos(2a+2bx)\,dx
\end{aligned}$$

$$= \frac{x^{1+m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(1+m)} - \frac{1}{4}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int e^{-i(2a+2bx)}x^m\,dx - \frac{1}{4}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int e^{i(2a+2bx)}x^m\,dx$$

$$= \frac{x^{1+m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(1+m)} + \frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\csc^2(a+bx)\Gamma(1+m,-2ibx)\left(c\sin^3(a+bx)\right)^{2/3}}{b} - \frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\csc^2(a+bx)\Gamma(1+m,2ibx)\left(c\sin^3(a+bx)\right)^{2/3}}{b}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.84

$$\int x^m\left(c\sin^3(a+bx)\right)^{2/3}\,dx = \frac{2^{-3-m}x^m(b^2x^2)^{-m}\csc^2(a+bx)\left(2^{2+m}bx(b^2x^2)^m - i(1+m)(-ibx)^m\Gamma(1+m,2ibx)(\cos(a)-\right.}{b(1+m)}$$

```
[In] Integrate[x^m*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (2^(-3 - m)*x^m*Csc[a + b*x]^2*(2^(2 + m)*b*x*(b^2*x^2)^m - I*(1 + m)*((-I)
*b*x)^m*Gamma[1 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 + I*(1 + m)*(I*b*x)^m
*Gamma[1 + m, (-2*I)*b*x]*(Cos[a] + I*Sin[a])^2)*(c*Sin[a + b*x]^3)^(2/3))/
(b*(1 + m)*(b^2*x^2)^m)
```

Maple [F]

$$\int x^m\left(c\left(\sin^3(bx+a)\right)\right)^{\frac{2}{3}}dx$$

```
[In] int(x^m*(c*sin(b*x+a)^3)^(2/3),x)
```

```
[Out] int(x^m*(c*sin(b*x+a)^3)^(2/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.66

$$\int x^m \left(c\sin^3(a+bx)\right)^{2/3}\,dx =$$

$$-\frac{\left(4\,bxx^m-(i\,m+i)e^{(-m\log(2i\,b)-2i\,a)}\Gamma(m+1,2i\,bx)-(-i\,m-i)e^{(-m\log(-2i\,b)+2i\,a)}\Gamma(m+1,-2i\,bx)\right)\left(-(c\,\mathrm{co}\right.}{8\left((bm+b)\cos\left(bx+a\right)^2-bm-b\right)}$$

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*x^m - (I*m + I)*e^(-m*log(2*I*b) - 2*I*a)*gamma(m + 1, 2*I*b*x)
 - (-I*m - I)*e^(-m*log(-2*I*b) + 2*I*a)*gamma(m + 1, -2*I*b*x))*(-(c*cos(b
*x + a)^2 - c)*sin(b*x + a))^(2/3)/((b*m + b)*cos(b*x + a)^2 - b*m - b)
```

Sympy [F]

$$\int x^m \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int x^m\left(c\sin^3\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate(x**m*(c*sin(b*x+a)**3)**(2/3),x)
```

```
[Out] Integral(x**m*(c*sin(a + b*x)**3)**(2/3), x)
```

Maxima [F]

$$\int x^m \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}}x^m\,dx$$

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/4*((m + 1)*integrate(x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + log(x)))*c^
(2/3)/(m + 1)
```

Giac [F]

$$\int x^m \left(c\sin^3(a+bx)\right)^{2/3} dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}} x^m \, dx$$

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a)^3)^(2/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(c\sin^3(a+bx)\right)^{2/3} dx = \int x^m \left(c\sin\left(a+b\,x\right)^3\right)^{2/3} dx$$

```
[In] int(x^m*(c*sin(a + b*x)^3)^(2/3),x)
[Out] int(x^m*(c*sin(a + b*x)^3)^(2/3), x)
```

3.335 $\int x^3 (c\sin^3(a+bx))^{2/3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 165

$$\int x^3 \left(c\sin^3(a+bx)\right)^{2/3} dx = -\frac{3(c\sin^3(a+bx))^{2/3}}{8b^4} + \frac{3x^2(c\sin^3(a+bx))^{2/3}}{4b^2} + \frac{3x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3} - \frac{x^3\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b} - \frac{3x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{8b^2} + \frac{1}{8}x^4\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

```
[Out] -3/8*(c*sin(b*x+a)^3)^(2/3)/b^4+3/4*x^2*(c*sin(b*x+a)^3)^(2/3)/b^2+3/4*x*co
t(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b^3-1/2*x^3*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/
3)/b-3/8*x^2*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)/b^2+1/8*x^4*csc(b*x+a)^2*(
c*sin(b*x+a)^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6852, 3392, 30, 3391}

$$\int x^3 \left(c\sin^3(a+bx)\right)^{2/3} dx = -\frac{3(c\sin^3(a+bx))^{2/3}}{8b^4} + \frac{3x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3} + \frac{3x^2(c\sin^3(a+bx))^{2/3}}{4b^2} - \frac{3x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{8b^2} + \frac{1}{8}x^4\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} - \frac{x^3\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}$$

```
[In] Int[x^3*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (-3*(c*Sin[a + b*x]^3)^(2/3))/(8*b^4) + (3*x^2*(c*Sin[a + b*x]^3)^(2/3))/(4
*b^2) + (3*x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3))/(4*b^3) - (x^3*Cot[a +
b*x]*(c*Sin[a + b*x]^3)^(2/3))/(2*b) - (3*x^2*Csc[a + b*x]^2*(c*Sin[a + b*x
]^3)^(2/3))/(8*b^2) + (x^4*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^3\sin^2(a+bx)\,dx\\ &= \frac{3x^2\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{x^3\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}\\ &\quad+\frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^3\,dx-\frac{\left(3\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x\sin^2(a+bx\end{aligned}$$

$$2b^2$$

$$= -\frac{3(c\sin^3(a+bx))^{2/3}}{8b^4} + \frac{3x^2(c\sin^3(a+bx))^{2/3}}{4b^2} + \frac{3x\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{4b^3} - \frac{x^3\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{2b} + \frac{1}{8}x^4\csc^2(a+bx)\,\left(c\sin^3(a+bx)\right)^{2/3} - \frac{\left(3\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}\right)\int x\,dx}{4b^2}$$

$$= -\frac{3(c\sin^3(a+bx))^{2/3}}{8b^4} + \frac{3x^2(c\sin^3(a+bx))^{2/3}}{4b^2} + \frac{3x\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{4b^3} - \frac{x^3\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{2b} - \frac{3x^2\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}}{8b^2} + \frac{1}{8}x^4\csc^2(a+bx)\,\left(c\sin^3(a+bx)\right)^{2/3}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.48

$$\int x^3\left(c\sin^3(a+bx)\right)^{2/3}\,dx = \frac{\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}\,(2b^4x^4+(3-6b^2x^2)\cos(2(a+bx))+(6bx-4b^3x^3)\sin(2(a+b}{16b^4}$$

```
[In] Integrate[x^3*(c*Sin[a + b*x]^3)^(2/3),x]
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(2*b^4*x^4 + (3 - 6*b^2*x^2)*Cos[2
*(a + b*x)] + (6*b*x - 4*b^3*x^3)*Sin[2*(a + b*x)]))/(16*b^4)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{x^4\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{2}{3}}\mathrm{e}^{2i(bx+a)}}{8\left(\mathrm{e}^{2i(bx+a)}-1\right)^2} - \frac{i(4b^3x^3+6ix^2b^2-6bx-3i)\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{2}{3}}\mathrm{e}^{4i(bx+a)}}{32b^4\left(\mathrm{e}^{2i(bx+a)}-1\right)^2} + \frac{i\left(ic\,\mathrm{e}^{-3\right.}$

```
[In] int(x^3*(c*sin(b*x+a)^3)^(2/3),x,method=_RETURNVERBOSE)
[Out] -1/8*x^4/(exp(2*I*(b*x+a))-1)^2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)
^3)^(2/3)*exp(2*I*(b*x+a))-1/32*I/b^4*(4*b^3*x^3+6*I*x^2*b^2-6*b*x-3*I)/(ex
p(2*I*(b*x+a))-1)^2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*ex
```

```
p(4*I*(b*x+a))+1/32*I*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)/
(exp(2*I*(b*x+a))-1)^2*(4*b^3*x^3-6*I*b^2*x^2-6*b*x+3*I)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.67

$$\int x^3\left(c\sin^3(a+bx)\right)^{2/3}\,dx =$$

$$-\frac{\left(2\,b^4x^4+6\,b^2x^2-6\left(2\,b^2x^2-1\right)\cos\left(bx+a\right)^2-4\left(2\,b^3x^3-3\,bx\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-3\right)\left(-\left(c\cos\left(b\right.\right.\right.}{16\left(b^4\cos\left(bx+a\right)^2-b^4\right)}$$

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
[Out] -1/16*(2*b^4*x^4 + 6*b^2*x^2 - 6*(2*b^2*x^2 - 1)*cos(b*x + a)^2 - 4*(2*b^3*
x^3 - 3*b*x)*cos(b*x + a)*sin(b*x + a) - 3)*(-(c*cos(b*x + a)^2 - c)*sin(b*
x + a))^(2/3)/(b^4*cos(b*x + a)^2 - b^4)
```

Sympy [F]

$$\int x^3\left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int x^3\left(c\sin^3\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate(x**3*(c*sin(b*x+a)**3)**(2/3),x)
[Out] Integral(x**3*(c*sin(a + b*x)**3)**(2/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(141) = 282.

Time = 0.33 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.73

$$\int x^3\left(c\sin^3(a+bx)\right)^{2/3}\,dx =$$

$$-\frac{32\left(c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)-\frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1}-\frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1}\right)a^3+6\left(2\left(bx+a\right)^2-2\left(bx+a\right)\sin\left(2\,bx+2\,a\right)\right.}{}$$

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/32*(32*(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b
*x + a)/(cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(
2*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4
 + 1))*a^3 + 6*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x +
2*a))*a^2*c^(2/3) - 2*(4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*
(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*c^(2/3) + (2*(b*x + a)^4 - 3*(2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2
*a))*c^(2/3))/b^4
```

Giac [F]

$$\int x^3\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\int\left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}}x^3\,dx$$

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a)^3)^(2/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\int x^3\,\left(c\sin\left(a+b\,x\right)^3\right)^{2/3}\,dx$$

```
[In] int(x^3*(c*sin(a + b*x)^3)^(2/3),x)
[Out] int(x^3*(c*sin(a + b*x)^3)^(2/3), x)
```

3.336 $\int x^2(c\sin^3(a+bx))^{2/3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 139

$$\int x^2\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\frac{x(c\sin^3(a+bx))^{2/3}}{2b^2}+\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}-\frac{x^2\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}-\frac{x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}+\frac{1}{6}x^3\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

```
[Out] 1/2*x*(c*sin(b*x+a)^3)^(2/3)/b^2+1/4*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b^3-
1/2*x^2*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b-1/4*x*csc(b*x+a)^2*(c*sin(b*x+a
)^3)^(2/3)/b^2+1/6*x^3*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6852, 3392, 30, 2715, 8}

$$\int x^2\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}+\frac{x(c\sin^3(a+bx))^{2/3}}{2b^2}-\frac{x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}+\frac{1}{6}x^3\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{x^2\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}$$

```
[In] Int[x^2*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (x*(c*Sin[a + b*x]^3)^(2/3))/(2*b^2) + (Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/
3))/(4*b^3) - (x^2*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3))/(2*b) - (x*Csc[a
+ b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/(4*b^2) + (x^3*Csc[a + b*x]^2*(c*Sin[a +
 b*x]^3)^(2/3))/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^2\sin^2(a+bx)\,dx \\
&= \frac{x\left(c\sin^3(a+bx)\right)^{2/3}}{2b^2}-\frac{x^2\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b} \\
&\quad +\frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^2\,dx-\frac{\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int\sin^2(a+bx)\,dx}{2b^2}
\end{aligned}$$

$$= \frac{x(c\sin^3(a+bx))^{2/3}}{2b^2} + \frac{\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{4b^3} - \frac{x^2\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{2b}$$
$$+ \frac{1}{6}x^3\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} - \frac{\left(\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}\right)\int 1\,dx}{4b^2}$$
$$= \frac{x(c\sin^3(a+bx))^{2/3}}{2b^2} + \frac{\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{4b^3}$$
$$- \frac{x^2\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{2b} - \frac{x\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}}{4b^2}$$
$$+ \frac{1}{6}x^3\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.50

$$\int x^2\left(c\sin^3(a+bx)\right)^{2/3}\,dx = \frac{\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}\,(4b^3x^3 - 6bx\cos(2(a+bx)) + (3-6b^2x^2)\sin(2(a+bx)))}{24b^3}$$

```
[In] Integrate[x^2*(c*Sin[a + b*x]^3)^(2/3),x]
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(4*b^3*x^3 - 6*b*x*Cos[2*(a + b*x)
] + (3 - 6*b^2*x^2)*Sin[2*(a + b*x)]))/(24*b^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{x^3\left(ic\,e^{-3i(bx+a)}\left(e^{2i(bx+a)}-1\right)^3\right)^{\frac{2}{3}}e^{2i(bx+a)}}{6\left(e^{2i(bx+a)}-1\right)^2} - \frac{i(2x^2b^2+2ibx-1)\left(ic\,e^{-3i(bx+a)}\left(e^{2i(bx+a)}-1\right)^3\right)^{\frac{2}{3}}e^{4i(bx+a)}}{16b^3\left(e^{2i(bx+a)}-1\right)^2} + \frac{i\left(ic\,e^{-3i(bx+a}\right.}{}$

```
[In] int(x^2*(c*sin(b*x+a)^3)^(2/3),x,method=_RETURNVERBOSE)
[Out] -1/6*x^3/(exp(2*I*(b*x+a))-1)^2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)
^3)^(2/3)*exp(2*I*(b*x+a))-1/16*I/b^3*(2*x^2*b^2+2*I*b*x-1)/(exp(2*I*(b*x+a
))-1)^2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*exp(4*I*(b*x+a
))+1/16*I*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)/(exp(2*I*(b*
x+a))-1)^2*(2*x^2*b^2-2*I*b*x-1)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int x^2\left(c\sin^3(a+bx)\right)^{2/3}\,dx =$$
$$-\frac{\left(2\,b^3x^3-6\,bx\cos\left(bx+a\right)^2-3\left(2\,b^2x^2-1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+3\,bx\right)\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx\right.\right.}{12\left(b^3\cos\left(bx+a\right)^2-b^3\right)}$$

[In] integrate(x^2*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")

[Out] -1/12*(2*b^3*x^3 - 6*b*x*cos(b*x + a)^2 - 3*(2*b^2*x^2 - 1)*cos(b*x + a)*si
n(b*x + a) + 3*b*x)*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(b^3*cos(b
*x + a)^2 - b^3)

Sympy [F]

$$\int x^2\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\int x^2\left(c\sin^3\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

[In] integrate(x**2*(c*sin(b*x+a)**3)**(2/3),x)

[Out] Integral(x**2*(c*sin(a + b*x)**3)**(2/3), x)

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.58

$$\int x^2\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\frac{48\left(c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)-\frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1}-\frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1}\right)a^2+6\left(2\left(bx+a\right)^2-2\left(bx+a\right)\sin\left(\right.\right.}{}$$

[In] integrate(x^2*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")

[Out] 1/48*(48*(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b*
x + a)/(cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(2
*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4
+ 1))*a^2 + 6*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2
*a))*a*c^(2/3) - (4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x
+ a)^2 - 1)*sin(2*b*x + 2*a))*c^(2/3))/b^3

Giac [F]

$$\int x^2 \left(c \sin^3(a+bx)\right)^{2/3} dx = \int \left(c \sin\left(bx+a\right)^3\right)^{\frac{2}{3}} x^2 \, dx$$

[In] integrate(x^2*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")

[Out] integrate((c*sin(b*x + a)^3)^(2/3)*x^2, x)

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c \sin^3(a+bx)\right)^{2/3} dx = \int x^2 \left(c \sin\left(a+b\,x\right)^3\right)^{2/3} dx$$

[In] int(x^2*(c*sin(a + b*x)^3)^(2/3),x)

[Out] int(x^2*(c*sin(a + b*x)^3)^(2/3), x)

3.337 $\int x(c\sin^3(a+bx))^{2/3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{4}x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

```
[Out] 1/4*(c*sin(b*x+a)^3)^(2/3)/b^2-1/2*x*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b+1/
4*x^2*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6852, 3391, 30}

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}+\frac{1}{4}x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}$$

```
[In] Int[x*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (c*Sin[a + b*x]^3)^(2/3)/(4*b^2) - (x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3)
)/(2*b) + (x^2*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x\sin^2(a+bx)\,dx \\
&= \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b} \\
&\quad+\frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x\,dx \\
&= \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{4}x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx = -\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}(\cos(2(a+bx))+2bx(-bx+\sin(2(a+bx))))}{8b^2}$$

```
[In] Integrate[x*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] -1/8*(Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(Cos[2*(a + b*x)] + 2*b*x*(-(
b*x) + Sin[2*(a + b*x)])))/b^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.20

method	result
risch	$-\frac{x^{2}\left(i c\,\mathrm{e}^{-3 i (b x+a)}\left(\mathrm{e}^{2 i (b x+a)}-1\right)^{3}\right)^{\frac{2}{3}} \mathrm{e}^{2 i (b x+a)}}{4\left(\mathrm{e}^{2 i (b x+a)}-1\right)^{2}}-\frac{i (2 b x+i)\left(i c\,\mathrm{e}^{-3 i (b x+a)}\left(\mathrm{e}^{2 i (b x+a)}-1\right)^{3}\right)^{\frac{2}{3}} \mathrm{e}^{4 i (b x+a)}}{16 b^{2}\left(\mathrm{e}^{2 i (b x+a)}-1\right)^{2}}+\frac{i\left(i c\,\mathrm{e}^{-3 i (b x+a)}\left(\mathrm{e}^{2 i (b x+a)}\right.\right.}{16\left(\mathrm{e}^{2 i (b x+a)}-\right.}$

```
[In] int(x*(c*sin(b*x+a)^3)^(2/3),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2/(exp(2*I*(b*x+a))-1)^2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)
^3)^(2/3)*exp(2*I*(b*x+a))-1/16*I/b^2*(2*b*x+I)/(exp(2*I*(b*x+a))-1)^2*(I*c
*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*exp(4*I*(b*x+a))+1/16*I*(I
*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)/(exp(2*I*(b*x+a))-1)^2*(
2*b*x-I)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx = -\frac{\left(2\,b^{2}x^{2}-4\,bx\cos\left(bx+a\right)\sin\left(bx+a\right)-2\,\cos\left(bx+a\right)^{2}+1\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{2}{3}}}{8\,\left(b^{2}\cos\left(bx+a\right)^{2}-b^{2}\right)}$$

```
[In] integrate(x*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
[Out] -1/8*(2*b^2*x^2 - 4*b*x*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 + 1)*(
-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(b^2*cos(b*x + a)^2 - b^2)
```

Sympy [F]

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int x\left(c\sin^{3}\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate(x*(c*sin(b*x+a)**3)**(2/3),x)
[Out] Integral(x*(c*sin(a + b*x)**3)**(2/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(67) = 134.

Time = 0.51 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.05

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx = -\frac{16\left(c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)-\frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1}-\frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1}\right)a+\left(2\,(bx+a)^2-2\,(bx+a)\sin\left(2\,bx+2\,a\right)-\right.}{16\,b^2}$$

```
[In] integrate(x*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/16*(16*(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b
*x + a)/(cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(
2*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4
 + 1))*a + (2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
)*c^(2/3))/b^2
```

Giac [F]

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}} x\,dx$$

```
[In] integrate(x*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int x\,\left(c\sin\left(a+b\,x\right)^3\right)^{2/3}\,dx$$

```
[In] int(x*(c*sin(a + b*x)^3)^(2/3),x)
```

```
[Out] int(x*(c*sin(a + b*x)^3)^(2/3), x)
```

3.338 $\int \left(c\sin^3(a+bx)\right)^{2/3} dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \left(c\sin^3(a+bx)\right)^{2/3} dx = -\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b} + \frac{1}{2}x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

```
[Out] -1/2*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b+1/2*x*csc(b*x+a)^2*(c*sin(b*x+a)^3
)^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3286, 2715, 8}

$$\int \left(c\sin^3(a+bx)\right)^{2/3} dx = \frac{1}{2}x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} - \frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}$$

```
[In] Int[(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] -1/2*(Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3))/b + (x*Csc[a + b*x]^2*(c*Sin[a
 + b*x]^3)^(2/3))/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \sin^2(a+bx)\,dx\\ &= -\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int 1\,dx\\ &= -\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{2}x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx=\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}(2(a+bx)-\sin(2(a+bx)))}{4b}$$

```
[In] Integrate[(c*Sin[a + b*x]^3)^(2/3),x]
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(2*(a + b*x) - Sin[2*(a + b*x)]))/
(4*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.87

method	result
risch	$-\frac{x\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{2}{3}}\mathrm{e}^{2i(bx+a)}}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}}-\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{2}{3}}\mathrm{e}^{4i(bx+a)}}{8b\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}}+\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{2}{3}}}{8\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}b}$

```
[In] int((c*sin(b*x+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/(exp(2*I*(b*x+a))-1)^2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3
)^(2/3)*exp(2*I*(b*x+a))-1/8*I/b/(exp(2*I*(b*x+a))-1)^2*(I*c*exp(-3*I*(b*x+
a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*exp(4*I*(b*x+a))+1/8*I*(I*c*exp(-3*I*(b*x
+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)/(exp(2*I*(b*x+a))-1)^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx = -\frac{(bx-\cos(bx+a)\sin(bx+a))\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{2}{3}}}{2\left(b\cos(bx+a)^2-b\right)}$$

```
[In] integrate((c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - cos(b*x + a)*sin(b*x + a))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a
))^(2/3)/(b*cos(b*x + a)^2 - b)
```

Sympy [F]

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int \left(c\sin^3\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((c*sin(b*x+a)**3)**(2/3),x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(2/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(47) = 94$.

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.11

$$\int \left(c\sin^3(a+bx)\right)^{2/3} dx = \frac{c^{\frac{2}{3}} \arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right) - \frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1} - \frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2} + \frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4} + 1}}{b}$$

```
[In] integrate((c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] (c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b*x + a)/(c
os(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(2*sin(b*x
+ a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4 + 1))/b
```

Giac [F]

$$\int \left(c\sin^3(a+bx)\right)^{2/3} dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}} dx$$

```
[In] integrate((c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c\sin^3(a+bx)\right)^{2/3} dx = \int \left(c\sin\left(a+b\,x\right)^3\right)^{2/3} dx$$

```
[In] int((c*sin(a + b*x)^3)^(2/3),x)
```

```
[Out] int((c*sin(a + b*x)^3)^(2/3), x)
```

3.339 $\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 99

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx = -\frac{1}{2}\cos(2a)\,\text{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} + \frac{1}{2}\csc^2(a+bx)\log(x)\left(c\sin^3(a+bx)\right)^{2/3} + \frac{1}{2}\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3}\text{Si}(2bx)$$

```
[Out] -1/2*Ci(2*b*x)*cos(2*a)*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)+1/2*csc(b*x+a)^
2*ln(x)*(c*sin(b*x+a)^3)^(2/3)+1/2*csc(b*x+a)^2*Si(2*b*x)*sin(2*a)*(c*sin(b
*x+a)^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6852, 3393, 3384, 3380, 3383}

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx = -\frac{1}{2}\cos(2a)\,\text{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} + \frac{1}{2}\sin(2a)\text{Si}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} + \frac{1}{2}\log(x)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

```
[In] Int[(c*Sin[a + b*x]^3)^(2/3)/x,x]
```

```
[Out] -1/2*(Cos[2*a]*CosIntegral[2*b*x]*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))
+ (Csc[a + b*x]^2*Log[x]*(c*Sin[a + b*x]^3)^(2/3))/2 + (Csc[a + b*x]^2*Sin[
2*a]*(c*Sin[a + b*x]^3)^(2/3)*SinIntegral[2*b*x])/2
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int\frac{\sin^2(a+bx)}{x}\,dx\\
&= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int\left(\frac{1}{2x}-\frac{\cos(2a+2bx)}{2x}\right)dx\\
&= \frac{1}{2}\csc^2(a+bx)\log(x)\left(c\sin^3(a+bx)\right)^{2/3}\\
&\quad-\frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int\frac{\cos(2a+2bx)}{x}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}\csc^2(a+bx)\log(x)\left(c\sin^3(a+bx)\right)^{2/3} \\
&\quad - \frac{1}{2}\left(\cos(2a)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int\frac{\cos(2bx)}{x}\,dx \\
&\quad + \frac{1}{2}\left(\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int\frac{\sin(2bx)}{x}\,dx \\
&= -\frac{1}{2}\cos(2a)\,\text{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} \\
&\quad + \frac{1}{2}\csc^2(a+bx)\log(x)\left(c\sin^3(a+bx)\right)^{2/3} \\
&\qquad + \frac{1}{2}\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3}\text{Si}(2bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.51

$$
\int\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}\,dx = \frac{1}{2}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-\cos(2a)\,\text{CosIntegral}(2bx)+\log(x)+\sin(2a)\text{Si}(2bx)\right)
$$

```
[In] Integrate[(c*Sin[a + b*x]^3)^(2/3)/x,x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-(Cos[2*a]*CosIntegral[2*b*x]) +
Log[x] + Sin[2*a]*SinIntegral[2*b*x]))/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.22

method	result	si
risch	$\frac{\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{2}{3}}\left(i\mathrm{e}^{2ibx}\pi\ \mathrm{csgn}(bx)-2i\mathrm{e}^{2ibx}\,\mathrm{Si}(2bx)-2\ln(x)\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2ibx}\,\mathrm{Ei}_1(-2ibx)-\mathrm{Ei}_1(-2ibx)\mathrm{e}^{2i(bx+2a)}\right)}{4\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}$	1

```
[In] int((c*sin(b*x+a)^3)^(2/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*(I*exp(2*I*b*x)*Pi
*csgn(b*x)-2*I*exp(2*I*b*x)*Si(2*b*x)-2*ln(x)*exp(2*I*(b*x+a))-exp(2*I*b*x)
*Ei(1,-2*I*b*x)-Ei(1,-2*I*b*x)*exp(2*I*(b*x+2*a)))/(exp(2*I*(b*x+a))-1)^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.64

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx = \frac{\left(\mathrm{Ei}(2i\,bx)\,e^{(2i\,a)} + \mathrm{Ei}(-2i\,bx)\,e^{(-2i\,a)} - 2\,\log(x)\right)\left(-\left(c\cos(bx+a)^2 - c\right)\sin(bx+ \cdots\right.}{4\left(\cos(bx+a)^2 - 1\right)}$$

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/4*(Ei(2*I*b*x)*e^(2*I*a) + Ei(-2*I*b*x)*e^(-2*I*a) - 2*log(x))*(-(c*cos(b
*x + a)^2 - c)*sin(b*x + a))^(2/3)/(cos(b*x + a)^2 - 1)
```

Sympy [F]

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx = \int \frac{(c\sin^3(a+bx))^{\frac{2}{3}}}{x}\,dx$$

```
[In] integrate((c*sin(b*x+a)**3)**(2/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(2/3)/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx =$$
$$-\frac{1}{8}\left((E_1(2i\,bx) + E_1(-2i\,bx))\cos(2\,a) + (-i\,E_1(2i\,bx) + i\,E_1(-2i\,bx))\sin(2\,a) + 2\,\log(bx)\right)c^{\frac{2}{3}}$$

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] -1/8*((exp_integral_e(1, 2*I*b*x) + exp_integral_e(1, -2*I*b*x))*cos(2*a) +
 (-I*exp_integral_e(1, 2*I*b*x) + I*exp_integral_e(1, -2*I*b*x))*sin(2*a) +
 2*log(b*x))*c^(2/3)
```

Giac [F]

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx = \int \frac{\left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}}}{x}\,dx$$

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x,x, algorithm="giac")
[Out] integrate((c*sin(b*x + a)^3)^(2/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx = \int \frac{\left(c\sin\left(a+b\,x\right)^3\right)^{2/3}}{x}\,dx$$

```
[In] int((c*sin(a + b*x)^3)^(2/3)/x,x)
[Out] int((c*sin(a + b*x)^3)^(2/3)/x, x)
```

3.340 $\int \frac{(c\sin^3(a+bx))^{2/3}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^2}\,dx = -\frac{(c\sin^3(a+bx))^{2/3}}{x} + b\,\text{CosIntegral}(2bx)\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3} + b\cos(2a)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\text{Si}(2bx)$$

```
[Out] -(c*sin(b*x+a)^3)^(2/3)/x+b*cos(2*a)*csc(b*x+a)^2*Si(2*b*x)*(c*sin(b*x+a)^3
)^(2/3)+b*Ci(2*b*x)*csc(b*x+a)^2*sin(2*a)*(c*sin(b*x+a)^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6852, 3394, 12, 3384, 3380, 3383}

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^2}\,dx = b\sin(2a)\,\text{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} + b\cos(2a)\text{Si}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} - \frac{(c\sin^3(a+bx))^{2/3}}{x}$$

```
[In] Int[(c*Sin[a + b*x]^3)^(2/3)/x^2,x]
```

```
[Out] -((c*Sin[a + b*x]^3)^(2/3)/x) + b*CosIntegral[2*b*x]*Csc[a + b*x]^2*Sin[2*a
]*(c*Sin[a + b*x]^3)^(2/3) + b*Cos[2*a]*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(
2/3)*SinIntegral[2*b*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin^2(a+bx)}{x^2}\,dx \\
&= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}+\left(2b\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin(2a+2bx)}{2x}\,dx \\
&= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}+\left(b\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin(2a+2bx)}{x}\,dx
\end{aligned}
$$

$$= -\frac{(c\sin^3(a+bx))^{2/3}}{x} + \left(b\cos(2a)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int\frac{\sin(2bx)}{x}\,dx + \left(b\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int\frac{\cos(2bx)}{x}\,dx$$

$$= -\frac{(c\sin^3(a+bx))^{2/3}}{x} + b\,\text{CosIntegral}(2bx)\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3} + b\cos(2a)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\text{Si}(2bx)$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int\frac{(c\sin^3(a+bx))^{2/3}}{x^2}\,dx = \frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}(-1+\cos(2(a+bx))+2bx\,\text{CosIntegral}(2bx)\sin}{2x}$$

```
[In] Integrate[(c*Sin[a + b*x]^3)^(2/3)/x^2,x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-1 + Cos[2*(a + b*x)] + 2*b*x*Cos
Integral[2*b*x]*Sin[2*a] + 2*b*x*Cos[2*a]*SinIntegral[2*b*x]))/(2*x)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{\left(ic\,e^{-3i(bx+a)}\left(e^{2i(bx+a)}-1\right)^3\right)^{\frac{2}{3}}\left(2i\,\text{Ei}_1(2ibx)e^{2ibx}bx-2i\,\text{Ei}_1(-2ibx)e^{2i(bx+2a)}bx-e^{4i(bx+a)}-1+2\,e^{2i(bx+a)}\right)}{4\left(e^{2i(bx+a)}-1\right)^2x}$	112

```
[In] int((c*sin(b*x+a)^3)^(2/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*(2*I*Ei(1,2*I*b*x)
*exp(2*I*b*x)*b*x-2*I*Ei(1,-2*I*b*x)*exp(2*I*(b*x+2*a))*b*x-exp(4*I*(b*x+a)
)-1+2*exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1)^2/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^2}\,dx = \frac{\left(i\,bx\mathrm{Ei}(2i\,bx)\,e^{(2i\,a)} - i\,bx\mathrm{Ei}(-2i\,bx)\,e^{(-2i\,a)} - 2\,\cos(bx+a)^2 + 2\right)\left(-(c\cos(bx+a$$

$$2\left(x\cos(bx+a)^2 - x\right)$$

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(I*b*x*Ei(2*I*b*x)*e^(2*I*a) - I*b*x*Ei(-2*I*b*x)*e^(-2*I*a) - 2*cos(b*
x + a)^2 + 2)*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(x*cos(b*x + a)^
2 - x)
```

Sympy [F]

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^2}\,dx = \int \frac{(c\sin^3(a+bx))^{\frac{2}{3}}}{x^2}\,dx$$

```
[In] integrate((c*sin(b*x+a)**3)**(2/3)/x**2,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(2/3)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.08

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^2}\,dx = \frac{\left(\left(\left(-i\sqrt{3}+1\right)E_2(2i\,bx) + \left(i\sqrt{3}+1\right)E_2(-2i\,bx)\right)\cos(2\,a)^3 - \left(\left(\sqrt{3}+i\right)E_2(2i\,bx)\right.\right.}{}$$

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*(((-I*sqrt(3) + 1)*exp_integral_e(2, 2*I*b*x) + (I*sqrt(3) + 1)*exp_in
tegral_e(2, -2*I*b*x))*cos(2*a)^3 - ((sqrt(3) + I)*exp_integral_e(2, 2*I*b*
x) + (sqrt(3) - I)*exp_integral_e(2, -2*I*b*x))*sin(2*a)^3 + (((-I*sqrt(3)
+ 1)*exp_integral_e(2, 2*I*b*x) + (I*sqrt(3) + 1)*exp_integral_e(2, -2*I*b*
x))*cos(2*a) - 4)*sin(2*a)^2 + ((I*sqrt(3) + 1)*exp_integral_e(2, 2*I*b*x)
+ (-I*sqrt(3) + 1)*exp_integral_e(2, -2*I*b*x))*cos(2*a) - 4*cos(2*a)^2 - (
((sqrt(3) + I)*exp_integral_e(2, 2*I*b*x) + (sqrt(3) - I)*exp_integral_e(2,
 -2*I*b*x))*cos(2*a)^2 - (sqrt(3) - I)*exp_integral_e(2, 2*I*b*x) - (sqrt(3
) + I)*exp_integral_e(2, -2*I*b*x))*sin(2*a))*b*c^(2/3)/(a*cos(2*a)^2 + a*s
in(2*a)^2 - (b*x + a)*(cos(2*a)^2 + sin(2*a)^2))
```

Giac [F]

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}}}{x^2}\,dx$$

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^2,x, algorithm="giac")
[Out] integrate((c*sin(b*x + a)^3)^(2/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(a+b\,x\right)^3\right)^{2/3}}{x^2}\,dx$$

```
[In] int((c*sin(a + b*x)^3)^(2/3)/x^2,x)
[Out] int((c*sin(a + b*x)^3)^(2/3)/x^2, x)
```

3.341 $\int \frac{(c\sin^3(a+bx))^{2/3}}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^3}\,dx = -\frac{(c\sin^3(a+bx))^{2/3}}{2x^2} - \frac{b\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{x} + b^2\cos(2a)\,\mathrm{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} - b^2\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3}\mathrm{Si}(2bx)$$

```
[Out] -1/2*(c*sin(b*x+a)^3)^(2/3)/x^2-b*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/x+b^2*C
i(2*b*x)*cos(2*a)*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)-b^2*csc(b*x+a)^2*Si(2
*b*x)*sin(2*a)*(c*sin(b*x+a)^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6852, 3395, 29, 3393, 3384, 3380, 3383}

$$\begin{aligned}\int \frac{(c\sin^3(a+bx))^{2/3}}{x^3}\,dx &= b^2\cos(2a)\,\mathrm{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} \\ &\quad - b^2\sin(2a)\mathrm{Si}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} \\ &\quad - \frac{(c\sin^3(a+bx))^{2/3}}{2x^2} - \frac{b\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{x}\end{aligned}$$

```
[In] Int[(c*Sin[a + b*x]^3)^(2/3)/x^3,x]
```

```
[Out] -1/2*(c*Sin[a + b*x]^3)^(2/3)/x^2 - (b*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3
))/x + b^2*Cos[2*a]*CosIntegral[2*b*x]*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2
/3) - b^2*Csc[a + b*x]^2*Sin[2*a]*(c*Sin[a + b*x]^3)^(2/3)*SinIntegral[2*b*
x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin^2(a+bx)}{x^3}\,dx$$

$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + \left(b^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{1}{x}\,dx - \left(2b^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin^2(a+bx)}{x}$$

$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + b^2\csc^2(a+bx)\log(x)\left(c\sin^3(a+bx)\right)^{2/3} - \left(2b^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \left(\frac{1}{2x} - \frac{\cos(2a+2bx)}{2x}\right)dx$$

$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + \left(b^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\cos(2a+2bx)}{x}\,dx$$

$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + \left(b^2\cos(2a)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\cos(2bx)}{x}\,dx - \left(b^2\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx\right.\right.$$

$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + b^2\cos(2a)\,\text{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} - b^2\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3}\text{Si}(2bx)$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx = \frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-1+\cos(2(a+bx))+4b^2x^2\cos(2a)\,\text{CosIntegral}(\right.}{4x^2}$$

```
[In] Integrate[(c*Sin[a + b*x]^3)^(2/3)/x^3,x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-1 + Cos[2*(a + b*x)] + 4*b^2*x^2
*Cos[2*a]*CosIntegral[2*b*x] - 2*b*x*Sin[2*(a + b*x)] - 4*b^2*x^2*Sin[2*a]*
SinIntegral[2*b*x]))/(4*x^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

method	result
risch	$\frac{\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{2}{3}}\left(4\,\mathrm{Ei}_1(-2ibx)\mathrm{e}^{2i(bx+2a)}x^2b^2+4\,\mathrm{e}^{2ibx}\,\mathrm{Ei}_1(2ibx)x^2b^2-2i\mathrm{e}^{4i(bx+a)}xb+2ibx+2\,\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{4i(bx+a)}-\right.}{8\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}x^2}$

```
[In] int((c*sin(b*x+a)^3)^(2/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*(4*Ei(1,-2*I*b*x)*
exp(2*I*(b*x+2*a))*x^2*b^2+4*exp(2*I*b*x)*Ei(1,2*I*b*x)*x^2*b^2-2*I*exp(4*I
*(b*x+a))*x*b+2*I*b*x+2*exp(2*I*(b*x+a))-exp(4*I*(b*x+a))-1)/(exp(2*I*(b*x+
a))-1)^2/x^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^3}\,dx =$$
$$-\frac{\left(b^2x^2\mathrm{Ei}(2i\,bx)\,e^{(2i\,a)}+b^2x^2\mathrm{Ei}(-2i\,bx)\,e^{(-2i\,a)}-2\,bx\cos\left(bx+a\right)\sin\left(bx+a\right)+\cos\left(bx+a\right)^2-1\right)\left(-(c\cos\right.}{2\left(x^2\cos\left(bx+a\right)^2-x^2\right)}$$

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2*Ei(2*I*b*x)*e^(2*I*a) + b^2*x^2*Ei(-2*I*b*x)*e^(-2*I*a) - 2*b
*x*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 - 1)*(-(c*cos(b*x + a)^2 - c)
*sin(b*x + a))^(2/3)/(x^2*cos(b*x + a)^2 - x^2)
```

Sympy [F]

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin^3\left(a+bx\right)\right)^{\frac{2}{3}}}{x^3}\,dx$$

```
[In] integrate((c*sin(b*x+a)**3)**(2/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(2/3)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.49

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx =$$
$$-\frac{\left(\left(\left(-i\sqrt{3}+1\right)E_3(2i\,bx)+\left(i\sqrt{3}+1\right)E_3(-2i\,bx)\right)\cos\left(2\,a\right)^3-\left(\left(\sqrt{3}+i\right)E_3(2i\,bx)+\left(\sqrt{3}-i\right)E_3(-2i\,bx)\right)\ldots}{}$$

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(((-I*sqrt(3) + 1)*exp_integral_e(3, 2*I*b*x) + (I*sqrt(3) + 1)*exp_i
ntegral_e(3, -2*I*b*x))*cos(2*a)^3 - ((sqrt(3) + I)*exp_integral_e(3, 2*I*b
*x) + (sqrt(3) - I)*exp_integral_e(3, -2*I*b*x))*sin(2*a)^3 + (((-I*sqrt(3)
 + 1)*exp_integral_e(3, 2*I*b*x) + (I*sqrt(3) + 1)*exp_integral_e(3, -2*I*b
*x))*cos(2*a) - 2)*sin(2*a)^2 + ((I*sqrt(3) + 1)*exp_integral_e(3, 2*I*b*x)
 + (-I*sqrt(3) + 1)*exp_integral_e(3, -2*I*b*x))*cos(2*a) - 2*cos(2*a)^2 -
(((sqrt(3) + I)*exp_integral_e(3, 2*I*b*x) + (sqrt(3) - I)*exp_integral_e(3
, -2*I*b*x))*cos(2*a)^2 - (sqrt(3) - I)*exp_integral_e(3, 2*I*b*x) - (sqrt(
3) + I)*exp_integral_e(3, -2*I*b*x))*sin(2*a))*b^2*c^(2/3)/(a^2*cos(2*a)^2
+ a^2*sin(2*a)^2 + (b*x + a)^2*(cos(2*a)^2 + sin(2*a)^2) - 2*(a*cos(2*a)^2
+ a*sin(2*a)^2)*(b*x + a))
```

Giac [F]

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}}}{x^3}\,dx$$

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin\left(a+b\,x\right)^3\right)^{2/3}}{x^3}\,dx$$

```
[In] int((c*sin(a + b*x)^3)^(2/3)/x^3,x)
```

```
[Out] int((c*sin(a + b*x)^3)^(2/3)/x^3, x)
```

3.342 $\int x^m(c\sin^3(a+bx^2))^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\int x^m\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\frac{x^{1+m}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{2(1+m)}+2^{-\frac{7}{2}-\frac{m}{2}}e^{2ia}x^{1+m}\left(-ibx^2\right)^{\frac{1}{2}(-1-m)}\csc^2\left(a+bx^2\right)\Gamma\left(\frac{1+m}{2},-2ibx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+2^{-\frac{7}{2}-\frac{m}{2}}e^{-2ia}x^{1+m}\left(ibx^2\right)^{\frac{1}{2}(-1-m)}\csc^2\left(a+bx^2\right)\Gamma\left(\frac{1+m}{2},2ibx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

```
[Out] 1/2*x^(1+m)*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)/(1+m)+2^(-7/2-1/2*m)*ex
p(2*I*a)*x^(1+m)*(-I*b*x^2)^(-1/2-1/2*m)*csc(b*x^2+a)^2*GAMMA(1/2+1/2*m,-2*
I*b*x^2)*(c*sin(b*x^2+a)^3)^(2/3)+2^(-7/2-1/2*m)*x^(1+m)*(I*b*x^2)^(-1/2-1/
2*m)*csc(b*x^2+a)^2*GAMMA(1/2+1/2*m,2*I*b*x^2)*(c*sin(b*x^2+a)^3)^(2/3)/exp
(2*I*a)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 3484, 3471, 2250}

$$\int x^m\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\frac{x^{m+1}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{2(m+1)}+e^{2ia}2^{-\frac{m}{2}-\frac{7}{2}}x^{m+1}\left(-ibx^2\right)^{\frac{1}{2}(-m-1)}\csc^2\left(a+bx^2\right)\Gamma\left(\frac{m+1}{2},-2ibx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+e^{-2ia}2^{-\frac{m}{2}-\frac{7}{2}}x^{m+1}\left(ibx^2\right)^{\frac{1}{2}(-m-1)}\csc^2\left(a+bx^2\right)\Gamma\left(\frac{m+1}{2},2ibx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

```
[In] Int[x^m*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (x^(1 + m)*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/(2*(1 + m)) + 2^(-7
/2 - m/2)*E^((2*I)*a)*x^(1 + m)*((-I)*b*x^2)^((-1 - m)/2)*Csc[a + b*x^2]^2*
Gamma[(1 + m)/2, (-2*I)*b*x^2]*(c*Sin[a + b*x^2]^3)^(2/3) + (2^(-7/2 - m/2)
*x^(1 + m)*(I*b*x^2)^((-1 - m)/2)*Csc[a + b*x^2]^2*Gamma[(1 + m)/2, (2*I)*b
*x^2]*(c*Sin[a + b*x^2]^3)^(2/3))/E^((2*I)*a)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int x^m\sin^2\left(a+bx^2\right)\,dx\\
&= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\left(\frac{x^m}{2}-\frac{1}{2}x^m\cos\left(2a+2bx^2\right)\right)dx\\
&= \frac{x^{1+m}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{2(1+m)}\\
&\quad-\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int x^m\cos\left(2a+2bx^2\right)\,dx
\end{aligned}$$

$$= \frac{x^{1+m}\csc^2(a+bx^2)\left(c\sin^3(a+bx^2)\right)^{2/3}}{2(1+m)} - \frac{1}{4}\left(\csc^2(a+bx^2)\left(c\sin^3(a+bx^2)\right)^{2/3}\right)\int e^{-2ia-2ibx^2}x^m\,dx - \frac{1}{4}\left(\csc^2(a+bx^2)\left(c\sin^3(a+bx^2)\right)^{2/3}\right)\int e^{2ia+2ibx^2}x^m\,dx$$

$$= \frac{x^{1+m}\csc^2(a+bx^2)\left(c\sin^3(a+bx^2)\right)^{2/3}}{2(1+m)} + 2^{-\frac{7}{2}-\frac{m}{2}}e^{2ia}x^{1+m}\left(-ibx^2\right)^{\frac{1}{2}(-1-m)}\csc^2(a+bx^2)\,\Gamma\left(\frac{1+m}{2},-2ibx^2\right)\left(c\sin^3(a+bx^2)\right)^{2/3} + 2^{-\frac{7}{2}-\frac{m}{2}}e^{-2ia}x^{1+m}\left(ibx^2\right)^{\frac{1}{2}(-1-m)}\csc^2(a+bx^2)\,\Gamma\left(\frac{1+m}{2},2ibx^2\right)\left(c\sin^3(a+bx^2)\right)^{2/3}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90

$$\int x^m\left(c\sin^3(a+bx^2)\right)^{2/3}dx = \frac{2^{\frac{1}{2}(-7-m)}x^{1+m}(b^2x^4)^{\frac{1}{2}(-1-m)}\csc^2(a+bx^2)\left(2^{\frac{5+m}{2}}(b^2x^4)^{\frac{1+m}{2}} + (1+m)(-ibx^2)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},2i\right.\right.}{}$$

```
[In] Integrate[x^m*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (2^((-7 - m)/2)*x^(1 + m)*(b^2*x^4)^((-1 - m)/2)*Csc[a + b*x^2]^2*(2^((5 +
m)/2)*(b^2*x^4)^((1 + m)/2) + (1 + m)*((-I)*b*x^2)^((1 + m)/2)*Gamma[(1 + m
)/2, (2*I)*b*x^2]*(Cos[2*a] - I*Sin[2*a]) + (1 + m)*(I*b*x^2)^((1 + m)/2)*G
amma[(1 + m)/2, (-2*I)*b*x^2]*(Cos[2*a] + I*Sin[2*a]))*(c*Sin[a + b*x^2]^3)
^(2/3))/(1 + m)
```

Maple [F]

$$\int x^m\left(c\left(\sin^3\left(b\,x^2+a\right)\right)\right)^{\frac{2}{3}}dx$$

```
[In] int(x^m*(c*sin(b*x^2+a)^3)^(2/3),x)
```

```
[Out] int(x^m*(c*sin(b*x^2+a)^3)^(2/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.62

$$\int x^m \left(c\sin^3\left(a+bx^2\right)\right)^{2/3} dx =$$
$$-\frac{\left(8\,bxx^m-(i\,m+i)e^{\left(-\frac{1}{2}\,(m-1)\log(2i\,b)-2i\,a\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},2i\,bx^2\right)-(-i\,m-i)e^{\left(-\frac{1}{2}\,(m-1)\log(-2i\,b)+2i\,a\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},\right.}{16\left((bm+b)\cos\left(bx^2+a\right)^2-bm-b\right)}$$

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/16*(8*b*x*x^m - (I*m + I)*e^(-1/2*(m - 1)*log(2*I*b) - 2*I*a)*gamma(1/2*
m + 1/2, 2*I*b*x^2) - (-I*m - I)*e^(-1/2*(m - 1)*log(-2*I*b) + 2*I*a)*gamma
(1/2*m + 1/2, -2*I*b*x^2))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)
/((b*m + b)*cos(b*x^2 + a)^2 - b*m - b)
```

Sympy [F]

$$\int x^m \left(c\sin^3\left(a+bx^2\right)\right)^{2/3} dx = \int x^m \left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}} dx$$

```
[In] integrate(x**m*(c*sin(b*x**2+a)**3)**(2/3),x)
```

```
[Out] Integral(x**m*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Maxima [F]

$$\int x^m \left(c\sin^3\left(a+bx^2\right)\right)^{2/3} dx = \int \left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}} x^m\, dx$$

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/4*(x*x^m - (m + 1)*integrate(x^m*cos(2*b*x^2 + 2*a), x))*c^(2/3)/(m + 1)
```

Giac [F]

$$\int x^m \left(c\sin^3\left(a+bx^2\right)\right)^{2/3} dx = \int \left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}} x^m \, dx$$

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(c\sin^3\left(a+bx^2\right)\right)^{2/3} dx = \int x^m \left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3} dx$$

```
[In] int(x^m*(c*sin(a + b*x^2)^3)^(2/3),x)
[Out] int(x^m*(c*sin(a + b*x^2)^3)^(2/3), x)
```

3.343 $\int x^3(c\sin^3(a+bx^2))^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 91

$$\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b^2}-\frac{x^2\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{8}x^4\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

```
[Out] 1/8*(c*sin(b*x^2+a)^3)^(2/3)/b^2-1/4*x^2*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(2
/3)/b+1/8*x^4*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 3460, 3391, 30}

$$\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b^2}-\frac{x^2\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{8}x^4\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

```
[In] Int[x^3*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (c*Sin[a + b*x^2]^3)^(2/3)/(8*b^2) - (x^2*Cot[a + b*x^2]*(c*Sin[a + b*x^2]^
3)^(2/3))/(4*b) + (x^4*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int x^3\sin^2\left(a+bx^2\right)\,dx \\
&= \frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\text{Subst}\left(\int x\sin^2(a+bx)\,dx,x,x^2\right) \\
&= \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b^2}-\frac{x^2\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b} \\
&\quad+\frac{1}{4}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\text{Subst}\left(\int x\,dx,x,x^2\right) \\
&= \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b^2}-\frac{x^2\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b} \\
&\quad+\frac{1}{8}x^4\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx = -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(\cos\left(2(a+bx^2)\right)+2bx^2(-bx^2+\sin\left(2(a+bx^2)\right))\right)}{16b^2}$$

```
[In] Integrate[x^3*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] -1/16*(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(Cos[2*(a + b*x^2)] + 2*
b*x^2*(-(b*x^2) + Sin[2*(a + b*x^2)])))/b^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.20

method	result
risch	$-\frac{x^4\left(ic\,e^{-3i\left(b\,x^2+a\right)}\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{2}{3}}e^{2i\left(b\,x^2+a\right)}}{8\left(e^{2i\left(b\,x^2+a\right)}-1\right)^2}-\frac{i(2b\,x^2+i)\left(ic\,e^{-3i\left(b\,x^2+a\right)}\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{2}{3}}e^{4i\left(b\,x^2+a\right)}}{32b^2\left(e^{2i\left(b\,x^2+a\right)}-1\right)^2}+\frac{i\left(ic\,e\right.}{}$

```
[In] int(x^3*(c*sin(b*x^2+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x^4/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+
a))-1)^3)^(2/3)*exp(2*I*(b*x^2+a))-1/32*I/b^2*(2*b*x^2+I)/(exp(2*I*(b*x^2+a
))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*exp(4*I*(b
*x^2+a))+1/32*I*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)/(e
xp(2*I*(b*x^2+a))-1)^2*(2*b*x^2-I)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx = -\frac{\left(2\,b^2x^4-4\,bx^2\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)-2\,\cos\left(bx^2+a\right)^2+1\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{2}{3}}}{16\left(b^2\cos\left(bx^2+a\right)^2-b^2\right)}$$

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/16*(2*b^2*x^4 - 4*b*x^2*cos(b*x^2 + a)*sin(b*x^2 + a) - 2*cos(b*x^2 + a)
^2 + 1)*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(b^2*cos(b*x^2 + a
)^2 - b^2)
```

Sympy [F]

$$\int x^3 \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} dx = \int x^3 \left(c \sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}} dx$$

```
[In] integrate(x**3*(c*sin(b*x**2+a)**3)**(2/3),x)
[Out] Integral(x**3*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int x^3 \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} dx = -\frac{\left(2\, b^2 x^4 - 2\, b x^2 \sin\left(2\, b x^2 + 2\, a\right) - \cos\left(2\, b x^2 + 2\, a\right)\right) c^{\frac{2}{3}}}{32\, b^2}$$

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
[Out] -1/32*(2*b^2*x^4 - 2*b*x^2*sin(2*b*x^2 + 2*a) - cos(2*b*x^2 + 2*a))*c^(2/3)
/b^2
```

Giac [F]

$$\int x^3 \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} dx = \int \left(c \sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}} x^3 \, dx$$

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(c \sin^3 \left(a+bx^2\right)\right)^{2/3} \, dx = \int x^3 \left(c \sin \left(b\,x^2+a\right)^3\right)^{2/3} dx$$

```
[In] int(x^3*(c*sin(a + b*x^2)^3)^(2/3),x)
[Out] int(x^3*(c*sin(a + b*x^2)^3)^(2/3), x)
```

3.344 $\int x^2 (c \sin^3(a+bx^2))^{2/3}\, dx$

Optimal result

Integrand size = 20, antiderivative size = 195

$$\int x^2 \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} dx = \frac{1}{6}x^3 \csc^2\left(a+bx^2\right)\left(c \sin^3\left(a+bx^2\right)\right)^{2/3} + \frac{\sqrt{\pi}\cos(2a)\csc^2\left(a+bx^2\right)\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}} + \frac{\sqrt{\pi}\csc^2\left(a+bx^2\right)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}} - \frac{x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\sin\left(2a+2bx^2\right)}{8b}$$

```
[Out] 1/6*x^3*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)-1/8*x*csc(b*x^2+a)^2*(c*sin
(b*x^2+a)^3)^(2/3)*sin(2*b*x^2+2*a)/b+1/16*cos(2*a)*csc(b*x^2+a)^2*FresnelS
(2*x*b^(1/2)/Pi^(1/2))*(c*sin(b*x^2+a)^3)^(2/3)*Pi^(1/2)/b^(3/2)+1/16*csc(b
*x^2+a)^2*FresnelC(2*x*b^(1/2)/Pi^(1/2))*sin(2*a)*(c*sin(b*x^2+a)^3)^(2/3)*
Pi^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used

$= \{6852, 3484, 3467, 3434, 3433, 3432\}$

$$\int x^2\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}dx = \frac{\sqrt{\pi}\sin(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}} + \frac{\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}} - \frac{x\sin\left(2a+2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b} + \frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

```
[In] Int[x^2*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (x^3*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/6 + (Sqrt[Pi]*Cos[2*a]*Cs
c[a + b*x^2]^2*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*(c*Sin[a + b*x^2]^3)^(2/3))
/(16*b^(3/2)) + (Sqrt[Pi]*Csc[a + b*x^2]^2*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]
*Sin[2*a]*(c*Sin[a + b*x^2]^3)^(2/3))/(16*b^(3/2)) - (x*Csc[a + b*x^2]^2*(c
*Sin[a + b*x^2]^3)^(2/3)*Sin[2*a + 2*b*x^2])/(8*b)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int x^2\sin^2\left(a+bx^2\right)\,dx\\
&= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\left(\frac{x^2}{2}-\frac{1}{2}x^2\cos\left(2a+2bx^2\right)\right)dx\\
&= \frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\\
&\quad-\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int x^2\cos\left(2a+2bx^2\right)\,dx\\
&= \frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\\
&\quad-\frac{x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\sin\left(2a+2bx^2\right)}{8b}\\
&\quad+\frac{\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\sin\left(2a+2bx^2\right)\,dx}{8b}\\
&= \frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\\
&\quad-\frac{x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\sin\left(2a+2bx^2\right)}{8b}\\
&\quad+\frac{\left(\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\sin\left(2bx^2\right)\,dx}{8b}\\
&\quad+\frac{\left(\csc^2\left(a+bx^2\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\cos\left(2bx^2\right)\,dx}{8b}
\end{aligned}$$

$$= \frac{1}{6}x^3 \csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} + \frac{\sqrt{\pi}\cos(2a)\csc^2\left(a+bx^2\right)\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}} + \frac{\sqrt{\pi}\csc^2\left(a+bx^2\right)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}} - \frac{x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\sin\left(2a+2bx^2\right)}{8b}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.58

$$\int x^2\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} dx = \frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(3\sqrt{\pi}\cos(2a)\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+3\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(\right.}{48b^{3/2}}$$

```
[In] Integrate[x^2*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(3*Sqrt[Pi]*Cos[2*a]*FresnelS[
(2*Sqrt[b]*x)/Sqrt[Pi]] + 3*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2
*a] + 2*Sqrt[b]*x*(4*b*x^2 - 3*Sin[2*(a + b*x^2)])))/(48*b^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.58

method	result
risch	$\frac{ix\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{2}{3}}}{16b\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^2} - \frac{i\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{2}{3}}\mathrm{e}^{2ib\,x^2}\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{ib}\,x\right)}{64\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^2 b\sqrt{ib}} + \frac{\left(ic\,\mathrm{e}^{-3i\left(b\,x^2\right.\right.}}{}$

```
[In] int(x^2*(c*sin(b*x^2+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*x/b/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^
2+a))-1)^3)^(2/3)-1/64*I*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)
^(2/3)/(exp(2*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)/b*Pi^(1/2)*2^(1/2)/(I*b)^(1/
2)*erf(2^(1/2)*(I*b)^(1/2)*x)+1/4/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b
*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*(-1/4*I*x/b*exp(4*I*(b*x^2+a))+1/8
*I/b*Pi^(1/2)/(-2*I*b)^(1/2)*erf((-2*I*b)^(1/2)*x)*exp(2*I*(b*x^2+2*a)))-1/
```

```
6*x^3/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))
-1)^3)^(2/3)*exp(2*I*(b*x^2+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

$$\int x^2 \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx = -\frac{\left(16\,b^2x^3 - 24\,bx\cos\left(bx^2+a\right)\sin\left(bx^2+a\right) + 3\left(-i\,\pi e^{(2i\,a)} + i\,\pi e^{(-2i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right) + 3\left(\pi e^{(2i\,a)} + \pi e^{(\cdots}\right.\right.}{96\left(b^2\cos\left(bx^2+a\right)^2 - b^2\right)}$$

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/96*(16*b^2*x^3 - 24*b*x*cos(b*x^2 + a)*sin(b*x^2 + a) + 3*(-I*pi*e^(2*I*
a) + I*pi*e^(-2*I*a))*sqrt(b/pi)*fresnel_cos(2*x*sqrt(b/pi)) + 3*(pi*e^(2*I
*a) + pi*e^(-2*I*a))*sqrt(b/pi)*fresnel_sin(2*x*sqrt(b/pi)))*(-(c*cos(b*x^2
 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(b^2*cos(b*x^2 + a)^2 - b^2)
```

Sympy [F]

$$\int x^2 \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx = \int x^2 \left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate(x**2*(c*sin(b*x**2+a)**3)**(2/3),x)
```

```
[Out] Integral(x**2*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.51

$$\int x^2 \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx = -\frac{3\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left((i+1)\cos\left(2\,a\right) - (i-1)\sin\left(2\,a\right)\right)\mathrm{erf}\left(\sqrt{2i\,b}x\right) + \left(-(i-1)\cos\left(2\,a\right) + (i+1)\sin\left(2\,a\right)\right)\mathrm{e}\cdots\right.}{768\,b^3}$$

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/768*(3*4^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*cos(2*a) - (I - 1)*sin(2*a))*e
rf(sqrt(2*I*b)*x) + (-(I - 1)*cos(2*a) + (I + 1)*sin(2*a))*erf(sqrt(-2*I*b)
*x))*b^(3/2)*c^(2/3) + 16*(4*b^3*x^3 - 3*b^2*x*sin(2*b*x^2 + 2*a))*c^(2/3))
/b^3
```

Giac [F]

$$\int x^2 \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} \, dx=\int \left(c \sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}} x^2 \, dx$$

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} \, dx=\int x^2 \left(c \sin\left(b\,x^2+a\right)^3\right)^{2/3} \,dx$$

```
[In] int(x^2*(c*sin(a + b*x^2)^3)^(2/3),x)
```

```
[Out] int(x^2*(c*sin(a + b*x^2)^3)^(2/3), x)
```

3.345 $\int x(c\sin^3(a+bx^2))^{2/3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=-\frac{\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{4}x^2\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

```
[Out] -1/4*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(2/3)/b+1/4*x^2*csc(b*x^2+a)^2*(c*sin(
b*x^2+a)^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6847, 3286, 2715, 8}

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\frac{1}{4}x^2\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}$$

```
[In] Int[x*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] -1/4*(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(2/3))/b + (x^2*Csc[a + b*x^2]^2*
(c*Sin[a + b*x^2]^3)^(2/3))/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\text{Subst}\left(\int \sin^2(a+bx)\,dx, x, x^2\right)\\
&= -\frac{\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}\\
&\quad+\frac{1}{4}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\text{Subst}\left(\int 1\,dx, x, x^2\right)\\
&= -\frac{\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{4}x^2\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(2(a+bx^2)-\sin\left(2(a+bx^2)\right)\right)}{8b}$$

```
[In] Integrate[x*(c*Sin[a + b*x^2]^3)^(2/3),x]
[Out] (Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(2*(a + b*x^2) - Sin[2*(a + b*
x^2)]))/(8*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.80

method	result
risch	$-\frac{x^2\left(ic\,e^{-3i(b\,x^2+a)}\left(e^{2i(b\,x^2+a)}-1\right)^3\right)^{\frac{2}{3}}e^{2i(b\,x^2+a)}}{4\left(e^{2i(b\,x^2+a)}-1\right)^2}-\frac{i\left(ic\,e^{-3i(b\,x^2+a)}\left(e^{2i(b\,x^2+a)}-1\right)^3\right)^{\frac{2}{3}}e^{4i(b\,x^2+a)}}{16b\left(e^{2i(b\,x^2+a)}-1\right)^2}+\frac{i\left(ic\,e^{-3i(b\,x^2}\right.}{16b\left(\right.}$

```
[In] int(x*(c*sin(b*x^2+a)^3)^(2/3),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+
a))-1)^3)^(2/3)*exp(2*I*(b*x^2+a))-1/16*I/b/(exp(2*I*(b*x^2+a))-1)^2*(I*c*e
xp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*exp(4*I*(b*x^2+a))+1/16*
I/b/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1
)^3)^(2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=$$
$$-\frac{\left(bx^2-\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{2}{3}}}{4\left(b\cos\left(bx^2+a\right)^2-b\right)}$$

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
[Out] -1/4*(b*x^2 - cos(b*x^2 + a)*sin(b*x^2 + a))*(-(c*cos(b*x^2 + a)^2 - c)*sin
(b*x^2 + a))^(2/3)/(b*cos(b*x^2 + a)^2 - b)
```

Sympy [F]

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int x\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate(x*(c*sin(b*x**2+a)**3)**(2/3),x)
[Out] Integral(x*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.43

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=-\frac{\left(2\,bx^2-\sin\left(2\,bx^2+2\,a\right)\right)c^{\frac{2}{3}}}{16\,b}$$

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
[Out] -1/16*(2*b*x^2 - sin(2*b*x^2 + 2*a))*c^(2/3)/b
```

Giac [F]

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}x\,dx$$

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int x\,\left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}\,dx$$

```
[In] int(x*(c*sin(a + b*x^2)^3)^(2/3),x)
[Out] int(x*(c*sin(a + b*x^2)^3)^(2/3), x)
```

3.346 $\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 148

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{\sqrt{\pi}\cos(2a)\csc^2\left(a+bx^2\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}}+\frac{\sqrt{\pi}\csc^2\left(a+bx^2\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}}$$

```
[Out] 1/2*x*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)-1/4*cos(2*a)*csc(b*x^2+a)^2*F
resnelC(2*x*b^(1/2)/Pi^(1/2))*(c*sin(b*x^2+a)^3)^(2/3)*Pi^(1/2)/b^(1/2)+1/4
*csc(b*x^2+a)^2*FresnelS(2*x*b^(1/2)/Pi^(1/2))*sin(2*a)*(c*sin(b*x^2+a)^3)^
(2/3)*Pi^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6852, 3438, 3435, 3433, 3432}

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3} dx = -\frac{\sqrt{\pi}\cos(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}} + \frac{\sqrt{\pi}\sin(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}} + \frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

```
[In] Int[(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (x*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/2 - (Sqrt[Pi]*Cos[2*a]*Csc[
a + b*x^2]^2*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*(c*Sin[a + b*x^2]^3)^(2/3))/(
4*Sqrt[b]) + (Sqrt[Pi]*Csc[a + b*x^2]^2*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Si
n[2*a]*(c*Sin[a + b*x^2]^3)^(2/3))/(4*Sqrt[b])
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \sin^2\left(a+bx^2\right)\,dx\\
&= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(2a+2bx^2\right)\right)dx\\
&= \frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\\
&\quad -\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \cos\left(2a+2bx^2\right)dx\\
&= \frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\\
&\quad -\frac{1}{2}\left(\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \cos\left(2bx^2\right)dx\\
&\quad +\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \sin\left(2bx^2\right)dx\\
&= \frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\\
&\quad -\frac{\sqrt{\pi}\cos(2a)\csc^2\left(a+bx^2\right)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}}\\
&\quad +\frac{\sqrt{\pi}\csc^2\left(a+bx^2\right)\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.63

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}dx=\frac{\csc^2\left(a+bx^2\right)\left(2\sqrt{b}x-\sqrt{\pi}\cos(2a)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)\right)\left(c\sin^3\right.}{4\sqrt{b}}$$

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x^2]^2*(2*Sqrt[b]*x - Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x)/S
qrt[Pi]] + Sqrt[Pi]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])*(c*Sin[a + b
*x^2]^3)^(2/3))/(4*Sqrt[b])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.51

method	result
risch	$\frac{\left(i c\,\mathrm{e}^{-3 i \left(b\,x^{2}+a\right)} \left(\mathrm{e}^{2 i \left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{2}{3}} \mathrm{e}^{2 i b\,x^{2}} \sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{i b}\,x\right)}{16 \left(\mathrm{e}^{2 i \left(b\,x^{2}+a\right)}-1\right)^{2} \sqrt{i b}}+\frac{\mathrm{erf}\left(\sqrt{-2 i b}\,x\right) \sqrt{\pi}\,\left(i c\,\mathrm{e}^{-3 i \left(b\,x^{2}+a\right)} \left(\mathrm{e}^{2 i \left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{2}{3}} \mathrm{e}^{2 i}}{8 \sqrt{-2 i b}\,\left(\mathrm{e}^{2 i \left(b\,x^{2}+a\right)}-1\right)^{2}}$

```
[In] int((c*sin(b*x^2+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)/(exp(2*I*(b*x
^2+a))-1)^2*exp(2*I*b*x^2)*Pi^(1/2)*2^(1/2)/(I*b)^(1/2)*erf(2^(1/2)*(I*b)^(
1/2)*x)+1/8*erf((-2*I*b)^(1/2)*x)/(-2*I*b)^(1/2)*Pi^(1/2)/(exp(2*I*(b*x^2+a
))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*exp(2*I*(b
*x^2+2*a))-1/2*x/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I
*(b*x^2+a))-1)^3)^(2/3)*exp(2*I*(b*x^2+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\frac{\left(\left(\pi e^{(2ia)}+\pi e^{(-2ia)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)+\left(i\,\pi e^{(2ia)}-i\,\pi e^{(-2ia)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(2\,x\sqrt{\frac{b}{\pi}}\right)-4\,bx\right)\left(-\left(c\,c\right.\right.}{8\left(b\cos\left(bx^2+a\right)^2-b\right)}$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/8*((pi*e^(2*I*a) + pi*e^(-2*I*a))*sqrt(b/pi)*fresnel_cos(2*x*sqrt(b/pi))
+ (I*pi*e^(2*I*a) - I*pi*e^(-2*I*a))*sqrt(b/pi)*fresnel_sin(2*x*sqrt(b/pi))
 - 4*b*x)*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(b*cos(b*x^2 + a
)^2 - b)
```

Sympy [F]

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int \left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((c*sin(b*x**2+a)**3)**(2/3),x)
[Out] Integral((c*sin(a + b*x**2)**3)**(2/3), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.51

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=$$

$$-\frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(\left(i-1\right)\cos\left(2\,a\right)+\left(i+1\right)\sin\left(2\,a\right)\right)\operatorname{erf}\left(\sqrt{2i\,b}x\right)+\left(-\left(i+1\right)\cos\left(2\,a\right)-\left(i-1\right)\sin\left(2\,a\right)\right)\operatorname{erf}\left(\ldots\right.}{64\,b^2}$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
[Out] -1/64*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*cos(2*a) + (I + 1)*sin(2*a))*erf(
sqrt(2*I*b)*x) + (-(I + 1)*cos(2*a) - (I - 1)*sin(2*a))*erf(sqrt(-2*I*b)*x)
)*b^(3/2)*c^(2/3) + 16*b^2*c^(2/3)*x)/b^2
```

Giac [F]

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int \left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}\,dx$$

```
[In] int((c*sin(a + b*x^2)^3)^(2/3),x)
[Out] int((c*sin(a + b*x^2)^3)^(2/3), x)
```

3.347 $\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x} dx = -\frac{1}{4}\cos(2a)\,\mathrm{CosIntegral}(2bx^2)\csc^2(a+bx^2)(c\sin^3(a+bx^2))^{2/3} + \frac{1}{2}\csc^2(a+bx^2)\log(x)(c\sin^3(a+bx^2))^{2/3} + \frac{1}{4}\csc^2(a+bx^2)\sin(2a)(c\sin^3(a+bx^2))^{2/3}\,\mathrm{Si}(2bx^2)$$

```
[Out] -1/4*Ci(2*b*x^2)*cos(2*a)*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)+1/2*csc(b
*x^2+a)^2*ln(x)*(c*sin(b*x^2+a)^3)^(2/3)+1/4*csc(b*x^2+a)^2*Si(2*b*x^2)*sin
(2*a)*(c*sin(b*x^2+a)^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6852, 3484, 3459, 3457, 3456}

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x} dx = -\frac{1}{4}\cos(2a)\,\mathrm{CosIntegral}(2bx^2)\csc^2(a+bx^2)(c\sin^3(a+bx^2))^{2/3} + \frac{1}{4}\sin(2a)\mathrm{Si}(2bx^2)\csc^2(a+bx^2)(c\sin^3(a+bx^2))^{2/3} + \frac{1}{2}\log(x)\csc^2(a+bx^2)(c\sin^3(a+bx^2))^{2/3}$$

```
[In] Int[(c*Sin[a + b*x^2]^3)^(2/3)/x,x]
```

```
[Out] -1/4*(Cos[2*a]*CosIntegral[2*b*x^2]*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(
2/3)) + (Csc[a + b*x^2]^2*Log[x]*(c*Sin[a + b*x^2]^3)^(2/3))/2 + (Csc[a + b
*x^2]^2*Sin[2*a]*(c*Sin[a + b*x^2]^3)^(2/3)*SinIntegral[2*b*x^2])/4
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \frac{\sin^2\left(a+bx^2\right)}{x}\,dx\\
&= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \left(\frac{1}{2x}-\frac{\cos\left(2a+2bx^2\right)}{2x}\right)dx\\
&= \frac{1}{2}\csc^2\left(a+bx^2\right)\log(x)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\\
&\quad -\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \frac{\cos\left(2a+2bx^2\right)}{x}\,dx
\end{aligned}
$$

$$= \frac{1}{2}\csc^2\left(a+bx^2\right)\log(x)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} - \frac{1}{2}\left(\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\frac{\cos\left(2bx^2\right)}{x}dx + \frac{1}{2}\left(\csc^2\left(a+bx^2\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\frac{\sin\left(2bx^2\right)}{x}dx$$

$$= -\frac{1}{4}\cos(2a)\operatorname{CosIntegral}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} + \frac{1}{2}\csc^2\left(a+bx^2\right)\log(x)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} + \frac{1}{4}\csc^2\left(a+bx^2\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\operatorname{Si}\left(2bx^2\right)$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}dx = \frac{1}{4}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(-\cos(2a)\operatorname{CosIntegral}\left(2bx^2\right)+2\log(x)+\sin(2a)\operatorname{Si}\left(2bx^2\right)\right)$$

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(2/3)/x,x]
```

```
[Out] (Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(-(Cos[2*a]*CosIntegral[2*b*x^
2]) + 2*Log[x] + Sin[2*a]*SinIntegral[2*b*x^2]))/4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

method	result
risch	$\frac{\left(ic\,e^{-3i\left(b\,x^2+a\right)}\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{2}{3}}\left(ie^{2ib\,x^2}\pi\operatorname{csgn}\left(b\,x^2\right)-2ie^{2ib\,x^2}\operatorname{Si}\left(2b\,x^2\right)-4\ln(x)e^{2i\left(b\,x^2+a\right)}-e^{2ib\,x^2}\operatorname{Ei}_1\left(-2ib\,x^2\right)-\operatorname{Ei}_1\left(-\right.\right.}{8\left(e^{2i\left(b\,x^2+a\right)}-1\right)^2}$

```
[In] int((c*sin(b*x^2+a)^3)^(2/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*(I*exp(2*I*b*x
^2)*Pi*csgn(b*x^2)-2*I*exp(2*I*b*x^2)*Si(2*b*x^2)-4*ln(x)*exp(2*I*(b*x^2+a)
)-exp(2*I*b*x^2)*Ei(1,-2*I*b*x^2)-Ei(1,-2*I*b*x^2)*exp(2*I*(b*x^2+2*a)))/(e
xp(2*I*(b*x^2+a))-1)^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x}\,dx = \frac{\left(\mathrm{Ei}(2i\,bx^2)\,e^{(2i\,a)} + \mathrm{Ei}(-2i\,bx^2)\,e^{(-2i\,a)} - 4\log(x)\right)\left(-\left(c\cos\left(bx^2+a\right)^2 - c\right)\sin\left(\right.\right.}{8\left(\cos\left(bx^2+a\right)^2 - 1\right)}$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(2*I*b*x^2)*e^(2*I*a) + Ei(-2*I*b*x^2)*e^(-2*I*a) - 4*log(x))*(-(c*c
os(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(cos(b*x^2 + a)^2 - 1)
```

Sympy [F]

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x}\,dx = \int \frac{(c\sin^3(a+bx^2))^{\frac{2}{3}}}{x}\,dx$$

```
[In] integrate((c*sin(b*x**2+a)**3)**(2/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(2/3)/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.48

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x}\,dx = \frac{1}{16}\left(\left(\mathrm{Ei}\left(2i\,bx^2\right) + \mathrm{Ei}\left(-2i\,bx^2\right)\right)\cos(2\,a) - \left(-i\,\mathrm{Ei}\left(2i\,bx^2\right) + i\,\mathrm{Ei}\left(-2i\,bx^2\right)\right)\sin(2\right.$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/16*((Ei(2*I*b*x^2) + Ei(-2*I*b*x^2))*cos(2*a) - (-I*Ei(2*I*b*x^2) + I*Ei(
-2*I*b*x^2))*sin(2*a) - 4*log(x))*c^(2/3)
```

Giac [F]

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x}\,dx = \int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}}{x}\,dx$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x,x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x}\,dx = \int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}}{x}\,dx$$

```
[In] int((c*sin(a + b*x^2)^3)^(2/3)/x,x)
[Out] int((c*sin(a + b*x^2)^3)^(2/3)/x, x)
```

3.348 $\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 132

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx = -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x} + \sqrt{b}\sqrt{\pi}\cos(2a)\csc^2\left(a+bx^2\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} + \sqrt{b}\sqrt{\pi}\csc^2\left(a+bx^2\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

```
[Out] -(c*sin(b*x^2+a)^3)^(2/3)/x+cos(2*a)*csc(b*x^2+a)^2*FresnelS(2*x*b^(1/2)/Pi
^(1/2))*(c*sin(b*x^2+a)^3)^(2/3)*b^(1/2)*Pi^(1/2)+csc(b*x^2+a)^2*FresnelC(2
*x*b^(1/2)/Pi^(1/2))*sin(2*a)*(c*sin(b*x^2+a)^3)^(2/3)*b^(1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6852, 3474, 4669, 3454, 3434, 3433, 3432}

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx = \sqrt{\pi}\sqrt{b}\sin(2a)\operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} + \sqrt{\pi}\sqrt{b}\cos(2a)\operatorname{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} - \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}$$

```
[In] Int[(c*Sin[a + b*x^2]^3)^(2/3)/x^2,x]
```

```
[Out] -((c*Sin[a + b*x^2]^3)^(2/3)/x) + Sqrt[b]*Sqrt[Pi]*Cos[2*a]*Csc[a + b*x^2]^
2*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*(c*Sin[a + b*x^2]^3)^(2/3) + Sqrt[b]*Sqr
t[Pi]*Csc[a + b*x^2]^2*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a]*(c*Sin[a +
 b*x^2]^3)^(2/3)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3454

```
Int[((a_.) + (b_.)*Sin[u_])^(p_.), x_Symbol] :> Int[(a + b*Sin[ExpandToSum[
u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ
[u, x]
```

Rule 3474

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[x^(m +
1)*(Sin[a + b*x^n]^p/(m + 1)), x] - Dist[b*n*(p/(m + 1)), Int[Sin[a + b*x^n
]^(p - 1)*Cos[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4669

```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin[2
*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\frac{\sin^2\left(a+bx^2\right)}{x^2}\,dx\\
&= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\\
&\quad+\left(4b\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\cos\left(a+bx^2\right)\sin\left(a+bx^2\right)\,dx\\
&= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}+\left(2b\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\sin\left(2\left(a+bx^2\right)\right)\,dx\\
&= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}+\left(2b\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\sin\left(2a+2bx^2\right)\,dx\\
&= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\\
&\quad+\left(2b\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\sin\left(2bx^2\right)\,dx\\
&\quad+\left(2b\csc^2\left(a+bx^2\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\cos\left(2bx^2\right)\,dx\\
&= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}+\sqrt{b}\sqrt{\pi}\cos(2a)\csc^2\left(a\right.\\
&\qquad\left.+bx^2\right)\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\\
&\quad+\sqrt{b}\sqrt{\pi}\csc^2\left(a+bx^2\right)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.81

$$
\int\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx=\frac{\csc^2\left(a+bx^2\right)\left(-1+\cos\left(2\left(a+bx^2\right)\right)+2\sqrt{b}\sqrt{\pi}x\cos(2a)\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+2\sqrt{b}\right.}{2x}
$$

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(2/3)/x^2,x]
```

```
[Out] (Csc[a + b*x^2]^2*(-1 + Cos[2*(a + b*x^2)] + 2*Sqrt[b]*Sqrt[Pi]*x*Cos[2*a]*
FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]] + 2*Sqrt[b]*Sqrt[Pi]*x*FresnelC[(2*Sqrt[b]
*x)/Sqrt[Pi]]*Sin[2*a])*(c*Sin[a + b*x^2]^3)^(2/3))/(2*x)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.28

method	result
risch	$-\frac{\left(ic\,\mathrm{e}^{-3i\left(b\,x^{2}+a\right)}\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{2}{3}}}{4x\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{2}}-\frac{i\left(ic\,\mathrm{e}^{-3i\left(b\,x^{2}+a\right)}\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{2}{3}}\mathrm{e}^{2ib\,x^{2}}b\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{ib}\,x\right)}{4\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{2}\sqrt{ib}}+\frac{\left(ic\,\mathrm{e}^{-3i(\right.}{}$

```
[In] int((c*sin(b*x^2+a)^3)^(2/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a)
)-1)^3)^(2/3)-1/4*I*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3
)/(exp(2*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)*b*Pi^(1/2)*2^(1/2)/(I*b)^(1/2)*er
f(2^(1/2)*(I*b)^(1/2)*x)+1/4/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+
a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*(-1/x*exp(4*I*(b*x^2+a))+2*I*b*Pi^(1/2)
/(-2*I*b)^(1/2)*erf((-2*I*b)^(1/2)*x)*exp(2*I*(b*x^2+2*a)))+1/2/x/(exp(2*I*
(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*ex
p(2*I*(b*x^2+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx=\frac{\left(\left(i\,\pi xe^{(2i\,a)}-i\,\pi xe^{(-2i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)-\left(\pi xe^{(2i\,a)}+\pi xe^{(-2i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(2\,x\right.}{2\left(x\cos\left(bx^2+a\right.\right.}$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*((I*pi*x*e^(2*I*a) - I*pi*x*e^(-2*I*a))*sqrt(b/pi)*fresnel_cos(2*x*sqrt
(b/pi)) - (pi*x*e^(2*I*a) + pi*x*e^(-2*I*a))*sqrt(b/pi)*fresnel_sin(2*x*sqr
t(b/pi)) - 2*cos(b*x^2 + a)^2 + 2)*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a
))^(2/3)/(x*cos(b*x^2 + a)^2 - x)
```

Sympy [F]

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x^2}\,dx = \int \frac{(c\sin^3(a+bx^2))^{\frac{2}{3}}}{x^2}\,dx$$

[In] integrate((c*sin(b*x**2+a)**3)**(2/3)/x**2,x)

[Out] Integral((c*sin(a + b*x**2)**3)**(2/3)/x**2, x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x^2}\,dx = \frac{\sqrt{2}\sqrt{bx^2}\left(\left(-(i+1)\,\sqrt{2}\Gamma\left(-\frac{1}{2},2i\,bx^2\right)+(i-1)\,\sqrt{2}\Gamma\left(-\frac{1}{2},-2i\,bx^2\right)\right)\cos(2\,a)+\left(\right.\right.}{32\,x}$$

[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^2,x, algorithm="maxima")

[Out] 1/32*(sqrt(2)*sqrt(b*x^2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*b*x^2) + (I - 1)*sqrt(2)*gamma(-1/2, -2*I*b*x^2))*cos(2*a) + ((I - 1)*sqrt(2)*gamma(-1/2, 2*I*b*x^2) - (I + 1)*sqrt(2)*gamma(-1/2, -2*I*b*x^2))*sin(2*a))*c^(2/3) + 8*c^(2/3))/x

Giac [F]

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}}{x^2}\,dx$$

[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^2,x, algorithm="giac")

[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}}{x^2}\,dx$$

```
[In] int((c*sin(a + b*x^2)^3)^(2/3)/x^2,x)
[Out] int((c*sin(a + b*x^2)^3)^(2/3)/x^2, x)
```

3.349 $\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx = -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2} + \frac{\cos\left(2(a+bx^2)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2} + \frac{1}{2}b\,\text{CosIntegral}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} + \frac{1}{2}b\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/...}$$

```
[Out] -1/4*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)/x^2+1/4*cos(2*b*x^2+2*a)*csc(b
*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)/x^2+1/2*b*cos(2*a)*csc(b*x^2+a)^2*Si(2*b
*x^2)*(c*sin(b*x^2+a)^3)^(2/3)+1/2*b*Ci(2*b*x^2)*csc(b*x^2+a)^2*sin(2*a)*(c
*sin(b*x^2+a)^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6852, 3484, 3461, 3378, 3384, 3380, 3383}

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx = \frac{1}{2}b\sin(2a)\,\text{CosIntegral}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} + \frac{1}{2}b\cos(2a)\text{Si}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} - \frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2} + \frac{\cos\left(2(a+bx^2)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}$$

```
[In] Int[(c*Sin[a + b*x^2]^3)^(2/3)/x^3,x]
```

```
[Out] -1/4*(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/x^2 + (Cos[2*(a + b*x^2)
]*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/(4*x^2) + (b*CosIntegral[2*b
*x^2]*Csc[a + b*x^2]^2*Sin[2*a]*(c*Sin[a + b*x^2]^3)^(2/3))/2 + (b*Cos[2*a]
*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*SinIntegral[2*b*x^2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\frac{\sin^2\left(a+bx^2\right)}{x^3}\,dx\\
&= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\left(\frac{1}{2x^3}-\frac{\cos\left(2a+2bx^2\right)}{2x^3}\right)dx\\
&= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}\\
&\quad-\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\frac{\cos\left(2a+2bx^2\right)}{x^3}\,dx\\
&= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}\\
&\quad-\frac{1}{4}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\text{Subst}\left(\int\frac{\cos(2a+2bx)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}+\frac{\cos\left(2(a+bx^2)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}\\
&\quad+\frac{1}{2}\left(b\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\text{Subst}\left(\int\frac{\sin(2a+2bx)}{x}\,dx,x,x^2\right)\\
&= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}+\frac{\cos\left(2(a+bx^2)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}\\
&\quad+\frac{1}{2}\left(b\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\text{Subst}\left(\int\frac{\sin(2bx)}{x}\,dx,x,x^2\right)+\frac{1}{2}\left(b\csc^2\left(a+bx^2\right)\right.\\
&= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}\\
&\quad+\frac{\cos\left(2(a+bx^2)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}\\
&\quad+\frac{1}{2}b\,\text{CosIntegral}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\\
&\quad+\frac{1}{2}b\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\text{Si}\left(2bx^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.49

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx = \frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(-1+\cos\left(2(a+bx^2)\right)+2bx^2\,\text{CosIntegral}\left(2\right.\right.}{4x^2}$$

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(2/3)/x^3,x]
```

```
[Out] (Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(-1 + Cos[2*(a + b*x^2)] + 2*b
*x^2*CosIntegral[2*b*x^2]*Sin[2*a] + 2*b*x^2*Cos[2*a]*SinIntegral[2*b*x^2])
)/(4*x^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{2}{3}}\left(2i\mathrm{e}^{2ib\,x^2}b\,\mathrm{Ei}_1\left(2ib\,x^2\right)x^2-2ib\,\mathrm{Ei}_1\left(-2ib\,x^2\right)\mathrm{e}^{2i\left(b\,x^2+2a\right)}x^2+2\,\mathrm{e}^{2i\left(b\,x^2+a\right)}-\mathrm{e}^{4i\left(b\,x^2+a\right)}-1\right.}{8x^2\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^2}$

```
[In] int((c*sin(b*x^2+a)^3)^(2/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*(2*I*exp(2*I*b
*x^2)*b*Ei(1,2*I*b*x^2)*x^2-2*I*b*Ei(1,-2*I*b*x^2)*exp(2*I*(b*x^2+2*a))*x^2
+2*exp(2*I*(b*x^2+a))-exp(4*I*(b*x^2+a))-1)/x^2/(exp(2*I*(b*x^2+a))-1)^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.62

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx = \frac{\left(i\,bx^2\mathrm{Ei}(2i\,bx^2)\,e^{(2i\,a)}-i\,bx^2\mathrm{Ei}(-2i\,bx^2)\,e^{(-2i\,a)}-2\,\cos\left(bx^2+a\right)^2+2\right)\left(-\left(c\,c\right.\right.}{4\left(x^2\cos\left(bx^2+a\right)^2-x^2\right)}$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(I*b*x^2*Ei(2*I*b*x^2)*e^(2*I*a) - I*b*x^2*Ei(-2*I*b*x^2)*e^(-2*I*a) -
2*cos(b*x^2 + a)^2 + 2)*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(x
^2*cos(b*x^2 + a)^2 - x^2)
```

Sympy [F]

$$\int \frac{\left(c \sin^3\left(a+b x^2\right)\right)^{2/3}}{x^3}\, dx = \int \frac{\left(c \sin^3\left(a+b x^2\right)\right)^{\frac{2}{3}}}{x^3}\, dx$$

```
[In] integrate((c*sin(b*x**2+a)**3)**(2/3)/x**3,x)
[Out] Integral((c*sin(a + b*x**2)**3)**(2/3)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.40

$$\int \frac{\left(c \sin^3\left(a+b x^2\right)\right)^{2/3}}{x^3}\, dx =$$
$$-\frac{\left(\left(\left(i\,\Gamma\left(-1, 2i\,bx^2\right) - i\,\Gamma\left(-1, -2i\,bx^2\right)\right)\cos\left(2\,a\right) + \left(\Gamma\left(-1, 2i\,bx^2\right) + \Gamma\left(-1, -2i\,bx^2\right)\right)\sin\left(2\,a\right)\right)bx^2 - 1\right)c^{\frac{2}{3}}}{8\,x^2}$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^3,x, algorithm="maxima")
[Out] -1/8*(((I*gamma(-1, 2*I*b*x^2) - I*gamma(-1, -2*I*b*x^2))*cos(2*a) + (gamma
(-1, 2*I*b*x^2) + gamma(-1, -2*I*b*x^2))*sin(2*a))*b*x^2 - 1)*c^(2/3)/x^2
```

Giac [F]

$$\int \frac{\left(c \sin^3\left(a+b x^2\right)\right)^{2/3}}{x^3}\, dx = \int \frac{\left(c \sin\left(b x^2+a\right)^3\right)^{\frac{2}{3}}}{x^3}\, dx$$

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^3,x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx=\int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}}{x^3}\,dx$$

```
[In] int((c*sin(a + b*x^2)^3)^(2/3)/x^3,x)
[Out] int((c*sin(a + b*x^2)^3)^(2/3)/x^3, x)
```

3.350 $\int x^m(c\sin^3(a+bx^n))^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 217

$$\int x^m\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx=\frac{x^{1+m}\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2(1+m)}+\frac{2^{-\frac{1+m+2n}{n}}e^{2ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\csc^2(a+bx^n)\,\Gamma\left(\frac{1+m}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{2^{-\frac{1+m+2n}{n}}e^{-2ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\csc^2(a+bx^n)\,\Gamma\left(\frac{1+m}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}$$

```
[Out] 1/2*x^(1+m)*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)/(1+m)+exp(2*I*a)*x^(1+m
)*csc(a+b*x^n)^2*GAMMA((1+m)/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/(2^((1+
m+2*n)/n))/n/((-I*b*x^n)^((1+m)/n))+x^(1+m)*csc(a+b*x^n)^2*GAMMA((1+m)/n,2*
I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/(2^((1+m+2*n)/n))/exp(2*I*a)/n/((I*b*x^n)
^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 3506, 3505, 2250}

$$\int x^m\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx=\frac{x^{m+1}\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2(m+1)}+\frac{e^{2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\csc^2(a+bx^n)\,\Gamma\left(\frac{m+1}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\csc^2(a+bx^n)\,\Gamma\left(\frac{m+1}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}$$

```
[In] Int[x^m*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^(1 + m)*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/(2*(1 + m)) + (E^((
2*I)*a)*x^(1 + m)*Csc[a + b*x^n]^2*Gamma[(1 + m)/n, (-2*I)*b*x^n]*(c*Sin[a
+ b*x^n]^3)^(2/3))/(2^((1 + m + 2*n)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + (x^(1
 + m)*Csc[a + b*x^n]^2*Gamma[(1 + m)/n, (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(
2/3))/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*n*(I*b*x^n)^((1 + m)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int x^m\sin^2\left(a+bx^n\right)\,dx\\
&= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\left(\frac{x^m}{2}-\frac{1}{2}x^m\cos\left(2a+2bx^n\right)\right)dx\\
&= \frac{x^{1+m}\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2(1+m)}\\
&\quad-\frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int x^m\cos\left(2a+2bx^n\right)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x^{1+m}\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2(1+m)} \\
&\quad - \frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int e^{-2ia-2ibx^n}x^m\,dx \\
&\quad - \frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int e^{2ia+2ibx^n}x^m\,dx \\
&= \frac{x^{1+m}\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2(1+m)} \\
&\quad + \frac{2^{-\frac{1+m+2n}{n}}e^{2ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\csc^2(a+bx^n)\,\Gamma\left(\frac{1+m}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n} \\
&\quad + \frac{2^{-\frac{1+m+2n}{n}}e^{-2ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\csc^2(a+bx^n)\,\Gamma\left(\frac{1+m}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.89

$$
\int x^m\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx = \frac{2^{-\frac{1+m+2n}{n}}e^{-2ia}x^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}}\csc^2(a+bx^n)\left(2^{\frac{1+m+n}{n}}e^{2ia}n(b^2x^{2n})^{\frac{1+m}{n}}+e^{4ia}(1+m)(ibx^n)^{\frac{1+}{}}\right.}{(1+m)n}
$$

```
[In] Integrate[x^m*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^(1 + m)*Csc[a + b*x^n]^2*(2^((1 + m + n)/n)*E^((2*I)*a)*n*(b^2*x^(2*n))^
((1 + m)/n) + E^((4*I)*a)*(1 + m)*(I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-
2*I)*b*x^n] + (1 + m)*((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (2*I)*b*x^n
])*(c*Sin[a + b*x^n]^3)^(2/3))/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*(1 + m)*n*(
b^2*x^(2*n))^((1 + m)/n))
```

Maple [F]

$$
\int x^m\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{2}{3}}dx
$$

```
[In] int(x^m*(c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int(x^m*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Fricas [F]

$$\int x^m \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^m\,dx$$

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\text{Timed out}$$

```
[In] integrate(x**m*(c*sin(a+b*x**n)**3)**(2/3),x)
[Out] Timed out
```

Maxima [F]

$$\int x^m \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^m\,dx$$

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
[Out] -1/4*(x*x^m - (m + 1)*integrate(x^m*cos(2*b*x^n + 2*a), x))*c^(2/3)/(m + 1)
```

Giac [F]

$$\int x^m \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^m\,dx$$

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(c \sin^3\left(a+bx^n\right)\right)^{2/3} \, dx = \int x^m \, \left(c \sin\left(a+b\,x^n\right)^3\right)^{2/3} dx$$

```
[In] int(x^m*(c*sin(a + b*x^n)^3)^(2/3),x)
[Out] int(x^m*(c*sin(a + b*x^n)^3)^(2/3), x)
```

3.351 $\int x^3(c\sin^3(a+bx^n))^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 188

$$\begin{aligned}\int x^3\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx &= \frac{1}{8}x^4\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\\ &+\frac{4^{-1-\frac{2}{n}}e^{2ia}x^4(-ibx^n)^{-4/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{4}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}\\ &+\frac{4^{-1-\frac{2}{n}}e^{-2ia}x^4(ibx^n)^{-4/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{4}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}\end{aligned}$$

```
[Out] 1/8*x^4*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)+4^(-1-2/n)*exp(2*I*a)*x^4*c
sc(a+b*x^n)^2*GAMMA(4/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n/((-I*b*x^n)^
(4/n))+4^(-1-2/n)*x^4*csc(a+b*x^n)^2*GAMMA(4/n,2*I*b*x^n)*(c*sin(a+b*x^n)^3
)^(2/3)/exp(2*I*a)/n/((I*b*x^n)^(4/n))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 3506, 3505, 2250}

$$\begin{aligned}\int x^3\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx &= \frac{1}{8}x^4\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\\ &+\frac{e^{2ia}4^{-\frac{2}{n}-1}x^4(-ibx^n)^{-4/n}\,\Gamma\left(\frac{4}{n},-2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}\\ &+\frac{e^{-2ia}4^{-\frac{2}{n}-1}x^4(ibx^n)^{-4/n}\,\Gamma\left(\frac{4}{n},2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}\end{aligned}$$

```
[In] Int[x^3*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^4*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/8 + (4^(-1 - 2/n)*E^((2*I
)*a)*x^4*Csc[a + b*x^n]^2*Gamma[4/n, (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/
3))/(n*((-I)*b*x^n)^(4/n)) + (4^(-1 - 2/n)*x^4*Csc[a + b*x^n]^2*Gamma[4/n,
(2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^(4/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int x^3\sin^2\left(a+bx^n\right)\,dx\\
&= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\left(\frac{x^3}{2}-\frac{1}{2}x^3\cos\left(2a+2bx^n\right)\right)dx\\
&= \frac{1}{8}x^4\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\\
&\quad-\frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int x^3\cos\left(2a+2bx^n\right)\,dx\\
&= \frac{1}{8}x^4\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\\
&\quad-\frac{1}{4}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int e^{-2ia-2ibx^n}x^3\,dx\\
&\quad-\frac{1}{4}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int e^{2ia+2ibx^n}x^3\,dx
\end{aligned}$$

$$= \frac{1}{8}x^4\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3} + \frac{4^{-1-\frac{2}{n}}e^{2ia}x^4(-ibx^n)^{-4/n}\csc^2(a+bx^n)\Gamma\left(\frac{4}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n} + \frac{4^{-1-\frac{2}{n}}e^{-2ia}x^4(ibx^n)^{-4/n}\csc^2(a+bx^n)\Gamma\left(\frac{4}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.86

$$\int x^3\left(c\sin^3(a+bx^n)\right)^{2/3}dx = \frac{2^{-3-\frac{4}{n}}e^{-2ia}x^4(b^2x^{2n})^{-4/n}\csc^2(a+bx^n)\left(16^{\frac{1}{n}}e^{2ia}n(b^2x^{2n})^{4/n}+2e^{4ia}(ibx^n)^{4/n}\Gamma\left(\frac{4}{n},-2ibx^n\right)+\right.}{n}$$

```
[In] Integrate[x^3*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (2^(-3 - 4/n)*x^4*Csc[a + b*x^n]^2*(16^n^(-1)*E^((2*I)*a)*n*(b^2*x^(2*n))^(
4/n) + 2*E^((4*I)*a)*(I*b*x^n)^(4/n)*Gamma[4/n, (-2*I)*b*x^n] + 2*((-I)*b*x
^n)^(4/n)*Gamma[4/n, (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)
*n*(b^2*x^(2*n))^(4/n))
```

Maple [F]

$$\int x^3\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{2}{3}}dx$$

```
[In] int(x^3*(c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int(x^3*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Fricas [F]

$$\int x^3\left(c\sin^3(a+bx^n)\right)^{2/3}dx = \int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^3\,dx$$

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(c\sin^3\left(a+bx^n\right)\right)^{2/3} dx = \text{Timed out}$$

```
[In] integrate(x**3*(c*sin(a+b*x**n)**3)**(2/3),x)
[Out] Timed out
```

Maxima [F]

$$\int x^3 \left(c\sin^3\left(a+bx^n\right)\right)^{2/3} dx = \int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}} x^3\, dx$$

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
[Out] -1/16*(x^4 - 4*integrate(x^3*cos(2*b*x^n + 2*a), x))*c^(2/3)
```

Giac [F]

$$\int x^3 \left(c\sin^3\left(a+bx^n\right)\right)^{2/3} dx = \int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}} x^3\, dx$$

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(c\sin^3\left(a+bx^n\right)\right)^{2/3} dx = \int x^3 \left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3} dx$$

```
[In] int(x^3*(c*sin(a + b*x^n)^3)^(2/3),x)
[Out] int(x^3*(c*sin(a + b*x^n)^3)^(2/3), x)
```

3.352 $\int x^2(c\sin^3(a+bx^n))^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 188

$$\int x^2\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx=\frac{1}{6}x^3\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}+\frac{2^{-2-\frac{3}{n}}e^{2ia}x^3(-ibx^n)^{-3/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{3}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{2^{-2-\frac{3}{n}}e^{-2ia}x^3(ibx^n)^{-3/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{3}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}$$

```
[Out] 1/6*x^3*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)+2^(-2-3/n)*exp(2*I*a)*x^3*c
sc(a+b*x^n)^2*GAMMA(3/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n/((-I*b*x^n)^
(3/n))+2^(-2-3/n)*x^3*csc(a+b*x^n)^2*GAMMA(3/n,2*I*b*x^n)*(c*sin(a+b*x^n)^3
)^(2/3)/exp(2*I*a)/n/((I*b*x^n)^(3/n))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 3506, 3505, 2250}

$$\int x^2\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx=\frac{1}{6}x^3\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}+\frac{e^{2ia}2^{-\frac{3}{n}-2}x^3(-ibx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}2^{-\frac{3}{n}-2}x^3(ibx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}$$

```
[In] Int[x^2*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^3*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/6 + (2^(-2 - 3/n)*E^((2*I
)*a)*x^3*Csc[a + b*x^n]^2*Gamma[3/n, (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/
3))/(n*((-I)*b*x^n)^(3/n)) + (2^(-2 - 3/n)*x^3*Csc[a + b*x^n]^2*Gamma[3/n,
(2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^(3/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int x^2\sin^2\left(a+bx^n\right)\,dx\\
&= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\left(\frac{x^2}{2}-\frac{1}{2}x^2\cos\left(2a+2bx^n\right)\right)dx\\
&= \frac{1}{6}x^3\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\\
&\quad-\frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int x^2\cos\left(2a+2bx^n\right)\,dx\\
&= \frac{1}{6}x^3\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\\
&\quad-\frac{1}{4}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int e^{-2ia-2ibx^n}x^2\,dx\\
&\quad-\frac{1}{4}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int e^{2ia+2ibx^n}x^2\,dx
\end{aligned}$$

$$= \frac{1}{6}x^3 \csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3} + \frac{2^{-2-\frac{3}{n}}e^{2ia}x^3(-ibx^n)^{-3/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{3}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n} + \frac{2^{-2-\frac{3}{n}}e^{-2ia}x^3(ibx^n)^{-3/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{3}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.89

$$\int x^2\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx = \frac{2^{-2-\frac{3}{n}}e^{-2ia}x^3(b^2x^{2n})^{-3/n}\csc^2(a+bx^n)\left(2^{\frac{3+n}{n}}e^{2ia}n(b^2x^{2n})^{3/n}+3e^{4ia}(ibx^n)^{3/n}\,\Gamma\left(\frac{3}{n},-2ibx^n\right)\right.}{3n}$$

```
[In] Integrate[x^2*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (2^(-2 - 3/n)*x^3*Csc[a + b*x^n]^2*(2^((3 + n)/n)*E^((2*I)*a)*n*(b^2*x^(2*n
))^(3/n) + 3*E^((4*I)*a)*(I*b*x^n)^(3/n)*Gamma[3/n, (-2*I)*b*x^n] + 3*((-I)
*b*x^n)^(3/n)*Gamma[3/n, (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(3*E^((2
*I)*a)*n*(b^2*x^(2*n))^(3/n))
```

Maple [F]

$$\int x^2\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{2}{3}}dx$$

```
[In] int(x^2*(c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int(x^2*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Fricas [F]

$$\int x^2\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx = \int \left(c\sin(bx^n+a)^3\right)^{\frac{2}{3}}x^2\,dx$$

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(c*sin(a+b*x**n)**3)**(2/3),x)
[Out] Timed out
```

Maxima [F]

$$\int x^2 \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx = \int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}} x^2\,dx$$

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
[Out] -1/12*(x^3 - 3*integrate(x^2*cos(2*b*x^n + 2*a), x))*c^(2/3)
```

Giac [F]

$$\int x^2 \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx = \int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}} x^2\,dx$$

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx = \int x^2\,\left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}\,dx$$

```
[In] int(x^2*(c*sin(a + b*x^n)^3)^(2/3),x)
[Out] int(x^2*(c*sin(a + b*x^n)^3)^(2/3), x)
```

3.353 $\int x(c\sin^3(a+bx^n))^{2/3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 188

$$\int x\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx=\frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}+\frac{4^{-1-\frac{1}{n}}e^{2ia}x^2(-ibx^n)^{-2/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{2}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{4^{-1-\frac{1}{n}}e^{-2ia}x^2(ibx^n)^{-2/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{2}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}$$

```
[Out] 1/4*x^2*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)+4^(-1-1/n)*exp(2*I*a)*x^2*c
sc(a+b*x^n)^2*GAMMA(2/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n/((-I*b*x^n)^
(2/n))+4^(-1-1/n)*x^2*csc(a+b*x^n)^2*GAMMA(2/n,2*I*b*x^n)*(c*sin(a+b*x^n)^3
)^(2/3)/exp(2*I*a)/n/((I*b*x^n)^(2/n))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6852, 3506, 3505, 2250}

$$\int x\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx=\frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}+\frac{e^{2ia}4^{-\frac{1}{n}-1}x^2(-ibx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}4^{-\frac{1}{n}-1}x^2(ibx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}$$

```
[In] Int[x*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^2*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/4 + (4^(-1 - n^(-1))*E^((
2*I)*a)*x^2*Csc[a + b*x^n]^2*Gamma[2/n, (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^
(2/3))/(n*((-I)*b*x^n)^(2/n)) + (4^(-1 - n^(-1))*x^2*Csc[a + b*x^n]^2*Gamma
[2/n, (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^(2/
n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int x\sin^2\left(a+bx^n\right)\,dx\\
&= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\left(\frac{x}{2}-\frac{1}{2}x\cos\left(2a+2bx^n\right)\right)\,dx\\
&= \frac{1}{4}x^2\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\\
&\quad-\frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int x\cos\left(2a+2bx^n\right)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3} \\
&\quad -\frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int e^{-2ia-2ibx^n}x\,dx \\
&\quad -\frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int e^{2ia+2ibx^n}x\,dx \\
&= \frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3} \\
&\quad +\frac{4^{-1-\frac{1}{n}}e^{2ia}x^2(-ibx^n)^{-2/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{2}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n} \\
&\quad +\frac{4^{-1-\frac{1}{n}}e^{-2ia}x^2(ibx^n)^{-2/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{2}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.85

$$
\int x\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx = \frac{4^{-\frac{1+n}{n}}e^{-2ia}x^2(b^2x^{2n})^{-2/n}\csc^2(a+bx^n)\left(4^{\frac{1}{n}}e^{2ia}n(b^2x^{2n})^{2/n}+e^{4ia}(ibx^n)^{2/n}\,\Gamma\left(\frac{2}{n},-2ibx^n\right)+(\cdots\right.}{n}
$$

```
[In] Integrate[x*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^2*Csc[a + b*x^n]^2*(4^n^(-1)*E^((2*I)*a)*n*(b^2*x^(2*n))^(2/n) + E^((4*I
)*a)*(I*b*x^n)^(2/n)*Gamma[2/n, (-2*I)*b*x^n] + ((-I)*b*x^n)^(2/n)*Gamma[2/
n, (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(4^((1 + n)/n)*E^((2*I)*a)*n*(
b^2*x^(2*n))^(2/n))
```

Maple [F]

$$
\int x\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{2}{3}}dx
$$

```
[In] int(x*(c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int(x*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Fricas [F]

$$\int x\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x\,dx$$

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x, x)
```

Sympy [F]

$$\int x\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int x\left(c\sin^3\left(a+bx^n\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate(x*(c*sin(a+b*x**n)**3)**(2/3),x)
[Out] Integral(x*(c*sin(a + b*x**n)**3)**(2/3), x)
```

Maxima [F]

$$\int x\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x\,dx$$

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
[Out] -1/8*(x^2 - 2*integrate(x*cos(2*b*x^n + 2*a), x))*c^(2/3)
```

Giac [F]

$$\int x\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x\,dx$$

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int x\,\left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}\,dx$$

```
[In] int(x*(c*sin(a + b*x^n)^3)^(2/3),x)
[Out] int(x*(c*sin(a + b*x^n)^3)^(2/3), x)
```

3.354 $\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 178

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\frac{1}{2}x\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}+\frac{2^{-2-\frac{1}{n}}e^{2ia}x(-ibx^n)^{-1/n}\csc^2\left(a+bx^n\right)\Gamma\left(\frac{1}{n},-2ibx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{n}+\frac{2^{-2-\frac{1}{n}}e^{-2ia}x(ibx^n)^{-1/n}\csc^2\left(a+bx^n\right)\Gamma\left(\frac{1}{n},2ibx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{n}$$

```
[Out] 1/2*x*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)+2^(-2-1/n)*exp(2*I*a)*x*csc(a
+b*x^n)^2*GAMMA(1/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n/((-I*b*x^n)^(1/n
))+2^(-2-1/n)*x*csc(a+b*x^n)^2*GAMMA(1/n,2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3
)/exp(2*I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6852, 3448, 3447, 2239}

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\frac{1}{2}x\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}+\frac{e^{2ia}2^{-\frac{1}{n}-2}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-2ibx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{n}+\frac{e^{-2ia}2^{-\frac{1}{n}-2}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},2ibx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{n}$$

```
[In] Int[(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/2 + (2^(-2 - n^(-1))*E^((2*
I)*a)*x*Csc[a + b*x^n]^2*Gamma[n^(-1), (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(
2/3))/(n*((-I)*b*x^n)^n^(-1)) + (2^(-2 - n^(-1))*x*Csc[a + b*x^n]^2*Gamma[n
^(-1), (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^n^
(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3448

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int \sin^2\left(a+bx^n\right)\,dx \\
&= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(2a+2bx^n\right)\right)\,dx \\
&= \frac{1}{2}x\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3} \\
&\quad -\frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int \cos\left(2a+2bx^n\right)\,dx \\
&= \frac{1}{2}x\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3} \\
&\quad -\frac{1}{4}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int e^{-2ia-2ibx^n}\,dx \\
&\quad -\frac{1}{4}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int e^{2ia+2ibx^n}\,dx
\end{aligned}
$$

$$= \frac{1}{2}x\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3} + \frac{2^{-2-\frac{1}{n}}e^{2ia}x(-ibx^n)^{-1/n}\csc^2(a+bx^n)\Gamma\left(\frac{1}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n} + \frac{2^{-2-\frac{1}{n}}e^{-2ia}x(ibx^n)^{-1/n}\csc^2(a+bx^n)\Gamma\left(\frac{1}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int \left(c\sin^3(a+bx^n)\right)^{2/3}dx = \frac{2^{-2-\frac{1}{n}}e^{-2ia}x(b^2x^{2n})^{-1/n}\csc^2(a+bx^n)\left(2^{1+\frac{1}{n}}e^{2ia}n(b^2x^{2n})^{\frac{1}{n}} + e^{4ia}(ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-2ibx^n\right) + (-i\right.}{n}$$

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (2^(-2 - n^(-1))*x*Csc[a + b*x^n]^2*(2^(1 + n^(-1))*E^((2*I)*a)*n*(b^2*x^(2
*n))^n^(-1) + E^((4*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-2*I)*b*x^n] + ((
-I)*b*x^n)^n^(-1)*Gamma[n^(-1), (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(
E^((2*I)*a)*n*(b^2*x^(2*n))^n^(-1))
```

Maple [F]

$$\int \left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{2}{3}}dx$$

```
[In] int((c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(2/3),x)
```

Fricas [F]

$$\int \left(c\sin^3(a+bx^n)\right)^{2/3}\,dx = \int \left(c\sin(bx^n+a)^3\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3), x)
```

Sympy [F]

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx = \int \left(c\sin^3\left(a+bx^n\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((c*sin(a+b*x**n)**3)**(2/3),x)
[Out] Integral((c*sin(a + b*x**n)**3)**(2/3), x)
```

Maxima [F]

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx = \int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
[Out] -1/4*c^(2/3)*(x - integrate(cos(2*b*x^n + 2*a), x))
```

Giac [F]

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx = \int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx = \int \left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}\,dx$$

```
[In] int((c*sin(a + b*x^n)^3)^(2/3),x)
[Out] int((c*sin(a + b*x^n)^3)^(2/3), x)
```

3.355 $\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 121

$$\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x}\,dx = -\frac{\cos(2a)\,\text{CosIntegral}(2bx^n)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2n} + \frac{1}{2}\csc^2(a+bx^n)\log(x)\left(c\sin^3(a+bx^n)\right)^{2/3} + \frac{\csc^2(a+bx^n)\sin(2a)\left(c\sin^3(a+bx^n)\right)^{2/3}\text{Si}(2bx^n)}{2n}$$

```
[Out] -1/2*Ci(2*b*x^n)*cos(2*a)*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)/n+1/2*csc
(a+b*x^n)^2*ln(x)*(c*sin(a+b*x^n)^3)^(2/3)+1/2*csc(a+b*x^n)^2*Si(2*b*x^n)*s
in(2*a)*(c*sin(a+b*x^n)^3)^(2/3)/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6852, 3506, 3459, 3457, 3456}

$$\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x}\,dx = -\frac{\cos(2a)\,\text{CosIntegral}(2bx^n)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2n} + \frac{\sin(2a)\text{Si}(2bx^n)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2n} + \frac{1}{2}\log(x)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}$$

```
[In] Int[(c*Sin[a + b*x^n]^3)^(2/3)/x,x]
```

```
[Out] -1/2*(Cos[2*a]*CosIntegral[2*b*x^n]*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(
2/3))/n + (Csc[a + b*x^n]^2*Log[x]*(c*Sin[a + b*x^n]^3)^(2/3))/2 + (Csc[a +
 b*x^n]^2*Sin[2*a]*(c*Sin[a + b*x^n]^3)^(2/3)*SinIntegral[2*b*x^n])/(2*n)
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\frac{\sin^2\left(a+bx^n\right)}{x}\,dx\\
&= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\left(\frac{1}{2x}-\frac{\cos\left(2a+2bx^n\right)}{2x}\right)dx\\
&= \frac{1}{2}\csc^2\left(a+bx^n\right)\log(x)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\\
&\quad-\frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\frac{\cos\left(2a+2bx^n\right)}{x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}\csc^2\left(a+bx^n\right)\log(x)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3} \\
&\quad - \frac{1}{2}\left(\cos(2a)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\frac{\cos\left(2bx^n\right)}{x}\,dx \\
&\quad + \frac{1}{2}\left(\csc^2\left(a+bx^n\right)\sin(2a)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\frac{\sin\left(2bx^n\right)}{x}\,dx \\
&= -\frac{\cos(2a)\,\mathrm{CosIntegral}\left(2bx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2n} \\
&\quad + \frac{1}{2}\csc^2\left(a+bx^n\right)\log(x)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3} \\
&\quad + \frac{\csc^2\left(a+bx^n\right)\sin(2a)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\mathrm{Si}\left(2bx^n\right)}{2n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$
\int\frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x}\,dx = \frac{\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\left(-\cos(2a)\,\mathrm{CosIntegral}\left(2bx^n\right)+n\log(x)+\sin\right.}{2n}
$$

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(2/3)/x,x]
```

```
[Out] (Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3)*(-(Cos[2*a]*CosIntegral[2*b*x^
n]) + n*Log[x] + Sin[2*a]*SinIntegral[2*b*x^n]))/(2*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.23

method	result
risch	$\frac{\left(ic\,\mathrm{e}^{-3i(a+b\,x^n)}\left(\mathrm{e}^{2i(a+b\,x^n)}-1\right)^3\right)^{\frac{2}{3}}\left(i\mathrm{e}^{2ib\,x^n}\pi\,\mathrm{csgn}(b\,x^n)-2i\mathrm{e}^{2ib\,x^n}\,\mathrm{Si}(2b\,x^n)-2\ln(x)\mathrm{e}^{2i(a+b\,x^n)}n-\mathrm{e}^{2ib\,x^n}\,\mathrm{Ei}_1(-2ib\,x^n)-\mathrm{Ei}_1(-2i\right.}{4\left(\mathrm{e}^{2i(a+b\,x^n)}-1\right)^2n}$

```
[In] int((c*sin(a+b*x^n)^3)^(2/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(I*c*exp(-3*I*(a+b*x^n))*(exp(2*I*(a+b*x^n))-1)^3)^(2/3)*(I*exp(2*I*b*x
^n)*Pi*csgn(b*x^n)-2*I*exp(2*I*b*x^n)*Si(2*b*x^n)-2*ln(x)*exp(2*I*(a+b*x^n)
)*n-exp(2*I*b*x^n)*Ei(1,-2*I*b*x^n)-Ei(1,-2*I*b*x^n)*exp(2*I*(b*x^n+2*a)))/
(exp(2*I*(a+b*x^n))-1)^2/n
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x}\,dx = \frac{\left(\mathrm{Ei}(2i\,bx^n)\,e^{(2i\,a)} + \mathrm{Ei}(-2i\,bx^n)\,e^{(-2i\,a)} - 2\,n\log(x)\right)\left(-\left(c\cos(bx^n+a)^2 - c\right)\sin\right.}{4\left(n\cos(bx^n+a)^2 - n\right)}$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/4*(Ei(2*I*b*x^n)*e^(2*I*a) + Ei(-2*I*b*x^n)*e^(-2*I*a) - 2*n*log(x))*(-(c
*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)/(n*cos(b*x^n + a)^2 - n)
```

Sympy [F]

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x}\,dx = \int \frac{(c\sin^3(a+bx^n))^{\frac{2}{3}}}{x}\,dx$$

```
[In] integrate((c*sin(a+b*x**n)**3)**(2/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x**n)**3)**(2/3)/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.26

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x}\,dx = \frac{\left(\left(\left(i\sqrt{3}+1\right)\mathrm{Ei}(2i\,bx^n) + \left(i\sqrt{3}+1\right)\mathrm{Ei}(-2i\,bx^n) + \left(-i\sqrt{3}+1\right)\mathrm{Ei}\left(2i\,be^{\left(n\overline{\log(}\right.}\right.\right.\right.}{}$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/16*(((I*sqrt(3) + 1)*Ei(2*I*b*x^n) + (I*sqrt(3) + 1)*Ei(-2*I*b*x^n) + (-I
*sqrt(3) + 1)*Ei(2*I*b*e^(n*conjugate(log(x)))) + (-I*sqrt(3) + 1)*Ei(-2*I*
b*e^(n*conjugate(log(x)))))*cos(2*a) - 4*n*log(x) - ((sqrt(3) - I)*Ei(2*I*b
*x^n) - (sqrt(3) - I)*Ei(-2*I*b*x^n) - (sqrt(3) + I)*Ei(2*I*b*e^(n*conjugat
e(log(x)))) + (sqrt(3) + I)*Ei(-2*I*b*e^(n*conjugate(log(x)))))*sin(2*a))*c
^(2/3)/n
```

Giac [F]

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x,x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x}\,dx = \int \frac{\left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}}{x}\,dx$$

```
[In] int((c*sin(a + b*x^n)^3)^(2/3)/x,x)
[Out] int((c*sin(a + b*x^n)^3)^(2/3)/x, x)
```

3.356 $\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 180

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x^2}\,dx = -\frac{\csc^2(a+bx^n)\,(c\sin^3(a+bx^n))^{2/3}}{2x} + \frac{2^{-2+\frac{1}{n}}e^{2ia}(-ibx^n)^{\frac{1}{n}}\csc^2(a+bx^n)\,\Gamma\left(-\frac{1}{n},-2ibx^n\right)(c\sin^3(a+bx^n))^{2/3}}{nx} + \frac{2^{-2+\frac{1}{n}}e^{-2ia}(ibx^n)^{\frac{1}{n}}\csc^2(a+bx^n)\,\Gamma\left(-\frac{1}{n},2ibx^n\right)(c\sin^3(a+bx^n))^{2/3}}{nx}$$

```
[Out] -1/2*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)/x+2^(-2+1/n)*exp(2*I*a)*(-I*b*
x^n)^(1/n)*csc(a+b*x^n)^2*GAMMA(-1/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n
/x+2^(-2+1/n)*(I*b*x^n)^(1/n)*csc(a+b*x^n)^2*GAMMA(-1/n,2*I*b*x^n)*(c*sin(a
+b*x^n)^3)^(2/3)/exp(2*I*a)/n/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 3506, 3505, 2250}

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x^2}\,dx = -\frac{\csc^2(a+bx^n)\,(c\sin^3(a+bx^n))^{2/3}}{2x} + \frac{e^{2ia}2^{\frac{1}{n}-2}(-ibx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-2ibx^n\right)\csc^2(a+bx^n)\,(c\sin^3(a+bx^n))^{2/3}}{nx} + \frac{e^{-2ia}2^{\frac{1}{n}-2}(ibx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},2ibx^n\right)\csc^2(a+bx^n)\,(c\sin^3(a+bx^n))^{2/3}}{nx}$$

```
[In] Int[(c*Sin[a + b*x^n]^3)^(2/3)/x^2,x]
```

```
[Out] -1/2*(Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/x + (2^(-2 + n^(-1))*E^(
(2*I)*a)*((-I)*b*x^n)^n^(-1)*Csc[a + b*x^n]^2*Gamma[-n^(-1), (-2*I)*b*x^n]*
(c*Sin[a + b*x^n]^3)^(2/3))/(n*x) + (2^(-2 + n^(-1))*(I*b*x^n)^n^(-1)*Csc[a
 + b*x^n]^2*Gamma[-n^(-1), (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*
I)*a)*n*x)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{\sin^2(a+bx^n)}{x^2}\,dx\\
&= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \left(\frac{1}{2x^2}-\frac{\cos(2a+2bx^n)}{2x^2}\right)dx\\
&= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2x}\\
&\quad -\frac{1}{2}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{\cos(2a+2bx^n)}{x^2}\,dx
\end{aligned}$$

$$= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2x} - \frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{e^{-2ia-2ibx^n}}{x^2}\,dx - \frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{e^{2ia+2ibx^n}}{x^2}\,dx$$

$$= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2x} + \frac{2^{-2+\frac{1}{n}}e^{2ia}(-ibx^n)^{\frac{1}{n}}\csc^2(a+bx^n)\,\Gamma\left(-\frac{1}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{nx} + \frac{2^{-2+\frac{1}{n}}e^{-2ia}(ibx^n)^{\frac{1}{n}}\csc^2(a+bx^n)\,\Gamma\left(-\frac{1}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{nx}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.69

$$\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^2}\,dx = \frac{e^{-2ia}\csc^2(a+bx^n)\left(-2e^{2ia}n + 2^{\frac{1}{n}}e^{4ia}(-ibx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-2ibx^n\right) + 2^{\frac{1}{n}}(ibx^n)^{\frac{1}{n}}\,\Gamma\right.}{4nx}$$

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(2/3)/x^2,x]
```

```
[Out] (Csc[a + b*x^n]^2*(-2*E^((2*I)*a)*n + 2^n^(-1)*E^((4*I)*a)*((-I)*b*x^n)^n^(
-1)*Gamma[-n^(-1), (-2*I)*b*x^n] + 2^n^(-1)*(I*b*x^n)^n^(-1)*Gamma[-n^(-1),
 (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(4*E^((2*I)*a)*n*x)
```

Maple [F]

$$\int \frac{\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{2}{3}}}{x^2}dx$$

```
[In] int((c*sin(a+b*x^n)^3)^(2/3)/x^2,x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(2/3)/x^2,x)
```

Fricas [F]

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^2}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^2,x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)/x^2, x)
```

Sympy [F]

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{\frac{2}{3}}}{x^2}\,dx$$

```
[In] integrate((c*sin(a+b*x**n)**3)**(2/3)/x**2,x)
[Out] Integral((c*sin(a + b*x**n)**3)**(2/3)/x**2, x)
```

Maxima [F]

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^2}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^2,x, algorithm="maxima")
[Out] 1/4*(x*integrate(cos(2*b*x^n + 2*a)/x^2, x) + 1)*c^(2/3)/x
```

Giac [F]

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^2}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^2,x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^2}\,dx=\int \frac{\left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}}{x^2}\,dx$$

```
[In] int((c*sin(a + b*x^n)^3)^(2/3)/x^2,x)
[Out] int((c*sin(a + b*x^n)^3)^(2/3)/x^2, x)
```

3.357 $\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 184

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x^3}\,dx = -\frac{\csc^2(a+bx^n)\,(c\sin^3(a+bx^n))^{2/3}}{4x^2} + \frac{4^{-1+\frac{1}{n}}e^{2ia}(-ibx^n)^{2/n}\csc^2(a+bx^n)\,\Gamma\left(-\frac{2}{n},-2ibx^n\right)(c\sin^3(a+bx^n))^{2/3}}{nx^2} + \frac{4^{-1+\frac{1}{n}}e^{-2ia}(ibx^n)^{2/n}\csc^2(a+bx^n)\,\Gamma\left(-\frac{2}{n},2ibx^n\right)(c\sin^3(a+bx^n))^{2/3}}{nx^2}$$

```
[Out] -1/4*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)/x^2+4^(-1+1/n)*exp(2*I*a)*(-I*
b*x^n)^(2/n)*csc(a+b*x^n)^2*GAMMA(-2/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)
/n/x^2+4^(-1+1/n)*(I*b*x^n)^(2/n)*csc(a+b*x^n)^2*GAMMA(-2/n,2*I*b*x^n)*(c*s
in(a+b*x^n)^3)^(2/3)/exp(2*I*a)/n/x^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 3506, 3505, 2250}

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x^3}\,dx = -\frac{\csc^2(a+bx^n)\,(c\sin^3(a+bx^n))^{2/3}}{4x^2} + \frac{e^{2ia}4^{\frac{1}{n}-1}(-ibx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},-2ibx^n\right)\csc^2(a+bx^n)\,(c\sin^3(a+bx^n))^{2/3}}{nx^2} + \frac{e^{-2ia}4^{\frac{1}{n}-1}(ibx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},2ibx^n\right)\csc^2(a+bx^n)\,(c\sin^3(a+bx^n))^{2/3}}{nx^2}$$

```
[In] Int[(c*Sin[a + b*x^n]^3)^(2/3)/x^3,x]
```

```
[Out] -1/4*(Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/x^2 + (4^(-1 + n^(-1))*E
^((2*I)*a)*((-I)*b*x^n)^(2/n)*Csc[a + b*x^n]^2*Gamma[-2/n, (-2*I)*b*x^n]*(c
*Sin[a + b*x^n]^3)^(2/3))/(n*x^2) + (4^(-1 + n^(-1))*(I*b*x^n)^(2/n)*Csc[a
+ b*x^n]^2*Gamma[-2/n, (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a
)*n*x^2)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\frac{\sin^2\left(a+bx^n\right)}{x^3}\,dx\\
&= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\left(\frac{1}{2x^3}-\frac{\cos\left(2a+2bx^n\right)}{2x^3}\right)dx\\
&= -\frac{\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{4x^2}\\
&\quad -\frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\frac{\cos\left(2a+2bx^n\right)}{x^3}\,dx
\end{aligned}
$$

$$= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{4x^2} - \frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int\frac{e^{-2ia-2ibx^n}}{x^3}\,dx - \frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int\frac{e^{2ia+2ibx^n}}{x^3}\,dx$$

$$= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{4x^2} + \frac{4^{-1+\frac{1}{n}}e^{2ia}(-ibx^n)^{2/n}\csc^2(a+bx^n)\,\Gamma\left(-\frac{2}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{nx^2} + \frac{4^{-1+\frac{1}{n}}e^{-2ia}(ibx^n)^{2/n}\csc^2(a+bx^n)\,\Gamma\left(-\frac{2}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{nx^2}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

$$\int\frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^3}\,dx = \frac{e^{-2ia}\csc^2(a+bx^n)\left(-e^{2ia}n+4^{\frac{1}{n}}e^{4ia}(-ibx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},-2ibx^n\right)+4^{\frac{1}{n}}(ibx^n)^{2/n}\,\Gamma\right.}{4nx^2}$$

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(2/3)/x^3,x]
```

```
[Out] (Csc[a + b*x^n]^2*(-(E^((2*I)*a)*n) + 4^n^(-1)*E^((4*I)*a)*((-I)*b*x^n)^(2/
n)*Gamma[-2/n, (-2*I)*b*x^n] + 4^n^(-1)*(I*b*x^n)^(2/n)*Gamma[-2/n, (2*I)*b
*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(4*E^((2*I)*a)*n*x^2)
```

Maple [F]

$$\int\frac{\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{2}{3}}}{x^3}dx$$

```
[In] int((c*sin(a+b*x^n)^3)^(2/3)/x^3,x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(2/3)/x^3,x)
```

Fricas [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^3}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^3,x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{\frac{2}{3}}}{x^3}\,dx$$

```
[In] integrate((c*sin(a+b*x**n)**3)**(2/3)/x**3,x)
[Out] Integral((c*sin(a + b*x**n)**3)**(2/3)/x**3, x)
```

Maxima [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^3}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^3,x, algorithm="maxima")
[Out] 1/8*(2*x^2*integrate(cos(2*b*x^n + 2*a)/x^3, x) + 1)*c^(2/3)/x^2
```

Giac [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^3}\,dx$$

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^3,x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^3}\,dx=\int \frac{\left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}}{x^3}\,dx$$

```
[In] int((c*sin(a + b*x^n)^3)^(2/3)/x^3,x)
[Out] int((c*sin(a + b*x^n)^3)^(2/3)/x^3, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```